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# RAILROAD CONSTRUCTION.

THEORY AND PRACTICE.

*A TEXT-BOOK FOR THE USE OF STUDENTS IN  
COLLEGES AND TECHNICAL SCHOOLS.*

BY

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## PREFACE.

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THE preparation of this book was begun several years ago, when much of the subject-matter treated was not to be found in print, or was scattered through many books and pamphlets, and was hence unavailable for student use. Portions of the book have already been printed by the mimeograph process or have been used as lecture-notes, and hence have been subjected to the refining process of classroom use.

The author would call special attention to the following features:

*a.* Transition curves; the multiform-compound-curve method is used, which has been followed by many railroads in this country; the particular curves here developed have the great advantage of being exceedingly simple, and although the method is not theoretically exact, it is demonstrable that the differences are so small that they may safely be neglected.

*b.* A system of earthwork computations by means of a slide-rule (which accompanies the volume) which enables one to compute readily the volume of the most complicated earthwork forms with an accuracy only limited by the precision of the cross-sectioning.

*c.* The "mass curve" in earthwork; the theory and use of this very valuable process.

*d.* Tables I, II, III, and IV have been computed *ab novo*. Tables I and II were checked (after computation) with other tables, which are generally considered as standard, and all discrepancies were further examined. They are believed to be perfect.

*e.* Tables V, VI, VII, and IX have been borrowed, by permission, from "Ludlow's Mathematical Tables." It is believed that five-place tables give as accurate results as actual field practice requires. Tables VIII and X have been compiled to conform with Ludlow's system.

The author wishes to acknowledge his indebtedness to Mr. Chas. A. Sims, civil engineer and railroad contractor, for reading and revising the portions relating to the cost of earthwork.

Since the book is written primarily for students of railroad engineering in technical institutions, the author has assumed the usual previous preparation in algebra, geometry, and trigonometry.

WALTER LORING WEBB.

UNIVERSITY OF PENNSYLVANIA,  
PHILADELPHIA,  
Jan. 1, 1900.

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# RAILROAD CONSTRUCTION.

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## CHAPTER I.

### RAILROAD SURVEYS.

THE proper conduct of railroad surveys presupposes an adequate knowledge of almost the whole subject of railroad engineering, and particularly of some of the complicated questions of Railroad Economics, which are not generally studied except at the latter part of a course in railroad engineering, if at all. This chapter will therefore be chiefly devoted to methods of instrumental work, and the problem of choosing a general route will be considered only as it is influenced by the topography or by the application of those elementary principles of Railroad Economics which are self-evident or which may be accepted by the student until he has had an opportunity of studying those principles in detail.

### RECONNOISSANCE SURVEYS.

**1. Character of a reconnoissance survey.** A reconnoissance survey is a very hasty examination of a belt of country to determine which of all possible or suggested routes is the most promising and best worthy of a more detailed survey. It is essentially very rough and rapid. It aims to discover those salient features which instantly stamp one route as distinctly superior to another and so narrow the choice to routes which are so nearly equal in value that a more detailed survey is necessary to decide between them.

**2. Selection of a general route.** The general question of running a railroad between two towns is usually a financial rather than an engineering question. Financial considerations usually determine that a road *must* pass through certain more or less important towns between its termini. When a railroad runs through a thickly settled and very flat country, where, from a topographical standpoint, the road may be run by any desired route, the "right-of-way agent" sometimes has a greater influence in locating the road than the engineer. But such modifications of alignment, on account of business considerations, are foreign to the engineer's side of the subject, and it will be hereafter assumed that topography alone determines the location of the line. The consideration of those larger questions combining finance and engineering (such as passing by a town on account of the necessary introduction of heavy grades in order to reach it) is likewise ignored.

**3. Valley route.** This is perhaps the simplest problem. If the two towns to be connected lie in the same valley, it is frequently only necessary to run a line which shall have a nearly uniform grade. The reconnoissance problem consists largely in determining the difference of elevation of the two termini of this division and the approximate horizontal distance so that the proper grade may be chosen. If there is a large river running through the valley, the road will probably remain on one side or the other throughout the whole distance, and both banks should be examined by the reconnoissance party to determine which is preferable. If the river may be easily bridged, both banks may be alternately used, especially when better alignment is thereby secured. A river valley has usually a steeper slope in the upper part than in the lower part. A uniform grade throughout the valley will therefore require that the road climbs up the side slopes in the lower part of the valley. In case the "ruling grade" \* for the whole road is as great as or greater

---

\* The *ruling grade* may here be loosely defined as the maximum grade which is permissible. This definition is not strictly true, as may be seen later when studying Railroad Economics, but it may here serve the purpose.

than the steepest natural valley slope, more freedom may be used in adopting that alignment which has the least cost—regardless of grade. The natural slope of large rivers is almost invariably so low that grade has no influence in determining the choice of location. When bridging is necessary, the river banks should be examined for suitable locations for abutments and piers. If the soil is soft and treacherous much difficulty may be experienced and the choice of route may be largely determined by the difficulty of bridging the river except at certain favorable places.

**4. Cross-country route.** A cross-country route always has one or more summits to be crossed. The problem becomes more complex on account of the greater number of possible solutions and the difficulty of properly weighing the advantages and disadvantages of each. The general aim should be to choose the lowest summits and the highest stream crossings, provided that by so doing the grades between these determining points shall be as low as possible and shall not be greater than the ruling grade of the road. Nearly all railroads combine cross-country and valley routes to some extent. Usually the steepest natural slopes are to be found on the cross-country routes, and also the greatest difficulty in securing a low through grade. An approximate determination of the ruling grade is usually made during the reconnoissance. If the ruling grade has been previously decided on by other considerations, the leading feature of the reconnoissance survey will be the determination of a general route along which it will be possible to survey a line whose maximum grade shall not exceed the ruling grade.

**5. Mountain route.** The streams of a mountainous region frequently have a slope exceeding the desired ruling grade. In such cases there is no possibility of securing the desired grade by following the streams. The penetration of such a region may only be accomplished by “development”—accompanied perhaps by tunneling. “Development” consists in deliberately increasing the length of the road between two extremes of elevation so that the rate of grade shall be as low as desired.

The usual method of accomplishing this is to take advantage of some convenient formation of the ground to introduce some lateral deviation. The methods may be somewhat classified as follows:

(a) Running the line up a convenient lateral valley, turning a sharp curve and working back up the opposite slope. As shown in Fig. 1, the considerable rise between *A* and *B* was

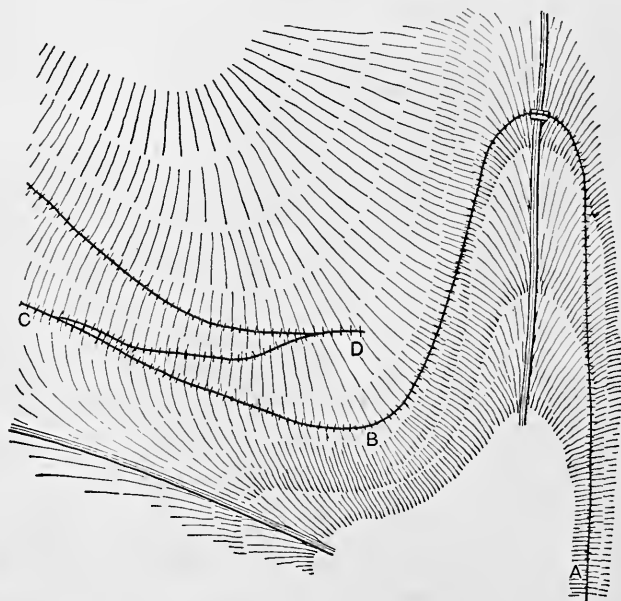


FIG. 1.

surmounted by starting off in a very different direction from the general direction of the road; then, when about one-half of the desired rise had been obtained, the line crossed the valley and continued the climb along the opposite slope. (b) *Switch-back*. On the steep side-hill *BCD* (Fig. 1) a very considerable gain in elevation was accomplished by the switchback *CD*. The gain in elevation from *B* to *D* is very great. On the other hand, the speed must always be slow; there are two complete stoppages of the train for each run; all trains must run backward from *C* to *D*. (c) *Bridge spiral*. When a valley is so narrow at some point that a bridge or viaduct of reasonable length can span the valley at a considerable elevation above the bottom of the valley, a bridge spiral may be desirable. In



Fig. 2 the line ascends the stream valley past *A*, crosses the stream at *B*, works back to the narrow place at *C*, and there crosses itself, having gained perhaps 100 feet in elevation. (d) *Tunnel spiral*. This is the reverse of the previous plan.

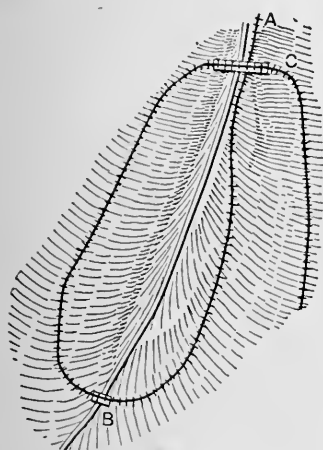


FIG. 2.

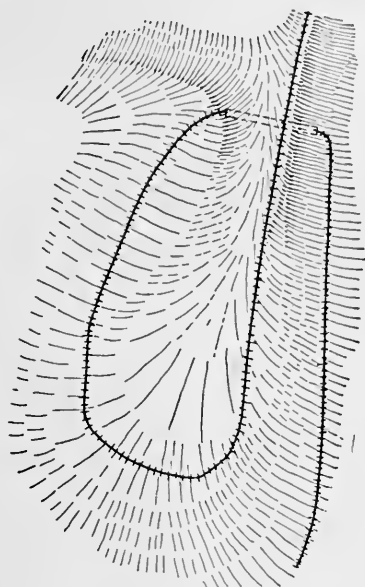


FIG. 3

It implies a thin steep ridge, so thin at some place that a tunnel through it will not be excessively long. Switchbacks and spirals are sometimes necessary in mountainous countries, but they should not be considered as normal types of construction. A region must be very difficult if these devices cannot be avoided.

Rack railways and cable roads, although types of mountain railroad construction, will not be here considered.

**6. Existing maps.** The maps of the U. S. Geological Survey are exceedingly valuable as far as they have been completed. So far as topographical considerations are concerned, they almost dispense with the necessity for the reconnoissance and "first preliminary" surveys. Some of the State Survey maps will give practically the same information. County and township maps can often be used for considerable information as to the relative *horizontal* position of governing points, and even some

approximate data regarding elevations may be obtained by a study of the streams. Of course such information will not dispense with surveys, but will assist in so planning them as to obtain the best information with the least work. When the relative horizontal positions of points are reliably indicated on a map, the reconnoissance may be reduced to the determination of the relative elevations of the governing points of the route.

**7. Determination of relative elevations.** A recent description of European methods includes spirit-leveling in the reconnoissance work. This may be due to the fact that, as indicated above, previous topographical surveys have rendered unnecessary the “exploratory” survey which is required in a new country, and that their reconnoissance really corresponds more nearly to our preliminary.

The perfection to which barometrical methods have been brought has rendered it possible to determine differences of elevation with sufficient accuracy for reconnoissance purposes by the combined use of a mercurial and an aneroid barometer. The mercurial barometer should be kept at “headquarters,” and readings should be taken on it at such frequent intervals that any fluctuation is noted, and throughout the period that observations with the aneroid are taken in the field. At each observation there should also be recorded the time, the reading of the attached thermometer, and the temperature of the external air. For uniformity, the mercurial readings should then be “reduced to 32° F.” Before starting out, a reading of the aneroid should be taken at headquarters coincident with a reading of the mercurial. The difference is one value of the correction to the aneroid. As soon as the aneroid is brought back another comparison of readings should be made. Even though there has been considerable rise or fall of pressure in the interval, the *difference* in readings (the correction) should be substantially the same provided the aneroid is a good instrument. The best aneroids read directly to  $\frac{1}{100}$  of an inch of mercury and may be estimated to  $\frac{1}{1000}$  of an inch—which corresponds to about 0.9 foot difference of elevation. In the field there should be read,

at each point whose elevation is desired, the aneroid, the time, and the temperature. These readings, corrected by the mean value of the correction between the aneroid and the mercurial, should then be combined with the reading of the mercurial (interpolated if necessary) for the times of the aneroid observations and the difference of elevation obtained. [See the author's "Problems in the Use and Adjustment of Engineering Instruments," Prob. 22.] Important points should be observed more than once if possible. Such duplicate observations will be found to give surprisingly concordant results even when a general fluctuation of atmospheric pressure so modifies the tabulated readings that an agreement is not at first apparent. Variations of pressure produced by high winds, thunder-storms, etc., will generally vitiate possible accuracy by this method. By "headquarters" is meant any place whose elevation above any given datum is known and where the mercurial may be placed and observed while observations within a range of several miles are made with the aneroid. If necessary the elevation of a new headquarters may be determined by the above method, but there should be if possible several independent observations whose accordance will give a fair idea of their accuracy.

The above method should be neither slighted nor used for more than it is worth. When properly used, the errors are compensating rather than cumulative. When used, for example, to determine that a pass *B* is 260 feet higher than a determined bridge crossing at *A* which is six miles distant, and that another pass *C* is 310 feet higher than *A* and is ten miles distant, the figures, even with all necessary allowances for inaccuracy, will give an engineer a good idea as to the choice of route especially as affected by ruling grade. There is no comparison between the time and labor involved in obtaining the above information by barometric and by spirit-leveling methods, and *for reconnaissance purposes* the added accuracy of the spirit-leveling method is hardly worth its cost.

**8. Horizontal measurements, bearings, etc.** When there is no map which may be depended on, or when only a skeleton

map is obtainable, a rapid survey, sufficiently accurate for the purpose, may be made by using a pocket compass for bearings and a telemeter, odometer, or pedometer for distances. The telemeter [stadia] is more accurate, but it requires a definite clear sight from station to station, which may be difficult through a wooded country. The odometer, which records the revolutions of a wheel of known circumference, may be used even in rough and wooded country, and the results may be depended on to a small percentage. The pedometer (or pace-measurer) depends for its accuracy on the actual movement of the mechanism for each pace and on the uniformity of the pacing. Its results are necessarily rough and approximate, but it may be used to fill in some intermediate points in a large skeleton map. A hand-level is also useful in determining the relative elevation of various topographical features which may have some bearing on the proper location of the road.

**9. Importance of a good reconnoissance.** The foregoing instruments and methods should be considered only as aids in exercising an educated common sense, without which a proper location cannot be made. The reconnoissance survey should command the best talent and the greatest experience available. If the general route is properly chosen, a comparatively low order of engineering skill can fill in a location which will prove a paying railroad property; but if the general route is so chosen that the ruling grades are high and the business obtained is small and subject to competition, no amount of perfection in detailed alignment or roadbed construction can make the road a profitable investment.

#### PRELIMINARY SURVEYS.

**10. Character of survey.** A preliminary railroad survey is properly a topographical survey of a belt of country which has been selected during the reconnoissance and within which it is estimated that the located line will lie. The width of this belt will depend on the character of the country. When a railroad

is to follow a river having very steep banks the choice of location is sometimes limited at places to a very few feet of width and the belt to be surveyed may be correspondingly narrowed. In very flat country the desired width may be only limited by the ability to survey points with sufficient accuracy at a considerable distance from what may be called the "backbone line" of the survey.

**11. Cross-section method.** This is the only feasible method in a wooded country, and is employed by many for all kinds of country. The *backbone* line is surveyed either by observing magnetic bearings with a compass or by carrying forward

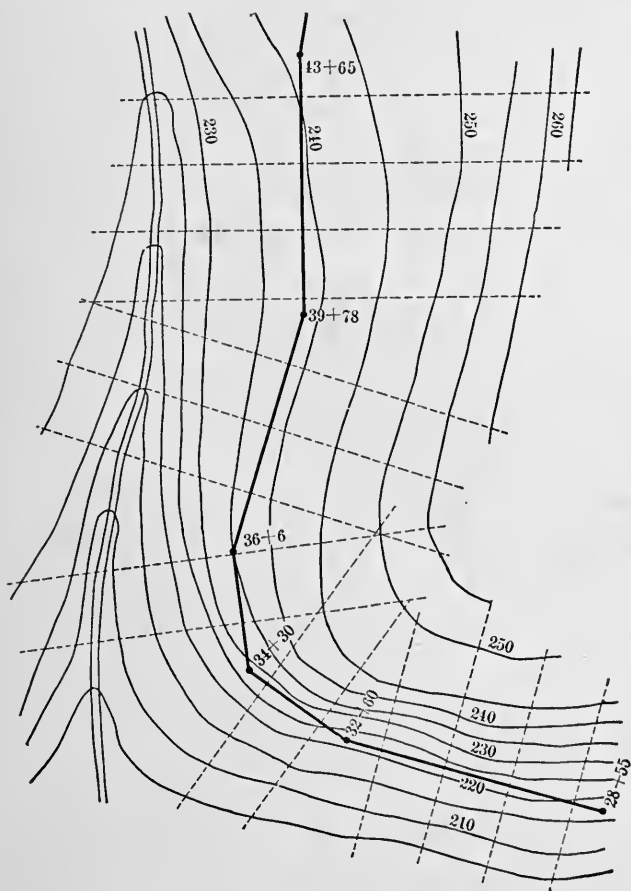


FIG. 4.

absolute azimuths with a transit. The compass method has the disadvantages of limited accuracy and the possibility of

considerable local error owing to local attraction. On the other hand there are the advantages of greater simplicity, no necessity for a back rodman, and the fact that the errors are purely local and not cumulative, and may be so limited, with care, that they will cause no vital error in the subsequent location survey. The transit method is essentially more accurate, but is liable to be more laborious and troublesome. If a large tree is encountered, either it must be cut down or a troublesome operation of offsetting must be used. If the compass is employed under these circumstances, it need only be set up on the far side of the tree and the former bearing produced. An error in reading a transit azimuth will be carried on throughout the survey. An error of only five minutes of arc will cause an offset of nearly eight feet in a mile. Large azimuth errors may, however, be avoided by immediately checking each new azimuth with a needle reading. It is advisable to obtain true azimuth at the beginning of the survey by an observation on the sun or Polaris, and to check the azimuths every few miles by azimuth observations. Distances along the backbone line should be measured with a chain or steel tape and stakes set every 100 feet. When a course ends at a substation, as is usually the case, the remaining portion of the 100 feet should be measured along the next course. The level party should immediately obtain the elevations (to the nearest tenth of a foot) of all stations, and also of the lowest points of all streams crossed and even of dry gullies which would require culverts.

**12. Cross-sectioning.** It is usually desirable to obtain contours at five-foot intervals. This may readily be done by the use of a Locke level (which should be held on top of a simple five-foot stick), a tape, and a rod ten feet in length graduated to feet and tenths. The method of use may perhaps be best explained by an example. Let Fig. 5 represent a section *perpendicular* to the survey line—such a section as would be made by the dotted lines in Fig. 4. *C* represents the station point. Its elevation as determined by the level is, say, 158.3 above datum. When the Locke level on its five-foot rod is placed at

$C$ , the level has an elevation of 163.3. Therefore when a point is found (as at  $a$ ) where the level will read 3.3 on the rod, that

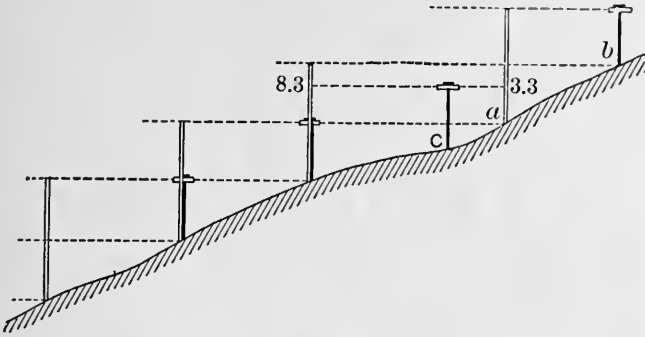


FIG. 5.

point has an elevation of 160.0 and its distance from the center gives the position of the 160-foot contour. Leaving the long rod at that point ( $a$ ), carry the level to some point ( $b$ ) such that the level will sight at the *top* of the rod.  $b$  is then on the 165-

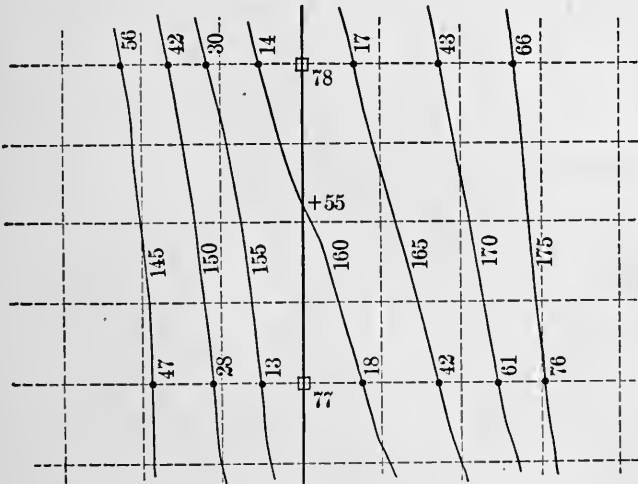


FIG. 6.

foot contour, and the *horizontal* distance  $ab$  added to the horizontal distance  $ac$  gives the position of that contour from the center. The contours on the lower side are found similarly. The first rod reading will be 8.3, giving the 155-foot contour. Plot the results in a note-book which is ruled in quarter-inch squares, using a scale of 100 feet per inch in both directions.

Plot the work up the page; then when looking ahead along the line, the work is properly oriented. When a contour crosses the survey line, the place of crossing may be similarly determined. If the ground flattens out so that five-foot contours are very far apart, the absolute elevations of points at even fifty-foot distances from the center should be determined. The method is exceedingly rapid. Whatever error or inaccuracy occurs is confined in its effect to the one station where it occurs. The work being thus plotted in the field, unusually irregular topography may be plotted with greater certainty and no great error can occur without detection. It would even be possible by this method to detect a gross error that might have been made by the level party.

**13. Stadia method.** This method is best adapted to fairly open country where a "shot" to any desired point may be taken without clearing. The *backbone* survey line is the same as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of such an error may be easily neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

Since the students taking this work are assumed to be familiar with the methods of stadia topographical surveys, this part of the subject will not be further elaborated.

**14. "First" and "second" preliminary surveys.** Some engineers advocate two preliminary surveys. When this is done,



the first is a very rapid survey, made perhaps with a compass, and is only a better grade of reconnoissance. Its aim is to rapidly develop the facts which will decide for or against any proposed route, so that if a route is found to be unfavorable another more or less modified route may be adopted without having wasted considerable time in the survey of useless details. By this time the student should have grasped the fundamental idea that both the reconnoissance and preliminary surveys are not surveys of *lines* but of *areas*; that their aim is to survey only those topographical features which would have a determining influence on any railroad line which might be constructed through that particular territory, and that the value of a locating engineer is largely measured by his ability to recognize those determining influences with the least amount of work from his surveying corps. Frequently too little time is spent on the comparative study of preliminary lines. A line will be hastily decided on after very little study; it will then be surveyed with minute detail and estimates carefully worked up, and the claims of any other suggested route will then be handicapped, if not disregarded, owing to an unwillingness to discredit and throw away a large amount of detailed surveying. The cost of two or three extra preliminary surveys (*at critical points* and not over the whole line) is utterly insignificant compared with the probable improvement in the "operating value" of a line located after such a comparative study of preliminary lines.

#### LOCATION SURVEYS.

**15. "Paper location."** When the preliminary survey has been plotted to a scale of 200 feet per inch and the contours drawn in, a study may be made for the location survey. Disregarding for the present the effect on location of transition curves, the alignment may be said to consist of straight lines (or "tangents") and circular curves. The "paper location" therefore consists in plotting on the preliminary map a succession of straight lines which are tangent to the circular curves connect-

ing them. The determining points should first be considered. Such points are the termini of the road, the lowest practicable point over a summit, a river-crossing, etc. So far as is possible, having due regard to other considerations, the road should be a "surface" road, i.e., the cut and fill should be made as small as possible. The maximum permissible grade must also have been determined and duly considered. The method of location differs radically according as the lines joining the determining points have a very low grade or have a grade that approaches the maximum permissible. With very low natural grades it is only necessary to strike a proper balance between the requirements for easy alignment and the avoidance of excessive earthwork. When the grade between two determined points approaches the maximum, a study of the location may be begun by finding a strictly surface line which will connect those points with a line at the given grade. For example, suppose the required grade is 1.6% and that the contours are drawn at 5-foot intervals. It will require 312 feet of 1.6% grade to rise 5 feet. Set a pair of dividers at 312 feet and step off this interval on successive contours. This line will in general be very irregular, but in an easy country it may lie fairly close to the proper location line, and even in difficult country such a surface line will assist greatly in selecting a suitable location. When the larger part of the line will evidently consist of tangents, the tangents should be first located and should then be connected by suitable curves. When the curves predominate, as they generally will in mountainous country, and particularly when the line is purposely lengthened in order to reduce the grade, the curves should be plotted first and the tangents may then be drawn connecting them. Considering the ease with which such lines may be drawn on the preliminary map, it is frequently advisable, after making such a paper location, to begin all over, draw a new line over some specially difficult section and compare results. Profiles of such lines may be readily drawn by noting their intersection with each contour crossed. Drawing on each profile the required grade line will furnish an approximate idea of the

*comparative* amount of earthwork required. After deciding on the paper location, the length of each tangent, the central angle (see § 21), and the radius of each curve should be measured as accurately as possible. Since a slight error made in such measurements, taken from a map with a scale of 200 feet per inch, would by accumulation cause serious discrepancies between the plotted location and the location as afterward surveyed in the field, frequent tie lines and angles should be determined between the plotted location line and the preliminary line, and the location should be altered, as may prove necessary, by changing the length of a tangent or changing the central angle or radius of a curve, so that the agreement of the check-points will be sufficiently close. The errors of an inaccurate preliminary survey may thus be easily neutralized (see § 33). When the preliminary line has been properly run, its "backbone" line will lie very near the location line and will probably cross it at frequent intervals, thus rendering it easy to obtain short and numerous tie lines.

**16. Surveying methods.** A transit should be used for alignment, and only precise work is allowable. The transit stations should be centered with tacks and should be tied to witness-stakes, which should be located outside of the range of the earthwork, so that they will neither be dug up nor covered up. All original property lines lying within the limits of the right of way should be surveyed with reference to the location line, so that the right-of-way agent may have a proper basis for settlement. When the property lines do not extend far outside of the required right of way they are frequently surveyed completely.

The leveler usually reads the target to the nearest thousandth of a foot on turning-points and bench-marks, but reads to the nearest tenth of a foot for the elevation of the ground at stations. Considering that  $\frac{1}{1000}$  of a foot has an angular value of only 7 seconds at a distance of 300 feet, and that one division of a level-bubble is usually about 30 seconds, it may be seen that it is a useless refinement to read to thousandths unless corresponding care is taken in the use of the level. The leveler

should also locate his bench-marks outside of the range of earthwork. A knob of rock protruding from the ground affords an excellent mark. A large nail, driven in the roots of a tree, which is not to be disturbed, is also a good mark. These marks should be clearly described in the note-book. The leveler should obtain the elevation of the ground at all station-points; also at all sudden breaks in the profile line, determining also the distance of these breaks from the previous even station. This will include the position and elevation of all streams, and even dry gullies, which are crossed.

**Measurements** should preferably be made with a steel tape, care being taken on steep ground to insure horizontal measurements. Stakes are set each 100 feet, and also at the beginning and end of all curves. Transit-points (sometimes called "plugs" or "hubs") should be driven flush with the ground, and a "witness-stake," having the "number" of the station, should be set three feet to the right. For example, the witness-stake might have on one side "137 + 69.92," and on the other side "P C 4° R," which would signify that the transit hub is 69.92 feet beyond station 137, or 13769.92 feet from the beginning of the line, and also that it is the "point of curve" of a "4°-curve" which turns to the *right*.

**Alignment.** The alignment is evidently a part of the location survey, but, on account of the magnitude and importance of the subject, it will be treated in a separate chapter.

**17. Form of Notes.** Although the Form of Notes cannot be thoroughly understood until after curves are studied, it is here introduced as being the most convenient place. The right-hand page should have a sketch showing all roads, streams, and property lines crossed with the bearings of those lines. This should be drawn to a scale of 100 feet per inch—the quarter-inch squares which are usually ruled in note-books giving convenient 25-foot spaces. This sketch will always be more or less distorted on curves, since the center line is always shown as *straight* regardless of curves. The station points ("Sta." in first column, left-hand page) should be placed opposite to their

sketched positions, which means that even stations will be recorded on every *fourth* line. This allows three intermediate lines for substations, which is ordinarily more than sufficient. The notes should read up the page, so that the sketch will be properly oriented when looking ahead along the line. The other columns on the left-hand page will be self-explanatory when the subject of curves is understood. If the “calculated bearings” are based on azimuthal observations, their agreement (or *constant* difference) with the needle readings will form a valuable check on the curve calculations and the instrumental work.

FORM OF NOTES.

[Left-hand page.]

[Right-hand page.]

| Sta.           | Align-<br>ment                                               | Vernier | Tang.<br>Defl. | Calc.<br>Bearing. | Needle.     |  |
|----------------|--------------------------------------------------------------|---------|----------------|-------------------|-------------|--|
| 54             |                                                              |         |                |                   |             |  |
| 53<br>⊙ + 72.2 | P.T.                                                         | 9° 11'  | 18° 22'        | N 54° 48' E       | N 62° 15' 1 |  |
| 52             |                                                              | 7 57    |                |                   |             |  |
| 51             |                                                              | 6 15    |                |                   |             |  |
| ⊙ 50           | [ 3° 24' curve to right for<br>18° 22'; tang. dist., 272.5 ] | 4 33    |                |                   |             |  |
| 49             |                                                              | 2 51    |                |                   |             |  |
| 48             |                                                              | 1 09    |                |                   |             |  |
| ⊙ + 32<br>47   | P.C.                                                         | 0°      |                |                   |             |  |
| 46             |                                                              |         |                | N 36° 26' E       | N 44° 0' 1  |  |

## CHAPTER II.

### ALIGNMENT.

IN this chapter the alignment of the *center line* only of a pair of rails is considered. When a railroad is crossing a summit in the grade line, although the horizontal projection of the alignment may be straight, the vertical projection will consist of two sloping lines joined by a curve. When a curve is on a grade, the center line is really a spiral, a curve of double curvature, although its horizontal projection is a circle. The center line therefore consists of straight lines and curves of single and double curvature. The simplest method of treating them is to consider their horizontal and vertical projections separately. In treating simple, compound, and transition curves, only the horizontal projections of those curves will be considered.

#### SIMPLE CURVES.

**18. Designation of curves.** A curve may be designated either by its radius or by the angle subtended by a chord of unit length. Such an angle is known as the “degree of curve” and is indicated by  $D$ . Since the curves that are practically used have very long radii, it is generally impracticable to make any use of the actual center, and the curve is located without reference to it. If

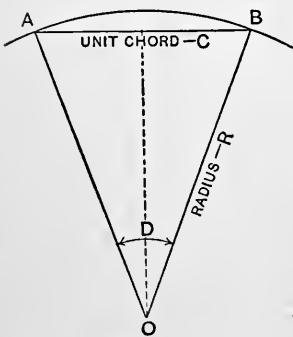


FIG. 7.

$AB$  in Fig. 7 represents a unit chord ( $C$ ) of a curve of radius  $R$ , then by the above defini-

tion the angle  $AOB$  equals  $D$ . Then  $AO \sin \frac{1}{2}D = \frac{1}{2}AB = \frac{1}{2}C$ .

$$\therefore R = \frac{\frac{1}{2}C}{\sin \frac{1}{2}D} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or, by inversion,

$$\sin \frac{1}{2}D = \frac{C}{2R} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The unit chord is variously taken throughout the world as 100 feet, 66 feet; and 20 meters. In the United States 100 feet is invariably used as the unit chord length, and throughout this work it will be so considered. Table I has been computed on this basis. It gives the radius, with its logarithm, of all curves from a  $0^{\circ} 01'$  curve up to a  $10^{\circ}$  curve, varying by single minutes. The sharper curves, which are seldom used, are given with larger intervals.

An approximate value of  $R$  may be readily found from the following simple rule, which should be memorized :

$$R = \frac{5730}{D}.$$

Although such values are not mathematically correct, since  $R$  does not strictly vary inversely as  $D$ , yet the resulting value is within a tenth of one per cent for all commonly used values of  $R$ , and is sufficiently close for many purposes, as will be shown later.

**19. Length of a sub-chord.** Since it is impracticable to measure along a curved arc, curves are always measured by laying off 100-foot chord lengths. This means that the actual arc is always a little longer than the chord. It also means that a *subchord* (a chord shorter than the unit length) will be a little longer than the ratio of the angles subtended would call for. The truth of this may be seen without calcu-

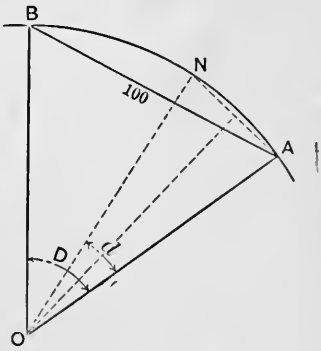


FIG. 8.

lation by noting that two equal subchords, each subtending the angle  $\frac{1}{2}D$ , will evidently be slightly longer than 50 feet each. If  $c$  be the length of a subchord subtending the angle  $d$ , then, as in Eq. (2),

$$\sin \frac{1}{2}d = \frac{c}{2R},$$

or, by inversion,

$$c = 2R \sin \frac{1}{2}d. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The *nominal length* of a subchord  $= 100 \frac{d}{D}$ . For example, a *nominal* subchord of 40 feet will subtend an angle of  $\frac{4.0}{100}$  of  $D^\circ$ ; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a  $10^\circ$  curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

**20. Length of a curve.** The length of a curve is always indicated by the quotient of  $100\Delta \div D$ . If the quotient of  $\Delta \div D$  is a whole number, the length as thus indicated is the true length—*measured in 100-foot chord lengths*. If it is an odd number or if the curve begins and ends with a subchord (even though  $\Delta \div D$  is a whole number), theoretical accuracy requires that the *true* subchord lengths shall be used, although the difference may prove insignificant. The length of the arc (or the mean length of the two rails) is therefore always in excess of the length as given above. Ordinarily the amount of this excess is of no practical importance. It simply adds an insignificant amount to the length of rail required.

*Example.* Required the nominal and true lengths of a  $3^\circ 45'$  curve having a central angle of  $17^\circ 25'$ . First reduce



the degrees and minutes to decimals of a degree.  $(100 \times 17^{\circ} 25') \div 3^{\circ} 45' = 1741.667 \div 3.75 = 464.444$ . The curve has four 100-foot chords and a nominal chord of 64.444. The true chord should be 64.451. The actual arc is

$$17^{\circ}.4167 \times \frac{\pi}{180^{\circ}} \times R = 464.527.$$

The excess is therefore  $464.527 - 464.451 = 0.076$  foot.

**21. Elements of a curve.** Considering the line as running from *A* toward *B*, the beginning of the curve, at *A*, is called the *point of curve* (*PC*). The other end of the curve, at *B*, is called the *point of tangency* (*PT*). The intersection of the tangents is called the *vertex* (*V*). The angle made by the tangents at *V*, which equals the angle made by the radii to the extremities of the curve, is called the *central angle* ( $\Delta$ ). *AV* and *BV*, the two equal tangents from the vertex to the *PC* and *PT*, are called the *tangent distances* (*T*). The chord *AB* is called the *long chord* (*LC*). The intercept *HG* from the middle of the long chord to the middle of the arc is called the *middle ordinate* (*M*). That part of the secant *GV* from the middle of the arc to the vertex is called the *external distance* (*E*). From the figure it is very easy to derive the following frequently used relations :

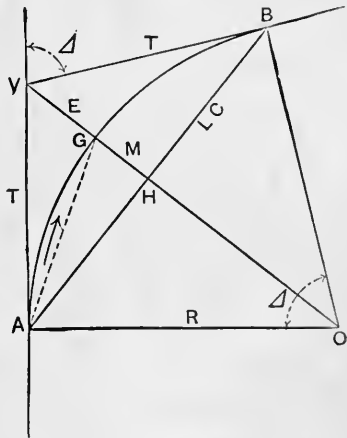


FIG. 9.

$$\begin{aligned} T &= R \tan \frac{1}{2} \Delta \quad . \quad . \quad . \quad . \quad . \quad (4) \\ LC &= 2R \sin \frac{1}{2} \Delta \quad . \quad . \quad . \quad . \quad . \quad (5) \\ M &= R \text{ vers } \frac{1}{2} \Delta \quad . \quad . \quad . \quad . \quad . \quad (6) \\ E &= R \text{ exsec } \frac{1}{2} \Delta \quad . \quad . \quad . \quad . \quad . \quad (7) \end{aligned}$$

**22. Relation between *T*, *E*, and  $\Delta$ .** Join *A* and *G* in Fig. 9. The angle  $VAG = \frac{1}{4} \Delta$ , since it is measured by one half of the

arc  $AG$  between the secant and tangent.  $AGO = 90^\circ - \frac{1}{4}\Delta$ .

$$AV:VG::\sin AGV:\sin VAG;$$

$$\sin AGV = \sin AGO = \cos \frac{1}{4}\Delta;$$

$$T:E::\cos \frac{1}{4}\Delta:\sin \frac{1}{4}\Delta;$$

$$T = E \cot \frac{1}{4}\Delta. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The same relation may be obtained by dividing Eq. 4 by Eq. 7, since  $\tan \alpha \div \text{exsec } \alpha = \cot \frac{1}{2}\alpha$ .

**23. Elements of a  $1^\circ$  curve.** From Eqs. 1 to 8 it is seen that the elements of a curve vary directly as  $R$ . It is also seen to be very nearly true that  $R$  varies inversely as  $D$ . If the elements of a  $1^\circ$  curve for various central angles are calculated and tabulated, the elements of a curve of  $D^\circ$  curvature may be approximately found by dividing by  $D$  the corresponding elements of a  $1^\circ$  curve having the same central angle. For small central angles and low degrees of curvature the errors involved by the approximation are insignificant, and even for larger angles the errors are so small that *for many purposes* they may be disregarded.

In Table II is given the value of the tangent distances, external distances, and long chords for a  $1^\circ$  curve for various central angles. The student should familiarize himself with the degree of approximation involved by solving a large number of cases under various conditions by the exact and approximate methods, in order that he may know when the approximate method is sufficiently exact for the intended purpose. The approximate method also gives a ready check on the exact method.

**24. Exercises.** (a) What is the tangent distance of a  $4^\circ 20'$  curve having a central angle of  $18^\circ 24'$ ?

(b) Given a  $3^\circ 30'$  curve and a central angle of  $16^\circ 20'$ , how far will the curve pass from the vertex? [Use Eq. 7.]

(c) An  $18^\circ$  curve is to be laid off using 25-foot (nominal) chord lengths. What is the true length of the subchords?

(d) Given two tangents making a central angle of  $15^\circ 24'$ . It is desired to connect these tangents by a curve which shall pass 16.2 feet from their intersection. How far down the tangent will the curve begin and what will be its radius? (Use Eq. 8 and then use Eq. 4 inverted.)

**25. Curve location by deflections.** The angle between a secant and a tangent (or between two secants intersecting on an arc) is measured by one half of the intercepted arc. Beginning at the  $PC$  ( $A$  in Fig. 10), if the first chord is to be a full chord we may deflect an angle  $V A a (= \frac{1}{2}D)$ , and the point  $a$ , which is 100 feet from  $A$ , is a point on the curve. For the next station,  $b$ , deflect an *additional* angle  $b A a (= \frac{1}{2}D)$  and, with one end of the tape at  $a$ , swing the other end until the 100-foot point is on the line  $Ab$ . The point  $b$  is then on the curve. If the final chord  $cB$  is a subchord, its *additional deflection* ( $\frac{1}{2}\alpha$ ) is something less than  $\frac{1}{2}D$ . The last *deflection* ( $BAV$ ) is of course  $\frac{1}{2}A$ . It is particularly important, when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection, to check the mathematical work by the agreement of this last computed deflection with  $\frac{1}{2}A$ .

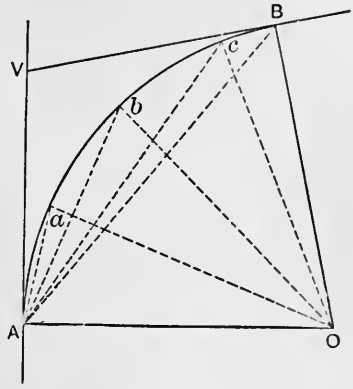


FIG. 10.

*Example.* Given a  $3^\circ 24'$  curve having a central angle of  $18^\circ 22'$  and beginning at sta.  $47 + 32$ , to compute the deflections. The nominal length of curve is  $18^\circ 22' \div 3^\circ 24' = 18.367 \div 3.40 = 5.402$  stations or 540.2 feet. The curve therefore ends at sta.  $52 + 72.2$ . The deflection for sta. 48 is  $\frac{68}{100} \times \frac{1}{2}(3^\circ 24') = 0.68 \times 1^\circ.7 = 1^\circ.156 = 1^\circ 09'$  nearly. For each additional 100 feet it is  $1^\circ 42'$  additional. The final additional deflection for the final subchord of 72.2 feet is

$$\frac{72.2}{100} \times \frac{1}{2}(3^\circ 24') = 1^\circ.2274 = 1^\circ 14' \text{ nearly.}$$

The deflections are

|                                     |        |                   |
|-------------------------------------|--------|-------------------|
| P. C . . . . Sta. 47 + 32 . . . . . | 0°     |                   |
| 48 . . . . .                        | 0°     | + 1° 09' = 1° 09' |
| 49 . . . . .                        | 1° 09' | + 1° 42' = 2° 51' |
| 50 . . . . .                        | 2° 51' | + 1° 42' = 4° 33' |
| 51 . . . . .                        | 4° 33' | + 1° 42' = 6° 15' |
| 52 . . . . .                        | 6° 15' | + 1° 42' = 7° 57' |
| P. T . . . . . 52 + 72.2 . . . .    | 7° 57' | + 1° 14' = 9° 11' |

As a check  $9^{\circ} 11' = \frac{1}{2}(18^{\circ} 22') = \frac{1}{2}\Delta$ . (See the Form of Notes in § 17.)

**26. Instrumental work.** It is generally impracticable to locate more than 500 to 600 feet of a curve from one station. Obstructions will sometimes require that the transit be moved up every 200 or 300 feet. There are two methods of setting off the angles when the transit has been moved up from the *PC*.

(a) The transit may be sighted at the previous transit station with a reading on the plates equal to the deflection angle from that station to the station occupied, but with the angle set off on the *other* side of  $0^{\circ}$ , so that when the telescope is turned to  $0^{\circ}$  it will sight along the tangent at the station occupied. Plunging the telescope, the forward stations may be set off by deflecting the proper deflections from the tangent at the station occupied. This is a very common method and, when the degree of curvature is an even number of degrees and when the transit is only set at even stations, there is but little objection to it. But the degree of curvature is sometimes an odd quantity, and the exigencies of difficult location frequently require that substations be occupied as transit stations. Method (a) will then require the recalculation of all deflections for each new station occupied. The mathematical work is largely increased and the probability of error is very greatly increased and not so easily detected. Method (b) is just as simple as method (a) even for the most simple cases, and for the more difficult cases just referred to the superiority is very great.

(b) Calculate the deflection for each station and substation throughout the curve as though the whole curve were to be located from the *PC*. The computations may thus be completed and *checked* (as above) before beginning the instrumental work. If it unexpectedly becomes necessary to introduce a substation at any point, its deflection from the *PC* may be readily interpolated. The stations actually set from the *PC* are located as usual. **RULE.** When the transit is set on any forward station, backsight to **ANY** previous station with the plates set at the deflection angle for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle originally computed for that station. When the plates read the deflection angle for the station occupied, the telescope is sighting along the tangent at that station—which is the method of getting the forward tangent when occupying the *PT*. Even though the station occupied is an unexpected substation, when the instrument is properly oriented at that station, the angle reading for *any* station, forward or back, is that originally computed for it from the *PC*. In difficult work, where there are obstructions, a valuable check on the accuracy may be found by sighting backward at *any* visible station and noting whether its deflection agrees with that originally computed. As a numerical illustration, assume a  $4^\circ$  curve, with  $28^\circ$  curvature, with stations 0, 2, 4, and 7 occupied. After setting stations 1 and 2, set up the transit at sta. 2 and backsight to sta. 0 with the deflection for sta. 0, which is  $0^\circ$ . The reading on sta. 1 is  $2^\circ$ ; when the reading is  $4^\circ$  the telescope is tangent to the curve, and when sighting at 3 and 4 the deflections will be  $6^\circ$  and  $8^\circ$ . Occupy 4; sight to 2 with a reading of  $4^\circ$ . When the reading is  $8^\circ$  the telescope is tangent to the curve and, by plunging the telescope, 5, 6, and 7 may be located with the originally com-

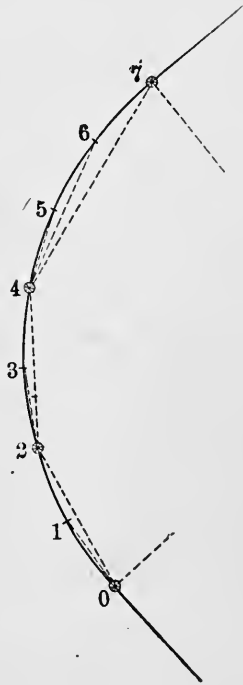


FIG. 11.

puted deflections of  $10^\circ$ ,  $12^\circ$ , and  $14^\circ$ . When occupying  $T$  a backsight may be taken to any visible station with the plates reading the deflection for that station; then when the plates read  $14^\circ$  the telescope will point along the forward tangent.

The location of curves by deflection angles is the normal method. A few other methods, to be described, should be considered as exceptional.

**27. Curve location by two transits.** A curve might be located more or less on a swamp where accurate chaining would be exceedingly difficult if not impossible. The long chord  $AB$  may be determined by triangulation or otherwise, and the elements of

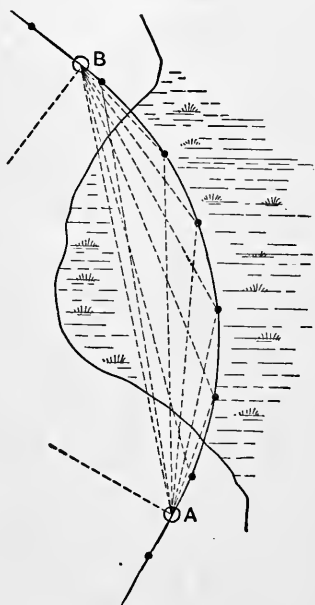


FIG. 12.

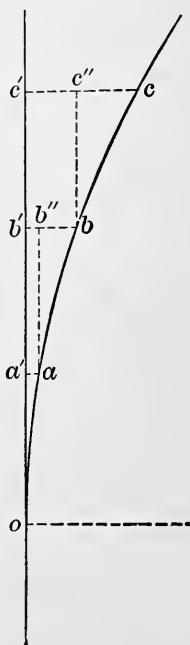


FIG. 13.

the curve computed, including (possibly) subchords at each end. The deflection from  $A$  and  $B$  to each point may be computed. A rodman may then be sent (by whatever means) to locate long stakes at points determined by the simultaneous sightings of the two transits.

**28. Curve location by tangential offsets.** When a curve is very flat and no transit is at hand the following method may be

used: Produce the back tangent as far forward as necessary. Compute the ordinates  $Oa'$ ,  $Ob'$ ,  $Oc'$ , etc., and the abscissæ  $a'a$ ,  $b'b$ ,  $c'e$ , etc. If  $Oa$  is a full station (100 feet), then

$$\left. \begin{aligned} Oa' &= Oa' &= 100 \cos \frac{1}{2}D, \text{ also } = R \sin D; \\ Ob' &= Oa' + a'b' &= 100 \cos \frac{1}{2}D + 100 \cos \frac{3}{2}D, \\ & &\text{also } = R \sin 2D; \\ Oc' &= Oa' + a'b' + b'e' &= 100(\cos \frac{1}{2}D + \cos \frac{3}{2}D + \cos \frac{5}{2}D), \\ & &\text{also } = R \sin 3D; \end{aligned} \right\} \quad (9)$$

etc.

$$\left. \begin{aligned} a'a &= &100 \sin \frac{1}{2}D, \text{ also } = R \text{ vers } D; \\ b'b &= a'a + b''b &= 100 \sin \frac{1}{2}D + 100 \sin \frac{3}{2}D, \\ & &\text{also } = R \text{ vers } 2D; \\ c'e &= b'b + c''c &= 100(\sin \frac{1}{2}D + \sin \frac{3}{2}D + \sin \frac{5}{2}D), \\ & &\text{also } = R \text{ vers } 3D; \end{aligned} \right\} \quad (10)$$

etc.

The functions  $\frac{1}{2}D$ ,  $\frac{3}{2}D$ , etc., may be more conveniently used *without* logarithms, by adding the several *natural* trigonometrical functions and pointing off two decimal places. It may also be noted that  $ob'$  (for example) is one half of the long chord for four stations; also that  $b'b$  is the middle ordinate for four stations. If the engineer is provided with tables giving the long chords and middle ordinates for various degrees of curvature, these quantities may be taken (perhaps by interpolation) from such tables.

If the curve begins or ends at a substation, the angles and terms will be correspondingly altered. The modifications may be readily deduced on the same principles as above, and should be worked out as an exercise by the student.

**29. Curve location by middle ordinates.** Take first the simpler case when the curve begins at an even station. If we consider (in Fig. 14) the curve produced back to  $z$ , the chord  $za = 2 \times 100 \cos \frac{1}{2}D$ ,  $A'a = 100 \cos \frac{1}{2}D$ , and  $A'A = am = zn = 100 \sin \frac{1}{2}D$ . Set off  $AA'$  perpendicular to the tangent and  $A'a$  parallel to the tangent.  $AA' = aa' = bb' = cc'$ , etc.  $= 100 \sin \frac{1}{2}D$ . Set off  $aa'$  perpendicular to  $a'A$ . Produce  $Aa'$

until  $a'b = A'a$ , thus determining  $b$ . Succeeding points of the curve may thus be determined indefinitely.

Suppose the curve begins with a subchord. As before  $ra = Am' = c' \cos \frac{1}{2}d'$ , and  $rA = am' = c' \sin \frac{1}{2}d'$ . Also  $sz = An' = c'' \cos \frac{1}{2}d''$ , and  $sA = zn' = c'' \sin \frac{1}{2}d''$ , in which

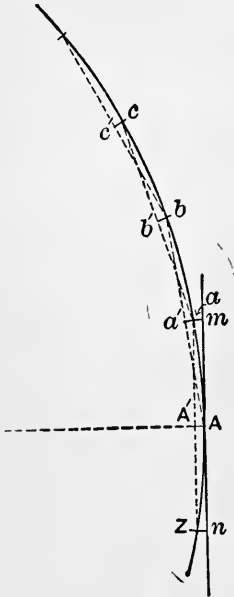


FIG. 14.

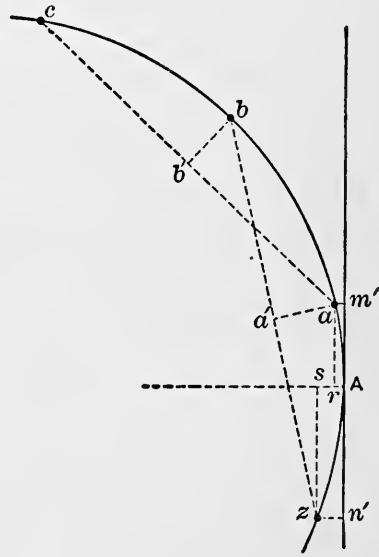


FIG. 15.

$(d' + d'') = D$ . The points  $z$  and  $a$  being determined on the ground,  $aa'$  may be computed and set off as before and the curve continued in full stations. A subchord at the end of the curve may be located by a similar process.

**30. Curve location by offsets from the long chord.** (Fig. 16.) Consider at once the general case in which the curve commences with a subchord (curvature,  $d'$ ), contains with one or more full chords (curvature of each,  $D$ ), and ends with a subchord with curvature  $d''$ . The numerical work consists in computing first  $AB$ , then the various abscissæ and ordinates.  $AB = 2R \sin \frac{1}{2}\Delta$ .

$$\begin{aligned}
 Aa' &= Aa' &= c' \cos \frac{1}{2}(\Delta - d'); \\
 Ab' &= Aa' + a'b' &= c' \cos \frac{1}{2}(\Delta - d') + 100 \cos \frac{1}{2}(\Delta - 2d' - D); \\
 Ac' &= Aa' + a'b' + b'c' &= c' \cos \frac{1}{2}(\Delta - d') + 100 \cos \frac{1}{2}(\Delta - 2d' - D) \\
 & & \quad + 100 \cos \frac{1}{2}(\Delta - 2d'' - D); \\
 \text{also} \\
 &= AB - Bc' &= 2R \sin \frac{1}{2}\Delta - c'' \cos \frac{1}{2}(\Delta - d'').
 \end{aligned}
 \tag{11}$$





the best method to use, and it is frequently advisable to devise a special solution for some particular case.

**a. When the vertex is inaccessible.** As shown in § 26, it is not absolutely essential that the vertex of a curve should be located on the ground. But it is very evident that the angle between the terminal tangents is determined with far less probable error if it is measured by a single measurement at the vertex rather than as the result of numerous angle measurements along the curve, involving several positions of the transit and comparatively short sights. Sometimes the location of the tangents is already determined on the ground (as by  $bn$  and  $am$ , Fig. 17), and it is required to join the tangents by a curve of given radius. *Method.* Measure  $ab$  and the angles  $Vba$  and  $baV$ .  $\Delta$  is the sum of these angles. The distances  $bV$  and  $aV$  are computable from the above data. Given  $\Delta$  and  $R$ , the tan-

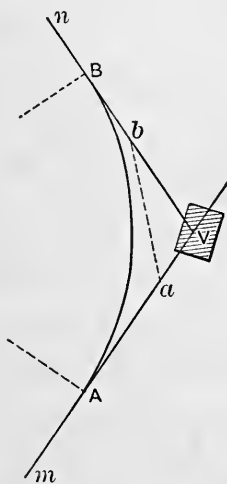


FIG. 17.

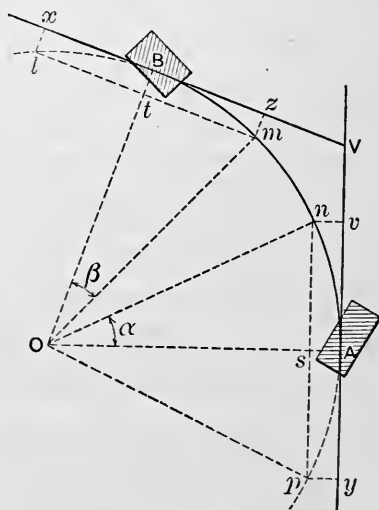


FIG. 18.

gent distances are computable, and then  $Bb$  and  $aA$  are found by subtracting  $bV$  and  $aV$  from the tangent distances. The curve may then be run from  $A$ , and the work may be checked by noting whether the curve as run ends at  $B$ —previously located from  $b$ .

**b. When the point of curve (or point of tangency) is inaccessible.** At some distance ( $As$ , Fig. 18) an unobstructed line  $pn$



a. To move the forward tangent parallel to itself a distance  $x$ , the point of curve ( $A$ ) remaining fixed. (Fig. 20.)

$$V'h = B'r = x'.$$

$$VV' = \frac{V'h}{\sin h} = \frac{x'}{\sin \Delta} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$AV' = AV + VV'.$$

The triangle  $BmB'$  is isosceles and  $Bm = B'm$ .

$$R' - R = O'O = mB = \frac{B'r}{\text{vers } B'mB} = \frac{x'}{\text{vers } \Delta}$$

$$\therefore R' = R + \frac{x'}{\text{vers } \Delta} \quad . \quad . \quad . \quad . \quad (14)$$

The solution is very similar in case the tangent is moved inward to  $V''B''$ . Note that this method necessarily changes the

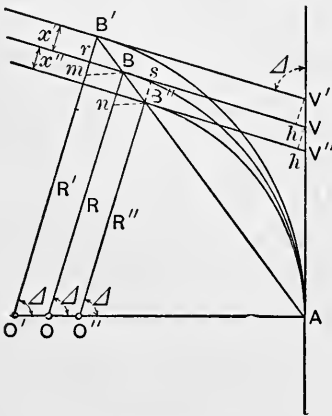


FIG. 20.

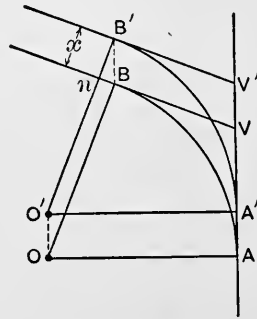


FIG. 21.

radius. If the radius is not to be changed, the point of curve must be altered as follows:

b. To move the forward tangent parallel to itself a distance  $x$ , the radius being unchanged. (Fig. 21.) In this case the whole



**34. Limitations in location.** It may be required to run a curve that shall join two given tangents and also pass through a given point. The point ( $P$ , Fig. 23) is assumed to be determined by its distance ( $VP$ ) from the vertex and by the angle  $\angle VP = \beta$ .

It is required to determine the radius ( $R$ ) and the tangent distance ( $AV$ ).  $\angle A$  is known.

$$PVG = \frac{1}{2}(180^\circ - \angle A) - \beta = 90^\circ - (\frac{1}{2}\angle A + \beta).$$

$$PP' = 2VP \sin PVG = 2VP \cos (\frac{1}{2}\angle A + \beta).$$

$$PSV = \frac{1}{2}\angle A. \quad \therefore SP = VP \frac{\sin \beta}{\sin \frac{1}{2}\angle A}.$$

$$AS = \sqrt{SP \times SP'} = \sqrt{SP(SP + PP')}.$$

$$= \sqrt{VP \frac{\sin \beta}{\sin \frac{1}{2}\angle A} \left[ VP \frac{\sin \beta}{\sin \frac{1}{2}\angle A} + 2VP \cos (\frac{1}{2}\angle A + \beta) \right]}$$

$$= VP \sqrt{\frac{\sin^2 \beta}{\sin^2 \frac{1}{2}\angle A} + \frac{2 \sin \beta \cos (\frac{1}{2}\angle A + \beta)}{\sin \frac{1}{2}\angle A}}.$$

$$SV = VP \frac{\sin (\frac{1}{2}\angle A + \beta)}{\sin \frac{1}{2}\angle A}.$$

$$AV = AS + SV$$

$$= \frac{VP}{\sin \frac{1}{2}\angle A} [\sin (\frac{1}{2}\angle A + \beta) + \sqrt{\sin^2 \beta + 2 \sin \beta \sin \frac{1}{2}\angle A \cos (\frac{1}{2}\angle A + \beta)}]. \quad (18)$$

$$R = AV \cot \frac{1}{2}\angle A.$$

In the special case in which  $P$  is on the median line  $OV$ ,  $\beta = 90^\circ - \frac{1}{2}\angle A$ , and  $(\frac{1}{2}\angle A + \beta) = 90^\circ$ . Eq. (18) then reduces to

$$AV = \frac{VP}{\sin \frac{1}{2}\angle A} (1 + \cos \frac{1}{2}\angle A) = VP \cot \frac{1}{4}\angle A,$$

as might have been immediately derived from Eq. (8).

In case the point  $P$  is given by the offset  $PK$  and by the distance  $VK$ , the triangle  $PKV$  may be readily solved, giving the distance  $VP$  and the angle  $\beta$ , and the remainder of the solution will be as above.

**35. Determination of the curvature of existing track.** (a) *Using a transit.* Set up the transit at any point in the center of the track. Measure in each direction 100 feet to points also in the center of the track. Sight on one point with the plates at  $0^\circ$ . Plunge the telescope and sight at the other point. The angle between the chords equals the degree of curvature.

(b) *Using a tape and string.* Stretch a string (say 50 feet long) between two points on the inside of the head of the outer rail. Measure the ordinate ( $x$ ) between the *middle* of the string and the head of the rail. Then

$$R = \frac{\text{chord}^2}{8x} \quad (\text{very nearly}). \quad . \quad . \quad . \quad . \quad (19)$$

For, in Fig. 24, since the triangles  $AOE$  and  $ADC$  are similar,  $AO : AE :: AD : DC$  or  $R = \frac{1}{2}AD^2 \div x$ . When, as is usual, the arc is very short compared with the radius,  $AD = \frac{1}{2}AB$ , very nearly. Making this substitution we have Eq. (19). With a chord of 50 feet and a  $10^\circ$  curve, the resulting difference in  $x$  is .0025 of an inch—far within the possible accuracy of such a method. The above method gives the radius of the inner head of the outer rail. It should be diminished by  $\frac{1}{2}g$  for the radius of the center of the track. With easy curvature, however, this will not affect the result by more than one or two tenths of one per cent.

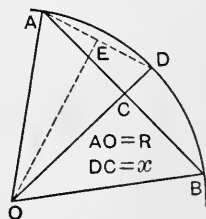


FIG. 24.

The inversion of this formula gives the required middle ordinate for a rail on a given curve. For example, the middle ordinate of a 30-foot rail, bent for a  $6^\circ$  curve, is

$$x = 900 \div (8 \times 955) = .118 \text{ foot} = 1.4 \text{ inches.}$$

Another much used rule is to require the foreman to have a string, knotted at the centre, of such length that the middle ordinate, measured in inches, equals the degree of curve. To find that length, substitute (in eq. (19))  $5730 \div D$  for  $R$  and  $D \div 12$  for  $x$ . Solving for *chord*, we obtain *chord* = 61.8 feet. The rule is not theoretically exact, but, considering the uncertain stretching of the string, the error is insignificant. In fact, the distance usually given is 62 feet, which is close enough for all purposes for which such a method should be used.

**36. Problems.** A systematic method of setting down the solution of a problem simplifies the work. Logarithms should always be used, and *all* the work should be so set down that a revision of the work to find a supposed error may be readily done. The value of such systematic work will become more apparent as the problems become more complicated. The two solutions given below will illustrate such work.

*a.* Given a  $3^\circ$  curve beginning at Sta. 27 + 60 and running to Sta. 32 + 45. Compute the ordinates and offsets used in locating the curve by tangential offsets.

*b.* With the same data as above, compute the distances to locate the curve by offsets from the long chord.

*c.* Assume that in Fig. 17  $ab$  is measured as 217.6 feet, the angle  $abV = 17^\circ 42'$ , and the angle  $baV = 21^\circ 14'$ . Join the tangents by a  $4^\circ 30'$  curve. Determine  $bB$  and  $aA$ .

*d.* Assume that in a case similar to Fig. 18 it was noted that a distance ( $As$ ) equal to 12 feet would clear the building. Assume that  $\angle = 38^\circ 20'$  and that  $D = 4^\circ 40'$ . Required the value of  $\alpha$  and the position of  $n$ . *Solution:*

|                           |                                 |                             |
|---------------------------|---------------------------------|-----------------------------|
| vers $\alpha = As \div R$ | $As = 12$                       | log = 1.07918               |
|                           | $R$ (for $4^\circ 40'$ curve)   | log = 3.08923               |
|                           | $\alpha = 8^\circ 01'$          | log vers $\alpha = 7.98994$ |
| $ns = R \sin \alpha$      |                                 | log sin $\alpha = 9.14445$  |
|                           |                                 | log $R = 3.08923$           |
|                           | <u><math>ns = 171.27</math></u> | log = 2.23369               |



*e.* Assume that the forward tangent of a  $3^{\circ} 20'$  curve having a central angle of  $16^{\circ} 50'$  must be moved 3.62 feet *inward*, without altering the *P.C.* Required the change in radius.

*f.* Given two tangents making an angle of  $36^{\circ} 18'$ . It is required to pass a curve through a point 93.2 feet from the vertex, the line from the vertex to the point making an angle of  $42^{\circ} 21'$  with the tangent. Required the radius and tangent distance. *Solution:* Applying eq. (18), we have

|                                                     |                                          |
|-----------------------------------------------------|------------------------------------------|
| $2$                                                 | $\log = 0.30103$                         |
| $\beta = 42^{\circ} 21'$                            | $\log \sin = 9.82844$                    |
| $\frac{1}{2}\Delta = 18^{\circ} 09'$                | $\log \sin = 9.49346$                    |
| $(\frac{1}{2}\Delta + \beta) = 60^{\circ} 30'$      | $\log \cos = 9.69234$                    |
| $.20667$                                            | <u><u>9.31527</u></u>                    |
| $\log \sin^2 \beta = 9.65688 \dots \dots .45382$    |                                          |
| $2 \mid 9.81987 \dots \dots .66049$                 |                                          |
| $9.90993 \dots \dots .81271$                        |                                          |
| $\text{nat sin } 60^{\circ} 30' = .8703$            |                                          |
| <u><math>1.6836</math></u>                          | $\log = \overline{0.22610}$              |
| $VP = 93.2$                                         | $\log = 1.96941$                         |
|                                                     | <u><u>2.19551</u></u>                    |
|                                                     | $\log \sin \frac{1}{2}\Delta = 9.49346$  |
| <u><math>\text{tang. dist. } AV = 503.56</math></u> | $\log = 2.70205$                         |
|                                                     | $\log \cot \frac{1}{2}\Delta = 10.48437$ |
| <u><math>R = 1536.1</math></u>                      | <u><u>3.18642</u></u>                    |
| $D = 3^{\circ} 44'$                                 |                                          |

#### COMPOUND CURVES.

**37. Nature and use.** Compound curves are formed by a succession of two or more simple curves of different curvature. The curves must have a common tangent at the point of compound curvature (*P.C.C.*). In mountainous regions there is frequently a necessity for compound curves having several changes of curvature. Such curves may be located separately as a succession of simple curves, but a combination of two

simple curves has special properties which are worth investigating and utilizing. In the following demonstrations  $R_2$  always represents the *longer* radius and  $R_1$  the *shorter*, no matter which succeeds the other.  $T_1$  is the tangent adjacent to the curve of shorter radius ( $R_1$ ), and is invariably the shorter tangent.  $\Delta_1$  is the central angle of the curve of radius  $R_1$ , but it may be greater or less than  $\Delta_2$ .

**38. Mutual relations of the parts of a compound curve having two branches.** In Fig. 25,  $AC$  and  $CB$  are the two branches of

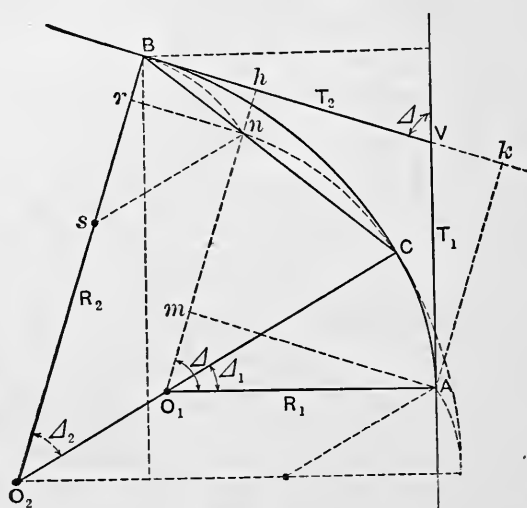


FIG 25.

the compound curve having radii of  $R_1$  and  $R_2$  and central angles of  $\Delta_1$  and  $\Delta_2$ . Produce the arc  $AC$  to  $n$  so that  $Ao_1n = \Delta$ . The chord  $Cn$  produced *must* intersect  $B$ . The line  $ns$ , parallel to  $CO_2$ , will intersect  $BO_2$  so that  $Bs = sn = O_2O_1 = R_2 - R_1$ . Draw  $Am$  perpendicular to  $O_1n$ . It will be parallel to  $hk$ .

$$Br = sn \text{ vers } Bsn = (R_2 - R_1) \text{ vers } \Delta_2;$$

$$mn = AO_1 \text{ vers } AO_1n = R_1 \text{ vers } \Delta;$$

$$Ak = AV \sin AVk = T_1 \sin \Delta;$$

$$Ak = hm = mn + nh = mn + Br.$$

$$\therefore T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2. \quad (20)$$

Similarly it may be shown that

$$T_2 \sin \Delta = R_2 \text{ vers } \Delta - (R_2 - R_1) \text{ vers } \Delta_1. \quad (21)$$

The mutual relations of the elements of compound curves may be solved by these two equations. For example, assume the tangents as fixed ( $\Delta$  therefore known) and that a curve of given radius  $R_1$  shall start from a given point at a distance  $T_1$  from the vertex, and that the curve shall continue through a given angle  $\Delta_1$ . Required the other parts of the curve. From Eq. (20) we have

$$R_2 - R_1 = \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } \Delta_2}.$$

$$\therefore R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } (\Delta - \Delta_1)}. \quad (22)$$

$T_2$  may then be obtained from Eq. (21).

As another problem, given the location of the two tangents, with the two tangent distances (thereby locating the  $PC$  and  $PT$ ), and the central angle of each curve; required the two radii. Solving Eq. (20) for  $R_1$ , we have

$$R_1 = \frac{T_1 \sin \Delta - R_2 \text{ vers } \Delta_2}{\text{vers } \Delta - \text{vers } \Delta_2}.$$

Similarly from Eq. (21) we may derive

$$R_1 = \frac{T_2 \sin \Delta - R_2 (\text{vers } \Delta - \text{vers } \Delta_1)}{\text{vers } \Delta_1}.$$

Equating these, reducing, and solving for  $R_2$ , we have

$$R_2 = \frac{T_1 \sin \Delta \text{ vers } \Delta_1 - T_2 \sin \Delta (\text{vers } \Delta - \text{vers } \Delta_2)}{\text{vers } \Delta_2 \text{ vers } \Delta_1 - (\text{vers } \Delta - \text{vers } \Delta_1)(\text{vers } \Delta - \text{vers } \Delta_2)}. \quad (23)$$

Although the various elements may be chosen as above with considerable freedom, there are limitations. For example, in Eq. (22), since  $R_2$  is always greater than  $R_1$ , the term to be added to  $R_1$  must be essentially positive—i.e.,  $T_1 \sin \Delta$  must be

greater than  $R_1 \text{ vers } \Delta$ . This means that  $T_1 > R_1 \frac{\text{vers } \Delta}{\sin \Delta}$ , or that  $T_1 > R_1 \tan \frac{1}{2}\Delta$ , or that  $T_1$  is greater than the corresponding tangent on a simple curve. Similarly it may be shown that  $T_2$  is *less* than  $R_2 \tan \frac{1}{2}\Delta$  or *less* than the corresponding tangent on a simple curve. Nevertheless  $T_2$  is always greater than  $T_1$ . In the limiting case when  $R_2 = R_1$ ,  $T_2 = T_1$  and  $\Delta_2 = \Delta_1$ .

**39. Modifications of location.** Some of these modifications may be solved by the methods used for simple curves. For example:

a. It is desired to move the tangent  $VB$ , Fig. 26, parallel to itself to  $V'B'$ . Run a new curve from the *P.C.C.* which shall reach the new tangent at  $B'$ , where the chord of the old curve

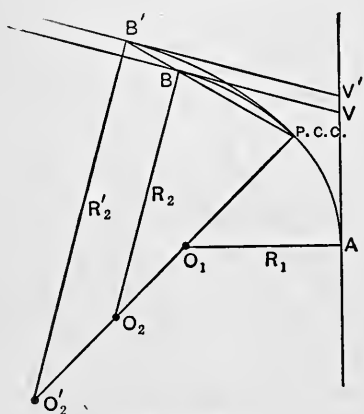


FIG. 26.

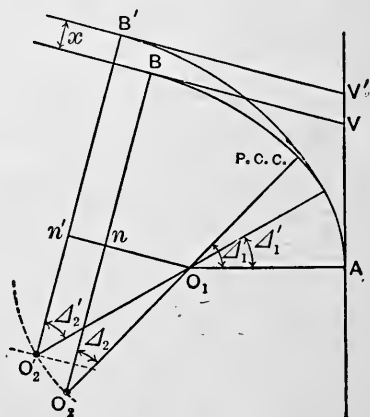


FIG. 27.

intersects the new tangent. The solution is almost identical with that in § 33, *a*.

b. Assume that it is desired to change the forward tangent (as above) but to retain the same radius. In Fig. 27

$$(R_2 - R_1) \cos \Delta_2 = O_2 n;$$

$$(R_2 - R_1) \cos \Delta_2' = O_2' n'.$$

$$x = O_2 n - O_2' n' = (R_2 - R_1)(\cos \Delta_2 - \cos \Delta_2').$$

$$\cos \Delta_2' = \cos \Delta_2 - \frac{x}{R_2 - R_1} \quad \cdot \quad \cdot \quad \cdot \quad (24)$$

The *P.C.C.* is moved *backward* along the sharper curve an angular distance of  $\Delta_2' - \Delta_2 = \Delta_1 - \Delta_1'$ .

In case the tangent is moved inward rather than outward, the solution will apply by transposing  $\Delta_2$  and  $\Delta_2'$ . Then we will have

$$\cos \Delta_2' = \cos \Delta_2 + \frac{x}{R_2 - R_1} \cdot \cdot \cdot \quad (25)$$

The *P.C.C.* is then moved *forward*.

c. Assume the same case as (b) except that the larger radius comes first and that the tangent adjacent to the smaller radius is moved. In Fig. 28

$$(R_2 - R_1) \cos \Delta_1 = O_1 n;$$

$$(R_2 - R_1) \cos \Delta_1' = O_1' n'.$$

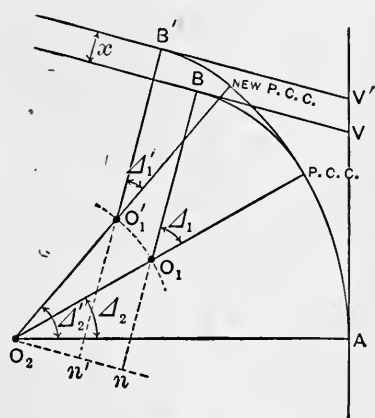


FIG. 28.

$$x = O_1' n' - O_1 n = (R_2 - R_1)(\cos \Delta_1' - \cos \Delta_1).$$

$$\cos \Delta_1' = \cos \Delta_1 + \frac{x}{R_2 - R_1} \cdot \cdot \cdot \quad (26)$$

The *P.C.C.* is moved *forward* along the easier curve an angular distance of  $\Delta_1' - \Delta_1 = \Delta_2 - \Delta_2'$ .

In case the tangent is moved *inward*, transpose as before and we have

$$\cos \Delta_1' = \cos \Delta_1 - \frac{x}{R_2 - R_1} \cdot \cdot \cdot \quad (27)$$

The *P.C.C.* is moved *backward*.

d. Assume that the radius of one curve is to be altered without changing either tangent. Assume conditions as in Fig. 29. For the diagrammatic solution assume that  $R_2$  is to be in-



As in §§ 32 and 33, the above problems are but a few, although perhaps the most common, of the problems the engineer may meet with in compound curves. All of the ordinary problems may be solved by these and similar methods.

**40. Problems.** *a.* Assume that the two tangents of a compound curve are to be 348 feet and 624 feet, and that  $\Delta_1 = 22^\circ 16'$  and  $\Delta_2 = 28^\circ 20'$ . Required the radii.

[Ans.  $R_1 = 326.92$ ;  $R_2 = 1574.85$ .]

*b.* A line crosses a valley by a compound curve which is first a  $6^\circ$  curve for  $46^\circ 30'$  and then a  $9^\circ 30'$  curve for  $84^\circ 16'$ . It is afterward decided that the last tangent should be 6 feet farther up the hill. What are the required changes? [Note. The second tangent is evidently moved *outward*. The solution corresponds to that in the first part of § 39, *c*. The *P.C.C.* is moved forward 16.39 feet. If it is desired to know how far the *P.T.* is moved in the direction of the tangent (i.e., the *projection* of  $BB'$ , Fig. 28, on  $V'B'$ ), it may be found by observing that it is equal to  $nn' = (R_2 - R_1)(\sin \Delta_1 - \sin \Delta_1')$ . In this case it equals 0.65 foot, which is very small because  $\Delta_1$  is nearly  $90^\circ$ . The value of  $\Delta_2$  ( $46^\circ 30'$ ) is not used, since the solution is independent of the value of  $\Delta_2$ . The student should learn to recognize which quantities are mutually related and therefore essential to a solution, and which are independent and non-essential.]

#### TRANSITION CURVES.

**41. Superelevation of the outer rail on curves.** When a mass is moved in a circular path it requires a centripetal force to keep it moving in that path. By the principles of mechanics we know that this force equals  $Gv^2 \div gR$ , in which  $G$  is the weight,  $v$  the velocity in feet per second,  $g$  the acceleration of gravity in feet per second in a second, and  $R$  the radius of curvature. If the two rails of a curved track were laid on a level (transversely), this centripetal force could only be furnished by the

pressure of the wheel-flanges against the rails. As this is very objectionable, the outer rail is elevated so that the reaction of

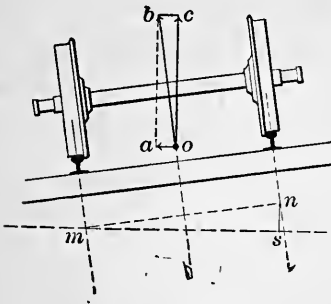


FIG. 30.

the rails against the wheels shall contain a horizontal component equal to the required centripetal force. In Fig. 30, if  $ob$  represents the reaction,  $oc$  will represent the weight  $G$ , and  $ao$  will represent the required centripetal force. From similar triangles we may write  $sn:sm::ao:oc$ . Call  $g = 32.17$ . Call  $R = 5730 \div D$ , which is sufficiently accurate

for this purpose (see § 19). Call  $v = 5280V \div 3600$ , in which  $V$  is the velocity in miles per hour.  $mn$  is the distance between rail centers, which, for an 80-lb. rail and standard gauge, is 4.916 feet.  $sm$  is slightly less than this. As an average value we may call it 4.900, which is its exact value when the superelevation is  $4\frac{3}{4}$  inches. Calling  $sn = e$ , we have

$$e = sm \frac{ao}{oc} = 4.9 \frac{Gv^2}{gR} \frac{1}{G} = \frac{4.9 \times 5280^2 V^2 D}{32.17 \times 3600^2 \times 5730}.$$

$$e = .0000572 V^2 D. \quad . \quad . \quad . \quad . \quad . \quad (30)$$

It should be noticed that, according to this formula, the required superelevation varies as the *square* of the velocity, which means that a change of velocity of only 10% would call for a change of superelevation of 21%. Since the velocities of trains over any road are extremely variable, it is impossible to adopt any superelevation which will fit all velocities even approximately. The above fact also shows why any over-refinement in the calculations is useless and why the above approximations, which are really small, are amply justifiable. For example, the above formula contains the approximation that  $R = 5730 \div D$ . In the extreme case of a  $10^\circ$  curve the error involved would be about 1%. A change of about  $\frac{1}{2}$  of 1% in



the velocity, or say from 40 to 40.2 miles per hour, would mean as much. The error in  $e$  due to the assumed constant value of  $sm$  is never more than a very small fraction of 1%. The rail-laying is not done closer than this. The following tabular form is based on Eq. 30:

SUPERELEVATION OF THE OUTER RAIL (IN FEET) FOR VARIOUS VELOCITIES AND DEGREES OF CURVATURE.

| Velocity<br>in<br>Miles<br>per<br>Hour. | Degree of Curve. |     |     |     |     |     |     |     |     |     |
|-----------------------------------------|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|                                         | 1°               | 2°  | 3°  | 4°  | 5°  | 6°  | 7°  | 8°  | 9°  | 10° |
| 30                                      | .05              | .10 | .15 | .20 | .26 | .31 | .36 | .41 | .46 | .51 |
| 40                                      | .09              | .18 | .27 | .37 | .46 | .55 | .64 | .73 | .82 |     |
| 50                                      | .14              | .29 | .43 | .57 | .71 | .86 |     |     |     |     |
| 60                                      | .20              | .41 | .62 | .82 |     |     |     |     |     |     |

**42. Practical rules for superelevation.** A much used rule for superelevation is to “elevate one half an inch for each degree of curvature.” The rule is rational in that  $e$  in Eq. 30 varies directly as  $D$ . The above rule therefore agrees with Eq. 30 when  $V$  is about 27 miles per hour. However applicable the rule may have been in the days of low velocities, the elevation thus computed is too small now.

Another (and better) rule is to “elevate for the speed of the fastest trains.” This rule is further justified by the fact that a four-wheeled truck, having two parallel axles, will always tend to run to the outer rail and will require considerable flange pressure to guide it along the curve. The effect of an excess of superelevation on the slower trains will only be to relieve this flange pressure somewhat. This rule is coupled with the limitation that the elevation should never exceed a limit of six inches—sometimes eight inches. This limitation implies that locomotive engineers must reduce the speed of fast trains around sharp curves until the speed does not exceed that for which the actual superelevation used is suitable. The heavy line in the tabular form (§ 41) shows the six-inch limitation.

Some roads furnish their track foremen with a list of the superelevations to be used on each curve in their sections. This method has the advantage that each location may be separately studied, and the proper velocity, as affected by local conditions (*e.g.*, proximity to a stopping-place for all trains), may be determined and applied.

Another method is to allow the foremen to determine the superelevation for each curve by a simple measurement taken at the curve. The rule is developed as follows: By an inversion of Eq. 19 we have

$$x = chord^2 \div 8R \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Putting  $x$  equal to  $e$  in Eq. 30 and solving for “*chord*,” we have

$$\begin{aligned} chord^2 &= .0000572 V^2 D 8R \\ &= 2.621 V^2. \\ chord &= 1.62 V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32) \end{aligned}$$

To apply the rule, assume that 50 miles per hour is fixed as the velocity from which the superelevation is to be computed. Then  $1.62 V = 1.62 \times 50 = 81$  feet, which is the distance given to the trackmen. Stretch a tape (or even a string) with a length of 81 feet between two points on the inside head of the outer rail or the outer head of the inner rail. The ordinate at the middle point then equals the superelevation. The values of this chord length for varying velocities are given in the accompanying tabular form.

|                                 |      |      |      |      |      |      |      |      |      |
|---------------------------------|------|------|------|------|------|------|------|------|------|
| Velocity in miles per hour..... | 20   | 25   | 30   | 35   | 40   | 45   | 50   | 55   | 60   |
| Chord length in feet .....      | 32.4 | 40.5 | 48.6 | 56.7 | 64.8 | 72.9 | 81.0 | 89.1 | 97.2 |

**43. Transition from level to inclined track.** On curves the track is inclined transversely; on tangents it is level. The transition from one condition to the other must be made gradu-

ally. If there is no transition curve, there must be either inclined track on the tangent or insufficiently inclined track on the curve or both. Sometimes the full superelevation is continued through the total length of the curve and the "run-off" (having a length of 100 to 200 feet) is located entirely on the tangents at each end. In other practice it is located partly on the tangent and partly on the curve. Whatever the method, the superelevation is correct at only one point of the run-off. At all other points it is too great or too small. This (and other causes) produces objectionable lurches and resistances when entering and leaving curves. The object of transition curves is to obviate these resistances.

**44. Fundamental principle of transition curves.** If a curve has variable curvature, beginning at the tangent with a curve of infinite radius, and the curvature gradually sharpens until it equals the curvature of the required simple curve and there becomes tangent to it, the superelevation of such a transition curve may begin at zero at the tangent, gradually increase to the required superelevation for the simple curve, and yet have at every point the superelevation required by the curvature at that point. Since in Eq. (30)  $e$  is directly proportional to  $D$ , the required curve must be one in which the degree of curve increases directly as the distance along the curve. The mathematical development of such a curve is quite complicated. It has, however, been developed, and tables have been computed for its use, by Prof. C. L. Crandall. The following method has the advantage of great simplicity, while its agreement with the true transition curve is as close as need be, as will be shown.

**45. Multiform compound curves.** If the transition curve commences with a very flat curve and at regular even chord lengths compounds into a curve of sharper curvature until the desired curvature is reached, the increase in curvature at each chord point being uniform, it is plain that such a curve is a close approximation to the true spiral, especially since the rails as laid will *gradually* change their curvature rather than maintain a uniform curvature throughout each chord length and

then abruptly change the curvature at the chord points. Such a curve, *as actually laid*, will be a much closer approximation to the true curve than the multiform compound curve by which it is set out. There will actually be a *gradual* increase in curvature which increases directly as the length of the curve.

**46. Required length of spiral.** The required length of spiral evidently depends on the amount of superelevation to be gained, and also depends somewhat on the speed. If the spiral is laid off in 25-foot chord lengths, with the first chord subtending a  $1^\circ$  curve, the second a  $2^\circ$  curve, etc., the fifth chord will subtend a  $5^\circ$  curve, and the increase from this last chord to a  $6^\circ$  curve is the same as the uniform increase of curvature between the chords. The same spiral extended would run on to a  $12^\circ$  curve in  $(12 - 1)25 = 275$  feet. The last chord of a spiral should have a smaller degree of curvature than the simple curve to which it is joined. If the curves are very sharp, such as are used in street work and even in suburban trolley work, an increase in degree of curvature of  $1^\circ$  per 25 feet will not be sufficiently rapid, as such a rate would require too long curves.  $2^\circ$ ,  $10^\circ$ , or even  $20^\circ$  increase per 25 feet may be necessary, but then the chords should be reduced to 5 feet. Such a rapid rate of increase is justified by the necessary reduction in speed. On the other hand, very high speed will make a lower rate of increase desirable, and therefore a spiral whose degree of curvature increases only  $0^\circ 30'$  per 25 feet may be used. Such a spiral would require a length of 375 feet to run on to an  $8^\circ$  curve, which is inconveniently long, but it might be used to run on to a  $4^\circ$  curve, where its length would be only 175 feet. Three spirals have been developed in Table IV, each with chords of 25 feet, the rate of increase in the degree of curvature being  $0^\circ 30'$ ,  $1^\circ$  and  $2^\circ$  per chord. One of these will be suitable for any curvature found on ordinary steam-railroads.

**47. To find the ordinates of a  $1^\circ$ -per-25-feet spiral.** Since the first chord subtends a  $1^\circ$  curve, its central angle is  $0^\circ 15'$  and the angle  $aQV$  (Fig. 31) is  $7' 30''$ . The tangent at  $a$  makes an angle of  $15'$  with  $VQ$ . The angle between the chord  $ba$  and

the tangent at  $a$  is  $\frac{1}{2}(30') = 15'$ , and the angle  $bab'' = \frac{1}{2}(30') + 15' = 30'$ . Similarly the angle  $cbc'' = \frac{1}{2}(45') + 30' + 15' = 67' 30' = 1^\circ 07' 30''$ , and the angle  $dcd''$  is  $2^\circ 0'$ . The ordinate  $aa' = 25 \sin 7' 30''$ , and  $Qa' = 25 \cos 7' 30''$ .  $Qb' = Qa' + a'b' = Qa' + ab'' = 25 (\cos 7' 30'' + \cos 30')$ .  $bb' = b'b'' + bb'' = 25 (\sin 7' 30'' + \sin 30')$ . Similarly the ordinates of  $c, d$ , etc., may be obtained.

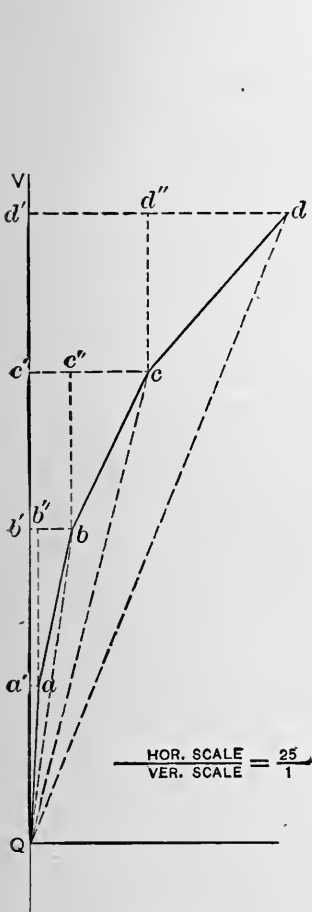


FIG. 31.

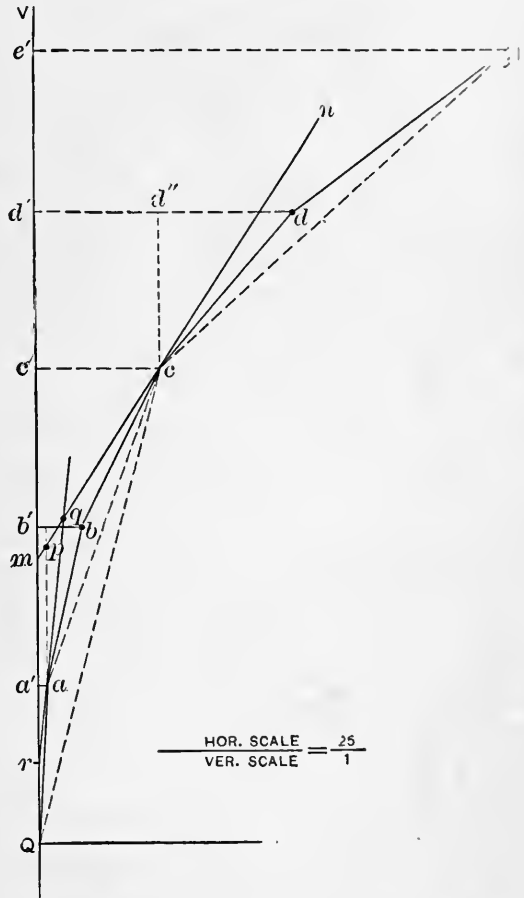


FIG. 32.

48. To find the deflections from any point of the spiral.  $aQV = 7' 30''$ .  $\tan bQV = bb' \div Qb'$ ;  $\tan cQV = cc' \div Qc'$ ; etc. Thus we are enabled to find the deflection angles from the tangent at  $Q$  to any point of the spiral.

The tangent to the curve at  $c$  (Fig. 32) makes an angle of  $1^\circ 30'$  with  $QV$ , or  $cmV = 1^\circ 30'$ .  $Qem = cmV - cQm$ . The



**49. Connection of spiral with circular curve and with tangent.** See Fig. 33.\* Let  $AV$  and  $BV$  be the tangents to be connected by a  $D^\circ$  curve, having a suitable spiral at each end. If no spirals were to be used, the problem would be solved as in simple curves giving the curve  $AMB$ . Introducing the spiral has the effect of throwing the curve away from the vertex a distance  $MM'$  and reducing the central angle of the  $D^\circ$  curve by  $2\phi$ . Continuing the curve beyond  $Z$  and  $Z'$  to  $A'$  and  $B'$ , we will have  $AA' = BB' = MM'$ .  $ZK$  = the  $x$  ordinate and is therefore known. Call  $MM' = m$ .  $A'N = x - R \text{ vers } \phi$ . Then

$$m = MM' = AA' = \frac{A'N}{\cos \frac{1}{2}\Delta} = \frac{x - R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \quad (33)$$

$$NA = AA' \sin \frac{1}{2}\Delta = (x - R \text{ vers } \phi) \tan \frac{1}{2}\Delta.$$

$$\begin{aligned} VQ &= QK - KN + NA + AV \\ &= y - R \sin \phi + (x - R \text{ vers } \phi) \tan \frac{1}{2}\Delta + R \tan \frac{1}{2}\Delta \\ &= y - R \sin \phi + x \tan \frac{1}{2}\Delta + R \cos \phi \tan \frac{1}{2}\Delta. \quad (34) \end{aligned}$$

When  $A'N$  has already been computed, it may be more convenient to write

$$VQ = y + R (\tan \frac{1}{2}\Delta - \sin \phi) + A'N \tan \frac{1}{2}\Delta. \quad (35)$$

$$\begin{aligned} VM' &= VM + MM' \\ &= R \text{ exsec } \frac{1}{2}\Delta + \frac{x}{\cos \frac{1}{2}\Delta} - \frac{R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \quad (36) \end{aligned}$$

$$\begin{aligned} AQ &= VQ - AV \\ &= y - R \sin \phi + (x - R \text{ vers } \phi) \tan \frac{1}{2}\Delta. \quad (37) \end{aligned}$$

*Example.* To join two tangents making an angle of  $34^\circ 20'$  by a  $5^\circ 40'$  curve and suitable spirals. Use  $1^\circ$ -per-25-feet

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\* The student should at once appreciate the fact of the necessary distortion of the figure. The distance  $MM'$  in Fig. 33 is perhaps 100 times its real proportional value.

spirals with five chords. Then  $\phi = 3^\circ 45'$ ,  $x = 2.999$ ,  $\frac{1}{2}\Delta = 17^\circ 10'$ , and  $y = 124.942$ .

|          |                         |                                       |                |
|----------|-------------------------|---------------------------------------|----------------|
| (Eq. 33) |                         | $R$                                   | 3.00497        |
|          |                         | vers $\phi$                           | 7.33063        |
|          | 2.166                   |                                       | <u>0.33560</u> |
|          | $x = 2.999$             |                                       |                |
|          | $A'N = 0.833$           |                                       | 9.92064        |
|          |                         | cos $\frac{1}{2}\Delta$               | 9.98021        |
|          | $m = MM' = AA' = 0.872$ |                                       | <u>9.94043</u> |
| (Eq. 36) |                         | $R$                                   | 3.00497        |
|          |                         | exsec $\frac{1}{2}\Delta$             | 8.66863        |
|          | $VM = 47.164$           |                                       | <u>1.67360</u> |
|          | $m = 0.872$             |                                       |                |
|          | $VM' = 48.036$          |                                       |                |
| (Eq. 35) | $y = 124.942$           | nat. tan $\frac{1}{2}\Delta = .30891$ |                |
|          |                         | nat. sin $\phi = .06540$              |                |
|          |                         | <u>.24351</u>                         | 9.38651        |
|          |                         | $R$                                   | 3.00497        |
|          | 246.314                 |                                       | <u>2.39148</u> |
|          | [See above]             | $A'N$                                 | 9.92064        |
|          |                         | tan $\frac{1}{2}\Delta$               | 9.48984        |
|          | 0.257                   | $AN$                                  | <u>9.41048</u> |
|          | $VQ = 371.513$          |                                       |                |
| (Eq. 37) |                         | $R$                                   | 3.00497        |
|          |                         | tan $\frac{1}{2}\Delta$               | 9.48984        |
|          | 312.471                 | $AV$                                  | 2.49481        |
|          | $AQ = 59.042$           |                                       |                |

**50. Field-work.** When the spiral is designed during the original location, the tangent distance  $VQ$  should be computed and the point  $Q$  located. It is hardly necessary to locate all of the points of the spiral until the track is to be laid. The extremities should be located, and as there will usually be one and perhaps two full station points on the spiral, these should



also be located.  $Z$  may be located by setting off  $QK = y$  and  $KZ = x$ , or else by the tabular deflection for  $Z$  from  $Q$  and the distance  $ZQ$ , which is the long chord. Setting up the instrument at  $Z$  and sighting back at  $Q$  with the proper deflection, the tangent at  $Z$  may be found and the circular curve located as usual, its central angle being  $\Delta - 2\phi$ . A similar operation will locate  $Q'$  from  $Z'$ .

**To locate points on the spiral.** Set up at  $Q$ , with the plates reading  $0^\circ$  when the telescope sights along  $VQ$ . Set off from  $Q$  the deflections given in Table IV for the instrument at  $Q$ , using a chord length of 25 feet, the process being like the method for simple curves except that the deflections are irregular. If a full station-point occurs within the spiral, interpolate between the deflections for the adjacent spiral-points. For example, a spiral begins at Sta.  $56 + 15$ . Sta. 57 comes 10 feet beyond the third spiral point. The deflection for the third point is  $35' 0''$ ; for the fourth it is  $56' 15''$ .  $\frac{10}{25}$  of the difference ( $21' 15''$ ) is  $8' 30''$ ; the deflection for Sta. 57 is therefore  $43' 30''$ . This method is not theoretically accurate, but the error is small. Arriving at  $z$ , the forward alignment may be obtained by sighting back at  $Q$  (or at any other point) with the given deflection for that point from the station occupied. Then when the plates read  $0^\circ$  the telescope will be tangent to the spiral and to the succeeding curve. All rear points should be checked from  $z$ . If it is necessary to occupy an intermediate station, use the deflections given for that station, orienting as just explained for  $z$ , checking the back points and locating all forward points up to  $z$  if possible.

After the center curve has been located and  $z'$  is reached, the other spiral must be located but *in reverse order*, i.e., the sharp curvature of the spiral is at  $z'$  and the curvature decreases toward  $Q'$ .

**51. To replace a simple curve by a curve with spirals.** This may be done by the method of § 49, but it involves shifting the whole track a distance  $m$ , which in the given example equals 0.87 foot. Besides this the track is appreciably shortened,



The length of the old curve from  $Q$  to  $Q' = 2A Q + 100 \frac{\Delta}{D}$ .

The length of the new curve from  $Q$  to  $Q' = 2L + 100 \frac{\Delta - 2\phi}{D}$ ,

in which  $L$  is the length of each spiral.

**Example.** Suppose the old curve is a  $7^\circ 30'$  curve with a central angle of  $38^\circ 40'$ . As a trial, compute the relative length of a new  $8^\circ$  curve with spirals of seven chords.  $\phi = 7^\circ 0'$ ;  $\frac{1}{2}\Delta = 19^\circ 20'$ ;  $R$  (for the  $7^\circ 30'$  curve) = 764.489;  $R'$  (for the  $8^\circ$  curve) = 716.779;  $x = 7.628$ .

[Eq. 38]

|                |                                   |         |
|----------------|-----------------------------------|---------|
|                | $R$                               | 2.88337 |
|                | $\text{exsec } \frac{1}{2}\Delta$ | 8.77642 |
| $R' = 716.779$ |                                   | 1.65979 |
| $762.466$      |                                   | <hr/>   |
|                | $R'$                              | 2.85538 |
|                | $\cos \phi$                       | 9.99675 |
|                | $\sec \frac{1}{2}\Delta$          | 0.02521 |

753.953

2.87734

$x$   
 $\sec \frac{1}{2}\Delta$

0.88241  
0.02521

8.084

0.90762

762.037 762.037

$m = 0.429$

[Eq. 39]

$y = 174.722$

$R'$   
 $\sin \phi$

2.85538  
9.08589

87.353

1.94128

$R'$   
 $\cos \phi$   
 $\tan \frac{1}{2}\Delta$

2.85538  
9.99675  
9.54512

249.606

2.39725

$R = 764.489$   
 $x = 7.628$

756.861  
 $\tan \frac{1}{2}\Delta$

2.87901  
9.54512

265.543

2.42413

424.328  
352.896

352.896

$AQ = 71.432$

The length of the old curve from  $Q$  to  $Q'$  is

$$\begin{array}{rcl}
 100 \frac{\Delta}{D} = 100 \frac{38.667}{7.5} & = & 515.556 \\
 2AQ = 2 \times 71.432 & = & 142.864 \\
 \hline
 & & 658.420 \\
 \text{New curve: } 100 \frac{\Delta - 2\phi}{D'} = 100 \frac{38.667 - 14.000}{8.0} & = & 308.333 \\
 2L = 2 \times 175 & = & 350.000 \\
 & & \hline
 & & 658.333 \quad 658.333 \\
 \text{Difference in length} & = & 0.087
 \end{array}$$

Considering that this difference may be divided among 22 joints (using 30-foot rails) no rail-cutting would be necessary. If the difference is too large, a slight variation in the value of the new radius  $R'$  will reduce the difference as much as necessary. A truer comparison of the lengths would be found by comparing the lengths of the arcs.

**52. Application of transition curves to compound curves.** Since compound curves are only employed when the location is limited by local conditions, the elements of the compound curve should be determined (as in §§ 38 and 39) regardless of the transition curves, depending on the fact that the lateral shifting of the curve when transition curves are introduced is very small. If the limitations are very close, an estimated allowance may be made for them.

Methods have been devised for inserting transition curves between the branches of a compound curve, but the device is complicated and usually needless, since when the train is once on a curve the wheels press against the outer rail steadily and a change in curvature will not produce a serious jar even though the superelevation is temporarily a little more or less than it should be.

If the easier curve of the compound curve is less than  $3^\circ$  or  $4^\circ$ , there may be no need for a transition curve off from that branch. This problem then has two cases according as transition curves are used at both ends or at one end only.

a. *With transition curves at both ends.* Adopting the method of § 49, calling  $\Delta_1 = \frac{1}{2}\Delta$ , we may compute  $m_1 = MM_1'$ . Similarly, calling  $\Delta_2 = \frac{1}{2}\Delta$ , we may compute  $m_2 = MM_2'$ . But

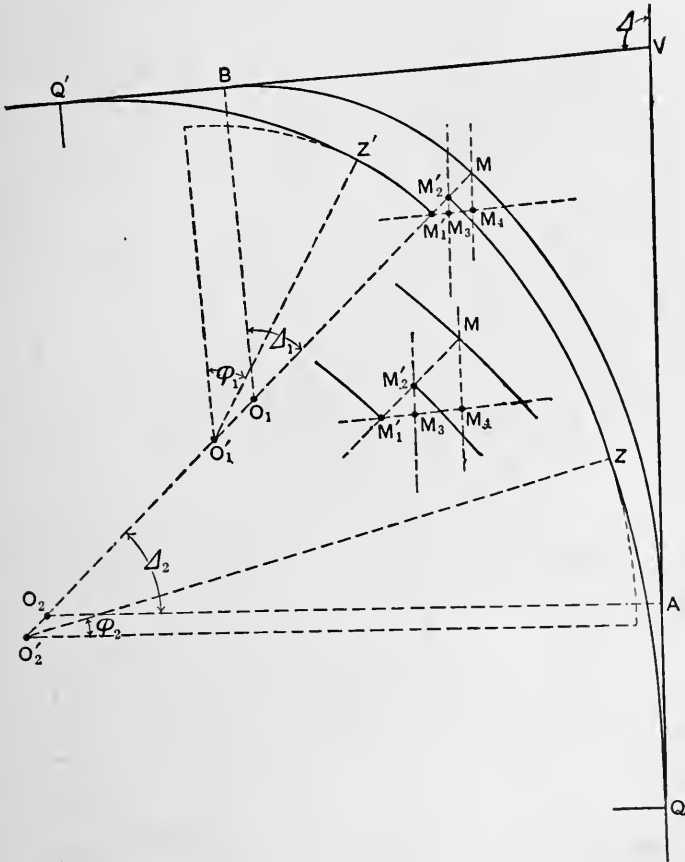


FIG. 35.

$M_1'$  and  $M_2'$  must be made to coincide. This may be done by moving the curve  $Z'M_1'$  and its transition curve parallel to  $Q'V$  a distance  $M_1'M_3$ , and the other curve parallel to  $QV$  a distance  $M_2'M_3$ . In the triangle  $M_1'M_3M_2'$ , the angle at  $M_1' = 90^\circ - \Delta_1$ , the angle at  $M_2' = 90^\circ - \Delta_2$ , and the angle at  $M_3 = \Delta$ .

$$\left. \begin{aligned} \text{Then } M_1'M_3 &= M_1'M_2' \frac{\sin(90^\circ - \Delta_2)}{\sin \Delta} = (m_1 - m_2) \frac{\cos \Delta_2}{\sin \Delta}. \\ \text{Similarly } M_2'M_3 &= M_1'M_2' \frac{\sin(90^\circ - \Delta_1)}{\sin \Delta} = (m_1 - m_2) \frac{\cos \Delta_1}{\sin \Delta}. \end{aligned} \right\} (40)$$

b. *With a transition curve on the sharper curve only.* Compute  $m_1 = MM_1'$  as before; then move the curve  $Z_1M_1'$  parallel to  $Q'V$  a distance of

$$M_1'M_4 = m_1 \frac{\cos \Delta_2}{\sin \Delta} \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The simple curve  $MA$  is moved parallel to  $VA$  a distance of

$$MM_4 = m_1 \frac{\cos \Delta_1}{\sin \Delta} \quad . \quad . \quad . \quad . \quad . \quad (42)$$

If  $\Delta_1$  and  $\Delta_2$  are both small,  $M_1'M_4$  and  $MM_4$  may be more than  $m_1$ , but the lateral deviation of the new curve from the old will always be less than  $m_1$ .

**53. To replace a compound curve by a curve with spirals.** The solution is somewhat analogous to that of § 51. Compute  $m_1$  for the sharper branch of the curve, placing  $\Delta_1 = \frac{1}{2}\Delta$  in Eq. 38. Since  $m_1$  and  $m_2$  for the two branches of the curve must be identical, a value for  $R_2'$  must be found which will satisfy the determined value of  $m_2 = m_1$ . Solving Eq. 38 for  $R'$ , we obtain

$$R' = \frac{R \text{ vers } \frac{1}{2}\Delta - m \cos \frac{1}{2}\Delta - x}{\cos \phi - \cos \frac{1}{2}\Delta} \quad . \quad . \quad . \quad . \quad (43)$$

Substituting in this equation the known value of  $m_1 (= m_2)$  and calling  $R' = R_2'$ ,  $R = R_1$ , and  $\Delta_2 = \frac{1}{2}\Delta$ , solve for  $R_2'$ . Obtain the value of  $AQ$  for each branch of the curve separately by Eq. 39, and compare the lengths of the old and new lines.

*Example.* Assume a compound curve with  $D_1 = 8^\circ$ ;  $D_2 = 4^\circ$ ;  $\Delta_1 = 36^\circ$  and  $\Delta_2 = 32^\circ$ . Use 1°-per-25-feet spirals;  $\phi_1 = 7^\circ 0'$ ;  $\phi_2 = 1^\circ 30'$ . Assume that the sharper curve is sharpened from  $8^\circ 0'$  to  $8^\circ 12'$ .

[Eq. 38]

$$R_1' = \frac{169.209}{868.535}$$

857.970

$$m_1 = \frac{867.399}{1.136}$$

[Eq. 43]

217.700

$$\frac{1.726}{215.974}$$

$$x_2 = \frac{0.963}{1.726}$$

$$R_2' = 1424.54$$

[4° 1' 22'']

[Eq. 39]

$$y_1 = 174.722$$

85.226

504.302

$$AQ_1 = \frac{679.024}{600.461} = 78.563$$

$$\begin{array}{r} R_1 \\ \text{exsec } 36^\circ \end{array} \quad \begin{array}{r} 2.85538 \\ 9.37303 \end{array}$$

2.22842

$$\begin{array}{r} R_1' \\ \cos \phi_1 \\ \sec \angle_1 \end{array} \quad \begin{array}{r} 2.84468 \\ 9.99675 \\ 0.09204 \end{array}$$

2.93347

$$\begin{array}{r} x_1 \\ \sec \angle_1 \end{array} \quad \begin{array}{r} 0.88241 \\ 0.09204 \end{array}$$

0.97445

$$\begin{array}{r} R_2 \\ \text{vers } 32^\circ \end{array} \quad \begin{array}{r} 3.15615 \\ 9.18176 \\ 2.33785 \end{array}$$

$$\begin{array}{r} m_1 = 1.136 \\ \cos 32^\circ \end{array} \quad \begin{array}{r} 0.05538 \\ 9.92842 \end{array}$$

9.98380

$$\begin{array}{l} \text{nat. cos } \phi = .99966 \\ \text{nat. cos } \angle_2 = .84805 \end{array}$$

.15161

2.33440

9.18073

3.15367

$$\begin{array}{r} R_1' \\ \sin \phi_1 \end{array} \quad \begin{array}{r} 2.84468 \\ 9.08586 \end{array}$$

1.93057

$$\begin{array}{r} R_1' \\ \cos \phi_1 \\ \tan \frac{1}{2} \angle_1 [ \angle_1 = 36^\circ ] \end{array} \quad \begin{array}{r} 2.84468 \\ 9.99675 \\ 9.86126 \end{array}$$

2.70269

$$\begin{array}{r} R_1 = 716.779 \\ x_1 = 7.628 \end{array}$$

$$\begin{array}{r} 709.151 \\ \tan \frac{1}{2} \angle \end{array} \quad \begin{array}{r} 2.85074 \\ 9.86126 \end{array}$$

2.71206

$$\frac{515.235}{600.461}$$

[Eq. 39]

$$y_2 = 74.994$$

$$37\ 290$$

$$389.843$$

$$AQ_2 = 32.777$$

$$\begin{array}{r} R_2' \\ \sin \phi_2 \end{array} \quad \begin{array}{r} 3.15367 \\ 8.41792 \end{array}$$

$$\underline{1.57159}$$

$$\begin{array}{r} R_2' \\ \cos \phi_2 \end{array} \quad \begin{array}{r} 3.15367 \\ 9.99985 \end{array}$$

$$\tan \frac{1}{2} \Delta (\Delta_2 = 32^\circ) \quad 9.79579$$

$$\underline{2.94931}$$

$$\begin{array}{r} R_2 = 1432.69 \\ x_2 = 0.76 \end{array}$$

$$1431.93 \quad 3.15592$$

$$\tan \frac{1}{2} \Delta \quad 9.79579$$

$$\underline{2.95171}$$

$$894.770$$

$$\begin{array}{r} 964.837 \\ 932.060 \end{array}$$

$$932.060$$

For the length of the old track we have :

$$100 \frac{\Delta_1}{D_1} = 100 \frac{36^\circ}{8^\circ} = 450.$$

$$100 \frac{\Delta_2}{D_2} = 100 \frac{32^\circ}{4^\circ} = 800.$$

$$AQ_1 = 78.563$$

$$AQ_2 = 32.777$$

$$\underline{1361.340}$$

For the length of the new track we have :

$$100 \frac{\Delta_1 - \phi_1}{D_1'} = 100 \frac{29^\circ}{8^\circ.20} = 353.659$$

$$100 \frac{\Delta_2 - \phi_2}{D_2'} = 100 \frac{30^\circ.5}{4^\circ.023} = 758.140$$

$$\text{Spiral on } 8^\circ 12' \text{ curve} \quad 175.000$$

$$\text{" " } 4^\circ 01' 22'' \text{ " } \quad 75.$$

$$\text{Length of new track} \quad = 1361.799$$

$$\text{" " old " } \quad = 1361.340$$

$$\text{Excess in length of new track} = 0.459 \text{ feet.}$$



Since the new track is slightly longer than the old, it shows that the new track runs too far *outside* the old track at the *P.C.C.* On the other hand the offset  $m$  is only 1.136. The maximum amount by which the new track comes *inside* of the old track at two points, presumably not far from  $Z'$  and  $Z$ , is very difficult to determine exactly. Since it is desirable that the maximum offsets (inside and outside) should be made as nearly equal as possible, this feature should not be sacrificed to an effort to make the two lines of precisely equal length so that the rails need not be cut. Therefore, if it is found that the offsets inside the old track are nearly equal to  $m$  (1.136), the above figures should stand. Otherwise  $m$  may be diminished (and the above excess in length of track diminished) by *increasing*  $R_1'$  very slightly and making the necessary consequent changes.

#### VERTICAL CURVES.

**54. Necessity for their use.** Whenever there is a change in the rate of grade, it is necessary to eliminate the angle that would be formed at the point of change and to connect the two grades by a curve. This is especially necessary at a sag between two grades, since the shock caused by abruptly forcing an upward motion to a rapidly moving heavy train is very severe both to the track and to the rolling stock.

**55. Required length.** Theoretically the length should depend on the change in the rate of grade, the greater change requiring a longer curve. The importance of this was greater in the days when link couplers were in universal use and the "slack" in a long train was very great. Under such circumstances, when a train was moving down a heavy grade the cars would crowd ahead against the engine. Reaching the sag, the engine would begin to pull out, rapidly taking out the slack. Six inches of slack on each car would amount to several feet on a long train, and the resulting jerk on the couplers, especially those near the rear of the train, has frequently resulted in



$\frac{1}{2}(163.4 + 163.8) = 163.6$ ;  $h$ ,  $\frac{1}{2}(163.6 + 162.6) = 163.1$ . Then  $eh = 0.5$ . The elevations of the points on the curve are:

|               |                                                   |        |
|---------------|---------------------------------------------------|--------|
| Sta. 15 + 20, | (A)                                               | 163.4  |
| “ 16 ,        | $163.4 - (.80 \times 0.8) + (.80^2 \times 0.5) =$ | 163.08 |
| “ 17 ,        | $162.6 + (.80 \times 1.2) + (.20^2 \times 0.5) =$ | 163.58 |
| “ 17 + 20,    | (C)                                               | 163.8  |

A theoretical inaccuracy in the above method lies in the fact that  $eh$  and all parallel lines are not truly vertical. In the above case the variation from the vertical is  $0^\circ 07'$ , while the effect of this variation on the elevations in this case (as in the most extreme cases) is absolutely inappreciable. The grades in the figure are necessarily very greatly exaggerated, which increases the apparent inaccuracy.

## CHAPTER III.

### EARTHWORK.

#### FORM OF EXCAVATIONS AND EMBANKMENTS.

58. Usual form of cross-section in cut or fill. The normal form of cross-section in cut is as shown in Fig. 37, in which  $e \dots g$  represents the natural surface of the ground, no matter how irregular;  $ab$  represents the position and width of the re-

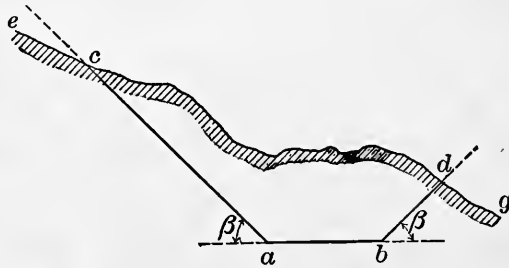


FIG. 37.

quired roadbed;  $ac$  and  $bd$  represent the “side slopes” which begin at  $a$  and  $b$  and which intersect the natural surface at such

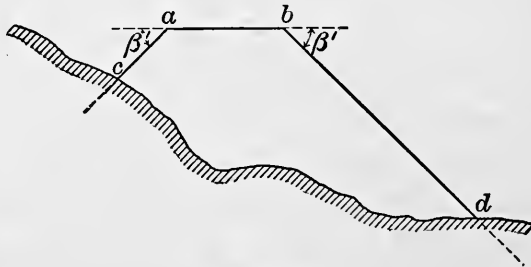


FIG. 38.

points ( $c$  and  $d$ ) as will be determined by the required slope angle ( $\beta$ ).

The normal section in fill is as shown in Fig. 38. The points  $c$  and  $d$  are likewise determined by the intersection of the required side slopes with the natural surface. In case the required roadbed ( $ab$  in Fig. 39) intersects the natural surface, both cut

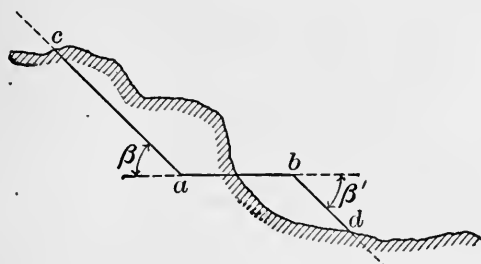


FIG. 39.

and fill are required, and the points  $c$  and  $d$  are determined as before. Note that  $\beta$  and  $\beta'$  are not necessarily equal. Their proper values will be discussed later.

**59. Terminal pyramids and wedges.** Fig. 40 illustrates the general form of cross-sections when there is a transition from cut to fill.  $a \dots g$  represents the grade line of the road which passes from cut to fill at  $d$ .  $sdt$  represents the surface profile. A cross-section taken at the point where either side of the roadbed *first* cuts the surface (the point  $m$  in this case) will usually be triangular if the ground is regular. A similar cross-section should be taken at  $o$ , where the other side of the roadbed cuts the surface. In general the earthwork of cut and fill terminates in two pyramids. In Fig. 40 the pyramid vertices are at  $n$  and  $k$ , and the bases are  $lhm$  and  $opq$ . The roadbed is generally wider in cut than in fill, and therefore the section  $lhm$  and the altitude  $ln$  are generally greater than the section  $opq$  and the altitude  $pk$ . When the line of intersection of the roadbed and natural surface ( $nodkm$ ) becomes perpendicular to the axis of the roadbed ( $ag$ ) the pyramids become wedges whose bases are the nearest convenient cross-sections.

**60. Slopes. a. Cuttings.** The required slopes for cuttings vary from perpendicular cuts, which may be used in hard rock which will not disintegrate by exposure, to a slope of perhaps

4 horizontal to 1 vertical in a soft material like quicksand or in a clayey soil which flows easily when saturated. For earthy materials a slope of 1 : 1 is the maximum allowable, and even this should only be used for firm material not easily affected by

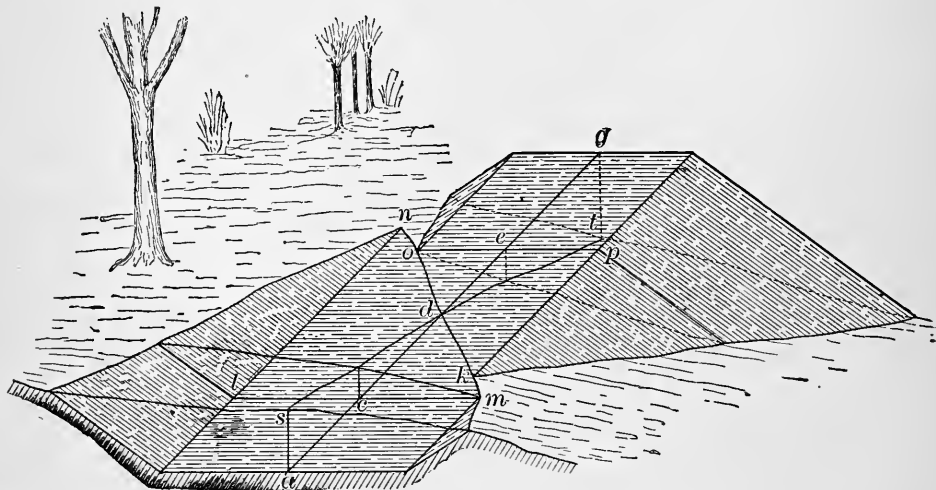


FIG. 40.

saturation. A slope of  $1\frac{1}{2}$  horizontal to 1 vertical is a safer slope for average earthwork. It is a frequent blunder that slopes in cuts are made too steep, and it results in excessive work in clearing out from the ditches the material that slides down, at a much higher cost per yard than it would have cost to take it out at first, to say nothing of the danger of accidents from possible landslides.

**b. Embankments.** The slopes of an embankment vary from 1 : 1 to 1.5 : 1. A rock fill will stand at 1 : 1, and if some care is taken to form the larger pieces on the outside into a rough dry wall, a much steeper slope can be allowed. This method is sometimes a necessity in steep side-hill work. Earthwork embankments generally require a slope of  $1\frac{1}{2}$  to 1. If made steeper at first, it generally results in the edges giving way, requiring repairs until the ultimate slope is nearly or quite  $1\frac{1}{2}$  : 1. The difficulty of incorporating the added material with the old embankment and preventing its sliding off frequently makes these repairs disproportionately costly.

**61. Compound sections.** When the cut consists partly of earth and partly of rock, a compound cross-section must be

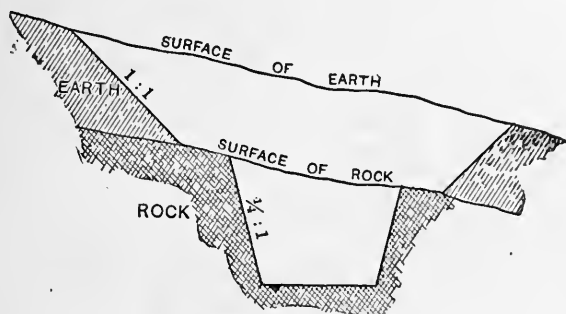


FIG. 41.

made. If borings have been made so that the contour of the rock surface is accurately known, then the true cross-section may be determined. The rock and earth should be calculated separately, and this will require an accurate knowledge of where the rock “runs out”—a difficult matter when it must be determined by boring. During construction the center part of the earth cut would be taken out first and the cut widened until a sufficient width of rock surface had been exposed so that the rock cut would have its proper width and side slopes. Then the earth slopes could be cut down at the proper angle. A “berm” of about three feet is usually left on the edges of the rock cut as a margin of safety against a possible sliding of the earth slopes. After the work is done, the amount of excavation that has been made is readily computable, but accurate preliminary estimates are difficult. The area of the cross-section of earth in the figure must be determined by a method similar to that developed for borrow-pits (see § 89).

**62. Width of roadbed.** Owing to the large and often disproportionate addition to volume of cut or fill caused by the addition of even one foot to the width of roadbed, there is a natural tendency to reduce the width until embankments become unsafe and cuts are too narrow for proper drainage. The cost of maintenance of roadbed is so largely dependent on the drainage of the roadbed that there is true economy in making an

ample allowance for it. The practice of some of the leading railroads of the country in this respect is given in the following table, in which are also given some data belonging more properly to the subject of superstructure.

WIDTH OF ROADBED FOR SINGLE AND DOUBLE TRACK—SLOPE RATIOS—  
DISTANCES BETWEEN TRACK CENTERS.

| Road.                  | Single Track.                            |                  | Double Track.    |          | Slope Ratios.                          |         | Dist. between Track Centers. |
|------------------------|------------------------------------------|------------------|------------------|----------|----------------------------------------|---------|------------------------------|
|                        | Cut.                                     | Fill.            | Cut.             | Fill.    | Cut.                                   | Fill.   |                              |
| A., T. & Santa Fé....  | { 38' earth<br>23' rock                  | 20               | .....            | .....    | 1 : 1<br>$\frac{1}{4}$ : 1             | 1.5 : 1 |                              |
| Chi., Burl. & Quincy   | 14 + (2 × .5) *                          | 16               | 28 + (2 × 5)     | 30       | 1.5 : 1                                | 1.5 : 1 | 14'                          |
| Chi., Mil. & St. Paul. | 18 + (2 × 6)                             | 20 to 24         | 31 + (2 × 6)     | 33 to 37 | 1.5 : 1                                | 1.5 : 1 | 13'                          |
| C., C., C. & St. Louis | 20 + (2 × 4)                             | 20               | 33 + (2 × 4)     | 33       | 1.5 : 1                                | 1.5 : 1 | 13'                          |
| Illinois Central.....  | 32.5                                     | 18               | .....            | .....    | 1.5 : 1                                | 1.5 : 1 |                              |
| Erie .....             | 20' 8½"                                  | 20' 8½"          | 33' 8½"          | 33' 8½"  | 1.5 : 1                                | 1.5 : 1 | 13'                          |
| Lehigh Valley.....     | 14 + (2 × 3.5)                           | 16               | 27 + (2 × 3.5)   | 30       | 1 : 1                                  | 1.5 : 1 | 13'                          |
| L. S. & Michigan So.   | .....                                    | .....            | 33 + (2 × 7.25)  | 32       | 1.5 : 1                                | 1.5 : 1 | 13'                          |
| Louisville & Nashv..   | 13 + (2 × 4.5)                           | 16               | .....            | .....    | 1 : 1                                  | 1.5 : 1 |                              |
| Michigan Central....   | .....                                    | .....            | 33 + (2 × 2.5)   | 33       | 1.5 : 1                                | 1.5 : 1 | 13'                          |
| N. Y. N. H. & H....    | .....                                    | .....            | 30               | 30       | 1.5 : 1                                | 1.5 : 1 | 12'                          |
| Norfolk & Western...   | { 21' 2" earth<br>16' rock               | 17' 2"           | 34' 2" earth     | 30' 2"   | 1.5 : 1                                | 1.5 : 1 | 13'                          |
|                        | .....                                    | .....            | 29' rock         | .....    | $\frac{1}{4}$ : 1<br>$\frac{1}{2}$ : 1 | { ....  | 13'                          |
| Pennsylvania.....      | { 19' 2" light traffic<br>27' 2" heavy " | 19' 2"<br>19' 2" | 31' 4" + (2 × 4) | 31' 4"   | 1.5 : 1                                | 1.5 : 1 | 12' 2"                       |
| Union Pacific....      | 14 + (2 × 3.5)                           | 16               | .....            | .....    | 1 : 1                                  | 1.5 : 1 |                              |

\* (2 × 5) signifies two ditches each 5 feet wide: the following cases should be interpreted similarly.

It may be noted from the above table that the average width for an *earthwork* cut, single track, is about 24.7 feet, with a minimum of 19 feet 2 inches. The widths of fills, single track, average over 18 feet, with numerous minimums of 16 feet. The widths for double track may be found by adding the distance between track centers, which is usually 13 feet.

**63. Form of subgrade.** The stability of the roadbed depends largely on preventing the ballast and subsoil from becoming saturated with water. The ballast must be porous so that it will not retain water, and the subsoil must be so constructed that it will readily drain off the rain-water that soaks through the ballast. This is accomplished by giving the subsoil a curved form, convex



upward, or a surface made up of two or three planes, the two outer planes having a slope of about 1 : 24 (sometimes more and sometimes less, depending on the soil) and the middle plane, if three are used, being level. When a circular form is used, a crowning of 6 inches in a total width of 17 or 18 feet is generally used. Occasionally the subgrade is made level, especially in rock-cuts, but if the subsoil is previously compressed by rolling, as required on the N. Y. C. & H. R. R., or if the subsoil is drained by tile drains laid underneath the ditches, the necessity for slopes is not so great. Rock cuts are generally required to be excavated to one foot below subgrade and then filled up again to subgrade with the same material, if it is suitable.

**64. Ditches.** . “The stability of the track depends upon the strength and permanence of the roadbed and structures upon which it rests; whatever will protect them from damage or prevent premature decay should be carefully observed. The worst enemy is WATER, and the further it can be kept away from the track, or the sooner it can be diverted from it, the better the track will be protected. Cold is damaging only by reason of the water which it freezes; therefore the first and most important provision for good track is drainage.” (Rules of the Road Department, Illinois Central R. R.)

The form of ditch generally prescribed has a flat bottom 12" to 24" wide and with sides having a minimum slope, except in rock-work, of 1 : 1, more generally 1.5 : 1 and sometimes 2 : 1. Sometimes the ditches are made V-shaped, which is objectionable unless the slopes are low. The best form is evidently that which will cause the greatest flow for a given slope, and this will evidently be the form in which the ratio of area to wetted perimeter is the largest. The semicircle fulfills this condition better than any other form, but the nearly vertical sides would be difficult to maintain. (See Fig. 42.) A ditch, with a flat bottom and such slopes as the soil requires, which approximates to the circular form will therefore be the best.

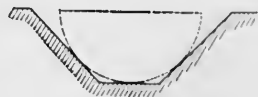


FIG. 42.

When the flow will probably be large and at times rapid it will be advisable to pave the ditches with stone, especially if the soil is easily washed away. Six-inch tile drains, placed 2' under the ditches, are prescribed on some roads. (See Fig. 43.) No better method could be devised to insure a dry subsoil. The ditches through cuts should be led off at the end of the cut so that the adjacent embankment will not be injured.

Wherever there is danger that the drainage from the land above a cut will drain down into the cut, a ditch should be made near the edge of the cut to intercept this drainage, and this ditch should be continued, and paved if necessary, to a point where the outflow will be harmless. Neglect of these simple and inexpensive precautions frequently causes the soil to be loosened on the shoulders of the slopes during the progress of a heavy rain, and results in a landslide which will cost more to repair than the ditches which would have prevented it for all time.

Ditches should be formed along the bases of embankments; they facilitate the drainage of water from the embankment, and may prevent a costly slip and disintegration of the embankment.

**65. Effect of sodding the slopes, etc.** Engineers are unanimously in favor of rounding off the shoulders and toes of embankments and slopes, sodding the slopes, paving the ditches, and providing tile drains for subsurface drainage, all to be put in during original construction. (See Fig. 43.) Some of the highest grade specifications call for the removal of the top layer of vegetable soil from cuts and from under proposed fills to some convenient place, from which it may be afterwards spread on the slopes, thus facilitating the formation of sod from grass-seed. But while engineers favor these measures and their economic value may be readily demonstrated, it is generally impossible to obtain the authorization of such specifications from railroad directors and promoters. The addition to the original cost of the roadbed is considerable, but is by no means as great as the capitalized value of the extra cost of maintenance resulting from the usual practice. Fig. 43 is a copy of

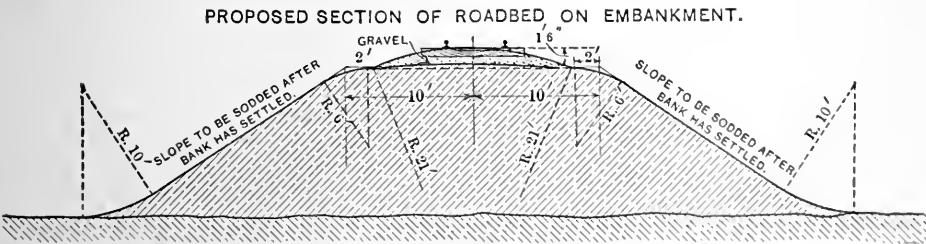
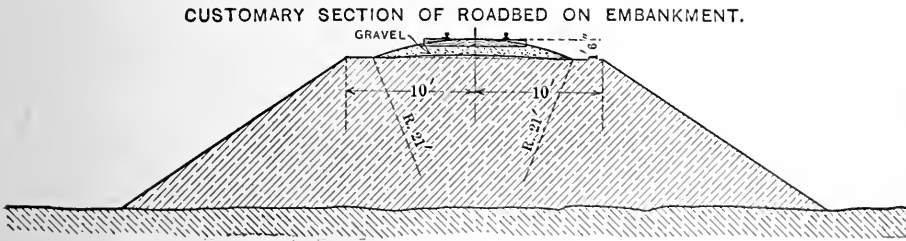
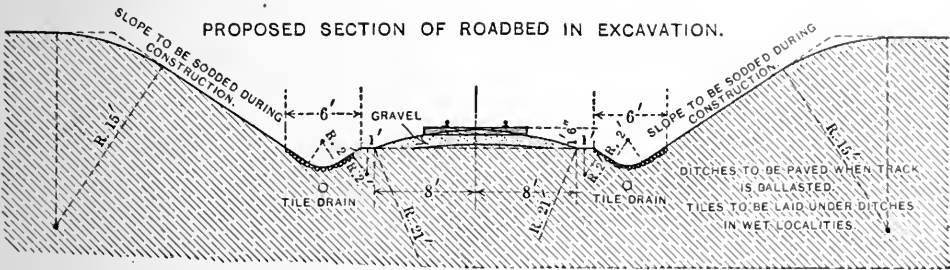
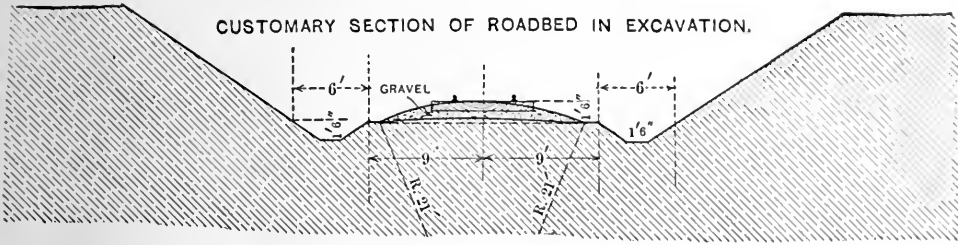


FIG. 43.—“WHITTEMORE ON RAILWAY EXCAVATION AND EMBANKMENTS,”  
Trans. Am. Soc. C. E., Sept. 1894

designs \* presented at a convention of the American Society of Civil Engineers by Mr. D. J. Whittemore, Past President of the Society and Chief Engineer of the Chi., Mil. & St. Paul R.R. The “customary sections” represent what is, with some variations of detail, the practice of many railroads. The “proposed sections” elicited unanimous approval. They should be adopted when not prohibited by financial considerations.

#### EARTHWORK SURVEYS.

**66. Relation of actual volume to the numerical result.** It should be realized at the outset that the accuracy of the result of computations of the volume of any given mass of earthwork has but little relation to the accuracy of the mere numerical work. The process of obtaining the volume consists of two distinct parts. In the first place it is assumed that the volume of the earthwork may be represented by a more or less complicated geometrical form, and then, secondly, the volume of such a geometrical form is computed. A desire for simplicity (or a frank willingness to accept approximate results) will often cause the cross-section men to assume that the volume may be represented by a very simple geometrical form which is really only a very rough approximation to the true volume. In such a case, it is only a waste of time to compute the volume with minute numerical accuracy. One of the first lessons to be learned is that economy of time and effort requires that the accuracy of the numerical work should be kept proportional to the accuracy of the cross-sectioning work, and also that the accuracy of both should be proportional to the use to be made of the results. The subject is discussed further in § 94.

**67. Prismoids.** To compute the volume of earthwork, it is necessary to assume that it has some geometric form whose volume is readily determinable. The general method is to consider

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\* Trans. Am. Soc. Civil Eng., Sept. 1894.

the volume as consisting of a series of *prismoids*, which are solids having parallel plane ends and bounded by surfaces which may be formed by lines moving continuously along the edges of the bases. These surfaces may also be considered as the surfaces generated by lines moving along the edges joining the corresponding points of the bases, these edges being the directrices, and the lines being always parallel to either base, which is a plane director. The surfaces thus developed may or may not be planes. The volume of such a prismoid is readily determinable (as explained in § 70 *et seq.*), while its definition is so very general that it may be applied to very rough ground. The “two plane ends” are sections perpendicular to the axis of the road. The roadbed and side slopes (also plane) form three of the side surfaces. The only approximation lies in the degree of accuracy with which the plane (or warped) surfaces coincide with the actual surface of the ground between these two sections. This accuracy will depend (*a*) on the number of points which are taken in each cross-section and the accuracy with which the lines joining these points coincide with the actual cross-sections; (*b*) on the skill shown in selecting places for the cross-sections so that the warped surfaces shall coincide as nearly as possible with the surface of the ground. In fairly smooth country, cross-sections every 100 feet, placed at the even stations, are sufficiently accurate, and such a method simplifies the computations greatly; but in rough country cross-sections must be interpolated as the surface demands. As will be explained later, carelessness or lack of judgment in cross-sectioning will introduce errors of such magnitude that all refinements in the computations are utterly wasted.

**68. Cross-sectioning.** The process of cross-sectioning consists in determining at any place the intersection by a vertical plane of the prism of earth lying between the roadbed, the side slopes, and the natural surface. The intersection with the roadbed and side slopes gives three straight lines. The intersection with the natural surface is in general an irregular line. On smooth regular ground or when approximate results are accept-

able this line is assumed to be straight. According to the irregularity of the ground and the accuracy desired more and more "intermediate points" are taken.

The distance ( $d$  in Fig. 44) of the roadbed below (or above) the natural surface at the center is known or determined from

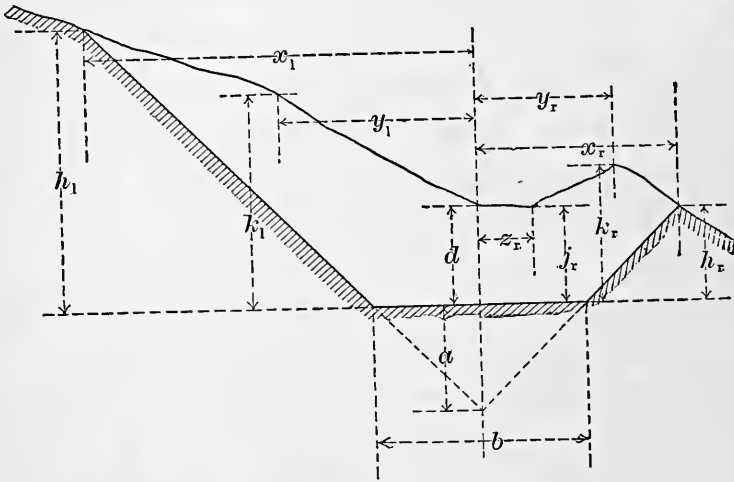


FIG. 44.

the profile or by the computed establishment of the grade line. The distances out from the center of all "breaks" are determined with a tape. To determine the elevations for a cut, set up a level at any convenient point so that the line of sight is higher than any point of the cross-section, and take a rod reading on the center point. This rod reading added to  $d$  gives the height of the instrument (H. I.) above the roadbed. Subtracting from H. I. the rod reading at any "break" gives the height of that point above the roadbed ( $h_1$ ,  $k_1$ ,  $h_2$ , etc.). This is true for all cases in excavation. For fill, the rod reading at center minus  $d$  equals the H. I., which may be positive or negative. When negative, add to the "H. I." the rod readings of the intermediate points to get their depths below "grade"; when positive, subtract the "H. I." from the rod readings.

The heights or depths of these intermediate points above or below grade need only be taken to the nearest tenth of a foot, and the distances out from the center will frequently be suffi-

ciently exact when taken to the nearest foot. The roughness of the surface of farming land or woodland generally renders useless any attempt to compute the volume with any greater accuracy than these figures would imply unless the form of the ridges and hollows is especially well defined. The position of the slope-stake points is considered in the next section. Additional discussion regarding cross-sectioning is found in § 82.

**69. Position of slope-stakes.** The slope-stakes are set at the intersection of the required side slopes with the natural surface, which depends on the center cut or fill ( $d$ ). The distance of

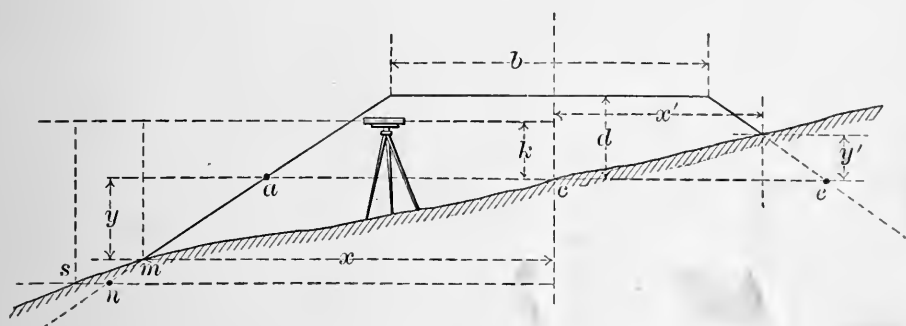


FIG. 45.

the slope-stake from the center for the lower side is  $x = \frac{1}{2}b + s(d + y)$ ; for the up-hill side it is  $x' = \frac{1}{2}b + s(d - y')$ .  $s$  is the "slope ratio" for the side slopes, the ratio of horizontal to vertical. In the above equation both  $x$  and  $y$  are unknown. Therefore some position must be found by trial which will satisfy the equation. As a preliminary, the value of  $x$  for the point  $a = \frac{1}{2}b + sd$ , which is the value of  $x$  for *level* cross-sections. In the case of fills on sloping ground the value of  $x$  on the *down-hill* side is *greater* than this; on the *up-hill* side it is *less*. The difference in distance is  $s$  times the difference of elevation. Take a numerical case corresponding with Fig. 45. The rod reading on  $c$  is 2.9;  $d = 4.2$ ; therefore the telescope is  $4.2 - 2.9 = 1.3$  *below* grade.  $s = 1.5 : 1$ ,  $b = 16$ . Hence for the point  $a$  (or for level ground)  $x = \frac{1}{2} \times 16 + 1.5 \times 4.2 = 14.3$ . At a distance out of 14.3 the ground is seen to be about 3 feet lower, which will not only require  $1.5 \times 3 = 4.5$  more, but

enough additional distance so that the added distance shall be 1.5 times the additional drop. As a first trial the rod may be held at 24 feet out and a reading of, say, 8.3 is obtained.  $8.3 + 1.3 = 9.6$ , the depth of the point below grade. The point on the slope line ( $n$ ) which has this depth below grade is at a distance from the center  $x = 8 + 1.5 \times 9.6 = 22.4$ . The point on the surface ( $s$ ) having that depth is 24 feet out. Therefore the true point ( $m$ ) is nearer the center. A second trial at 20.5 feet out gives a rod reading of, say, 7.1 or a depth of 8.4 below grade. This corresponds to a distance out of 20.6. Since the natural soil (especially in farming lands or woods) is generally so rough that a difference of elevation of a tenth or so may be readily found by slightly varying the location of the rod (even though the distance from the center is the same), it is useless to attempt too much refinement, and so in a case like the above the combination of 8.4 below grade and 20.6 out from center may be taken to indicate the proper position of the slope-stake. This is usually indicated in the form of a fraction, the distance out being the denominator and the height above (or below) grade being the numerator; the fact of *cut* or *fill* may be indicated by  $C$  or  $F$ . Ordinarily a second trial will be sufficient to determine with sufficient accuracy the true position of the slope-stake. Experienced men will frequently estimate the required distance out to within a few tenths at the first trial. The left-hand pages of the note-book should have the station number, surface elevation, grade elevation, center cut or fill, and rate of grade. The right-hand pages should be divided in the center and show the distances out and heights above grade of all points, as is illustrated in § 84. The notes should read up the page, so that when looking ahead along the line the figures are in their proper relative position. The "fractions" farthest from the center line represent the slope-stake points.

#### COMPUTATION OF VOLUME.

**70. Prismoidal formula.** Let Fig. 46 represent a triangular prismoid. The two triangles forming the ends lie in *parallel*



planes, but since the angles of one triangle are not equal to the corresponding angles of the other triangle, at least two of the surfaces must be *warped*. If a section, parallel to the bases, is

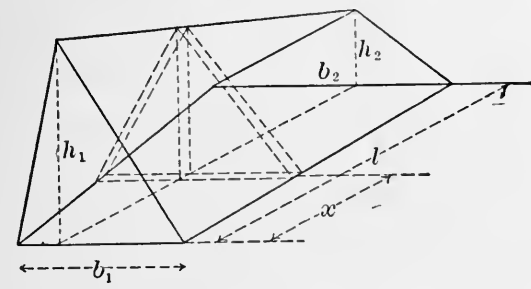


FIG. 46.

made at any point at a distance  $x$  from one end, the area of the section will evidently be

$$A_x = \frac{1}{2}b_x h_x = \frac{1}{2} \left[ b_1 + (b_2 - b_1) \frac{x}{l} \right] \left[ h_1 + (h_2 - h_1) \frac{x}{l} \right].$$

The volume of a section of infinitesimal length will be  $A_x dx$ , and the total volume of the prismoid will be \*

$$\begin{aligned} \int_0^l A_x dx &= \frac{1}{2} \int_0^l \left[ b_1 + (b_2 - b_1) \frac{x}{l} \right] \left[ h_1 + (h_2 - h_1) \frac{x}{l} \right] dx \\ &= \frac{1}{2} \left[ b_1 h_1 x + (b_2 - b_1) h_1 \frac{x^2}{2l} + b_1 (h_2 - h_1) \frac{x^2}{2l} \right. \\ &\quad \left. + (b_2 - b_1)(h_2 - h_1) \frac{x^3}{3l^2} \right]_0^l \\ &= \frac{1}{2} \left\{ b_1 h_1 l + [(b_2 - b_1) h_1 + b_1 (h_2 - h_1)] \frac{l}{2} + (b_2 - b_1)(h_2 - h_1) \frac{l}{3} \right\}, \end{aligned}$$

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\* Students unfamiliar with the Integral Calculus may take for granted the fundamental formulæ that  $\int dx = x$ , that  $\int x dx = \frac{1}{2}x^2$ , and that  $\int x^2 dx = \frac{1}{3}x^3$ ; also that in integrating between the limits of  $l$  and 0 (zero), the value of the integral may be found by simply substituting  $l$  for  $x$  after integration.

$$\begin{aligned}
\int_0^l A_x dx &= \frac{l}{2} \left[ \frac{1}{3} b_1 h_1 + \frac{1}{6} b_1 h_2 + \frac{1}{6} b_2 h_1 + \frac{1}{3} b_2 h_2 \right] \\
&= \frac{l}{6} \left[ \frac{1}{2} b_1 h_1 + \frac{1}{2} b_1 (h_1 + h_2) + \frac{1}{2} b_2 (h_1 + h_2) + \frac{1}{2} b_2 h_2 \right] \\
&= \frac{l}{6} \left[ \frac{1}{2} b_1 h_1 + 4 \left( \frac{1}{2} \cdot \frac{b_1 + b_2}{2} \cdot \frac{h_1 + h_2}{2} \right) + \frac{1}{2} b_2 h_2 \right] \\
&= \frac{l}{6} [A_1 + 4A_m + A_2], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)
\end{aligned}$$

in which  $A_1$ ,  $A_2$ , and  $A_m$  are the areas respectively of the two bases and of the middle section. Note that  $A_m$  is *not* the *mean* of  $A_1$  and  $A_2$ , although it does not necessarily differ very greatly from it.

The above proof is absolutely independent of the values, absolute or relative, of  $b_1$ ,  $b_2$ ,  $h_1$  or  $h_2$ . For example,  $h_2$  may be zero and the second base reduces to a line and the prismoid becomes wedge-shaped; or  $b_2$  and  $h_2$  may both vanish, the second base becoming a point and the prismoid reduces to a pyramid. Since every prismoid (as defined in § 67) may be reduced to a combination of triangular prismoids, wedges, and pyramids, and since the formula is true for any one of them individually, it is true for all collectively; therefore it may be stated that \*

*The volume of a prismoid equals one sixth of the perpendicular distance between the bases multiplied by the sum of the areas of the two bases plus four times the area of the middle section.*

While it is always possible to compute the volume of any prismoid by the above method, it becomes an extremely complicated and tedious operation to compute the true value of the middle section if the end sections are complicated in form. It

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\* The student should note that the derivation of equation (45) does not complete the proof, but that the statements in the following paragraph are logically necessary for a general proof.

therefore becomes a simpler operation to compute volumes by approximate formulæ and apply, if necessary, a correction. The most common methods are as follows:

**71. Averaging end areas.** The volume of the triangular prismoid (Fig. 46), computed by averaging end areas, is  $\frac{l}{2}[\frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2]$ . Subtracting this from the true volume (as given in the equation above, Eq. (45)), we obtain the correction

$$\frac{l}{12}[(b_1 - b_2)(h_2 - h_1)]. \quad . \quad . \quad . \quad . \quad (46)$$

This shows that if either the  $h$ 's or  $b$ 's are equal, the correction vanishes; it also shows that if the bases are roughly similar and  $b$  varies roughly with  $h$  (which *usually* occurs, as will be seen later), the correction will be *negative*, which means that the method of averaging end areas *usually* gives *too large* results.

**72. Middle areas.** Sometimes the middle area is computed and the volume is assumed to be equal to the length times the middle area. This will equal  $\frac{l}{2} \times \frac{b_1 + b_2}{2} \times \frac{h_1 + h_2}{2}$ . Subtracting this from the true volume, we obtain the correction

$$\frac{l}{24}(b_1 - b_2)(h_1 - h_2). \quad . \quad . \quad . \quad . \quad (47)$$

As before, the form of the correction shows that if either the  $h$ 's or  $b$ 's are equal, the correction vanishes; also under the *usual* conditions, as before, the correction is *positive* and only one-half as large as by averaging end areas. Ordinarily the labor involved in the above method is no less than that of applying the exact prismoidal formula.

**73. Two-level ground.** When *approximate* computations of earthwork are sufficiently exact the field-work may be materially reduced by observing simply the center cut (or fill) and the

natural slope  $\alpha$ , measured with a clinometer. The area of such a section (see Fig. 48) equals

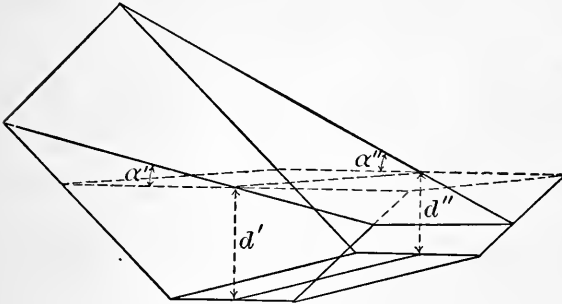


FIG. 47.

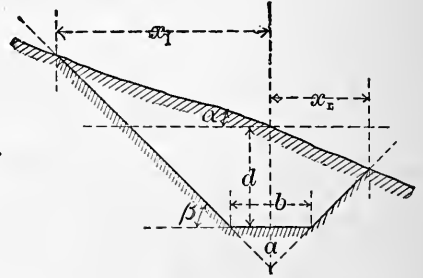


FIG. 48.

$$\frac{1}{2}(a + d)(x_l + x_r) - \frac{ab}{2}.$$

But

$$x_l \tan \beta = a + d + x_l \tan \alpha,$$

from which

$$x_l = \frac{a + d}{\tan \beta - \tan \alpha}.$$

Similarly,

$$x_r = \frac{a + d}{\tan \beta + \tan \alpha}.$$

Substituting,

$$\text{Area} = (a + d)^2 \frac{\tan \beta}{\tan^2 \beta - \tan^2 \alpha} - \frac{ab}{2}. \quad (48)$$

The values  $\alpha$ ,  $\tan \beta$ ,  $\tan^2 \beta$  are constant for all sections, so that it requires but little work to find the area of any section. As this method of cross-sectioning implies considerable approximation, it is generally a useless refinement to attempt to compute the volume with any greater accuracy than that obtained by averaging end areas. It may be noted that it may be easily proved that the correction to be applied is of the same form as that found in § 71 and equals

$$\frac{l}{12}[(x_l' + x_r') - (x_l'' + x_r'')][(d'' + a) - (d' + a)],$$

which reduces to

$$\text{Correction} = \frac{l}{6} \left\{ \left[ (a+d') \frac{\tan \beta}{\tan^2 \beta - \tan^2 \alpha'} - (a+d'') \frac{\tan \beta}{\tan^2 \beta - \tan^2 \alpha''} \right] [d'' - d'] \right\}. \quad (49)$$

When  $d'' = d'$  the correction vanishes. This shows that when the center heights are equal there is no correction—regardless of the slope. If the slope is uniform throughout, the form of the correction is simplified and is invariably *negative*. Under the usual conditions the correction is *negative*, i.e., the method *generally* gives *too large* results.

**74. Level sections.** When the country is very level or when only approximate preliminary results are required, it is sometimes assumed that the cross-sections are level. The method of level sections is capable of easy and rapid computation. The area may be written as

$$(a + d)^2 s - \frac{ab}{2}. \quad . \quad . \quad . \quad . \quad (50)$$

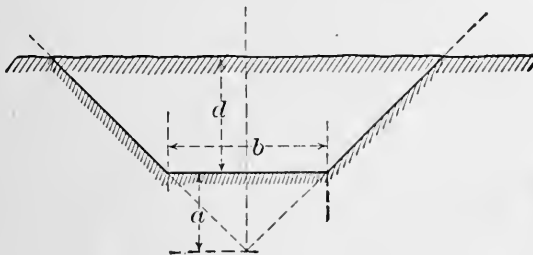


FIG. 49.

This also follows from Eq. (48) when  $\alpha = 0$  and  $\tan \beta = \frac{1}{s}$ .  $s$  here represents the “slope ratio,” i.e., the ratio of the horizontal projection of the slope to the vertical. A table is very readily formed giving the area in square feet of a section of given depth and for any given width of roadbed and ratio of side-slopes. The area may also be readily determined (as illustrated in the following example) without the use of such a table; a table of squares will facilitate the work. Assuming

the cross-sections at equal distances  $(=l)$  apart, the total approximate volume for any distance will be

$$\frac{l}{2}[A_0 + 2(A_1 + A_2 + \dots A_{n-1}) + A_n]. \quad (51)$$

The prismoidal correction may be directly derived from Eq. (46) as  $\frac{l}{12}[2(a + d')s - 2(a + d'')s][(a + d'') - (a + d')]$ , which reduces to

$$-\frac{ls}{6}(d' - d'')^2 \quad \text{or} \quad -\frac{l}{12}\frac{b}{a}(d' - d'')^2. \quad (52)$$

This may also be derived from Eq. (49), since  $\alpha = 0$ ,  $\tan \alpha = 0$ , and  $\tan \beta = 2a \div b$ . This correction is *always* negative, showing that the method of averaging end areas, when the sections are level, always gives too large results. The prismoidal correction for any one prismoid is therefore a constant times the *square* of a difference. The squares are always positive whether the differences are positive or negative. The correction therefore becomes

$$-\frac{l}{12}\frac{b}{a}\Sigma(d' \sim d'')^2. \quad (53)$$

**75. Numerical example: level sections.** Given the following center heights for the same number of consecutive stations 100 feet apart; width of roadbed 18 feet; slope  $1\frac{1}{2}$  to 1.

The products in the fifth column may be obtained very readily and with sufficient accuracy by the use of the slide-rule described in § 79. The products should be considered as  $(a + d)(a + d) \div \frac{1}{s}$ . In this problem  $s = 1\frac{1}{2}, \frac{1}{s} = .6667$ .

To apply the rule to the first case above, place 6667 on scale *B* over 89 on scale *A*, then opposite 89 on scale *B* will be found

118.8 on scale *A*. The position of the decimal point will be evident from an approximate mental solution of the problem.

| Sta. | Center Height. | $a + d$ | $(a + d)^2$ | $(a + d)^2s$ | Areas.                                                                               | $d' \sim d''$ | $(d' \sim d'')^2$ |
|------|----------------|---------|-------------|--------------|--------------------------------------------------------------------------------------|---------------|-------------------|
| 17   | 2.9            | 8.9     | 79.21       | 118.81       | 118.81                                                                               |               |                   |
| 18   | 4.7            | 10.7    | 114.49      | 171.74       | $\times 2 = \begin{cases} 343.48 \\ 491.52 \\ 939.86 \\ 312.12 \\ 86.64 \end{cases}$ | 1.8           | 3.24              |
| 19   | 6.8            | 12.8    | 163.84      | 245.76       |                                                                                      | 2.1           | 4.41              |
| 20   | 11.7           | 17.7    | 313.29      | 469.93       |                                                                                      | 4.9           | 24.01             |
| 21   | 4.2            | 10.2    | 104.04      | 156.06       |                                                                                      | 7.5           | 56.25             |
| 22   | 1.6            | 7.6     | 57.76       | 86.64        |                                                                                      | 2.6           | 6.76              |

$$\frac{ab}{2} = \frac{6 \times 18}{2} = 54$$
$$10 \times 54 = \frac{540}{1752.43}$$
$$\frac{1752.43 \times 100}{2 \times 27} = 3245 \text{ cub. yards} = \text{approx. vol.}$$
$$\text{Corr.} = - \frac{100 \times 18}{12 \times 6 \times 27} \times 94.67 = - 91 \text{ cub. yds.}$$
$$3245 - 91 = 3154 \text{ cub. yds.} = \text{exact volume.}$$

The above demonstration of the correction to be applied to the approximate volume, found by averaging end areas, is introduced mainly to give an idea of the amount of that correction. Absolutely level sections are practically unknown, and the error involved in assuming any given sections as truly level will ordinarily be greater than the computed correction. If greater accuracy is required, more points should be obtained in the cross-sectioning, which will generally show that the sections are not truly level.

**76. Equivalent sections.** When sections are very irregular the following method may be used, especially if great accuracy is not required. The sections are plotted to scale and then a uniform slope line is obtained by stretching a thread so that the undulations are averaged and an *equivalent section* is obtained. The *center depth* ( $d$ ) and the *slope angle* ( $\alpha$ ) of this line can be obtained from the drawing, but it is more convenient to measure the distances ( $x_l$  and  $x_r$ ) from the center. The area

may then be obtained independent of the center depth as follows: Let  $s$  = the slope ratio of the side slopes =  $\cot \beta = \frac{b}{2a}$ . (See Fig. 48.) Then the

$$\begin{aligned} \text{Area} &= \frac{1}{2} \left( \frac{x_l + x_r}{s} \right) (x_l + x_r) - \frac{x_r x_r}{s} \frac{1}{2} - \frac{x_l x_l}{s} \frac{1}{2} - \frac{ab}{2} \\ &= \frac{x_r x_r}{s} - \frac{ab}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54) \end{aligned}$$

The true volume, according to the prismoidal formula, of a length of the road measured in this way will be

$$\frac{l}{6} \left[ \frac{x'_l x'_r}{s} - \frac{ab}{2} + 4 \left( \frac{x'_l + x''_l}{2} \frac{x'_r + x''_r}{2} \frac{1}{s} - \frac{ab}{2} \right) + \frac{x''_l x''_r}{s} - \frac{ab}{2} \right].$$

If computed by averaging end areas, the approximate volume will be

$$\frac{l}{2} \left[ \frac{x'_l x'_r}{s} - \frac{ab}{2} + \frac{x''_l x''_r}{s} - \frac{ab}{2} \right].$$

Subtracting this result from the true volume, we obtain as the correction

$$\text{Correction} = \frac{l}{6s} (x''_l - x'_l)(x'_r - x''_r). \quad . \quad . \quad (55)$$

This shows that if the side distances to either the right or left are equal at adjacent stations the correction is *zero*, and also that if the difference is small the correction is also small and very probably within the limit of accuracy obtainable by that method of cross-sectioning. In fact, as has already been shown in the latter part of § 75, it will usually be a useless refinement to compute the prismoidal correction when the method of cross-sectioning is as rough and approximate as this method generally is.



**77. Equivalent level sections.** These sloping “two-level” sections are sometimes transformed into “level sections of equal area,” and the volume computed by the method of level sections (§ 74). But the true volume of a prismoid with sloping ends does not agree with that of a prismoid with equivalent bases and level ends except under special conditions, and when this method is used a correction must be applied if accuracy is desired, although, as intimated before, the assumption that the sections have uniform slopes will frequently introduce greater inaccuracies than that of this method of computation. The following demonstration is therefore given to show the scope and limitations of the errors involved in this much used method.

In Fig. 50, let  $d_1$  be the center height which gives an

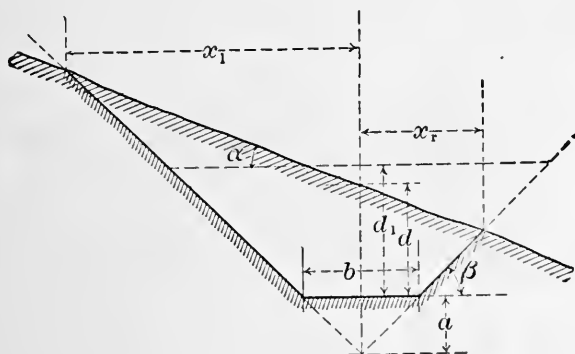


FIG. 50.

equivalent level section. The area will equal  $(a + d_1)^2 s - \frac{ab}{2}$ ,

which must equal the area given in § 76,  $\frac{x_1 x_2}{s} - \frac{ab}{2}$ .  $s = \frac{b}{2a}$ .

$$\therefore (a + d_1)^2 s = \frac{x_1 x_2}{s},$$

$$\text{or } a + d_1 = \frac{\sqrt{x_1 x_2}}{s}. \quad . \quad . \quad . \quad (56)$$

To obtain  $d_1$  directly from notes, given in terms of  $d$  and  $\alpha$ ,

we may substitute the values of  $x_l$  and  $x_r$  given in § 73, which gives

$$a + d_l = (a + d) \frac{\tan \beta}{\sqrt{\tan^2 \beta - \tan^2 \alpha}} = \frac{a + d}{\sqrt{1 - s^2 \tan^2 \alpha}}. \quad (57)$$

The *true* volume of the equivalent section may be represented by

$$\frac{ls}{6} \left[ (a + d_l')^2 + 4 \left( \frac{a + d_l'}{2} + \frac{a + d_l''}{2} \right)^2 + (a + d_l'')^2 \right].$$

From this there should be subtracted the volume of the “grade prism” under the roadbed to obtain the volume of the cut that would be actually excavated, but in the following comparison, as well as in other similar comparisons elsewhere made, the volume of the grade prism invariably cancels out, and so for the sake of simplicity it will be disregarded. This expression for volume may be transposed to

$$\frac{ls}{6} \left[ \frac{x_l' x_r'}{s^2} + 4 \left( \frac{\sqrt{x_l' x_r'}}{2s} + \frac{\sqrt{x_l'' x_r''}}{2s} \right)^2 + \frac{x_l'' x_r''}{s^2} \right].$$

The true volume of the prismoid with sloping ends is (see § 76)

$$\frac{l}{6} \left[ \frac{x_l' x_r'}{s} + 4 \left( \left( \frac{x_l' + x_l''}{2} \right) \left( \frac{x_r' + x_r''}{2} \right) \frac{1}{s} \right) + \frac{x_l'' x_r''}{s} \right].$$

The difference of the two volumes

$$\begin{aligned} &= \frac{l}{6s} (x_l' x_r' + x_l'' x_r' + x_l' x_r'' + x_l'' x_r'' - x_l' x_r' - 2\sqrt{x_l' x_r' x_l'' x_r''} - x_l'' x_r'') \\ &= \frac{l}{6s} (\sqrt{x_l' x_r''} - \sqrt{x_l'' x_r'})^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58) \end{aligned}$$

This shows that “equivalent level sections” do *not* in general give the true volume, there being an exception when

$x'_i x''_r = x'_i x'_r$ . This condition is fulfilled when the slope is uniform, i.e., when  $\alpha' = \alpha''$ . When this is nearly so the error is evidently not large. On the other hand, if the slopes are inclined in opposite directions the error may be very considerable, particularly if the angles of slope are also large.

**78. Three-level sections.** The next method of cross-sectioning in the order of complexity, and therefore in the order of

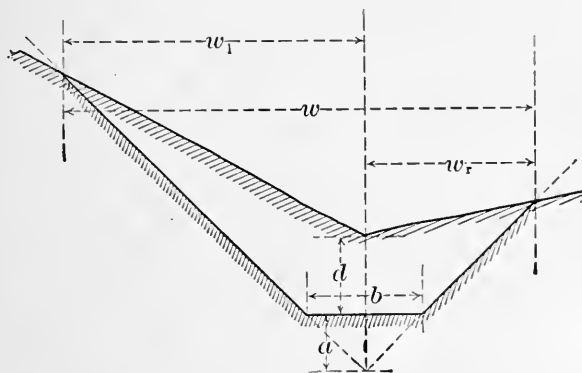


FIG. 51.

accuracy, is the method of three-level sections. The area of the section is  $\frac{1}{2}(a + d)(w_r + w_l) - \frac{ab}{2}$ , which may be written  $\frac{1}{2}(a + d)w - \frac{ab}{2}$ , in which  $w = w_r + w_l$ . If the volume is computed by averaging end areas, it will equal

$$\frac{l}{4}[(a + d')w' - ab + (a + d'')w'' - ab]. \quad (59)$$

If we divide by 27 to reduce to cubic yards, we have, when  $l = 100$ ,

$$\text{Vol}_{(\dots)} = \frac{2.5}{7}(a + d')w' - \frac{2.5}{7}ab + \frac{2.5}{7}(a + d'')w'' - \frac{2.5}{7}ab.$$

For the next section

$$\text{Vol}_{(\dots)} = \frac{2.5}{7}(a + d'')w'' - \frac{2.5}{7}ab + \frac{2.5}{7}(a + d''')w''' - \frac{2.5}{7}ab.$$

For a partial station length compute as usual and multiply result by  $\frac{\text{length in feet}}{100}$ . The prismoidal correction may be obtained by applying Eq. (46) to each side in turn. For the left side we have

$$\frac{l}{12}[(a + d') - (a + d'')](w_l'' - w_l'), \quad \text{which equals}$$

$$\frac{l}{12}(d' - d'')(w_l'' - w_l').$$

For the right side we have, similarly,

$$\frac{l}{12}(d' - d'')(w_r'' - w_r').$$

The total correction therefore equals

$$\begin{aligned} & \frac{l}{12}(d' - d'')[(w_l'' + w_r'') - (w_l' + w_r')] \\ &= \frac{l}{12}(d' - d'')(w'' - w'). \end{aligned}$$

Reduced to cubic yards, and with  $l = 100$ ,

$$\text{Pris. Corr.} = \frac{25}{81}(d' - d'')(w'' - w'). \quad . \quad . \quad (60)$$

When this result is compared with that given in Eq. (55) there is an apparent inconsistency. If two-level ground is considered as but a special case of three-level ground, it would seem as if the same laws should apply. If, in Eq. (55),  $x_r' = x_r''$ , and  $x_l''$  is different from  $x_l'$ , the equation reduces to zero; but in this case  $d'$  would also be different from  $d''$ ; and since  $x_l' + x_r'$  would  $= w'$ , and  $x_l'' + x_r'' = w''$  in Eq. (60),  $w'' - w'$  would not equal zero and the correction would be some finite quantity and not zero. The explanation lies in the difference in the form and volume of the prismoids, according to the method of the

formation of the warped surfaces. If the surface is supposed to be generated by the locus of a line moving parallel to the ends as plane directors and along two *straight* lines lying in the side-slopes, then  $x_l^{\text{mid.}}$  will equal  $\frac{1}{2}(x_l' + x_l'')$ , and  $x_r^{\text{mid.}}$  will equal  $\frac{1}{2}(x_r' + x_r'')$ , but the profile of the center line will *not* be straight and  $d^{\text{mid.}}$  will *not* equal  $\frac{1}{2}(d' + d'')$ . On the other hand, if the surfaces be generated by *two* lines moving parallel to the ends as plane directors and along a *straight* center line and straight side lines lying in the slopes, a warped surface will be generated each side of the center line, which will have uniform slopes on each side of the center at the two ends and *nowhere else*. This shows that when the upper surface of earth-work is warped (as it generally is), two-level ground should not be considered as a special case of three-level ground. This discussion, however, is only valuable to explain an apparent inconsistency and error. The method of two-level ground should only be used when such refinements as are here discussed are of no importance as affecting the accuracy.

The following example is given to illustrate the method of three-level sections.

| Station. | Center. | Left.                | Right.              | $a + d$ | $w$  | Yards.  | $d' - d''$ | $w'' - w'$ | Pris. Corr. | $x_l \sim x_r$ | $\frac{V(x_l \sim x_r)}{3R}$ | Curv. Corr.* |
|----------|---------|----------------------|---------------------|---------|------|---------|------------|------------|-------------|----------------|------------------------------|--------------|
| 17       | 2.6F    | $\frac{10.6F}{22.9}$ | $\frac{0.8F}{8.2}$  | 7.3     | 31.1 | 210     |            |            |             | 14.7           | +1                           |              |
| 18       | 8.1F    | $\frac{15.8F}{30.7}$ | $\frac{3.4F}{12.1}$ | 12.8    | 42.8 | 507 595 | -5.5       | +11.7      | -20         | 18.6           | +3                           | +4           |
| +40      | 10.7F   | $\frac{20.2F}{37.3}$ | $\frac{4.8F}{14.2}$ | 15.4    | 51.5 | 734 448 | -2.6       | + 8.7      | - 3         | 23.1           | +6                           | +4           |
| 19       | 6.4F    | $\frac{14.0F}{28.0}$ | $\frac{2.1F}{10.1}$ | 11.1    | 38.1 | 392 602 | +4.3       | -13.4      | -11         | 17.9           | +2                           | +5           |
| 20       | 3.7F    | $\frac{5.8F}{15.7}$  | $\frac{0.2F}{7.3}$  | 8.4     | 23.0 | 179 449 | +2.7       | -15.1      | -13         | 8.4            | +1                           | +3           |

Roadbed, 14' wide in fill.      Approx. Vol.=2094      -47      +16  
Slope 1½ to 1.      Pris. corr. = 47  
 $a = \frac{b}{2s} = \frac{14}{3} = 4.7;$       True Vol. =2047 (disregarding curv. corr).\*

$\frac{25}{27}ab = 61.$       \* For the Derivation of the curvation correction, see § 93.

In the first column of yards

$$210 = \frac{25}{27}(a + d)w = \frac{25}{27} \times 7.3 \times 31.1;$$

507, 734, etc., are found similarly;

$$595 = 210 - 61 + 507 - 61;$$

$$448 = \frac{40}{100}(507 - 61 + 734 - 61);$$

$$602 = \frac{60}{100}(734 - 61 + 392 - 61);$$

$$449 = 392 - 61 + 179 - 61.$$

For the prismoidal correction,

$$\begin{aligned} -20 &= \frac{25}{81}(d' - d'')(w'' - w') = \frac{25}{81}(2.6 - 8.1)(42.8 - 31.1) \\ &= \frac{25}{81}(-5.5)(+11.7). \end{aligned}$$

For the next line,  $-3 = \frac{40}{100}[\frac{25}{81}(-2.8)(+8.7)]$ , and similarly for the rest. The “*F*” in the columns of center heights, as well as in the columns of “right” and “left,” are inserted to indicate *fill* for all those points. Cut would be indicated by “*C*.”

**79. Computation of products.** The quantities  $\frac{25}{27}(a + d)w$  and  $\frac{25}{81}ab$  represent in each case the product of two variable terms and a constant. These products are sometimes obtained from tables which are calculated for all ordinary ranges of the variable terms as arguments. A similar table computed for  $\frac{25}{81}(d' - d'')(w'' - w')$  will assist similarly in computing the prismoidal correction. Prof. Charles L. Crandall, of Cornell University, is believed to be the first to prepare such a set of tables, which were first published in 1886 in “Tables for the Computation of Railway and Other Earthwork.” Another

easy method of obtaining these products is by the use of a slide-rule. A slide-rule has been designed by the author to accompany this volume. It is designed particularly for this special work, although it may be utilized for many other purposes for which slide-rules are valuable. To illustrate its use, suppose  $(a + d) = 28.2$ , and  $w = 62.4$ ; then

$$\frac{25}{27}(a + d)w = \frac{28.2 \times 62.4}{1.08}.$$

Set 108 (which, being a constant of frequent use, is specially marked) on the sliding scale (*B*) opposite 282 on the other scale (*A*), and then opposite 624 on scale *B* will be found 1629 on scale *A*, the 162 being read directly and the 9 read by estimation. Although strict rules may be followed for pointing off the final result, it only requires a very simple mental calculation to know that the result must be 1629 rather than 162.9 or 16290. For products less than 1000 cubic yards the result may be read directly from the scale; for products between 1000 and 5000 the result may be read directly to the nearest 10 yards, and the tenths of a division estimated. Between 5000 and 10,000 yards the result may be read directly to the nearest 20 yards, and the fraction estimated; but prisms of such volume will never be found as simple triangular prisms—at least, an assumption that any mass of ground was as regular as this would probably involve more error than would occur from faulty estimation of fractional parts. Facilities for reading as high as 10,000 cubic yards would not have been put on the scale except for the necessity of finding such products as  $\frac{25}{27}(9.1 \times 9.5)$ , for example. This product would be read off from the same part of the rule as  $\frac{25}{27}(91 \times 95)$ . In the first case the product (80.0) could be read directly to the nearest .2 of a cubic yard, which is unnecessarily accurate. In the other case, the product (8004) could only be obtained by estimating  $\frac{4}{20}$  of a division.

The computation for the prismoidal correction may be made

similarly except that the divisor is 3.24 instead of 1.08. For example,  $\frac{25}{81}(5.5 \times 11.7) = \frac{5.5 \times 11.7}{3.24}$ . Set the 324 on scale

*B* (also specially marked like 108) opposite 55 on scale *A*, and proceed as before.

**80. Five-level sections.** Sometimes the elevations over each edge of the roadbed are observed when cross-sectioning. These are distinctively termed “five-level sections.” If the center, the slope-stakes, and *one* intermediate point on each side (*not* necessarily over the edge of the roadbed) are observed, it is termed an “irregular section.” The field-work of cross-sectioning five-level sections is no less than for irregular sections with one intermediate point; the computations, although capable of peculiar treatment on account of the location of the intermediate point, are no easier, and in some respects more laborious; the cross-sections obtained will not in general represent the actual cross-sections as truly as when there is perfect freedom in locating the intermediate point; as it is generally inadvisable or unnecessary to employ five-level sections throughout the length of a road, the change from one method to another adds a possible element of inaccuracy and loses the advantage of uniformity of method, particularly in the notes and *form* of computations. On these accounts the method will not be further developed, except to note that this case, as well as any other, may be solved by dividing the whole prismoid into triangular prismoids, computing the volume by averaging end areas, and computing the prismoidal correction by adding the computed corrections for each elementary triangular prismoid.

**81. Irregular sections.** In cross-sectioning irregular sections, the distance from the center and the elevation above “grade” of every “break” in the cross-section must be observed. The area of the irregular section may be obtained by computing the area of the trapezoids (*five*, in Fig. 44) and subtracting the two external triangles. For Fig. 44 the area would be



$$\begin{aligned} & \frac{h_l + k_l}{2}(x_l - y_l) + \frac{k_l + d}{2}y_l + \frac{d + j_r}{2}z_r + \frac{j_r + k_r}{2}(y_r - z_r) \\ & + \frac{k_r + h_r}{2}(x_r - y_r) - \frac{h_l}{2}\left(x_l - \frac{b}{2}\right) - \frac{h_r}{2}\left(x_r - \frac{b}{2}\right). \end{aligned}$$

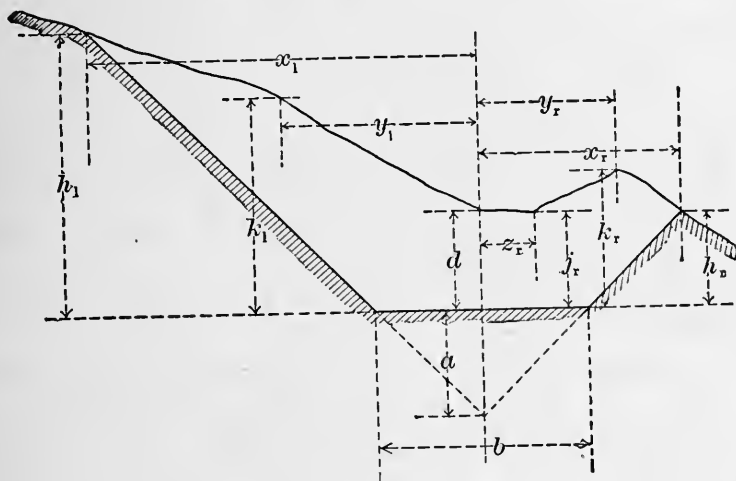


FIG. 44.

Expanding this and collecting terms, of which many will cancel, we obtain

$$\begin{aligned} \text{AREA} = \frac{1}{2} \bigg[ & x_l k_l + y_l (d - h_l) + x_r k_r + y_r (j_r - h_r) \\ & + z_r (d - k_r) + \frac{b}{2} (h_l + h_r) \bigg]. \quad (61) \end{aligned}$$

An examination of this formula will show a perfect regularity in its formation which will enable one to write out a similar formula for any section, no matter how irregular or how many points there are, without any of the preliminary work. The formula may be expressed in words as follows:

*AREA equals one-half the sum of products obtained as follows:*  
*the distance to each slope-stake times the height above grade of the point next inside the slope-stake;*

*the distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;*

*finally, one-half the width of the roadbed times the sum of the slope-stake heights.*

If one of the sides is perfectly regular from center to slope-stake, it is easy to show that the rule holds literally good. The “point next inside the slope-stake” in this case is the center; the intermediate terms for that side vanish. The *last term* must always be used. The rule holds good for three-level sections, in which case there are three terms, which may be reduced to two. Since these two terms are both variable quantities for each cross-section, the special method, given in § 78, in which one term  $\left(\frac{ab}{2}\right)$  is a constant for all sections, is preferable. In the general method, each intermediate “break” adds another term.

**82. Volume of an irregular prismoid.** If there is a break at one cross-section which is not represented at the next, the ridge (or hollow) implied by that break is supposed to “vanish” at the next section. In fact, the volume will not be correctly

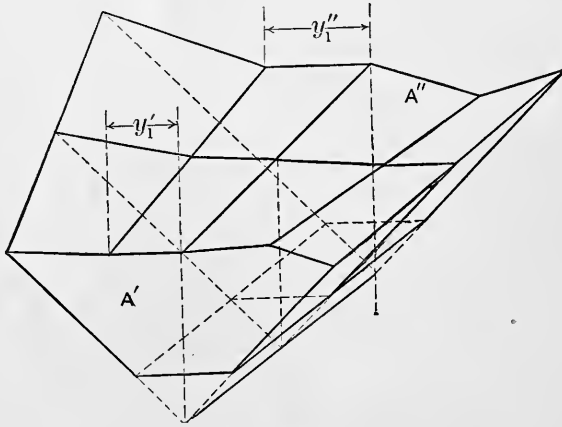


FIG. 52.

represented unless a cross-section is taken at the point where the ridge or hollow “vanishes” or “runs out.” To obtain the true prismoidal correction it is necessary to observe on the ground the place where a break in an adjacent section, which is not represented in the section being taken, runs out. For example, in Fig. 52, the break on the left of section  $A''$ , at a

distance of  $y_i''$  from the center, is observed to run out in section  $A'$  at a distance of  $y_i'$  from the center. The volume of the prismoid, computed by the prismoidal formula as in § 70, will involve the midsection, to obtain the dimension of which will require a laborious computation. A simpler process is to compute the volume by averaging end areas as in § 81 and apply a prismoidal correction. To do this write out an expression for each end area similar to that given in Eq. 61. The sum of

these areas times  $\frac{l}{2}$  gives the approximate volume. As before,

for partial station lengths, multiply the result by  $\frac{\text{length in feet}}{100}$ .

There will be no constant subtractive term,  $\frac{2}{7}ab$ , as in § 78. The *true* prismoidal correction may be computed, as in § 83, or the following approximate method may be used: Consider the irregular section to be three-level ground *for the purpose of computing the correction* only. This has the advantage of less labor in computation than the use of the true prismoidal correction, and although the error involved may be considerable in individual sections, the error is as likely to be positive as negative, and in the long run the error will not be large and generally will be much less than would result by the neglect of any prismoidal correction.

**83. True prismoidal correction for irregular prismoids.** As intimated in § 82, each cross-section should be assumed to have the same number of sides as the adjacent cross-section when computing the prismoidal correction. This being done, it permits the division of the whole prismoid into elementary triangular prismoids, the dimensions of the bases of which being given in each case by a vertical distance above grade line and by the horizontal distance between two adjacent breaks. The summation of the prismoidal corrections for each of the elementary triangular prismoids will give the true prismoidal correction. Assuming for an example the cross-section of Fig. 44, with a cross-section of the same number of sides, and with dimensions

similarly indicated, for the other end, the prismoidal correction becomes (see Eq. 46)

$$\frac{l}{12} \left[ (h_l' - h_l'')(x_l'' - y_l'') - (x_l' - y_l') + (k_l' - k_l'')(x_l'' - y_l'') - (x_l' - y_l') \right. \\
+ (k_l' - k_l'')(y_l'' - y_l') + (d' - d'')(y_l'' - y_l') + (d' - d'')(z_r'' - z_r') \\
+ (j_r' - j_r'')(z_r'' - z_r') + (j_r' - j_r'')(y_r'' - z_r'') - (y_r' - z_r') \\
+ (k_r' - k_r'')(y_r'' - z_r'') - (y_r' - z_r') \\
+ (k_r' - k_r'')(x_r'' - y_r'') - (x_r' - y_r') + (h_r' - h_r'')(x_r'' - y_r'') - (x_r' - y_r') \\
\left. - (h_l' - h_l'') \left[ \left( x_l'' - \frac{b}{2} \right) - \left( x_l' - \frac{b}{2} \right) \right] - (h_r' - h_r'') \left[ \left( x_r'' - \frac{b}{2} \right) - \left( x_r' - \frac{b}{2} \right) \right] \right].$$

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{Pris. Corr.} = \frac{l}{12} \left[ (x_l'' - x_l')(k_l' - k_l'') + (y_l'' - y_l')[d' - h_l'] - (d'' - h_l'') \right. \\
+ (x_r'' - x_r')(k_r' - k_r'') + (y_r'' - y_r')[j_r' - h_r'] - (j_r'' - h_r'') \\
\left. + (z_r'' - z_r')[d' - k_r'] - (d'' - k_r'') \right]. \quad . . . . . (62)$$

By comparing this equation with Eq. 61 a remarkable coincidence in the law of formation may be seen, which enables this formula to be written by mere inspection and to be applied numerically with a minimum of labor from the computations for end areas, as will be shown (§ 84) by a numerical example. For each term in Eq. 61, as, for example,  $y_r(j_r - h_r)$ , there is a correction term in Eq. 62 of the form

$$(y_r'' - y_r')(j_r' - h_r') - (j_r'' - h_r'').$$

Each one of these terms ( $y_r''$ ,  $y_r'$ ,  $(j_r' - h_r')$ , and  $(j_r'' - h_r'')$ ) has been previously used in finding the end areas and has its place in the computation sheet. The summation of the products of these differences times a constant gives the total true prismoidal correction in cubic yards for the whole prismoid considered.

The *constant* is the same as that computed in § 78, i.e.,  $\frac{2.5}{81}$ .

**84. Numerical example; irregular sections; volume, with true prismoidal correction.**

| Sta. | Center {<br>cut<br>or<br>fill. | Left.                |                                   |                                 | Right.                          |                     |
|------|--------------------------------|----------------------|-----------------------------------|---------------------------------|---------------------------------|---------------------|
| 19   | 0.6c                           | $\frac{3.6c}{14.4}$  | $\left(\frac{2.3c}{8.2}\right)$   | $\left(\frac{1.8c}{6.0}\right)$ | $\frac{0.1c}{4.2}$              | $\frac{0.4c}{9.6}$  |
| 18   | 2.3c                           | $\frac{4.2c}{15.3}$  | $\frac{6.8c}{8.4}$                | $\frac{3.2c}{5.2}$              | $\left(\frac{1.9c}{3.6}\right)$ | $\frac{1.2c}{10.8}$ |
| 17   | 7.6c                           | $\frac{8.2c}{21.3}$  | $\frac{10.2c}{17.4}$              | $\frac{8.0c}{6.1}$              | $\left(\frac{5.8c}{8.0}\right)$ | $\frac{4.2c}{15.3}$ |
| + 42 | 10.2c                          | $\frac{12.2c}{27.3}$ | $\left(\frac{12.3c}{22.0}\right)$ | $\frac{12.6c}{8.2}$             | $\frac{6.2c}{7.5}$              | $\frac{8.4c}{21.6}$ |
| 16   | 6.8c                           | $\frac{8.9c}{22.4}$  |                                   | $\frac{7.6c}{12.0}$             | $\frac{3.2c}{4.1}$              | $\frac{2.6c}{12.9}$ |

Roadbed 18 feet wide in cut; slope  $1\frac{1}{2}$  to 1.

The figures in the bracket  $\left(\frac{12.3c}{22.0}\right)$  mean that it was noted in the field that the break, indicated at Sta. 17 as being 17.4 to the left, ran out at Sta. 16 + 42 at 22.0 to the left. By interpolation between 8.2 and 27.3 the height of this point is *computed* as 12.3. The quantities in the other brackets are obtained similarly. These quantities are only used when the computation of the true prismoidal correction is desired. They are not needed in computing the volume by averaging end areas, nor are they used at all if the prismoidal correction is to be obtained by assuming (*for this purpose*) the ground to be *three-level* ground.

In the tabular form on page 98 the figures within the braces (—) are not used in computing the volume, but are only used to obtain the *differences* of widths or heights with which to compute the *true* prismoidal correction. It may be noted, as a check, that the volume, computed from these figures in the braces, is the same as that computed from the other figures.

## VOLUME OF IRREGULAR PRISMOID, WITH TRUE PRISMOIDAL CORRECTION.

| Sta.                           | Width.                                                                                       | Height.                                                                                 | Yards.                                                          |     | True pris. corr.                                                   |                                                                    |                                                                |
|--------------------------------|----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|-----------------------------------------------------------------|-----|--------------------------------------------------------------------|--------------------------------------------------------------------|----------------------------------------------------------------|
|                                |                                                                                              |                                                                                         |                                                                 |     | $w'' - w'$                                                         | $h' - h''$                                                         | Yards.                                                         |
| 16                             | L $\begin{bmatrix} 22.4 \\ 12.0 \\ 12.9 \\ 4.1 \\ 9.0 \end{bmatrix}$ R                       | $\begin{matrix} 7.6 \\ -2.1 \\ 3.2 \\ 4.2 \\ 11.5 \end{matrix}$                         | $\begin{matrix} 158 \\ -23 \\ 40 \\ 16 \\ 96 \end{matrix}$      |     |                                                                    |                                                                    |                                                                |
| + 42                           | L $\begin{bmatrix} 27.3 \\ 8.2 \\ 27.3 \\ 22.0 \\ 8.2 \\ 21.6 \\ 7.5 \\ 9.0 \end{bmatrix}$ R | $\begin{matrix} 12.6 \\ -2.0 \\ 12.3 \\ 0.4 \\ -2.1 \\ 6.2 \\ 1.8 \\ 20.6 \end{matrix}$ | $\begin{matrix} 319 \\ -15 \\ \\ 124 \\ 13 \\ 172 \end{matrix}$ | 378 | $\begin{matrix} +4.9 \\ -3.8 \\ \\ +8.7 \\ +3.4 \end{matrix}$      | $\begin{matrix} -5.0 \\ -0.1 \\ \\ -3.0 \\ +2.4 \end{matrix}$      | $\begin{matrix} -7 \\ 0 \\ \\ -8 \\ +3 \\ (-5) \end{matrix}$   |
| 17                             | L $\begin{bmatrix} 21.3 \\ 17.4 \\ 6.1 \\ 15.3 \\ 8.0 \\ 15.3 \\ 9.0 \end{bmatrix}$ R        | $\begin{matrix} 10.2 \\ -0.2 \\ -2.6 \\ 5.8 \\ 3.4 \\ 7.6 \\ 12.4 \end{matrix}$         | $\begin{matrix} 201 \\ -3 \\ -14 \\ \\ 107 \\ 103 \end{matrix}$ | 584 | $\begin{matrix} -6.0 \\ -4.6 \\ -2.1 \\ -6.3 \\ +0.5 \end{matrix}$ | $\begin{matrix} +2.1 \\ +0.6 \\ +0.5 \\ +0.4 \\ -1.6 \end{matrix}$ | $\begin{matrix} -4 \\ -1 \\ 0 \\ -1 \\ 0 \\ (-3) \end{matrix}$ |
| 18                             | L $\begin{bmatrix} 15.3 \\ 8.4 \\ 5.2 \\ 10.8 \\ 10.8 \\ 3.6 \\ 9.0 \end{bmatrix}$ R         | $\begin{matrix} 6.8 \\ -1.0 \\ -4.5 \\ 2.3 \\ 1.9 \\ 1.1 \\ 5.4 \end{matrix}$           | $\begin{matrix} 95 \\ -7 \\ -22 \\ 23 \\ \\ 45 \end{matrix}$    | 528 | $\begin{matrix} -6.0 \\ -9.0 \\ -0.9 \\ -4.5 \end{matrix}$         | $\begin{matrix} +3.4 \\ +0.8 \\ +1.9 \\ +5.3 \end{matrix}$         | $\begin{matrix} -6 \\ -2 \\ -1 \\ -7 \\ (-16) \end{matrix}$    |
| 19                             | L $\begin{bmatrix} 14.4 \\ 14.4 \\ 8.2 \\ 6.0 \\ 9.6 \\ 4.2 \\ 9.0 \end{bmatrix}$ R          | $\begin{matrix} 0.6 \\ 2.3 \\ -1.8 \\ -1.7 \\ 0.1 \\ 0.2 \\ 4.0 \end{matrix}$           | $\begin{matrix} 8 \\ \\ 1 \\ 1 \\ 33 \end{matrix}$              | 177 | $\begin{matrix} -0.9 \\ -0.2 \\ +0.8 \\ -1.2 \\ +0.6 \end{matrix}$ | $\begin{matrix} +4.5 \\ +0.8 \\ -2.8 \\ +1.8 \\ +0.9 \end{matrix}$ | $\begin{matrix} -1 \\ 0 \\ -1 \\ -1 \\ 0 \\ (-3) \end{matrix}$ |
| Approx. vol. = 1667            |                                                                                              |                                                                                         |                                                                 |     | - 27                                                               |                                                                    |                                                                |
| True pris. corr. = - 27        |                                                                                              |                                                                                         |                                                                 |     |                                                                    |                                                                    |                                                                |
| True volume = 1640 cubic yards |                                                                                              |                                                                                         |                                                                 |     |                                                                    |                                                                    |                                                                |

The figures within each brace (or bracket) constitute a group which must be used in connection with a group which has the same number of points, on the same side of the center, in the *next* cross-section, previous or succeeding. In the column of

“Yards” under “True pris. corr.,” we have, for example,  
 $(-5) = \frac{42}{100}(-7 + 0 - 8 + 3)$ .

**85. Volume of irregular prismoid, with approximate prismoidal correction.** If the prismoidal correction is obtained approximately, by the method outlined in § 82, the process will be as shown in the tabular form. Not only is the numerical work considerably less than the exact method, but the discrepancy in cubic yards is almost insignificant.

| Sta. | Width. | Height. | Yards. |     | Cen.<br>Height. | Total<br>width. | $d' - d''$ | $w'' - w'$ | Approx.<br>pris. corr. |
|------|--------|---------|--------|-----|-----------------|-----------------|------------|------------|------------------------|
| 16   | 22.4   | 7.6     | 158    |     | + 6.8           | 35.3            |            |            |                        |
|      | 12.0   | - 2.1   | - 23   |     |                 |                 |            |            |                        |
|      | 12.9   | 3.2     | 40     |     |                 |                 |            |            |                        |
|      | 4.1    | 4.2     | 16     |     |                 |                 |            |            |                        |
|      | 9.0    | 11.5    | 96     |     |                 |                 |            |            |                        |
| + 42 | 27.3   | 12.6    | 319    |     | + 10.2          | 48.9            | - 3.4      | + 13.6     | - 14                   |
|      | 8.2    | - 2.0   | - 15   |     |                 |                 |            |            |                        |
|      | 21.6   | 6.2     | 124    |     |                 |                 |            |            |                        |
|      | 7.5    | 1.8     | 13     |     |                 |                 |            |            |                        |
|      | 9.0    | 20.6    | 172    | 378 |                 |                 |            |            |                        |
| 17   | 21.3   | 10.2    | 201    |     | + 7.6           | 36.6            | + 2.6      | - 12.3     | - 10                   |
|      | 17.4   | - 0.2   | - 3    |     |                 |                 |            |            |                        |
|      | 6.1    | - 2.6   | - 14   |     |                 |                 |            |            |                        |
|      | 15.3   | 7.6     | 107    |     |                 |                 |            |            |                        |
|      | 9.0    | 12.4    | 103    | 584 |                 |                 |            |            |                        |
| 18   | 15.3   | 6.8     | 95     |     | + 2.3           | 26.1            | + 5.3      | - 10.5     | - 17                   |
|      | 8.4    | - 1.0   | - 7    |     |                 |                 |            |            |                        |
|      | 5.2    | - 4.5   | - 22   |     |                 |                 |            |            |                        |
|      | 10.8   | 2.3     | 23     |     |                 |                 |            |            |                        |
|      | 9.0    | 5.4     | 45     | 528 |                 |                 |            |            |                        |
| 19   | 14.4   | 0.6     | 8      |     | + 0.6           | 24.0            | + 1.7      | - 2.1      | - 1                    |
|      | 9.6    | 0.1     | 1      |     |                 |                 |            |            |                        |
|      | 4.2    | 0.2     | 1      |     |                 |                 |            |            |                        |
|      | 9.0    | 4.0     | 33     | 177 |                 |                 |            |            |                        |

Approx. volume = 1667

- 30

Approx. pris. corr. = - 30

Corrected volume = 1637 cubic yards

**86. Illustration of value of approximate rules.** The accompanying tabulation shows that when the volume of an irregular prismoid is computed by averaging end areas and is corrected by considering the ground as three-level ground (*for the pur-*

poses of the correction only), the error for the different sections is sometimes positive and sometimes negative, and in this case

| Sections.      | True volume. | Approx. vol.<br>by averaging<br>end areas. | Difference or<br><i>true pris.</i><br>corr. | Approx. pris.<br>corr. on basis<br>of three-level<br>ground. | Error. | Approx. vol.,<br>computed<br>from center<br>and side<br>heights <i>only</i> . | Error. |
|----------------|--------------|--------------------------------------------|---------------------------------------------|--------------------------------------------------------------|--------|-------------------------------------------------------------------------------|--------|
| 16.....16 + 42 | 373          | 378                                        | - 5                                         | - 6                                                          | - 1    | 396                                                                           | + 23   |
| 16 + 42...17   | 581          | 584                                        | - 3                                         | - 6                                                          | - 3    | 577                                                                           | - 4    |
| 17.....18      | 512          | 528                                        | - 16                                        | - 17                                                         | - 1    | 463                                                                           | - 49   |
| 18 .....19     | 174          | 177                                        | - 3                                         | - 1                                                          | + 2    | 147                                                                           | - 27   |
|                | 1640         | 1667                                       | - 27                                        | - 30                                                         | - 3    | 1583                                                                          | - 57   |

amounts to only 3 yards in 1640—less than  $\frac{1}{5}$  of 1%. If the prismoidal correction had been neglected, the error would have been 27 yards—nearly 2%. The approximate results are here *too large* for each section—as is usually the case. If points between the center and slope stakes are omitted and the volume computed as if the ground were *three-level* ground, the error is quite large in individual sections, but the errors are both positive and negative and therefore compensating.

**87. Cross-sectioning irregular sections.** The prismoids considered have *straight* lines joining corresponding points in the two cross-sections. The center line must be straight between two cross-sections. If a ridge or valley is found lying diagonally across the roadbed, a cross-section *must* be interpolated at the lowest (or highest) point of the profile. Therefore a “break” at any section cannot be said to run out at the other section on the *opposite* side of the center. It must run out on the *same* side of the center or possibly *at* the center. Very frequently complicated cross-sectioning may be avoided by computing the volume, by some special method, of a mound or hollow when the ground is comparatively regular except for the irregularity referred to.

**88. Side-hill work.** When the natural slope cuts the roadbed there is a necessity for both cut and fill at the same cross-section. When this occurs the cross-sections of both cut and fill are often so nearly triangular that they may be considered as such without



great error, and the volumes may be computed separately as triangular prismoids without adopting the more elaborate form of computation so necessary for complicated irregular sections. When the ground is too irregular for this the best plan is to follow the uniform system. In computing the cut, as in Fig. 53,

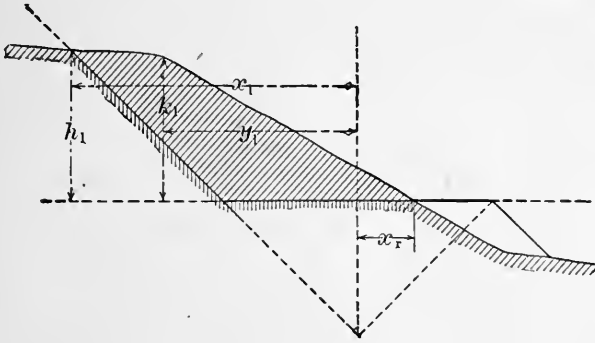


FIG. 53.

the left side would be as usual; there would be a small center cut and an ordinate of zero at a short distance to the right of the center. Then, *ignoring the fill*, and applying Eq. 61 strictly, we have two terms for the left side, one for the right, and the term involving  $\frac{1}{2}b$ , which will be  $\frac{1}{2}bh_l$  in this case, since  $h_r = 0$ , and the equation becomes

$$\text{Area} = \frac{1}{2}[x_l k_l + y_l(d - h_l) + x_r d + \frac{1}{2}bh_l].$$

The area for fill may also be computed by a strict application

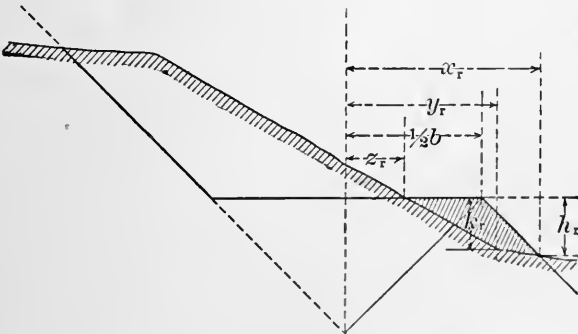


FIG. 54.

of Eq. 61, but for Fig. 54 all distances for the left side are zero and the elevation for the first point out is zero.  $d$  also must be

considered as zero. Following the rule, § 81, literally, the equation becomes

$$\text{Area}_{(\text{Fill})} = \frac{1}{2}[x_r k_r + y_r(o - h_r) + z_r(o - k_r) + \frac{1}{2}b(o + h_r)],$$

which reduces to

$$\frac{1}{2}[x_r k_r - y_r h_r - z_r k_r + \frac{1}{2}b h_r].$$

(*Note* that  $x_r$ ,  $h_r$ , etc., have different significations and values in this and in the preceding paragraphs.) The “terminal pyramids” illustrated in Fig. 40 are instances of side-hill work for very short distances. Since side-hill work always implies *both* cut and fill at the same cross-section, whenever either the cut or fill disappears and the earthwork becomes wholly cut or wholly fill, that point marks the end of the “side-hill work,” and a cross-section should be taken at this point.

**89. Borrow-pits.** The cross-sections of borrow-pits will vary not only on account of the undulations of the surface of the

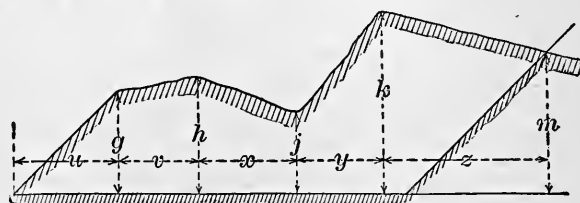


FIG. 55.

ground, but also on the sides, according to whether they are made by widening a convenient cut (as illustrated in Fig. 55) or simply by digging a pit. The sides should always be properly sloped and the cutting made cleanly, so as to avoid unsightly roughness. If the slope ratio on the right-hand side (Fig. 55) is  $s$ , the area of the triangle is  $\frac{1}{2}sm^2$ . The area of the section is  $\frac{1}{2}[ug + (g + h)v + (h + j)x + (j + k)y + (k + m)z - sm^2]$ . If all the horizontal measurements were referred to one side as an origin, a formula similar to Eq. 61 could readily be developed, but little or no advantage would be gained on account of any simplicity of computation. Since the *exact* volume of the earth borrowed is frequently necessary, the prismoidal correc-

tion should be computed; and since such a section as Fig. 55 does not even approximate to a three-level section, the method suggested in § 82 cannot be employed. It will then be necessary to employ the exact method, § 83, by dividing the volume into triangular prismoids and taking the summation of their corrections, found according to the general method of § 71.

**90. Correction for curvature.** The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. If the centers of gravity of all cross-sections lie in the center of the road, where the length of the road is measured, there is absolutely no necessary correction for curvature. If all the cross-sections in any given length were exactly the same and therefore had the same eccentricity, the correction for curvature would be very readily computed according to the above principle. But when both the areas and the eccentricities vary from point to point, as is generally the case, a theoretically exact solution is quite complex, both in its derivation and application. Suppose, for simplicity, a curved section of the road, of uniform cross-sections and with the center of gravity of every cross-section at the same distance  $e$  from the center line of the road. The length of the path of the center of gravity will be to the length of the center line as  $R \pm e : R$ . Therefore we have *True vol.* : *nominal vol.* ::  $R \pm e : R$ .  
 $\therefore \text{True vol.} = lA \frac{R \pm e}{R}$  for a volume of uniform area and eccentricity. For any other area and eccentricity we have, similarly,  $\text{True vol.}' = lA' \frac{R \pm e'}{R}$ . This shows that the effect of curvature is the same as increasing (or diminishing) the area by a quantity depending on the area and eccentricity, the increased (or diminished) area being found by multiplying the actual area by the ratio  $\frac{R \pm e}{R}$ . This being independent of the value of  $l$ , it is true for infinitesimal lengths. If the eccen-

tricity is assumed to vary uniformly between two sections, the *equivalent area* of a cross-section located midway between the

two end cross-sections would be  $A_m \frac{\left(R \pm \frac{e' + e''}{2}\right)}{R}$ . Therefore the volume of a solid which, when straight, would be  $\frac{l}{6}(A' + 4A_m + A'')$ , would then become

$$\text{True vol.} = \frac{l}{6R} \left[ A'(R \pm e') + 4A_m \left( R \pm \frac{e' + e''}{2} \right) + A''(R \pm e'') \right].$$

Subtracting the nominal volume (the true volume when the prismoid is straight), the

$$\text{Correction} = \pm \frac{l}{6R} \left[ (A' + 2A_m)e' + (2A_m + A'')e'' \right]. \quad (63)$$

Another demonstration of the same result is given by Prof. C. L. Crandall in his "Tables for the Computation of Railway and other Earthwork," in which is obtained by calculus methods the summation of elementary volumes having variable areas with variable eccentricities. The exact application of Eq. (63) requires that  $A_m$  be known, which requires laborious computations, but no error worth considering is involved if the equation is written approximately

$$\text{Curv. corr.} = \frac{l}{2R} (A'e' + A''e''), \quad . \quad . \quad . \quad (64)$$

which is the equation generally used. The approximation consists in assuming that the difference between  $A'$  and  $A_m$  equals the difference between  $A_m$  and  $A''$  but with opposite sign. The error due to the approximation is always utterly insignificant.

**91. Eccentricity of the center of gravity.** The determination of the true positions of the centers of gravity of a long series of irregular cross-sections would be a very laborious operation, but fortunately it is generally sufficiently accurate to

consider the cross-sections as three-level ground, or, for side-hill work, to be triangular, *for the purpose of this correction*. The

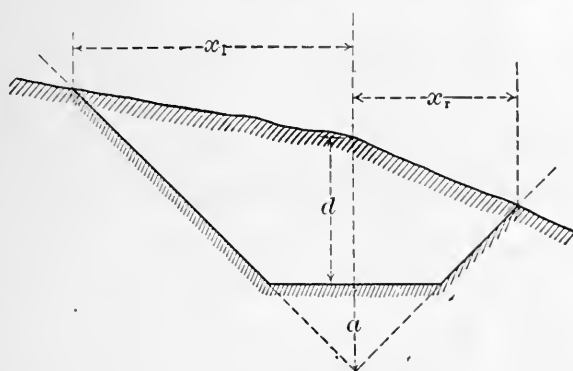


FIG. 56.

eccentricity of the cross-section of Fig. 56 (including the grade triangle) may be written

$$e = \frac{\frac{(a+d)x_l}{2} \cdot \frac{x_l}{3} - \frac{(a+d)x_r}{2} \cdot \frac{x_r}{3}}{\frac{(a+d)x_l}{2} + \frac{(a+d)x_r}{2}} = \frac{1}{3} \frac{x_l^2 - x_r^2}{x_l + x_r} = \frac{1}{3}(x_l - x_r). \quad (65)$$

The side toward  $x_r$  being considered positive in the above demonstration, if  $x_r > x_l$ ,  $e$  would be negative, i.e., the center of gravity would be on the left side. Therefore, for three-level ground, the correction for curvature (see Eq. 64) may be written

$$\text{Correction} = \frac{l}{6R} [A'(x_l' - x_r') + A''(x_l'' - x_r'')].$$

Since the approximate volume of the prismoid is

$$\frac{l}{2}(A + A') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V'',$$

in which  $V'$  and  $V''$  represent the number of cubic yards corresponding to the area at each station, we may write

$$\text{Corr. in cub. yds.} = \frac{1}{3R} [V'(x_l' - x_r') + V''(x_l'' - x_r'')]. \quad (66)$$

It should be noted that the value of  $e$ , derived in Eq. 65, is the eccentricity of the whole area including the triangle under the roadbed. The eccentricity of the true area is greater than this and equals

$$e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}} = e_1.$$

The required quantity ( $A'e'$  of Eq. 64) equals  $\text{true area} \times e_1$ , which equals  $(\text{true area} + \frac{1}{2}ab) \times e$ . Since the value of  $e$  is very simple, while the value of  $e_1$  would, in general, be a complex quantity, it is easier to use the simple value of Eq. 65 and add  $\frac{1}{2}ab$  to the area. Therefore, in the case of three-level ground the subtractive term  $\frac{2}{7}\frac{5}{7}ab$  (§ 78) should *not* be subtracted in computing this correction. For irregular ground, when computed by the method given in §§ 81 and 82, which does not involve the grade triangle, a term  $\frac{2}{7}\frac{5}{7}ab$  must be *added* at every station when computing the quantities  $V'$  and  $V''$  for Eq. 66.

It should be noted that the factor  $1 \div 3R$ , which is constant for the length of the curve, may be computed with all necessary accuracy and without resorting to tables by remembering that

$$R = \frac{5730}{\text{degree of curve}}.$$

Since it is useless to attempt the computation of railroad earthwork closer than the nearest cubic yard, it will frequently be possible to write out all curvature corrections by a simple mental process upon a mere inspection of the computation sheet. Eq. 66 shows that the correction for each station is of the form

$\frac{V(x_l - x_r)}{3R}$ .  $3R$  is generally a large quantity—for a  $6^\circ$  curve it is 2865.  $(x_l - x_r)$  is generally small. It may frequently be seen by inspection that the product  $V(x_l - x_r)$  is roughly twice or three times  $3R$ , or perhaps less than half of  $3R$ , so that the corrective term for that station may be written 2, 3, or 0 cubic yards, the fraction being disregarded. For much larger absolute

amounts the correction must be computed with a correspondingly closer percentage of accuracy.

The algebraic sign of the curvature correction is best determined by noting that the center of gravity of the cross-section is on the right or left side of the center according as  $x_r$  is greater or less than  $x_l$ , and that the correction is *positive* if the center of gravity is on the *outside* of the curve, and *negative* if on the *inside*.

It is frequently found that  $x_l$  is uniformly greater (or uniformly less) than  $x_r$  throughout the length of the curve. Then the curvature correction for each station is uniformly positive or negative. But in irregular ground the center of gravity is apt to be irregularly on the outside or on the inside of the curve, and the curvature correction will be correspondingly positive or negative. If the curve is to the *right*, the correction will be positive or negative according as  $(x_l - x_r)$  is positive or negative; if the curve is to the *left*, the correction will be positive or negative according as  $(x_r - x_l)$  is positive or negative. Therefore when computing curves to the *right* use the form  $(x_l - x_r)$  in Eqs. 66 and 68; when computing curves to the *left* use the form  $(x_r - x_l)$  in these equations; the algebraic sign of the correction will then be strictly in accordance with the results thus obtained.

**92. Center of gravity of side-hill sections.** In computing the

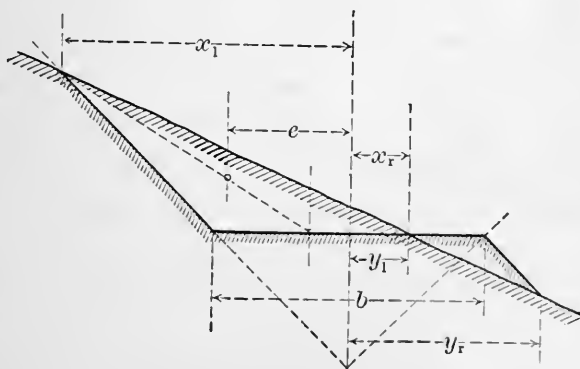


FIG. 57.

correction for side-hill work the cross section would be treated as triangular unless the error involved would evidently be too





which reduces to

$$e = -\frac{1}{3} \left[ \frac{b}{2} + y_l + y_r \right]. \quad . \quad . \quad . \quad . \quad . \quad (69)$$

As the algebraic signs tend to create confusion in these formulæ, it is more simple to remember that for a triangle lying on *both* sides of the center  $e$  is always numerically equal to  $\frac{1}{3} \left[ \frac{b}{2} + (x_l \sim x_r) \right]$ , and for a triangle entirely on one side,  $e$  is numerically equal to  $\frac{1}{3} \left[ \frac{b}{2} + \text{the numerical sum of the two distances out} \right]$ . The algebraic sign of  $e$  is readily determinable as in § 91.

**93. Example of curvature correction.** Assume that the fill in § 78 occurred on a  $6^\circ$  curve to the *right*.  $\frac{1}{3R} = \frac{1}{2865}$ . The quantities 210, 507, etc., represent the quantities  $V'$ ,  $V''$ , etc., since they include in each case the 61 cubic yards due to the grade prism. Then

$$\frac{V(x_l \sim x_r)}{3R} = \frac{210(22.9 - 8.2)}{2865} = \frac{3101.7}{2865} = + 1.$$

The sign is plus since the center of gravity of the cross-section is on the left side of the center and the road curves to the right, thus making the true volume larger. For Sta. 18 the correction, computed similarly, is  $+ 3$ , and the correction for the whole section is  $1 + 3 = 4$ . For Sta. 18 + 40 the correction is computed as 6 yards. Therefore, for the 40 feet, the correction is  $\frac{40}{100}(3 + 6) = 3.6$ , which is called 4. Computing the others similarly we obtain a total correction of  $+ 16$  cubic yards.

**94. Accuracy of earthwork computations.** The preceding methods give the *precise volume* (except where approximations are distinctly admitted) of the prismoids which are *supposed* to represent the volume of the earthwork. To appreciate the accuracy necessary in cross-sectioning to obtain a given accuracy

in volume, consider that a fifteen-foot length of the cross-section, which is assumed to be straight, really sags 0.1 foot, so that the cross-section is in error by a triangle 15 feet wide and 0.1 foot high. This sag 0.1 foot high would hardly be detected by the eye, but in a length of 100 feet in each direction it would make an error of volume of 1.4 cubic yards in *each* of the two prismoids, assuming that the sections at the other ends were perfect. If the cross-sections at both ends of a prismoid were in error by this same amount, the volume of that prismoid would be in error by 2.8 cubic yards if the errors of area were both plus or both minus. If one were plus and one minus, the errors would neutralize each other, and it is the compensating character of these errors which permits any confidence in the results as obtained by the usual methods of cross-sectioning. It demonstrates the utter futility of attempting any closer accuracy than the nearest cubic yard. It will thus be seen that if an error really exists at *any* cross-section it involves the prismoids on *both* sides of the section, even though all the other cross-sections are perfect. As a further illustration, suppose that cross-sections were taken by the method of slope angle and center depth (§ 73), and that a cross-section, assumed as uniform, sags 0.4 foot in a width of 20 feet. Assume an equal error (of same sign) at the other end of a 100-foot section. The error of volume for that one prismoid is 38 cubic yards.

The computations further assume that the warped surface, passing through the end sections, coincides with the surface of the ground. Suppose that the cross-sectioning had been done with mathematical perfection; and, to assume a simple case, suppose a sag of 0.5 foot between the sections, which causes an error equal to the volume of a pyramid having a base of 20 feet (in each cross-section) times 100 feet (between the cross-sections) and a height of 0.5 foot. The volume of this pyramid is  $\frac{1}{3}(20 \times 100) \times 0.5 = 333$  cub. ft. = 12 cub. yds. And yet this sag or hump of 6 inches would generally be utterly unnoticed, or at least disregarded.

When the ground is very rough and broken it is sometimes

practically impossible, even with frequent cross-sections, to locate warped surfaces which will closely coincide with all the sudden irregularities of the ground. In such cases the computations are necessarily more or less approximate and dependence must be placed on the compensating character of the errors.

**95. Approximate computations from profiles.** As a means of comparing the relative amounts of earthwork on two or more proposed routes which have been surveyed by preliminary surveys, it will usually be sufficiently accurate to compare the *areas* of cutting (assuming that the cut and fill are approximately balanced) as shown by the several profiles. The errors involved may be large in individual cases and for certain small sections, but fortunately the errors (in comparing two lines) will be largely compensated. The errors are much larger on side-hill work than when the cross-sections are comparatively level. The errors become large when the depth of cut or fill is very great. If the lines compared have the same general character as to the slope of the cross-sections, the proportion of side-hill work, and the average depth of cut or fill, the error involved in considering their relative volumes of cutting to be as the relative areas of cutting on the profiles (obtained perhaps by a planimeter) will probably be small. If the volume in each case is computed by assuming the sections as *level*, with a depth equal to the center cut, the error involved will depend only on the amount of side-hill work and the degree of the slope. If these features are about the same on the two lines compared, the error involved is still less.

#### FORMATION OF EMBANKMENTS.

**96. Shrinkage of earthwork.** The evidence on this subject as to the amount of shrinkage is very conflicting, a fact which is probably due to the following causes:

1. The various kinds of earthy material act very differently as respects shrinkage. There has been but little uniformity in the *classification of earths* in the tests and experiments that have been made.

2. Very much depends on the *method* of forming an embankment (as will be shown later). Different reports have been based on different methods—often without mention of the method.

3. An embankment requires considerable *time* to shrink to its final volume, and therefore much depends on the time elapsed between construction and the measurement of what is supposed to be the settled volume.

P. J. Flynn quotes some experiments (*Eng. News*, May 1, 1886) made in India in which pits were dug, having volumes of 400 to 600 cubic feet. The material, when piled into an embankment, measured largely in excess of the original measurement—as is the universal experience. The pits were refilled with the same material. As the rains, very heavy in India, settled the material in the pits, more was added to keep the pits full. Even after the rainy season was over, there was in every case material in excess. This would seem to indicate a permanent *expansion*, although it is possible that the observations were not continued for a sufficient time to determine the final settled volume.

On the contrary, notes made by Mr. Elwood Morris many years ago on the behavior of embankments of several thousand cubic yards, formed in layers by carts and scrapers, one winter intervening between commencement and completion, showed in each case a permanent *contraction* averaging about 10%.

All authorities agree that rockwork *expands* permanently when formed into an embankment, but the percentages of expansion given by different authorities differ even more than with earth—varying from 8 to 90%. Of course this very large range in the coefficient is due to differences in the character of the rock. The softer the rock and the closer its similarity to earth, the less will be its expansion. On account of the conflicting statements made, and particularly on account of the influence of methods of work, but little confidence can be felt in any given coefficient, especially when given to a fraction of a per

cent, but the consensus of American practice seems to average about as follows:

- Permanent contraction of earth..... about 10%
- “ expansion of rock..... 40 to 60%

These values for rock should be materially reduced, according to judgment, when the rock is soft and liable to disintegrate. The hardest rocks, loosely piled, may occasionally give even higher results. The following is given by several authors as the permanent contraction of several grades of earth:

- Gravel or sand..... about 8%
- Clay..... “ 10%
- Loam..... “ 12%
- Loose vegetable surface soil..... “ 15%

It may be noticed from the above table that the harder and cleaner the material the less is the contraction. Perfectly clean gravel or sand would not probably change volume appreciably. The above coefficients of shrinkage and expansion may be used to form the following convenient table.

| Material.                 | To make 1000 cubic yards of embankment will require | 1000 cubic yards measured in excavation will make |
|---------------------------|-----------------------------------------------------|---------------------------------------------------|
| Gravel or sand.....       | 1087 cubic yards                                    | 920 cubic yards                                   |
| Clay.....                 | 1111 “ “                                            | 900 “ “                                           |
| Loam.....                 | 1136 “ “                                            | 880 “ “                                           |
| Loose vegetable soil..... | 1176 “ “                                            | 850 “ “                                           |
| Rock, large pieces.....   | 714 “ “                                             | 1400 “ “                                          |
| “ small “ .....           | 625 “ “                                             | 1600 “ “                                          |
|                           | measured in excavation                              | of embankment.                                    |

**97. Allowance for shrinkage.** On account of the initial expansion and subsequent contraction of earth, it becomes necessary to form embankments higher than their required ultimate form in order to allow for the subsequent shrinkage. As the shrinkage appears to be all vertical (practically), the embankment must be formed as shown in Fig. 58. The effect

of shrinkage should not be confounded with that of slipping of the sides, which is especially apt to occur if the embankment is subjected to heavy rains very soon after being formed, and also when the embankments are originally steep. It is often difficult

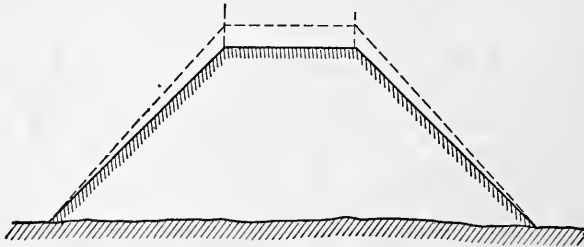


FIG. 58.

to form an embankment at a slope of 1 : 1 which will not slip more or less before it hardens.

Very high embankments shrink a greater percentage than lower ones. Various rules giving the relation between shrinkage and height have been suggested, but they vary as badly as the suggested coefficients of contraction, probably for the same causes. As the fact is unquestionable, however, the extra height of the embankment must be varied somewhat as in Fig. 59, which represents a longitudinal section of an embankment.

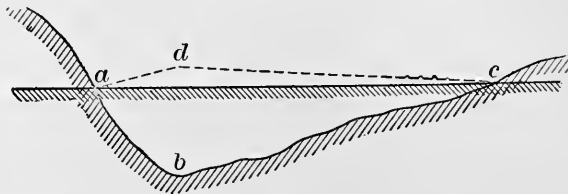


FIG. 59.

As considerable time generally elapses between the completion of the embankment and the actual running of trains, the grade *ad* will generally be nearly flattened down to its ultimate form before traffic commences, but such grades are occasionally objectionable if added to what is already a ruling grade. With some kinds of soil the time required for complete settlement may be as much as two or three years, but, even in such cases, it is

probable that one-half of the settlement will take place during the first six months. The engineer should therefore require the contractor to make all fills about 8 to 15% (according to the material) higher than the profiles call for, in order that subsequent shrinkage may not reduce it to less than the required volume.

**98. Methods of forming embankments.** When the method is not otherwise objectionable, a high embankment can be formed very cheaply (assuming that carts or wheelbarrows are used) by dumping over the end and building to the full height (or even higher, to allow for shrinkage) as the embankment proceeds. This allows more time for shrinkage, saves nearly all the cost of spreading (see Item 4, § 111), and reduces the cost of roadways (Item 5). Of course this method is especially applicable when the material comes from a place as high as or higher than grade, so that no up-hill hauling is required.

Another method is to spread it in layers two or three feet thick (see Fig. 60), which are made concave upwards to avoid

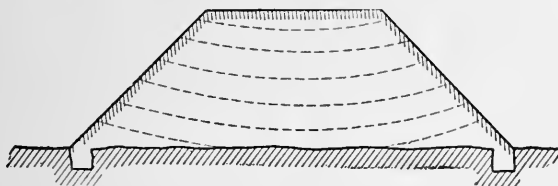


FIG. 60.

possible sliding on each other. Spreading in layers has the advantage of partially ramming each layer, so that the subsequent shrinkage is very small. Sometimes small trenches are dug along the lines of the toes of the embankment. This will frequently prevent the sliding of a large mass of the embankment, which will then require extensive and costly repairs, to say nothing of possible accidents if the sliding occurs after the road is in operation. Incidentally these trenches will be of value in draining the subsoil. When circumstances require an embankment on a hillside, it is advisable to cut out "steps" to prevent a possible sliding of the whole embankment. Merely

ploughing the side-hill will often be a cheaper and sufficiently effective method.

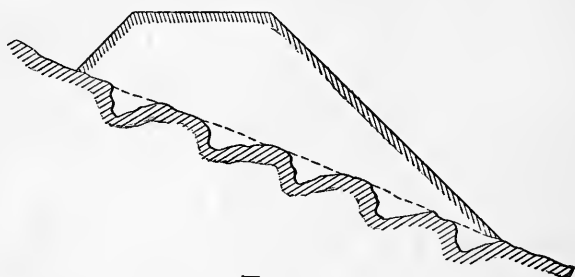


FIG. 61.

Occasionally the formation of a very high and long embankment may be most easily and cheaply accomplished by building a trestle to grade and opening the road. Earth can then be procured where most convenient, perhaps several miles away, loaded on cars with a steam-shovel, hauled by the trainload, and dumped from the cars with a patent unloader. On such a large scale, the cost per yard would be very much less than by ordinary methods—enough less sometimes to more than pay for the temporary trestle, besides allowing the road to be opened for traffic very much earlier, which is often a matter of prime financial importance. It may also obviate the necessity for extensive borrow-pits in the immediate neighborhood of the heavy fill and also utilize material which would otherwise be wasted.

#### COMPUTATION OF HAUL.

**99. Nature of subject.** As will be shown later when analyzing the cost of earthwork, the most variable item of cost is that depending on the distance hauled. As it is manifestly impracticable to calculate the exact distance to which every individual cartload of earth has been moved, it becomes necessary to devise a means which will give at least an equivalent of the haulage of all the earth moved. Evidently the *average* haul for any mass of earth moved is equal to the distance from the center of gravity of the excavation to the center of gravity of the embank-



ment formed by the excavated material. As a rough approximation the center of gravity of a cut (or fill) may sometimes be considered to coincide with the center of gravity of that part of the profile representing it, but the error is frequently very large. The center of gravity may be determined by various methods, but the method of the "mass diagram" accomplishes the same ultimate purpose (the determination of the haul) with all-sufficient accuracy and also furnishes other valuable information.

**100. Mass diagram.** In Fig. 62 let  $A'B' \dots G'$  represent a profile and grade line drawn to the usual scales. Assume  $A'$

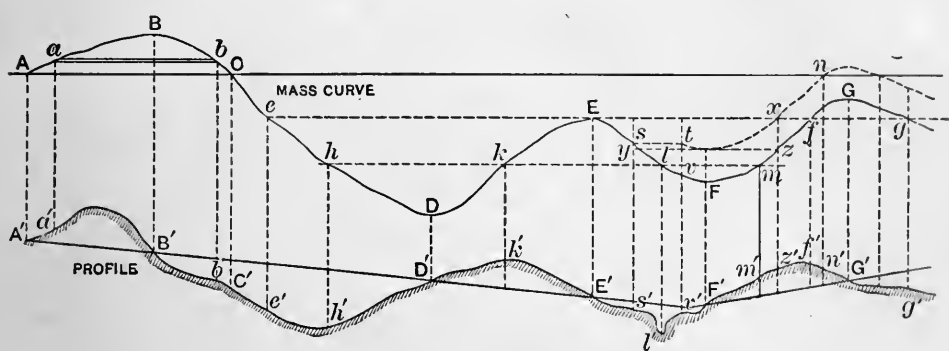


FIG. 62.—MASS DIAGRAM.

to be a point past which no earthwork will be hauled. Above every station point in the profile draw an ordinate which will represent the algebraic sum of the cubic yards of cut and fill (calling cut  $+$  and fill  $-$ ) from the point  $A'$  to the point considered. In doing this shrinkage must be allowed for by considering how much embankment would actually be made by so many cubic yards of excavation of such material. For example, it will be found that 1000 cubic yards of sand or gravel, measured in place (see § 97), will make about 920 cubic yards of embankment; therefore all cuttings in sand or gravel should be discounted in about this proportion. Excavations in rock should be increased in the proper ratio. In short, all excavations should be valued according to the amount of *settled* embankment that could be made from them. The computations may be made systematically as shown in the tabular form. Place

in the first column a list of the stations; in the second column, the number of cubic yards of cut or fill between each station and the preceding station; in the third and fourth columns, the kind of material and the proper shrinkage factor; in the fifth column, a repetition of the quantities in cubic yards, except that the excavations are diminished (or increased, in the case of rock) to the number of cubic yards of settled embankment which may be made from them. In the sixth column, place the *algebraic sum* of the quantities in the fifth column (calling cuts + and fills -) from the starting-point to the station considered. These algebraic sums at each station will be the ordinates, drawn to some scale, of the mass curve. The scale to be used will depend somewhat on whether the work is heavy or light, but for ordinary cases a scale of 5000 cubic yards per inch may be used. Drawing these ordinates to scale, a curve *A, B, . . . G* may be obtained by joining the extremities of the ordinates.

| Sta.    | Yards { cut +<br>fill - | Material.   | Shrinkage<br>factor. | Yards, reduced<br>for shrinkage. | Ordinate in<br>mass curve. |
|---------|-------------------------|-------------|----------------------|----------------------------------|----------------------------|
| 46 + 70 | .....                   | .....       | .....                | .....                            | 0                          |
| 47      | + 195                   | Clayey soil | -10 per cent         | + 175                            | + 175                      |
| 48      | + 1792                  | " "         | -10 "                | + 1613                           | + 1788                     |
| + 60    | + 614                   | " "         | -10 "                | + 553                            | + 2341                     |
| 49      | - 143                   | .....       | .....                | - 143                            | + 2198                     |
| 50      | - 906                   | .....       | .....                | - 906                            | + 1292                     |
| 51      | - 1985                  | .....       | .....                | - 1985                           | - 693                      |
| 52      | - 1721                  | .....       | .....                | - 1721                           | - 2414                     |
| + 30    | - 112                   | .....       | .....                | - 112                            | - 2526                     |
| 53      | + 177                   | Hard rock   | +60 per cent         | + 283                            | - 2243                     |
| + 70    | + 180                   | " "         | +60 "                | + 289                            | - 1954                     |
| 54      | - 52                    | .....       | .....                | - 52                             | - 2006                     |
| + 42    | - 71                    | .....       | .....                | - 71                             | - 2077                     |
| 55      | + 276                   | Clayey soil | -10 per cent         | + 249                            | - 1828                     |
| 56      | + 1242                  | " "         | -10 "                | + 1118                           | - 710                      |
| 57      | + 1302                  | " "         | -10 "                | + 1172                           | + 462                      |

### 101. Properties of the mass curve.

1. The curve will be rising while over cuts and falling while over fills.

2. A tangent to the curve will be horizontal (as at *B, D, E, F, and G*) when passing from cut to fill or from fill to cut.

3. When the curve is *below* the “zero line” it shows that material must be drawn *backward* (to the left); and *vice versa*, when the curve is *above* the zero line it shows that material must be drawn *forward* (to the right).

4. When the curve crosses the zero line (as at  $A$  and  $C$ ) it shows (in this instance) that the cut between  $A'$  and  $B'$  will just provide the material required for the fill between  $B'$  and  $C'$ , and that no material should be hauled past  $C'$ , or, in general, past any intersection of the mass curve and the zero line.

5. If any horizontal line be drawn (as  $ab$ ), it indicates that the cut and fill between  $a'$  and  $b'$  will just balance.

6. When the center of gravity of a given volume of material is to be moved a given distance, it makes no difference (at least theoretically) how far each individual load may be hauled or how any individual load may be disposed of. The summation of the products of each load times the distance hauled will be a constant, whatever the method, and will equal the total volume times the movement of the center of gravity. The *average haul*, which is the movement of the center of gravity, will therefore equal the summation of these products divided by the total volume. If we draw two horizontal parallel lines at an infinitesimal distance  $dx$  apart, as at  $ab$ , the small increment of cut  $dx$  at  $a'$  will fill the corresponding increment of fill at  $b'$ , and this material must be hauled the distance  $ab$ . Therefore the product of  $ab$  and  $dx$ , which is the product of distance times volume, is represented by the area of the infinitesimal rectangle at  $ab$ , and the total area  $ABC$  represents the summation of volume times distance for all the earth movement between  $A'$  and  $C'$ . This summation of products divided by the total volume gives the average haul.

7. The horizontal line, tangent at  $E$  and cutting the curve at  $e, f$ , and  $g$ , shows that the cut and fill between  $e'$  and  $E'$  will just balance, and that a *possible* method of hauling (whether desirable or not) would be to “borrow” earth for the fill between  $C'$  and  $e'$ , use the material between  $D'$  and  $E'$  for the

fill between  $e'$  and  $D'$ , and similarly balance cut and fill between  $E'$  and  $f'$  and also between  $f'$  and  $g'$ .

8. Similarly the horizontal line  $hklm$  may be drawn cutting the curve, which will show another *possible* method of hauling. According to this plan, the fill between  $C'$  and  $h'$  would be made by borrowing; the cut and fill between  $h'$  and  $k'$  would balance; also that between  $k'$  and  $l'$  and between  $l'$  and  $m'$ . Since the area  $ehDkE$  represents the measure of haul for the earth between  $e'$  and  $E'$ , and the other areas measure the corresponding hauls similarly, it is evident that the sum of the areas  $ehDkE$  and  $ElFmf$ , which is the measure of haul of all the material between  $e'$  and  $f'$ , is largely in excess of the sum of the areas  $hDk$ ,  $kEl$ , and  $lFm$ , plus the somewhat uncertain measures of haul due to borrowing material for  $e'h'$  and wasting the material between  $m'$  and  $f'$ . Therefore to make the measure of haul a minimum a line should be drawn which will make the sum of the areas between it and the mass curve a minimum. Of course this is not necessarily the cheapest plan, as it implies more or less borrowing and wasting of material, which *may* cost more than the amount saved in haul. The comparison of the two methods is quite simple, however. Since the amount of fill between  $e'$  and  $h'$  is represented by the *difference* of the ordinates at  $e$  and  $h$ , and similarly for  $m'$  and  $f'$ , it follows that the amount to be borrowed between  $e'$  and  $h'$  will exactly equal the amount wasted between  $m'$  and  $f'$ . By the first of the above methods the haul is excessive, but is definitely known from the mass diagram, and all of the material is utilized; by the second method the haul is reduced to about one-half, but there is a known quantity in cubic yards wasted at one place and the same quantity borrowed at another. The length of haul necessary for the borrowed material would need to be ascertained; also the haul necessary to waste the other material at a place where it would be unobjectionable. Frequently this is best done by widening an embankment beyond its necessary width. The computation of the relative cost of the above methods will be discussed later (§ 116).

9. Suppose that it were deemed best, after drawing the mass curve, to introduce a trestle between  $s'$  and  $v'$ , thus saving an amount in fill equal to  $tv$ . If such had been the original design, the mass curve would have been a straight horizontal line between  $s$  and  $t$  and would continue as a curve which would be at all points a distance  $tv$  above the curve  $vFmzfGg$ . If the line  $Ef$  is to be used as a zero line, its intersection with the new curve at  $x$  will show that the material between  $E'$  and  $z'$  will just balance if the trestle is used, and that the amount of haul will be measured by the area between the line  $Ex$  and the broken line  $Estx$ . The same computed result may be obtained without drawing the auxiliary curve  $txn \dots$  by drawing the horizontal line  $zy$  at a distance  $xz (= tv)$  below  $Ex$ . The amount of the haul can then be obtained by adding the triangular area between  $Es$  and the horizontal line  $Ex$ , the rectangle between  $st$  and  $Ex$ , and the irregular area between  $vFz$  and  $y \dots z$  (which last is evidently equal to the area between  $tx$  and  $E \dots x$ ). The disposal of the material at the right of  $z'$  would then be governed by the indications of the profile and mass diagram which would be found at the right of  $g'$ . In fact it is difficult to decide with the best of judgment as to the proper disposal of material without having a mass diagram extending to a considerable distance each side of that part of the road under immediate consideration.

**102. Area of the mass curve.** The area may be computed most readily by means of a planimeter, which is capable with reasonable care of measuring such areas with as great accuracy as is necessary for this work. If no such instrument is obtainable, the area may be obtained by an application of "Simpson's rule." The ordinates will usually be spaced 100 feet apart. Select an *even* number of such spaces, leaving, if necessary, one or more triangles or trapezoids at the ends for separate and independent computation. Let  $y_0 \dots y_n$  be the ordinates, i.e., the number of cubic yards at each station of the mass curve, or the figures of "column six" referred to in § 100. Let the uniform distance between ordinates ( $= 100$  feet) be called 1, i.e.,

one station. Then the units of the resulting area will be cubic yards hauled one station. Then the

$$\text{Area} = \frac{1}{2}[y_0 + 4(y_1 + y_3 + \dots y_{(n-1)}) + 2(y_2 + y_4 + \dots y_{(n-2)}) + y_n]. \quad (70)$$

When an ordinate occurs at a substation, the best plan is to ignore it at first and calculate the area as above. Then, if the difference involved is too great to be neglected, calculate the area of the triangle having the extremity of the ordinate at the substation as an apex, and the extremities of the ordinates at the adjacent stations as the ends of the base. This may be done by finding the ordinate at the substation that would be a proportional between the ordinates at the adjacent full stations. Subtract this from the real ordinate (or *vice versa*) and multiply the difference by  $\frac{1}{2} \times 1$ . An inspection will often show that the correction thus obtained would be too small to be worthy of consideration. If there is more than one substation between two full stations, the corrective area will consist of two triangles and one or more trapezoids which may be similarly computed, if necessary.

When the zero line (Fig. 62) is shifted to  $eE$ , the drop from  $AC$  (produced) to  $E$  is known in the same units, cubic yards. This constant may be subtracted from the numbers ("column 4," § 100) representing the ordinates, and will thus give, without any scaling from the diagram, the exact value of the modified ordinates.

**103. Value of the mass diagram.** The great value of the mass diagram lies in the readiness with which different plans for the disposal of material may be examined and compared. When the mass curve is once drawn, it will generally require only a shifting of the horizontal line to show the disposal of the material by any proposed method. The mass diagram also shows the extreme length of haul that will be required by any proposed method of disposal of material. This brings into consideration the "limit of profitable haul," which will be fully discussed in § 116. For the present it may be said that with each method of carrying material there is some limit beyond which the expense

of hauling will exceed the loss resulting from borrowing and wasting. With wheelbarrows and scrapers the limit of profitable haul is comparatively short, with carts and tram-cars it is much longer, while with locomotives and cars it may be several miles. If, in Fig. 62,  $eE$  or  $Ef$  exceeds the limit of profitable haul, it shows at once that some such line as  $hklm$  should be drawn and the material disposed of accordingly.

**104. Changing the grade line.** The formation of the mass curve and the resulting plans as to the disposal of material are based on the mutual relations of the grade line and the surface profile and the amounts of cut and fill which are thereby implied. If the grade line is altered, every cross-section is altered, the amount of cut and fill is altered, and the mass curve is also changed. At the farther limit of the actual change of the grade line the revised mass curve will have (in general) a different ordinate from the previous ordinate at that point. From that point on, the revised mass curve will be parallel to its former position, and the revised curve may be treated similarly to the case previously mentioned in which a trestle was introduced. Since it involves tedious calculations to determine accurately how much the volume of earthwork is altered by a change in grade line, especially through irregular country, the effect on the mass curve of a change in the grade line cannot therefore be readily determined except in an approximate way. Raising the grade line will evidently increase the fills and diminish the cuts, and *vice versa*. Therefore if the mass curve indicated, for example, either an excessively long haul or the necessity for borrowing material (implying a fill) and wasting material farther on (implying a cut), it would be *possible* to diminish the fill (and hence the amount of material to be borrowed) by lowering the grade line near that place, and diminish the cut (and hence the amount of material to be wasted) by raising the grade line at or near the place farther on. Whether the advantage thus gained would compensate for the possibly injurious effect of these changes on the grade line would require patient investigation. But the method outlined shows how the

mass curve might be used to indicate a possible change in grade line which might be demonstrated to be profitable.

**105. Limit of free haul.** It is sometimes specified in contracts for earthwork that *all* material shall be entitled to free haul up to some specified limit, say 500 or 1000 feet, and that all material drawn farther than that shall be entitled to an allowance on the *excess* of distance. It is manifestly impracticable to measure the excess for each load, as much so as to measure the actual haul of each load. The mass diagram also solves this problem very readily. Let Fig. 63 represent a pro-

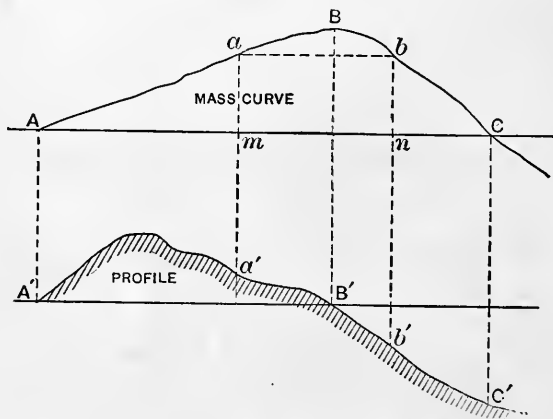


FIG. 63.

file and mass diagram of about 2000 feet of road, and suppose that 800 feet is taken as the limit of free haul. Find two points,  $a$  and  $b$ , in the mass curve *which are on the same horizontal line* and which are 800 feet apart. Project these points down to  $a'$  and  $b'$ . Then the cut and fill between  $a'$  and  $b'$  will just balance, and the cut between  $A'$  and  $a'$  will be needed for the fill between  $b'$  and  $C'$ . In the mass curve, the area between the horizontal line  $ab$  and the curve  $aBb$  represents the haulage of the material between  $a'$  and  $b'$ , which is all free. The rectangle  $abmn$  represents the haulage of the material in the cut  $A'a'$  across the 800 feet from  $a'$  to  $b'$ . This is also free. The sum of the two areas  $Aam$  and  $bnC$  represents the haulage entitled to an allowance, since it is the summation of the products of cubic yards times the *excess* of distance hauled.



If the amount of cut and fill was symmetrical about the point  $B'$ , the mass curve would be a symmetrical curve about the vertical line through  $B$ , and the two limiting lines of free haul would be placed symmetrically about  $B$  and  $B'$ . In general there is no such symmetry, and frequently the difference is considerable. The area  $aBbnm$  will be materially changed according as the two vertical lines  $am$  and  $bn$ , always 800 feet apart, are shifted to the right or left. It is easy to show that the area  $aBbnm$  is a *maximum* when  $ab$  is horizontal. The minimum value would be obtained either when  $m$  reached  $A$  or  $n$  reached  $C$ , depending on the exact form of the curve. Since the position for the minimum value is manifestly unfair, the best definite value obtainable is the maximum, which must be obtained as above described. Since  $aBbnm$  is made maximum, the remainder of the area, which is the allowance for overhaul, becomes a minimum. The areas  $Aam$  and  $bCn$  may be obtained as in § 102. If the whole area  $AaBbCA$  has been previously computed, it may be more convenient to compute the area  $aBbnm$  and subtract it from the total area.

Since the intersections of the mass curve and the "zero line" mark limits past which no material is drawn, it follows that there will be no allowance for overhaul except where the distance between consecutive intersections of the zero line and mass curve exceeds the limit of free haul.

Frequently all allowances for overhaul are disregarded; the profiles, estimates of quantities, and the required disposal of material are shown to bidding contractors, and they must then make their own allowances and bid accordingly. This method has the advantage of avoiding possible disputes as to the amount of the overhaul allowance, and is popular with railroad companies on this account. On the other hand the facility with which different plans for the disposal of material may be studied and compared by the mass-curve method facilitates the adoption of the most economical plan, and the elimination of uncertainty will frequently lead to a safe reduction of the bid, and so the method is valuable to both the railroad company and the contractor.

## ELEMENTS OF THE COST OF EARTHWORK.

(The following analysis of the cost of earthwork follows the general method given in the well-known papers published by Ellwood Morris, C.E., in the Journal of the Franklin Institute in September and October, 1841. Numerous corroborative data have been obtained from various other sources, and also figures on methods not then in vogue.)

**106. General divisions of the subject.** The variations in the cost of earthwork are caused by the greatly varying conditions under which the work is done, chief among which is character of material, method of carriage, and length of haul. Any general system of computation must therefore differentiate the total cost into such elementary items that all differences due to variations in conditions may be allowed for. The variations due to character of material will be allowed for by an estimate on loose light sandy soil, and also an estimate on the heaviest soils, such as stiff clay and hard-pan. These represent the extremes (excluding rock, which will be treated separately), and the cost of intermediate grades must be estimated by interpolating between the extreme values. The general divisions of the subject will be : \*

1. Loosening.
2. Loading.
3. Hauling.
4. Spreading.
5. Keeping roadways in order.
6. Repairs, wear, depreciation, and interest on cost of plant.
7. Superintendence and incidentals.
8. Contractor's profit.

By making the estimates on the basis of \$1 per day for the cost of common labor, it is a simple matter to revise the estimates according to the local price of labor by multiplying the final estimate of cost by the price of labor in dollars per day.

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\* Trautwine.

**107. Item 1. LOOSENING. (a) Ploughs.** Very light sandy soils can frequently be shovelled without any previous loosening, but it is generally economical, even with very light material, to use a plough. Morris quotes, as the results of experiments, that a three-horse plough would loosen from 250 to 800 cubic yards of earth per day, which at a valuation of \$5 per day would make the cost per yard vary from 2 cents to 0.6 cent. Trautwine estimates the cost on the basis of two men handling a two-horse plough at a total cost of \$3.87 per day, being \$1 each for the men, 75 c. for each horse, and an allowance of 37 c. for the plough, harness, etc. From 200 to 600 cubic yards is estimated as a fair day's work, which makes a cost of 1.9 c. to 0.65 c. per yard, which is substantially the same estimate as above. Extremely heavy soils have sometimes been loosened by means of special ploughs operated by traction-engines.

**(b) Picks.** When picks are used for loosening the earth, as is frequently necessary and as is often done when ploughing would perhaps be really cheaper, an estimate\* for a fair day's work is from 14 to 60 cubic yards, the 14 yards being the estimate for stiff clay or cemented gravel, and the 60 yards the estimate for the lightest soil that would require loosening. At \$1 per day this means about 7 c. to 1.7 c. per cubic yard, which is about three times the cost of ploughing. Five feet of the face is estimated† as the least width along the face of a bank that should be allowed to enable each laborer to work with freedom and hence economically.

**(c) Blasting.** Although some of the softer shaly rocks may be loosened with a pick for about 15 to 20 c. per yard, yet rock in general, frozen earth, and sometimes even compact clay is most economically loosened by blasting. The subject of blasting will be taken up later, §§ 117–123.

**(d) Steam-shovels.** The items of loosening and loading merge together with this method, which will therefore be treated in the next section.

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\* Trautwine.

† Hurst.

**108. Item 2. LOADING. (a) Hand-shovelling.** Much depends on proper management, so that the shovellers need not wait unduly either for material or carts. With the best of management considerable time is thus lost, and yet the intervals of rest need not be considered as entirely lost, as it enables the men to work, while actually loading, at a rate which it would be physically impossible for them to maintain for ten hours. Seven shovellers are sometimes allowed for each cart; otherwise there should be five, two on each side and one in the rear. Economy requires that the number of loads per cart per day should be made as large as possible, and it is therefore wise to employ as many shovellers as can work without mutual interference and without wasting time in waiting for material or carts. The figures obtainable for the cost of this item are unsatisfactory on account of their large disagreements. The following are quoted as the number of cubic yards that can be loaded into a cart by an average laborer in a working day of ten hours, the lower estimate referring to heavy soils, and the higher to light sandy soils: 10 to 14 cubic yards (Morris), 12 to 17 cubic yards (Haskoll), 18 to 22 cubic yards (Hurst), 17 to 24 cubic yards (Trautwine), 16 to 48 cubic yards (Ancelin). As these estimates are generally claimed to be based on actual experience, the discrepancies are probably due to differences of management. If the average of 15 to 25 cubic yards be accepted, it means, on the basis of \$1 per day, 6.7 c. to 4 c. per cubic yard. These estimates apply only to earth. *Rockwork* costs more, not only because it is harder to handle, but because a cubic yard of solid rock, measured in place, occupies about 1.8 cubic yards when broken up, while a cubic yard of earth will occupy about 1.2 cubic yards. Rockwork will therefore require about 50% more loads to haul a given volume, *measured in place*, than will the same nominal volume of earthwork. The above authorities give estimates for loading rock varying from 6.9 c. to 10 c. per cubic yard. The above estimates apply only to the loading of carts or cars with shovels or by hand (loading masses of rock). The

cost of loading wheelbarrows and the cost of scraper work will be treated under the item of hauling.

(b) **Steam-shovels.\*** Whenever the magnitude of the work will warrant it there is great economy in the use of steam-shovels. These have a "bucket" or "dipper" on the end of a long beam, the bucket having a capacity varying from  $\frac{1}{2}$  to  $2\frac{1}{2}$  cubic yards. Steam-shovels handle all kinds of material from the softest earth to shale rock, earthy material containing large boulders, tree-stumps, etc. The capacity of the larger sizes is about 3000 cubic yards in 10 hours. They perform all the work of loosening and loading. Their economical working requires that the material shall be hauled away as fast as it can be loaded, which usually means that cars on a track, hauled by horses or mules, or still better by a locomotive, shall be used. The expenses for a steam-shovel, costing about \$5000, will average about \$1000 per month. Of this the engineer will get \$100; the fireman \$50; the cranesman \$90; repairs perhaps \$250 to \$300; coal, from 15 to 25 tons, cost very variable on account of expensive hauling; water, a very uncertain amount, sometimes costing \$100 per month; about five laborers and a foreman, the laborers getting \$1.25 per day and the foreman \$2.50 per day, which will amount to \$227.50 per month. This gang of laborers is employed in shifting the shovel when necessary, taking up and relaying tracks for the cars, shifting loaded and unloaded cars, etc. In shovelling through a deep cut, the shovel is operated so as to undermine the upper parts of the cut, which then fall down within reach of the shovel, thus increasing the amount of material handled for each new position of the shovel. If the material is too tough to fall down by its own weight, it is sometimes found economical to employ a gang of men to loosen it or even blast it rather than shift the shovel so frequently. Non-condensing engines of 50 horse-power use so much water that the cost of water-supply becomes a serious

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\* For a thorough treatment of the capabilities, cost, and management of steam-shovels the reader is referred to "Steam-shovels and Steam-shovel Work," by E. A. Hermann. D. Van Nostrand Co., New York.

matter if water is not readily obtainable. The lack of water facilities will often justify the construction of a pipe line from some distant source and the installation of a steam-pump. Hence the seemingly large estimate of \$100 per month for water-supply, although under favorable circumstances the cost may almost vanish. The larger steam-shovels will consume nearly a ton of coal per day of 10 hours. The expense of hauling this coal from the nearest railroad or canal to the location of the cut is often a very serious item of expense and may easily double the cost per ton. Some steam-shovels have been constructed to be operated by electricity obtained from a plant perhaps several miles away. Such a method is especially advantageous when fuel and water are difficult to obtain.

**109. Item 3. HAULING.** The cost of hauling depends on the number of round trips per day that can be made by each vehicle employed. As the cost of each vehicle is practically the same whether it makes many trips or few, it becomes important that the number of trips should be made a maximum, and to that end there should be as little delay as possible in loading and unloading. Therefore devices for facilitating the passage of the vehicles have a real money value.

(a) **Carts.** The average speed of a horse hauling a two-wheeled cart has been found to be 200 feet per minute, a little slower when hauling the load and a little faster when returning empty. This figure has been repeatedly verified. It means an allowance of one minute for each 100 feet (or "station") of "lead—the lead being the distance the earth is hauled." The time lost in loading, dumping, waiting to load, etc., has been found to average 4 minutes per load. Representing the number of stations (100 feet) of lead by  $s$ , the number of loads handled in 10 hours (600 minutes) would be  $600 \div (s + 4)$ . The number of loads per cubic yard, measured in the bank, is differentiated by Morris into three classes, viz.:

3 loads per cubic yard in descending hauling;  
 $3\frac{1}{2}$  " " " " " level hauling; and  
 4 " " " " " ascending hauling.

Attempts have been made to estimate the effect of the grade of the roadway by a theoretical consideration of its rate, and of the comparative strength of a horse on a level and on various grades. While such computations are always practicable on a railway (even on a temporary construction track), the traction on a temporary earth roadway is always very large and so very variable that any refinements are useless. On railroad earthwork the hauling is generally nearly level or it is *descending*—forming embankments on low ground with material from cuts in high ground. The only common exception occurs when an embankment is formed from borrow-pits on low ground. One method of allowing for ascending grade is to add to the horizontal distance 14 times the difference of elevation for work with carts and 24 times the difference of elevation for work with wheelbarrows, and use that as the lead. For example, using carts, if the lead is 300 feet and there is a difference of elevation of 20 feet, the lead would be considered equivalent to  $300 + (14 \times 20) = 580$  feet on a level.

Trautwine assumes the average load for all classes of work to be  $\frac{1}{3}$  cubic yard, which figure is justified by large experience. Using one figure for all classes of work simplifies the calculations and gives the number of cubic yards carried per day of 10 hours equal to  $\frac{600}{3(s + 4)}$ . Dividing the cost of a cart per day by the number of cubic yards carried gives the cost of hauling per yard. In computing the cost of a cart per day, Trautwine refers to the practice of having one driver manage four carts, thus making a charge of 25 c. per day for each cart for the driver. 75 c. is allowed for the horse, which is supposed to be the total cost, including that for Sundays and rainy days. 25 c. more is allowed for the cart, harness, repairs, etc., thus making a total cost of \$1.25 per day. Some contractors employ a greater number of drivers and expect each to assist in loading. There is found to be no saving in total cost per yard, while the chances of loafing are perhaps greater. Morris instances five actual cases in which the cost of the cart (reduced to the basis of

\$1 per day for labor) varied from \$1.37 to \$1.48. The items of these costs were not given.

Since the time required for loading loose rock is greater than for earthwork, less loads will be hauled per day. The time allowance for loading, etc., is estimated by Trautwine as 6 minutes instead of 4 as for earth. Considering the great expansion of rock when broken up (see § 97), one cubic yard of solid rock, measured in place, would furnish the equivalent of five loads of earthwork of  $\frac{1}{5}$  cubic yard. Therefore, on the basis of five loads per cubic yard, the number of cubic yards handled per day per cart would be  $\frac{600}{5(s+6)}$ .

$$\text{Cost per yard in cents} = \frac{125 \times 5(s+6)}{600}. \quad (71)$$

(b) **Wagons.** For longer leads (i.e., from  $\frac{1}{3}$  to  $\frac{2}{3}$  of a mile) wagons drawn by two horses have been found most economical. The wagons have bottoms of loose thick narrow boards and are unloaded very easily and quickly by lifting the individual boards and breaking up the continuity of the bottom, thus depositing the load directly underneath the wagon. The capacity is about one cubic yard. The cost may be estimated on the same principles as that for carts.

(c) **Wheelbarrows.** According to Trautwine, the speed of moving wheelbarrows may be considered the same as for carts, 200 feet per minute; the time spent in loading and dumping is  $1\frac{1}{4}$  minutes, and in addition about  $\frac{1}{10}$  of the time is wasted in short rests, adjusting the wheeling plants, etc. On the basis of \$1 per day for labor, an allowance of 5 c. for the barrow, and 14 loads per cubic yard, the cost of hauling per cubic yard (computed on the same principles as above) will be

$$\frac{105 \times 14(s+1.25)}{600 \times 0.9} \cdot \cdot \cdot \cdot \cdot (72)$$



For rockwork the number of loads per cubic yard is estimated as 24, and the time spent in loading, etc., estimated at 1.6 minutes instead of 1.25 minutes, which makes the estimate

$$\text{Cost per cubic yard} = \frac{105 \times 24(s + 1.6)}{600 \times 0.9}. \quad (73)$$

(d) **Scrapers.\*** Scrapers, or scoops, are especially useful in canal work, and also for railroad work when a low embankment is to be formed from borrow-pits at the sides, when the distance does not exceed 100 feet, nor the vertical height 15 feet. The slope should not exceed 1.5 to 1. Under these conditions scraper work is cheaper than any other method. Scooping may be done all in one direction, in which case two half-turns are made for each load moved; or it may be done in both directions (from both sides on to a bank, or, in canal work, from the center to each bank), in which case one load is hauled to each half-turn. The capacity of the scoops (the “drag” variety) is  $\frac{1}{10}$  cubic yard; the time lost in loading, unloading, and all other ways per load (except in turning) will average  $\frac{2}{3}$  minute; the time lost in each half-turn (semi-circle) is  $\frac{1}{3}$  minute; the speed of the horses may be estimated as 70 feet of *lead* per minute, the lead being here considered as the *sum* of the vertical and horizontal distances, and the estimate including the time of going and returning. If  $a$  represents the sum of the horizontal and vertical distances, the number of cubic yards handled per day of 10 hours by “side-scooping” will be

$$0.1 \left( \frac{600}{\frac{a}{70} + 1\frac{1}{3}} \right), \text{ which equals } \frac{4200}{a + 93\frac{1}{3}}.$$

For “double-scooping” the formula becomes

$$0.1 \left( \frac{600}{\frac{a}{70} + 1} \right), \text{ which equals } \frac{4200}{a + 70}.$$

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\* Condensed from Journ. Franklin Inst., Oct. 1841, by Morris.

Dividing the cost of a scraper per day (estimated at \$2.75) by the number of yards handled per day gives the average cost per yard.

Except in very loose sandy soil it is best to plough the earth first, which will cost *about* 1 c. per yard. (See § 107.) Drag-scrapers are now made chiefly of steel, and their capacity is more nearly 0.15 cubic yard. Wheeled scrapers, having a capacity of about 0.5 cubic yard, are frequently used with even greater economy and for greater distances, as they are cheaper than carts up to 250 or 300 feet of lead. Both drag- and wheel-scrapers are best operated in gangs of perhaps 10, using extra or "snap" teams to help load, and a few extra men to help in loading and unloading. The average cost of one scraper per day may thus be easily calculated and the average number of cubic yards handled per day computed as above, from which the cost per yard may be estimated.

(e) **Cars and horses.** The items of cost by this method are (a) charge for horses employed, (b) charge for men employed strictly in hauling, (c) charge for shifting rails when necessary, (d) repairs, depreciation, and interest on cost of cars and track. Part of this cost should strictly be classified under items 5 and 6, mentioned in § 106, but it is perhaps more convenient to estimate them as follows.

The traction of a car on rails is so very small and constant that grade resistance constitutes a very large part of the total resistance if the grade is 1% or more. For all ordinary grades it is sufficiently accurate to say that the grade resistance is to the gross weight as the rise is to the distance. If the distance is supposed to be measured along the slope, the proportion is strictly true; i.e., on a 1% grade the grade resistance is 1 lb. per 100 of weight or 20 lbs. per ton. If the resistance on a level at the usual velocity is  $\frac{1}{120}$ , a grade of 1:120 (0.83%) will exactly double it. If the material is hauled *down* a grade of 1:120, the cars will run by gravity after being started. The work of hauling will then consist practically of hauling the empty cars up the grade. The grade resistance depends only

on the rate of grade and the weight, but the tractive resistance will be *greater per ton of weight* for the unloaded than for the loaded cars. The tractive power of a horse is less on a grade than on a level, not only because the horse raises his own weight in addition to the load, but is anatomically less capable of pulling on a grade than on a level. In general it will be possible to plan the work so that loaded cars need not be hauled *up* a grade, unless an embankment is to be formed from a low borrow-pit, in which case another method would probably be advisable. These computations are chiefly utilized in designing the method of work—the proportion of horses to cars. An example may be quoted from English practice (Hurst), in which the cars had a capacity of  $3\frac{1}{2}$  cubic yards, weighing 30 cwt. empty. Two horses took five “*wagons*”  $\frac{3}{4}$  of a mile on a level railroad and made 15 journeys per day of 10 hours, i.e., they handled 250 yards per day. In addition to those on the “*straight road*,” another horse was employed to make up the train of loaded wagons. With a short lead the straight-road horses were employed for this purpose. In the above example the number of men required to handle these cars, shift the tracks, etc., is not given, and so the exact cost of the above work cannot be analyzed. It may be noticed that the two horses travelled  $22\frac{1}{2}$  miles per day, drawing in one direction a load, including the weight of the cars, of about 57,300 lbs. or 28.65 net tons. Allowing  $\frac{1}{1\frac{1}{2}0}$  as the necessary tractive force, it would require a pull of 477.5 lbs., or 239 lbs. for each horse. With a velocity of 220 feet per minute this would amount to  $1\frac{1}{2}$  horse-power per horse, exerted for only a short time, however, and allowing considerable time for rest and for drawing only the empty cars. The cars generally used in this country have a capacity of  $1\frac{1}{2}$  cubic yards and cost about \$65 apiece. Besides the shovellers and dumping-gang, several men and a foreman will be required to keep the track in order and to make the constant shifts that are necessary. Two trains are generally used, one of which is loaded while the other is run to the dump. Some passing-place is necessary, but this is generally

provided by having a switch at the cut and running the trains on each track alternately. This insures a train of cars always at the cut to keep the shovellers employed. The cost of hauling per cubic yard can only be computed when the number of laborers, cars, and horses employed are known, and these will depend on the lead, on the character of the excavation, on the grade, if any, etc., and must be so proportioned that the shovellers need not wait for cars to fill, nor the dumping-gang for material to handle, nor the horses and drivers for cars to haul. Much skill is necessary to keep a large force in smooth running order.

(f) **Cars and locomotives.** 30-lb. rails are the lightest that should be used for this work, and 35- or 40-lb. rails are better. One or two narrow-gauge locomotives (depending on the length of haul), costing about \$2500 each, will be necessary to handle two trains of about 15 cars each, the cars having a capacity of about 2 cubic yards and costing about \$100 each. Some cars can be obtained as low as \$70. A force of about five men and a foreman will be required to shift the tracks. The track-shifters, except the foreman, may be common laborers. The dumping-gang will require about seven men. Even when the material is all taken *down* grade the grades may be too steep for the safe hauling of loaded cars down the grade, or for hauling empty cars up the grade. Under such circumstances temporary trestles are necessary to reduce the grade. When these are used, the uprights and bracing are left in the embankment—only the stringers being removed. This is largely a necessity, but is partially compensated by the fact that the trestle forms a core to the embankment which prevents lateral shifting during settlement. The average speed of the trains may be taken as 10 miles per hour or 5 miles of lead per hour. The time lost in loading and unloading is estimated (Trautwine) as 9 minutes or .15 of an hour. The number of trips per day of 10 hours will equal  $\frac{10}{\frac{1}{3} \text{ (miles of lead)} + .15}$  or  $\frac{50}{\text{(miles of lead)} + .75}$ . Of course this quotient *must* be a whole number. Knowing the

number of trains and their capacity, the total number of cubic yards handled is known, which, divided into the total daily cost of the trains, will give the cost of hauling per yard. The daily cost of a train will include

- (a) Wages of engineer, who frequently fires his own engine;
- (b) Fuel, about  $\frac{1}{4}$  to 1 ton of bituminous coal, depending on work done;
- (c) Water, a very variable item, frequently costing \$3 to \$5 per day;
- (d) Repairs, variable, frequently at rate of 50 to 60% per year;
- (e) Interest on cost and depreciation, 16 to 40%.

To these must be added, to obtain the total cost of the haul,

- (f) Wages of the gang employed in shifting track.

#### 110. Choice of method of haul dependent on distance.

In light side-hill work in which material need not be moved more than 12 or 15 feet, i.e., moved *laterally* across the road-bed, the earth may be moved most cheaply by mere shovelling. Beyond 12 feet scrapers are more economical. At about 100 feet drag-scrapers and wheelbarrows are equally economical. Between 100 and 200 feet wheelbarrows are generally cheaper than either carts or drag-scrapers, but wheeled scrapers are always cheaper than wheelbarrows. Beyond 500 feet two-wheeled carts become the most economical up to about 1700 feet; then four-wheeled wagons become more economical up to 3500 feet. Beyond this cars on rails, drawn by horses or by locomotives, become cheaper. The economy of cars on rails becomes evident for distances as small as 300 feet provided the volume of the excavation will justify the outlay. Locomotives will always be cheaper than horses and mules providing the work to be done is of sufficient magnitude to justify the purchase of the necessary plant and risk the loss in selling the plant ultimately as second-hand equipment, or keeping the plant on hand and idle for an indefinite period waiting for other work. Horses will not be economical for distances much over a mile. For greater distances locomotives are more economical, but the

question of "limit of profitable haul" (§ 116) must be closely studied, as the circumstances are certainly not common when it is advisable to haul material much over a mile.

**111. Item 4. SPREADING.** The cost of spreading varies with the method employed in dumping the load. When the earth is tipped over the edge of an embankment there is little if any necessary work. Trautwine allows about  $\frac{1}{4}$  c. per cubic yard for keeping the dumping-places clear and in order. This would represent the wages of one man at \$1 per day attending to the unloading of 1200 two-wheeled carts each carrying  $\frac{1}{3}$  cubic yard. 1200 carts in 10 hours would mean an average of two per minute, which implies more rapid and efficient work than may be depended on. The allowance is probably too small. When the material is dumped in layers some levelling is required, for which Trautwine allows 50 to 100 cubic yards as a fair day's work, costing from 1 to 2 cents per cubic yard. The cost of spreading will not ordinarily exceed this and is frequently nothing—all depending on the method of unloading. It should be noted that Mr. Morris's examples and computations (Jour. Franklin Inst., Sept. 1841) disregard altogether any special charge for this item.

**112. Item 5. KEEPING ROADWAYS IN ORDER.** This feature is important as a measure of true economy, whatever the system of transportation, but it is often neglected. A petty saving in such matters will cost many times as much in increased labor in hauling and loss of time. With some methods of haul the cost is best combined with that of other items.

(a) **Wheelbarrows.** Wheelbarrows should generally be run on planks laid on the ground. The adjusting and shifting of these planks is done by the wheelers, and the time for it is allowed for in the 10% allowance for "short rests, adjusting the wheeling plank, etc." The actual cost of the planks must be added, but it would evidently be a very small addition per cubic yard in a large contract. When the wheelbarrows are run on planks placed on "horses" or on trestles the cost is very appreciable; but the method is frequently used with great economy. The

variations in the requirements render any general estimate of such cost impracticable.

(b) **Carts and wagons.** The cost of keeping roadways in order for carts and wagons is sometimes estimated merely as so much per cubic yard, but it is evidently a function of the *lead*. The work consists in draining off puddles, filling up ruts, picking up loose stones that may have fallen off the loads, and in general doing everything that will reduce the traction as much as possible. Temporary inclines, built to avoid excessive grade at some one point, are often measures of true economy. Trautwine suggests  $\frac{1}{10}$  c. per cubic yard per 100 feet of lead for earthwork and  $\frac{2}{10}$  c. for rockwork, as an estimate for this item when carts are used.

(c) **Cars.** When cars are used a shifting-gang, consisting of a foreman and several men (say five), are constantly employed in shifting the track so that the material may be loaded and unloaded where it is desired. The average cost of this item may be estimated by dividing the total daily cost of this gang by the number of cubic yards handled in one day.

**113. Item 6. REPAIRS, WEAR, DEPRECIATION, AND INTEREST ON COST OF PLANT.** The amount of this item evidently depends upon the character of the soil—the harder the soil the worse the wear and depreciation. The *interest on cost* depends on the current borrowing value of money. The estimate for this item has already been included in the allowances for horses, carts, ploughs, harness, wheelbarrows, steam-shovels, etc. Trautwine estimates  $\frac{1}{4}$  c. per cubic yard for picks and shovels. Depreciation is generally a large percentage of the cost of earth-working tools, the life of all being limited to a few years, and of many tools to a few months.

**114. Item 7. SUPERINTENDENCE AND INCIDENTALS.** The incidentals include water-carriers, trimming cuts to grade, digging the side ditches, trimming up the sides of borrow-pits to prevent their becoming unsightly, etc. These last operations yield but little earth and cost far more than the price paid per cubic yard. Morris allows 1 c. per cubic yard for this item; Trautwine

allows  $1\frac{3}{4}$  to 2 c. for it; while others combine items 6 and 7 and call them 5% of the total cost, which method has the merit of making the cost of items 6 and 7 a function of the character of soil and length of lead.

**115. Item 8. CONTRACTOR'S PROFIT.** This is usually estimated at from 6 to 15%, according to the sharpness of the competition and the possible uncertainty as to true cost owing to unfavorable circumstances. The contractor's real profit may vary considerably from this. He often pays clerks, boards and lodges the laborers in shanties built for the purpose, or keeps a supply-store, and has various other items both of profit and expense. His profit is largely dependent on skill in so handling the men that all can work effectively without interference or delays in waiting for others. An unusual season of bad weather will often affect the cost very seriously. It is a common occurrence to find that two contractors may be working on the same kind of material and under precisely similar conditions and at the same price, and yet one may be making money and the other losing it—all on account of difference of management.

**116. Limit of profitable haul.** As intimated in §§ 103 and 110, there is with every method of haul a limit of distance beyond which the expense for excessive hauling will exceed the loss resulting from borrowing and wasting. This distance is somewhat dependent on local conditions, thus requiring an independent solution for each particular case, but the general principles involved will be about as follows: Assume that it has been determined, as in Fig. 62, that the cut and fill will exactly balance between two points, as between  $e$  and  $x$ , assuming that, as indicated in § 101 (9), a trestle has been introduced between  $s$  and  $t$ , thus altering the mass curve to *Estæn* . . . Since there is a balance between  $A'$  and  $C'$ , the material for the fill between  $C'$  and  $e'$  must be obtained either by "borrowing" in the immediate neighborhood or by transportation from the excavation between  $z'$  and  $n'$ . If cut and fill have been approximately balanced in the selection of grade line, as is ordinarily done, borrowing material for the fill  $C'e'$  implies a wastage of material



at the cut  $z'n'$ . To compare the two methods, we may place against the plan of borrowing and wasting, (a) cost, if any, of extra right of way that may be needed from which to obtain earth for the fill  $C'e'$ ; (b) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the borrow-pit and of the fill, and the other expenses incidental to borrowing  $M$  cubic yards for the fill  $C'e'$ ; (c) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the cut  $z'n'$  and of the spoil-bank, and the other expenses incidental to wasting  $M$  cubic yards at the cut  $z'n'$ ; (d) cost, if any, of land needed for the spoil-bank. The cost of the other plan will be the cost of loosening, loading, hauling (the hauling being represented by the trapezoidal figure  $Cean$ ), and the other expenses incidental to making the fill  $C'e'$  with the material from the cut  $z'n'$ , the amount of material being  $M$  cubic yards, which is represented in the figure by the vertical ordinate from  $e$  to the line  $Cn$ . The difference between these costs will be the cost, if any, of land for borrow-pit and spoil-bank *plus* the cost of loosening, loading, etc. (except hauling and roadways) of  $M$  cubic yards, *minus* the difference in cost of the excessive haul from  $Ce$  to  $an$  and the comparatively short hauls from borrow-pit and to spoil-bank.

As an illustration, taking some of the estimates previously given for operating with average material, the cost of all items, except hauling and roadways, would be about as follows: loosening, with plough, 1.2 c., loading 5.0 c., spreading 1.5 c., wear, depreciation, etc., .25 c., superintendence, etc., 1.5 c.; total 8.95 c. Suppose that the haul for both borrowing and wasting averages 100 feet or 1 station. Then the cost of haul per yard, using carts, would be (§ 109, a)  $[125 \times 3(1 + 4)] \div 600 = 3.125$  c. The cost of roadways would be about 0.1 c. per yard, making a total of 3.225 c. per cubic yard. Assume  $M = 10000$  cubic yards and the area  $Cean = 180000$  yards-stations or the equivalent of 10000 yards hauled 1800 feet. This haul would cost  $[125 \times 3(18 + 4)] \div 600 = 13.75$  c. per cubic yard. The cost of roadways will be  $18 \times .1$  or 1.8 c.,

making a total of 15.55 c. for hauling and roadways. The difference of cost of hauling and roadways will be 15.55 —  $(2 \times 3.225) = 9.10$  c. per yard or \$910 for the 10000 yards. Offsetting this is the cost of loosening, etc., 10000 yards, at 8.95 c., costing \$895. These figures may be better compared as follows:

|                           |   |                                                    |                       |
|---------------------------|---|----------------------------------------------------|-----------------------|
| LONG HAUL.                | { | Loosening, etc., 10000 yards, @ 8.95 c.            | \$ 895.               |
|                           |   | Hauling, " 10000 " @ 15.55 c.                      | 1555.                 |
|                           |   |                                                    | <hr/> \$2450. <hr/>   |
| BORROWING<br>AND WASTING. | { | Loosening, etc., 10000 yards (borrowed), @ 8.95 c. | \$895.                |
|                           |   | " " 10000 " (wasted), @ 8.95 c.                    | 895.                  |
|                           |   | Hauling, etc., 10000 " (borrowed), @ 3.225 c.      | 322.50                |
|                           |   | " " 10000 " (wasted), @ 3.225 c.                   | 322.50                |
|                           |   |                                                    | <hr/> \$2435.00 <hr/> |

These costs are practically balanced, but no allowance has been made for right of way. If any considerable amount had to be paid for that, it would decide this particular case in favor of the long haul. This shows that *under these conditions* 1800 feet is *about* the limit of profitable haul, the land costing nothing extra.

#### BLASTING.

**117. Explosives.** The effect of blasting is due to the extremely rapid expansion of a gas which is developed by the decomposition of a very small amount of solid matter. Blasting compounds may be divided into two general classes, (a) slow-burning and (b) detonating. Gunpowder is a type of the slow-burning compounds. These are generally ignited by heat; the ignition proceeds from grain to grain; the heat and pressure produced are comparatively low. Nitro-glycerine is a type of the detonating compounds. They are exploded by a shock which *instantaneously* explodes the whole mass. The heat and pressure developed are far in excess of that produced by the explosion of powder. Nitro-glycerine is so easily exploded that it is very dangerous to handle. It was discovered that if the nitro-glycerine was absorbed by a spongy material like infu-

social earth, it was much less liable to explode, while its power when actually exploded was practically equal to that of the amount of pure nitro-glycerine contained in the dynamite, which is the name given to the mixture of nitro-glycerine and infusorial earth. Nitro-glycerine is expensive; many other explosive chemical compounds which properly belong to the slow-burning class are comparatively cheap. It has been conclusively demonstrated that a mixture of nitro-glycerine and some of the cheaper chemicals has a greater explosive force than the sum of the strengths of the component parts when exploded separately. Whatever the reason, the fact seems established. The reason is possibly that the explosion of the nitro-glycerine is sufficiently powerful to produce a *detonation* of the other chemicals, which is impossible to produce by ordinary means, and that this explosion caused by detonation is more powerful than an ordinary explosion. The majority of the explosive compounds and "powders" on the market are of this character—a mixture of 20 to 60 per cent. of nitro-glycerine with variable proportions of one or more of a great variety of explosive chemicals.

The choice of the explosive depends on the character of the rock. A hard brittle rock is most effectively blasted by a detonating compound. The rapidity with which the full force of the explosive is developed has a shattering effect on a brittle substance. On the contrary, some of the softer tougher rocks and indurated clays are but little affected by dynamite. The result is but little more than an enlargement of the blast-hole. Quarrying must generally be done with blasting-powder, as the quicker explosives are too shattering. Although the results obtained by various experimenters are very variable, it may be said that pure nitro-glycerine is eight times as powerful as black powder, dynamite (75% nitro-glycerine) six times, and gun-cotton four to six times as powerful. For open work where time is not particularly valuable, black powder is by far the cheapest, but in tunnel-headings, whose progress determines the progress of the whole work, dynamite is so much more effective and so expedites the work that its use becomes economical.

**118. Drilling.** Although many very complicated forms of drill-bars have been devised, the best form (with slight modifications to suit circumstances) is as shown in Fig. 64, (a) and (b).

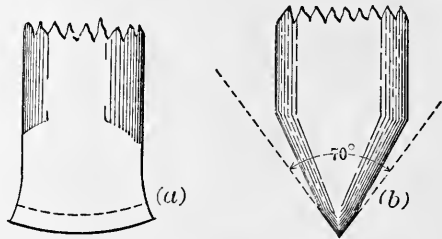


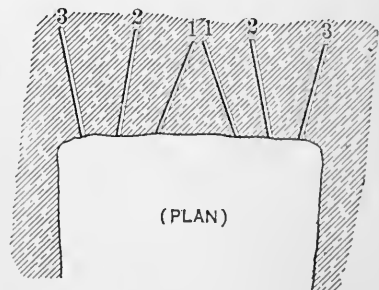
FIG. 64.

The width should flare at the bottom (a) about 15 to 30%. For hard rock the curve of the edge should be somewhat flatter and for soft rock somewhat more curved than shown, Fig. 64, (a). Sometimes the angle of the two faces is varied from that given, Fig. 64, (b), and occasionally the edge is purposely blunted so as to give a crushing rather than a cutting effect. The drills will require sharpening for each 6 to 18 inches depth of hole, and will require a new edge to be worked every 2 to 4 days. For drilling vertical holes the *churn-drill* is the most economical. The drill-bar is of iron, about 6 to 8 feet long,  $1\frac{1}{4}$ " in diameter, weighs about 25 to 30 lbs., and is shod with a piece of steel welded on. The bar is lifted a few inches between each blow, turned partially around, and allowed to fall, the impact doing the work. From 5 to 15 feet of holes, depending on the character of the rock, is a fair day's work—10 hours. In very soft rocks even more than this may be done. This method is inapplicable for inclined holes or even for vertical holes in confined places, such as tunnel-headings. For such places the only practical *hand* method is to use hammers. This may be done by light drills and light hammers (one-man work), or by heavier drills held by one man and struck by one or two men with heavy hammers. The conclusion of an exhaustive investigation as to the relative economy of light or heavy hammers is that the light-hammer method is more economical for the softer rocks, the heavy-hammer method is more economical for the harder

rocks, but that the light-hammer method is always more expeditious and hence to be preferred when time is important.

The subject of machine rock-drills is too vast to be treated here. The method is only practicable when the amount of work to be done is large, and especially when time is valuable. The machines are generally operated by compressed air for tunnel-work, thus doing the additional service of supplying fresh air to the tunnel-headings where it is most needed. The cost per foot of hole drilled is quite variable, but is usually somewhat less than that of hand-drilling—sometimes but a small fraction of it.

**119. Position and direction of drill-holes.** As the cost of drilling holes is the largest single item in the total cost of blasting, it is necessary that skill and judgment should be used in so locating the holes that the blasts will be most effective. The greatest effect of a blast will evidently be in the direction of the “line of least resistance.” In a strictly homogeneous material this will be the shortest line from the center of the explosive to the surface. The variations in homogeneity on account of laminations and seams require that each case shall be judged according to experience. In open-pit blasting it is generally easy to obtain two and sometimes three exposed faces to the rock, making it a simple matter to drill holes so that a blast will do effective work. When a solid face of rock must be broken into, as in a tunnel-heading, the work is necessarily ineffectual and expensive. A conical or wedge-shaped mass will first be blown out by simultaneous blasts in the holes marked 1, Fig. 65; blasts in the holes marked 2 and 3 will then complete the cross-section of the heading. A great saving in cost may often be secured by skilfully taking advantage of seams, breaks, and irregularities. When the work is economically done there is but little noise or throwing of rock,



DRILL HOLES IN TUNNEL HEADING

FIG. 65.

a covering of old timbers and branches of trees generally sufficing to confine the smaller pieces which would otherwise fly up.

**120. Amount of explosive.** The amount of explosive required varies as the cube of the line of least resistance. The best results are obtained when the line of least resistance is  $\frac{2}{4}$  of the depth of the hole; also when the powder fills about  $\frac{1}{3}$  of the hole. For average rock the amount of powder required is as follows:

|                               |                   |        |                     |         |
|-------------------------------|-------------------|--------|---------------------|---------|
| Line of least resistance..... | 2 ft.             | 4 ft.  | 6 ft.               | 8 ft.   |
| Weight of powder.....         | $\frac{1}{4}$ lb. | 2 lbs. | $6\frac{3}{4}$ lbs. | 16 lbs. |

Strict compliance with all of the above conditions would require that the diameter of the hole should vary for every case. While this is impracticable, there should evidently be some variation in the size of the hole, depending on the work to be done. For example, a 1" hole, drilled 2' 8" deep, with its line of least resistance 2', and loaded with  $\frac{1}{4}$  lb. of powder, would be filled to a depth of  $9\frac{1}{2}$ ", which is nearly  $\frac{1}{3}$  of the depth. A 2" hole, drilled 8' deep, with its line of least resistance 6', and loaded with  $6\frac{3}{4}$  lbs. of powder, would be filled to a depth of over 28", which is also nearly  $\frac{1}{3}$  of the depth. One pound of blasting-powder will occupy about 28 cubic inches. Quarrying necessitates the use of numerous and sometimes repeated light charges of powder, as a heavy blast or a powerful explosive like dynamite is apt to shatter the rock. This requires more powder to the cubic yard than blasting for mere excavation, which may usually be done by the use of  $\frac{1}{4}$  to  $\frac{1}{3}$  lb. of powder per cubic yard of easy open blasting. On account of the great resistance offered by rock when blasted in headings in tunnels, the powder used per cubic yard will run up to 2, 4, and even 6 lbs. per cubic yard. As before stated, nitro-glycerine is about eight times (and dynamite about six times) as powerful as the same *weight* of powder.

**121. Tamping.** Blasting-powder and the slow-burning explosives require thorough tamping. Clay is probably the best,

but sand and fine powdered rock are also used. Wooden plugs, inverted expansive cones, etc., are periodically reinvented by enthusiastic inventors, only to be discarded for the simpler methods. Owing to the extreme rapidity of the development of the force of a nitro-glycerine or dynamite explosion, tamping is not so essential with these explosives, although it unquestionably adds to their effectiveness. Blasting under water has been effectively accomplished by merely pouring nitro-glycerine into the drilled holes through a tube and then exploding the charge without any tamping except that furnished by the superincumbent water. It has been found that air-spaces about a charge make a material reduction in the effectiveness of the explosion. It is therefore necessary to carefully ram the explosive into a solid mass. Of course the liquid nitro-glycerine needs no ramming, but dynamite should be rammed with a *wooden* rammer. Iron should be carefully avoided in ramming gunpowder. A copper bar is generally used.

**122. Exploding the charge.** Black powder is generally exploded by means of a fuse which is essentially a cord in which there is a thin vein of gunpowder, the cord being protected by tar, extra linings of hemp, cotton, or even gutta-percha. The fuse is inserted into the middle of the charge, and the tamping carefully packed around it so that it will not be injured. To produce the detonation required to explode nitro-glycerine and dynamite, there must be an initial explosion of some easily ignited explosive. This is generally accomplished by means of caps containing fulminating-powder which are exploded by electricity. The electricity (in one class of caps) heats a very fine platinum wire to redness, thereby igniting the sensitive powder, or (in another class) a spark is caused to jump through the powder between the ends of two wires suitably separated. Dynamite can also be exploded by using a small cartridge of gunpowder which is itself exploded by an ordinary fuse.

**123. Cost.** Trautwine estimates the cost of blasting (for mere excavation) as averaging 45 cents per cubic yard, falling as low as 30 cents for easy but *brittle* rock, and running up to

60 cents and even \$1 when the cutting is shallow, the rock especially tough, and the strata unfavorably placed. Soft tough rock frequently requires more powder than harder brittle rock.

**124. Classification of excavated material.** The classification of excavated material is a fruitful source of dispute between contractors and railroad companies, owing mainly to the fact that the variation between the softest earth and the hardest rock is so gradual that it is very difficult to describe distinctions between different classifications which are unmistakable and indisputable. The classification frequently used is (*a*) earth, (*b*) loose rock, and (*c*) solid rock. As blasting is frequently used to loosen "loose rock" and even "earth" (if it is frozen), the fact that blasting is employed cannot be used as a criterion, especially as this would (if allowed) lead to unnecessary blasting for the sake of classifying material as rock.

**Earth.** This includes clay, sand, gravel, loam, decomposed rock and slate, boulders or loose stones not greater than 1 cubic foot (3 cubic feet, P. R. R.), and sometimes even "hard-pan." In general it will signify material which *can* be loosened by a plough with two horses, or with which one picker can keep one shoveller busy.

**Loose rock.** This includes boulders and loose stones of more than one cubic foot and less than one cubic yard; stratified rock, not more than six inches thick, separated by a stratum of clay; also all material (not classified as earth) which may be loosened by pick or bar and which "*can* be quarried without blasting, although blasting may occasionally be resorted to."

**Solid rock** includes all rock found in masses of over one cubic yard which cannot be removed except by blasting.

It is generally specified that the engineer of the railroad company shall be the judge of the classification of the material, but frequently an appeal is taken from his decisions to the courts.

**125. Specifications for earthwork.** The following specifications, issued by the Norfolk and Western R. R., represent the average requirements. It should be remembered that very strict



specifications invariably increase the cost of the work, and frequently add to the cost more than is gained by improved quality of work.

1. The grading will be estimated and paid for by the cubic yard, and will include clearing and grubbing, and all open excavations, channels, and embankments required for the formation of the roadbed, and for turnouts and sidings; cutting all ditches or drains about or contiguous to the road; digging the foundation-pits of all culverts, bridges, or walls; reconstructing turnpikes or common roads in cases where they are destroyed or interfered with; changing the course or channel of streams; and all other excavations or embankments connected with or incident to the construction of said Railroad.

2. All grading, except where otherwise specified, whether for cuts or fills, will be measured in the excavations and will be classified under the following heads, viz.: Solid Rock, Loose Rock, Hard-pan, and Earth.

SOLID ROCK shall include all rock occurring in masses which, in the judgment of the said Engineer Maintenance of Way, may be best removed by blasting.

LOOSE ROCK shall include all kinds of shale, soapstone, and other rock which, in the judgment of the said Engineer Maintenance of Way, can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be occasionally resorted to; also, detached stone of less than one (1) cubic yard and more than one (1) cubic foot.

HARD-PAN shall consist of tough indurated clay or cemented gravel, which requires blasting or other equally expensive means for its removal, or which cannot be ploughed with less than four horses and a railroad plough, or which requires two pickers to a shoveller, the said Engineer Maintenance of Way to be the judge of these conditions.

EARTH shall include all material of an earthy nature, of whatever name or character, not unquestionably loose rock or hard-pan as above defined.

POWDER. The use of powder in cuts will not be considered

as a reason for any other classification than earth, unless the material in the cut is clearly other than earth under the above specifications.

3. Earth, gravel, and other materials taken from the excavations, except when otherwise directed by the said Engineer Maintenance of Way or his assistant, shall be deposited in the adjacent embankment; the cost of removing and depositing which, when the distance necessary to be hauled is not more than sixteen hundred (1600) feet, shall be included in the price paid for the excavation.

4. EXTRA HAUL will be estimated and paid for as follows: whenever material from excavations is necessarily hauled a greater distance than sixteen hundred (1600) feet, there shall be paid in addition to the price of excavation the price of extra haul per 100 feet, or part thereof, after the first 1600 feet; the necessary haul to be determined in each case by the said Engineer Maintenance of Way or his assistant, from the profile and cross-sections, and the estimates to be in accordance therewith.

5. All embankments shall be made in layers of such thickness and carried on in such manner as the said Engineer Maintenance of Way or his assistant may prescribe, the stone and heavy materials being placed in slopes and top. And in completing the fills to the proper grade such additional heights and fulness of slope shall be given them, to provide for their settlement, as the said Engineer Maintenance of Way, or his assistant, may direct. Embankments about masonry shall be built at such times and in such manner and of such materials as the said Engineer Maintenance of Way or his assistant may direct.

6. In procuring materials for embankments from without the line of the road, and in wasting materials from cuttings, the place and manner of doing it shall in each case be indicated by the Engineer Maintenance of Way or his assistant; and care must be taken to injure or disfigure the land as little as possible. Borrow-pits and spoil-banks must be left by the Contractor in regular and sightly shape.

7. The lands of the said Railroad Company shall be cleared

to the extent required by the said Engineer Maintenance of Way, or his assistant, of all trees, brushes, logs, and other perishable materials, which shall be destroyed by burning or deposited in heaps as the said Engineer Maintenance of Way, or his assistant, may direct. Large trees must be cut not more than two and one-half ( $2\frac{1}{2}$ ) feet from the ground, and under embankments less than four (4) feet high they shall be cut close to the ground. All small trees and bushes shall be cut close to the ground.

8. Clearing shall be estimated and paid for by the acre or fraction of an acre.

9. All stumps, roots, logs, and other obstructions shall be grubbed out, and removed from all places where embankments occur less than two (2) feet in height; also, from all places where excavations occur and from such other places as the said Engineer Maintenance of Way or his assistant may direct.

10. Grubbing shall be estimated and paid for by the acre or fraction of an acre.

11. Contractors, when directed by the said Engineer Maintenance of Way or his assistant in charge of the work, will deposit on the side of the road, or at such convenient points as may be designated, any stone, rock, or other materials that they may excavate; and all materials excavated and deposited as above, together with all timber removed from the line of the road, will be considered the property of the Railroad Company, and the Contractors upon the respective sections will be responsible for its safe-keeping until removed by said Railroad Company, or until their work is finished.

12. Contractors will be accountable for the maintenance of safe and convenient places wherever public or private roads are in any way interfered with by them during the progress of the work. They will also be responsible for fences thrown down, and for gates and bars left open, and for all damages occasioned thereby.

13. Temporary bridges and trestles, erected to facilitate the progress of the work, in case of delays at masonry structures

from any cause, or for other reasons, will be at the expense of the Contractor.

14. The line of road or the gradients may be changed in any manner, and at any time, if the said Engineer Maintenance of Way or his assistant shall consider such a change necessary or expedient; but no claim for an increase in prices of excavation or embankment on the part of the Contractor will be allowed or considered unless made in writing before the work on that part of the section where the alteration has been made shall have been commenced. The said Engineer Maintenance of Way or his assistant may also, on the conditions last recited, increase or diminish the length of any section for the purpose of more nearly equalizing or balancing the excavations and embankments, or for any other reason.

15. The roadbed will be graded as directed by the said Engineer Maintenance of Way or his assistant, and in conformity with such breadths, depths, and slopes of cutting and filling as he may prescribe from time to time, and no part of the work will be finally accepted until it is properly completed and dressed off at the required grade.

## CHAPTER IV.

### TRESTLES.

**126. Extent of use.** Trestles constitute from 1 to 3% of the length of the average railroad. It was estimated in 1889 that there was then about 2400 miles of single-track railway trestle in the United States, divided among 150,000 structures and estimated to cost about \$75,000,000. The annual charge for maintenance, estimated at  $\frac{1}{8}$  of the cost, therefore amounted to about \$9,500,000 and necessitated the annual use of perhaps 300,000,000 ft. B.M. of timber. The corresponding figures at the present time must be somewhat in excess of this. The magnitude of this use, which is causing the rapid disappearance of forests, has resulted in endeavors to limit the use of timber for this purpose. Trestles may be considered as justifiable under the following conditions:

*a. Permanent trestles.*

1. Those of *extreme* height—then called viaducts and frequently constructed of iron or steel, as the Kinzua viaduct, 302 ft. high.

2. Those across waterways—*e.g.*, that across Lake Pontchartrain, near New Orleans, 22 miles long.

3. Those across swamps of soft deep mud, or across a river-bottom, liable to occasional overflow.

*b. Temporary trestles.*

1. To open the road for traffic as quickly as possible—often a reason of great financial importance.

2. To quickly replace a more elaborate structure, destroyed

by accident, on a road already in operation, so that the interruption to traffic shall be a minimum.

3. To form an earth embankment with earth brought from a distant point by the train-load, when such a measure would cost less than to borrow earth in the immediate neighborhood.

4. To bridge an opening temporarily and thus allow time to learn the regimen of a stream in order to better proportion the size of the waterway and also to facilitate bringing *suitable* stone for masonry from a distance. In a new country there is always the double danger of either building a culvert too small, requiring expensive reconstruction, perhaps after a disastrous washout, or else wasting money by constructing the culvert unnecessarily large. Much masonry has been built of a very poor quality of stone because it could be conveniently obtained and because good stone was unobtainable except at a prohibitive cost for transportation. Opening the road for traffic by the use of temporary trestles obviates both of these difficulties.

**127. Trestles vs. embankments.** Low embankments are very much cheaper than low trestles both in first cost and maintenance. Very high embankments are very expensive to construct, but cost comparatively little to maintain. A trestle of equal height may cost much less to construct, but will be expensive to maintain—perhaps  $\frac{1}{8}$  of its cost per year. To determine the height beyond which it will be cheaper to maintain a trestle rather than build an embankment, it will be necessary to allow for the cost of maintenance. The height will also depend on the relative cost of timber, labor, and earthwork. At the present average values, it will be found that for less heights than 25 feet the *first cost* of an embankment will *generally* be less than that of a trestle; this implies that a permanent trestle should never be constructed with a height less than 25 feet except for the reasons given in § 126. The height at which a permanent trestle is certainly cheaper than earthwork is more uncertain. A high grade line joining two hills will invariably imply at least a culvert if an embankment is used. If the culvert is built of masonry, the cost of the embankment will be

so increased that the height at which a trestle becomes economical will be materially reduced. The cost of an embankment increases much more rapidly than the height—with very high embankments more nearly as the square of the height—while the cost of trestles does not increase as rapidly as the height. Although local circumstances may modify the application of any set rules, it is probably seldom that it will be cheaper to build an embankment 40 or 50 feet high than to permanently maintain a wooden trestle of that height. A steel viaduct would probably be the best solution of such a case. These are frequently used for permanent structures, especially when very high. The cost of maintenance is much less than that of wood, which makes the use of iron or steel preferable for permanent trestles unless wood is abnormally cheap. Neither the cost nor the construction of iron or steel trestles will be considered in this chapter.

**128. Two principal types.** There are two principal types of wooden trestles—pile trestles and framed trestles. The great objection to pile trestles is the rapid rotting of the portion of the pile which is underground, and the difficulty of renewal. The maximum height of pile trestles is about 30 feet, and even this height is seldom reached. Framed trestles have been constructed to a height of considerably over 100 feet. They are frequently built in such a manner that any injured piece may be readily taken out and renewed without interfering with traffic. Trestles consist of two parts—the supports called “bents,” and the stringers and floor system. As the stringers and floor system are the same for both pile and framed trestles, the “bents” are all that need be considered separately.

#### PILE TRESTLES.

**129. Pile bents.** A pile bent consists generally of four piles driven into the ground deep enough to afford not only sufficient vertical resistance but also lateral resistance. On top of these piles is placed a horizontal “cap.” The caps are fastened to the tops of the piles by methods illustrated in Fig. 66. The

method of fastening shown in each case should not be considered as applicable only to the particular type of pile bent used to illustrate it. Fig. 66 (*a* and *d*) illustrates a mortise-joint with a hard-

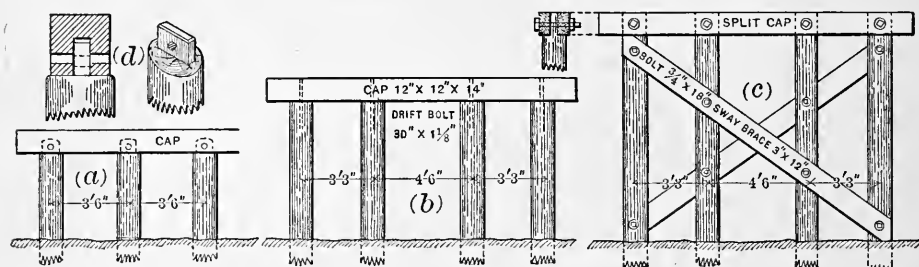


FIG. 66.

wood pin about  $1\frac{1}{4}$ " in diameter. The hole for the pin should be bored separately through the cap and the mortise, and the hole through the cap should be at a slightly higher level than that through the mortise, so that the cap will be drawn down tight when the pin is driven. Occasionally an iron dowel (an iron pin about  $1\frac{1}{2}$ " in diameter and about 6" long) is inserted partly in the cap and partly in the pile. The use of drift-bolts, shown in Fig. 66 (*b*), is cheaper in first cost, but renders repairs and renewals very troublesome and expensive. "Split caps," shown in Fig. 66 (*c*), are formed by bolting two half-size strips on each side of a tenon on top of the pile. Repairs are very easily and cheaply made without interference with the traffic and without injuring other pieces of the bent. The smaller pieces are more easily obtainable in a sound condition; the decay of one does not affect the other, and the first cost is but little if any greater than the method of using a single piece. For further discussion, see § 136.

For very light traffic and for a height of about 5 feet three vertical piles will suffice, as shown in Fig. 66 (*a*). Up to a height of 8 or 10 feet four piles may be used without sway-bracing, as in Fig. 66 (*b*), if the piles have a good bearing. For heights greater than 10 feet sway-bracing is generally necessary. The outside piles are frequently driven with a batter varying from 1 : 12 to 1 : 4.



Piles are made, if possible, from timber obtained in the vicinity of the work. Durability is the great requisite rather than strength, for almost any timber is strong enough (except as noted below) and will be suitable if it will resist rapid decay. The following list is quoted as being in the order of preference on account of durability:

|                |               |              |               |
|----------------|---------------|--------------|---------------|
| 1. Red cedar   | 5. White pine | 9. White oak | 12. Black oak |
| 2. Red cypress | 6. Redwood    | 10. Post-oak | 13. Hemlock   |
| 3. Pitch-pine  | 7. Elm        | 11. Red oak  | 14. Tamarac   |
| 4. Yellow pine | 8. Spruce     |              |               |

Red-cedar piles are said to have an average life of 27 years with a possible maximum of 50 years, but the timber is rather weak, and if exposed in a river to flowing ice or driftwood is apt to be injured. Under these circumstances oak is preferable, although its life may be only 13 to 18 years.

**130. Methods of driving piles.** The following are the principal methods of driving piles:

*a.* A hammer weighing 2000 to 3000 lbs. or more, sliding in guides, is drawn up by horse-power or a portable engine, and allowed to fall *freely*.

*b.* The same as above except that the hammer does not fall freely, but drags the rope and revolving drum as it falls and is thus quite materially retarded. The mechanism is a little more simple, but is less effective, and is sometimes made deliberately deceptive by a contractor by retarding the blow, in order to apparently indicate the requisite resistance on the part of the pile.

The above methods have the advantage that the mechanism is cheap and can be transported into a new country with comparative ease, but the work done is somewhat ineffective and costly compared with some of the more elaborate methods given below.

*c. Gunpowder pile-drivers*, which automatically explode a cartridge every time the hammer falls. The explosion not only forces the pile down, but throws up the hammer for the next blow. For a given height of fall the effect is therefore doubled. It has been shown by experience, however, that when it is at-

tempted to use such a pile-driver rapidly the mechanism becomes so heated that the cartridges explode prematurely, and the method has therefore been abandoned.

*d. Steam pile-drivers*, in which the hammer is operated directly by steam. The hammer falls freely a height of about 40 inches and is raised again by steam. The effectiveness is largely due to the rapidity of the blows, which does not allow time between the blows for the ground to settle around the pile and increase the resistance, which does happen when the blows are infrequent. "The hammer-cylinder weighs 5500 lbs., and with 60 to 75 lbs. of steam gives 75 to 80 blows per minute. With 41 blows a large unpointed pile was driven 35 feet into a hard clay bottom in half a minute." Such a driver would cost about \$800.

The above four methods are those usual for dry earth. In very soft wet or sandy soils, where an unlimited supply of water is available, the *water-jet* is sometimes employed. A pipe is fastened along the side of the pile and extends to the pile-point. If water is forced through the pipe, it loosens the sand around the point and, rising along the sides, decreases the side resistance so that the pile sinks by its own weight, aided perhaps by extra weights loaded on. This loading may be accomplished by connecting the top of the pile and the pile-driver by a block and tackle so that a portion of the weight of the pile-driver is continually thrown on the pile.

Excessive driving frequently fractures the pile below the surface and thereby greatly weakens its bearing power. To prevent excessive "brooming" of the top of the pile, owing to the action of the hammer, the top should be protected by an iron ring fitted to the top of the pile. The "brooming" not only renders the driving ineffective and hence uneconomical, but vitiates the value of any test of the bearing power of the pile by noting the sinking due to a given weight falling a given distance. If the pile is so soft that brooming is unavoidable, the top should be adzed off

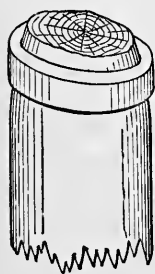


FIG. 67.

frequently, and especially should it be done just before the final blows which are to test its bearing-power.

In a new country judgment and experience will be required to decide intelligently whether to employ a simple drop-hammer machine, operated by horse-power and easily transported but uneconomical in operation, or a more complicated machine working cheaply and effectively after being transported at greater expense.

**131. Pile-driving formulæ.** If  $R$  = the resistance of a pile, and  $s$  the set of the pile during the last blow,  $w$  the weight of the pile-hammer, and  $h$  the fall during the last blow, then we may state the approximate relation that  $Rs = wh$ , or  $R = \frac{wh}{s}$ .

This is the basic principle of all rational formulæ, but the maximum weight which a pile will sustain after it has been driven some time is by no means equal to the resistance of the pile during the last blow. There are also many other modifying elements which have been variously allowed for in the many proposed formulæ. The formulæ range from the extreme of empirical simplicity to very complicated attempts to allow properly for all modifying causes. As the simplest rule, specifications sometimes require that the piles shall be driven until the pile will not sink more than 5 inches under five consecutive blows of a 2000 lb., hammer falling 25 feet. The "*Engineering News* formula"\* gives the safe load as  $\frac{2wh}{s+1}$ , in which  $w$  = weight of hammer,  $h$  = fall in feet,  $s$  = set of pile in inches under the last blow. This formula is derived from the above basic formula by calling the safe load  $\frac{1}{6}$  of the final resistance, and by adding (arbitrarily) 1 to the final set ( $s$ ) as a compensation for the extra resistance caused by the settling of earth around the pile between each blow. This formula is used only for ordinary hammer-driving. When the piles are driven by a steam pile-driver the formula becomes

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\* *Engineering News*, Nov. 17, 1892.

safe load  $= \frac{2wh}{s + 0.1}$ . For the "gunpowder pile-driver," since the explosion of the cartridge drives the pile in with the same force with which it throws the hammer upward, the effect is *twice* that of the fall of the hammer, and the formula becomes

safe load  $= \frac{4wh}{s + 0.1}$ . In these last two formulæ the constant

in the denominator is changed from  $s + 1$  to  $s + 0.1$ . The constant (1.0 or 0.1) is supposed to allow, as before stated, for the effect of the extra resistance caused by the earth settling around the pile between each blow. The more rapid the blows the less the opportunity to settle and the less the proper value of the constant.

The above formulæ have been given on account of their simplicity and their practical agreement with experience. Many other formulæ have been proposed, the majority of which are more complicated and attempt to take into account the weight of the pile, resistance of the guides, etc. While these elements, as well as many others, have their influence, their effect is so overshadowed by the indeterminable effect of other elements—as, for example, the effect of the settlement of earth around the pile between blows—that it is useless to attempt to employ anything but a purely empirical formula.

**132. Pile-points and pile-shoes.** Piles are generally sharpened to a blunt point. If the pile is liable to strike boulders, sunken logs, or other obstructions which are liable to turn the point, it

is necessary to protect the point by some form of shoe. Several forms in cast iron have been used, also a wrought-iron shoe, having four "straps" radiating from the apex, the straps being nailed on to the pile, as shown in Fig. 68 (b). The cast-iron form shown in Fig. 68 (a) has a base cast around a drift-bolt. The recess on the top of the base receives the bottom of the pile and prevents a tendency to split the bottom

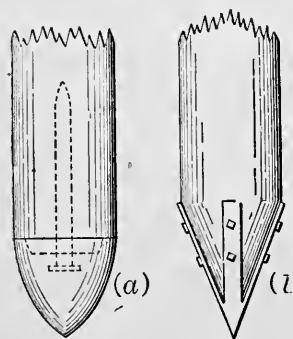


FIG. 68.

of the pile or to force the shoe off laterally.

**133. Details of design.** No theoretical calculations of the strength of pile bents need be attempted on account of the extreme complication of the theoretical strains, the uncertainty as to the real strength of the timber used, the variability of that strength with time, and the insignificance of the economy that would be possible even if exact sizes could be computed. The piles are generally required to be not less than 10" or 12" in diameter at the large end. The P. R. R. requires that they shall be "not less than 14 and 7 inches in diameter at butt and small end respectively, exclusive of bark, which must be removed." The removal of the bark is generally required in good work. Soft *durable* woods, such as are mentioned in § 129, are best for the piles, but the caps are generally made of oak or yellow pine. The caps are generally 14 feet long (for single track) with a cross-section 12"  $\times$  12" or 12"  $\times$  14". "Split caps" would consist of two pieces 6"  $\times$  12". The sway-braces, never used for less heights than 6', are made of 3"  $\times$  12" timber, and are spiked on with  $\frac{3}{8}$ " spikes 8" long. The floor system will be the same as that described later for framed trestles.

**134. Cost of pile trestles.** The cost, per linear foot, of piling depends on the method of driving, the scarcity of suitable timber, the price of labor, the length of the piles, and the amount of shifting of the pile-driver required. The cost of soft-wood piles varies from 8 to 15 c. per lineal foot, and the cost of oak piles varies from 10 to 30 c. per foot according to the length, the longer piles costing more per foot. The cost of driving will average about \$2.50 per pile, or 7.5 to 10 c. per lineal foot. Since the cost of shifting the pile-driver is quite an item in the total cost, the cost of driving a long pile would be *less* per foot than for a short pile, but on the other hand the cost of the pile is *greater* per foot, which tends to make the total cost per foot constant. Specifications generally say that the piling will be paid for per lineal foot of piling *left in the work*. The wastage of the tops of piles sawed off is always something, and is frequently very large. Sometimes a small amount per foot of piling sawed off is allowed the contractor as compensation for

his loss. This reduces the contractor's risk and possibly reduces his bid by an equal or greater amount than the extra amount actually paid him.

#### FRAMED TRESTLES.

**135. Typical Design.** A typical design for a framed trestle bent is given in Fig. 69. This represents, with slight variations of detail, the plan according to which a large part of the framed

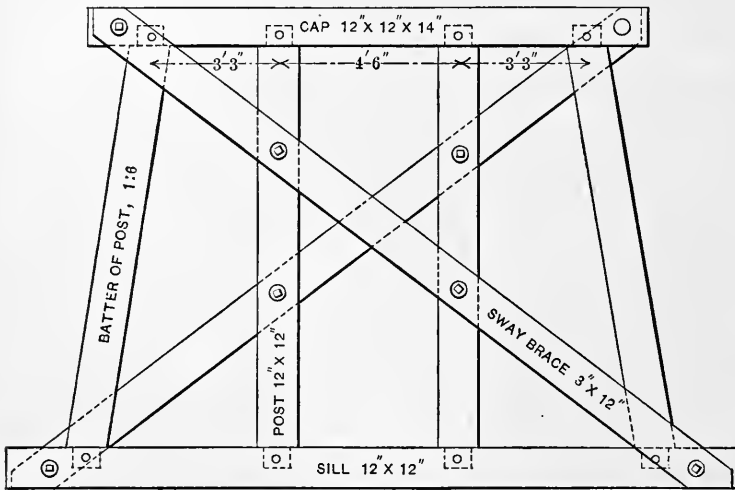


FIG. 69.

trestle bents of the country have been built—i.e., of those less than 20 or 30 feet in height, not requiring multiple-story construction.

**136. Joints.** (a) The mortise-and-tenon joint is illustrated in Fig. 69 and also in Fig. 66 (a). The tenon should be about 3" thick, 8" wide, and  $5\frac{1}{2}$ " long. The mortise should be cut a little deeper than the tenon. "Drip-holes" from the mortise

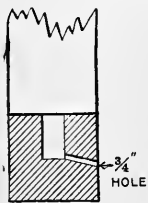


FIG. 70.

to the outside will assist in draining off water that may accumulate in the joint and thus prevent the rapid decay that would otherwise ensue. These joints are very troublesome if a single post decays and requires renewal. It is generally required that

the mortise and tenon should be thoroughly daubed with paint before putting them together. This will tend to

make the joint water-tight and prevent decay from the accumulation and retention of water in the joint.

(b) **The plaster joint.** This joint is made by bolting and spiking a 3"  $\times$  12" plank on both sides of the joint. The cap and sill should be notched to receive the posts. Repairs are greatly facilitated by the use of these joints. This method has been used by the Delaware and Hudson Canal Co. [R. R.].

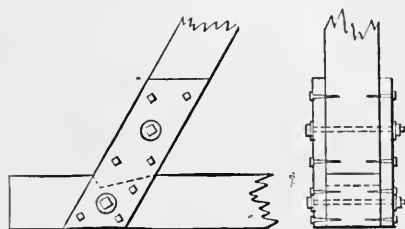


FIG. 71.

(c) **Iron plates.** An iron plate of the form shown in Fig. 72 (b) is bent and used as shown in Fig. 72 (a). Bolts passing through the bolt-holes shown secure the plates to the timbers and make a strong joint which may be readily loosened for repairs. By slight modifications in the design the method may be used for inclined posts and complicated joints.

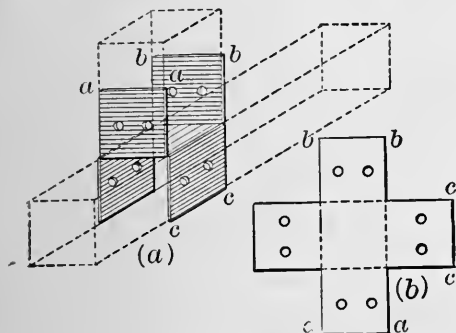


FIG. 72.

(d) **Split caps and sills.** These are described in § 129. Their

advantages apply with even greater force to framed trestles.

(e) **Dowels and drift-bolts.** These joints facilitate cheap and rapid construction, but renewals and repairs are very difficult, it being almost impossible to extract a drift-bolt which has been driven its full length without splitting open the pieces containing it. Notwithstanding this objection they are extensively used, especially for temporary work which is not expected to be used long enough to need repairs.

**137. Multiple-story construction.** Single-story framed trestle bents are used for heights up to 18 or 20 feet and exceptionally up to 30 feet. For greater heights some such construction as is illustrated in a skeleton design in Fig. 73 is used. By using split sills between each story and separate vertical and batter posts in each story, any piece may readily be removed and

renewed if necessary. The height of these stories varies, in different designs, from 15 to 25 and even 30 feet. In some designs the structure of each story is independent of the stories above and below. This greatly facilitates both the original construction and subsequent repairs. In other designs the verticals and batter-posts are made continuous through two consecutive stories. The structure is somewhat stiffer, but is much more difficult to repair.

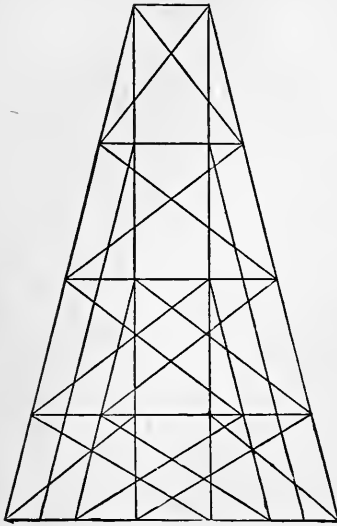


FIG. 73.

Since the bents of any trestle are usually of variable height and those heights are not always an even multiple of the uniform height desired for the stories, it becomes necessary to make the upper stories of uniform height and let

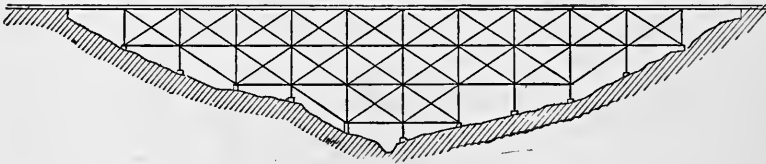


FIG. 74.

the odd amount go to the lowest story, as shown in Figs. 73 and 74.

**138. Span.** The shorter the span the greater the number of trestle bents; the longer the span the greater the required strength of the stringers supporting the floor. Economy demands the adoption of a span that shall make the sum of these requirements a minimum. The higher the trestle the greater the cost of each bent, and the greater the span that would be justifiable. Nearly all trestles have bents of variable height, but the advantage of employing uniform standard sizes is so great that many roads use the same span and sizes of timber not only for the panels of any given trestle, but also for all trestles



regardless of height. The spans generally used vary from 10 to 16 feet. The Norfolk and Western R. R. uses a span of 12' 6'' for all single-story trestles, and a span of 25' for all multiple-story trestles. The stringers are the same in both cases, but when the span is 25 feet, knee-braces are run

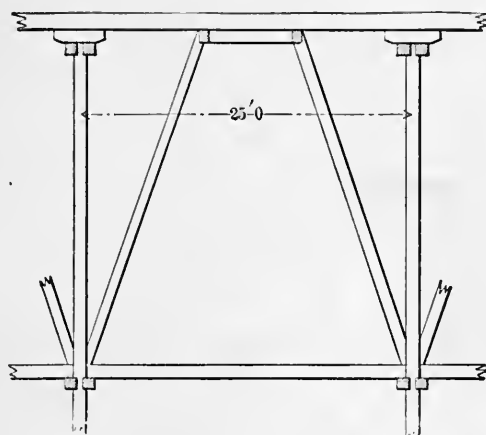


FIG. 75.

from the sill of the first story below to near the middle of each set of stringers. These knee-braces are connected at the top by a "straining-beam" on which the stringers rest, thus supporting the stringer in the center and virtually reducing the span about one-half.

**139. Foundations.** (a) **Piles.** Piles are frequently used as a foundation, as in Fig. 76, particularly in soft ground, and also for temporary structures. These foundations are cheap, quickly constructed, and are particularly valuable

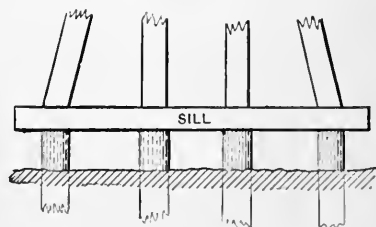


FIG. 76.

when it is financially necessary to open the road for traffic as soon as possible and with the least expenditure of money; but there is the disadvantage of inevitable decay within a few years unless the piles are chemically treated, as will be discussed later. Chemical treatment, however, increases the cost so that such a foundation would often cost more than a foundation of stone. A pile should be driven under each post as shown in Fig. 76.

(b) **Mud-sills.** Fig. 77 illustrates the use of mud-sills as built by the Louisville and Nashville R. R. Eight blocks  $12'' \times 12'' \times 6'$  are used under each bent. When the ground is very soft, two additional timbers ( $12'' \times 12'' \times$  length of bent-sill), as shown by the dotted lines, are placed underneath. The number required evidently depends on the nature of the ground.

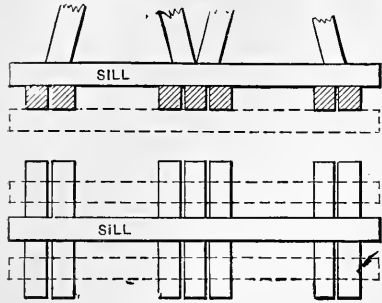


FIG. 77.

(c) **Stone foundations.** Stone foundations are the best and the most expensive. For very high trestles the Norfolk and Western R. R. employs foundations as shown in Fig. 78, the



FIG. 78.

walls being 4 feet thick. When the height of the trestle is 72 feet or less (the plans requiring for 72' in height a foundation-wall 39' 6' long) the foundation is made continuous. The sill of the trestle should rest on several short lengths of  $3'' \times 12''$  plank, laid transverse to the sill on top of the wall.

**140. Longitudinal bracing.** This is required to give the structure longitudinal stiffness and also to reduce the columnar length of the posts. This bracing generally consists of horizontal "waling-strips" and diagonal braces. Sometimes the braces are placed wholly on the outside posts unless the trestle is very high. For single-story trestles the P. R. R. employs the "laced" system, i.e., a line of posts joining the cap of one bent with the sill of the next, and the sill of that bent with the cap of the next. Some plans employ braces forming an  $\times$  in alternate panels. Connecting these braces in the center more than doubles their columnar strength. Diagonal braces, when bolted to posts, should be fastened to them as near the ends of

the posts as possible. The sizes employed vary largely, depending on the clear length and on whether they are expected to act by tension or compression.  $3'' \times 12''$  planks are often used when the design would require tensile strength only, and  $8'' \times 8''$  posts are often used when compression may be expected.

**141. Lateral bracing.** Several of the more recent designs of trestles employ diagonal lateral bracing between the caps of adjacent bents. It adds greatly to the stiffness of the trestle and better maintains its alignment.  $6'' \times 6''$  posts, forming an  $\times$  and connected at the center, will answer the purpose.

**142. Abutments.** When suitable stone for masonry is at hand and a suitable subsoil for a foundation is obtainable without too much excavation, a masonry abutment will be the best. Such an abutment would probably be used when masonry footings for trestle bents were employed (§ 139, *c*).

Another method is to construct a "crib" of  $10'' \times 12''$  timber, laid horizontally, drift-bolted together, securely braced and embedded into the ground. Except for temporary construction such a method is generally objectionable on account of rapid decay.

Another method, used most commonly for pile trestles, and for framed trestles having pile foundations (§ 139, *a*), is to use a pile bent at such a place that the natural surface on the *up-hill* side is not far below the cap, and the thrust of the material, filled in to bring the surface to grade, is insignificant.  $3'' \times 12''$  planks are placed behind the piles, cap, and stringers to retain the filled material.

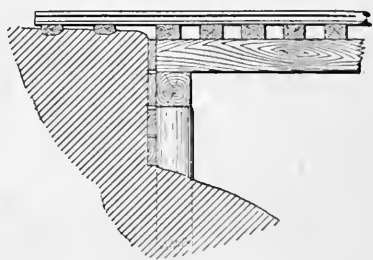


FIG. 79.

#### FLOOR SYSTEMS.

**143. Stringers.** The general practice is to use two, three, and even four stringers under each rail. Sometimes a stringer

is placed under each guard-rail. Generally the stringers are made of two panel lengths and laid so that the joints alternate. A few roads use stringers of only one panel length, but this practice is strongly condemned by many engineers. The stringers should be separated to allow a circulation of air around them and prevent the decay which would occur if they were placed close together. This is sometimes done by means of 2" planks, 6' to 8' long, which are placed over each trestle bent. Several bolts, passing through all the stringers forming a group and through the separators, bind them all into one solid construction. Cast-iron "spools" or washers, varying from 4" to  $\frac{3}{4}$ " in length (or thickness), are sometimes strung on each bolt so as to separate the stringers. Sometimes washers are used between the separating planks and the stringers, the object of the separating planks then being to bind the stringers, especially abutting stringers, and increase their stiffness.

The most common size for stringers is 8"  $\times$  16". The Pennsylvania Railroad varies the width, depth, and number of stringers under each rail according to the clear span. It may be noticed that, assuming a uniform load per running foot, both

| Clear span. | No. of pieces<br>under each rail. | Width.   | Depth.    |
|-------------|-----------------------------------|----------|-----------|
| 10 feet     | 2                                 | 8 inches | 15 inches |
| 12 "        | 2                                 | 8 "      | 16 "      |
| 14 "        | 2                                 | 10 "     | 17 "      |
| 16 "        | 3                                 | 8 "      | 17 "      |

the pressure per square inch at the ends of the stringers (the caps having a width of 12") and also the stress due to transverse strain are kept *approximately* constant for the variable gross load on these varying spans.

**144. Corbels.** A corbel (in trestle-work) is a stick of timber (perhaps two placed side by side), about 3' to 6' long, placed underneath and along the stringers and resting on the cap. There are strong prejudices for and against their use, and a

corresponding diversity in practice. They are bolted to the stringers and thus stiffen the joint. They certainly reduce the objectionable crushing of the fibers at each end of the stringer, but if the corbel is no wider than the stringers, as is generally the case, the area of pressure between the corbels and the cap is



FIG. 80.

no greater and the pressure per square inch on the cap is no less than the pressure on the cap if no corbels were used. If the corbels and cap are made of hard wood, as is recommended by some, the danger of crushing is lessened, but the extra cost and the frequent scarcity of hard wood, and also the extra cost and labor of using corbels, may often neutralize the advantages obtained by their use.

**145. Guard-rails.** These are frequently made of  $5'' \times 8''$  stuff, notched  $1''$  for each tie. The sizes vary up to  $8'' \times 8''$ , and the depth of notch from  $\frac{3}{4}''$  to  $1\frac{1}{2}''$ . They are generally bolted to every third or fourth tie. It is frequently specified that they shall be made of oak, white pine, or yellow pine. The joints are made over a tie, by halving each piece, as illustrated in Fig. 81. The joints on opposite sides of the trestle should be

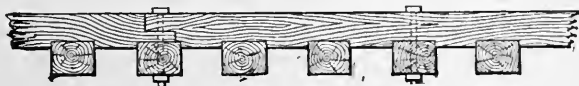


FIG. 81.

“staggered.” Some roads fasten every tie to the guard-rail, using a bolt, a spike, or a lag-screw.

Guard-rails were originally used with the idea of preventing the wheels of a derailed truck from running off the ends of the ties. But it has been found that an outer guard-rail alone (without an inner guard-rail) becomes an actual element of danger, since it has frequently happened that a derailed wheel has caught

on the outer guard-rail, thus causing the truck to slew around and so produce a dangerous accident. The true function of the *outside* guard-rail is thus changed to that of a tie-spacer, which keeps the ties from spreading when a derailment occurs. The inside guard-rail generally consists of an ordinary steel rail spiked about 10 inches inside of the running rail. These inner guard-rails should be bent inward to a point in the center of the track about 50 feet from the end of the bridge or trestle. If the inner guard-rails are placed with a clear space of 10 inches inside the running rail, the outer guard-rails should be *at least* 6' 10" apart. They are generally much farther apart than this.

**146. Ties on trestles.** If a car is derailed on a bridge or trestle, the heavily loaded wheels are apt to force their way between the ties by displacing them unless the ties are closely spaced and fastened. The clear space between ties is generally equal to or less than their width. Occasionally it is a little more than their width. 6"  $\times$  8" ties, spaced 14" to 16" from center to center, are most frequently used. The length varies from 9' to 12' for single track. They are generally notched  $\frac{1}{2}$ " deep on the under side where they rest on the stringers. Oak ties are generally required even when cheaper ties are used on the other sections of the road. Usually every third or fourth tie is bolted to the stringers. When stringers are placed underneath the guard-rails, bolts are run from the top of the guard-rail to the under side of the stringer. The guard-rails thus hold down the whole system of ties, and no direct fastening of the ties to the stringers is needed.

**147. Superelevation of the outer rail on curves.** The location of curves on trestles should be avoided if possible, especially when the trestle is high. Serious additional strains are introduced especially when the curvature is sharp or the speed high. Since such curves are sometimes practically unavoidable, it is necessary to design the trestle accordingly. If a train is stopped on a curved trestle, the action of the train on the trestle is evidently vertical. If the train is moving with a considerable velocity, the resultant of the weight and the cen-

trifugal action is a force somewhat inclined from the vertical. Both of these conditions may be expected to exist at times. If the *axis* of the system of posts is vertical (as illustrated in methods *a*, *b*, *c*, *d*, and *e*), any lateral force, such as would be produced by a moving train, will tend to rack the trestle bent. If the stringers are set vertically, a centrifugal force likewise tends to tip them sidewise. If the axis of the system of posts (or of the stringers) is inclined so as to coincide with the pressure of the train on the trestle when the train is moving at its normal velocity, there is no tendency to rack the trestle when the train is moving at that velocity, but there will be a tendency to rack the trestle or twist the stringers when the train is stationary. Since a moving train is usually the normal condition of affairs, as well as the condition which produces the maximum stress, an inclined axis is evidently preferable from a theoretical standpoint; but whatever design is adopted, the trestle should evidently be sufficiently cross-braced for either a moving or a stationary load, and any proposed design must be studied as to the effect of *both* of these conditions. Some of the various methods of securing the requisite superelevation may be described as follows:

(a) **Framing the outer posts longer than the inner posts, so that the cap is inclined at the proper angle; axis of posts vertical.** (Fig. 82.) The method requires more work in framing the trestle, but simplifies subsequent track-laying and maintenance, unless it should be found that the superelevation adopted is unsuitable, in which case it could be corrected by one of the other methods given below. The stringers tend to twist when the train is stationary.

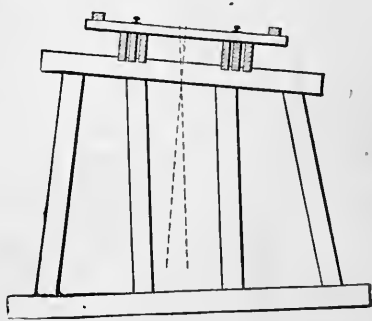


FIG. 82

(b) **Notching the cap so that the stringers are at a different elevation.** (Fig. 83.) This weakens the cap and requires that all ties shall be notched to a bevelled surface to fit the stringers,

which also weakens the ties. A centrifugal force will tend to twist the stringers and rack the trestle.

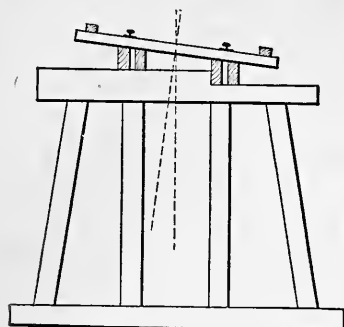


FIG. 83.

(c) **Placing wedges underneath the ties at each stringer.** These wedges are fastened with two bolts. Two or more wedges will be required for each tie. The additional number of pieces required for a long curve will be immense, and the work of inspection and keeping the nuts tight will greatly in-

crease the cost of maintenance.

(d) **Placing a wedge under the outer rail at each tie.** This requires but one extra piece per tie. There is no need of a wedge under the inner tie in order to make the rail normal to the tread. The resulting inward inclination is substantially that produced by some forms of rail-chairs or tie-plates. The spikes (a little longer than usual) are driven through the wedge into the tie. Sometimes "lag-screws" are used instead of spikes. If experience proves that the superelevation is too much or too little, it may be changed by this method with less work than by any other.

(e) **Corbels of different heights.** When corbels are used (see § 144) the required inclination of the floor system may be obtained by varying the depth of the corbels.

(f) **Tipping the whole trestle.** This is done by placing the trestle on an inclined foundation. If very much inclined, the trestle bent must be secured against the possibility of slipping sidewise, for the slope would be considerable with a sharp curve, and the vibration of a moving train would reduce the coefficient of friction to a comparatively small quantity.

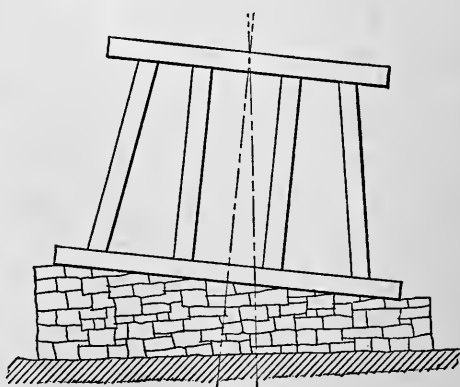


FIG. 84.

(g) **Framing the outer posts longer.** This case is identical



with case (a) except that the axis of the system of posts is inclined, as in case (f), but the sill is horizontal.

The above-described plans will suggest a great variety of methods which are possible and which differ from the above only in minor details.

**148. Protection from fire.** Trestles are peculiarly subject to fire, from passing locomotives, which may not only destroy the trestle, but perhaps cause a terrible disaster. This danger is sometimes reduced by placing a strip of galvanized iron along the top of each set of stringers and also along the tops of the caps. Still greater protection was given on a long trestle on the Louisville and Nashville R. R. by making a solid flooring of timber, covered with a layer of ballast on which the ties and rails were laid as usual.

Barrels of water should be provided and kept near all trestles, and on very long trestles barrels of water should be placed every two or three hundred feet along its length. A place for the barrels may be provided by using a few ties which have an extra length of about four feet, thus forming a small platform, which should be surrounded by a railing. The track-walkers should be held accountable for the maintenance of a supply of water in these barrels, renewals being frequently necessary on account of evaporation. Such platforms should also be provided as REFUGE-BAYS for track-walkers and trackmen working on the trestle. On very long trestles such a platform is sometimes provided with sufficient capacity for a hand-car.

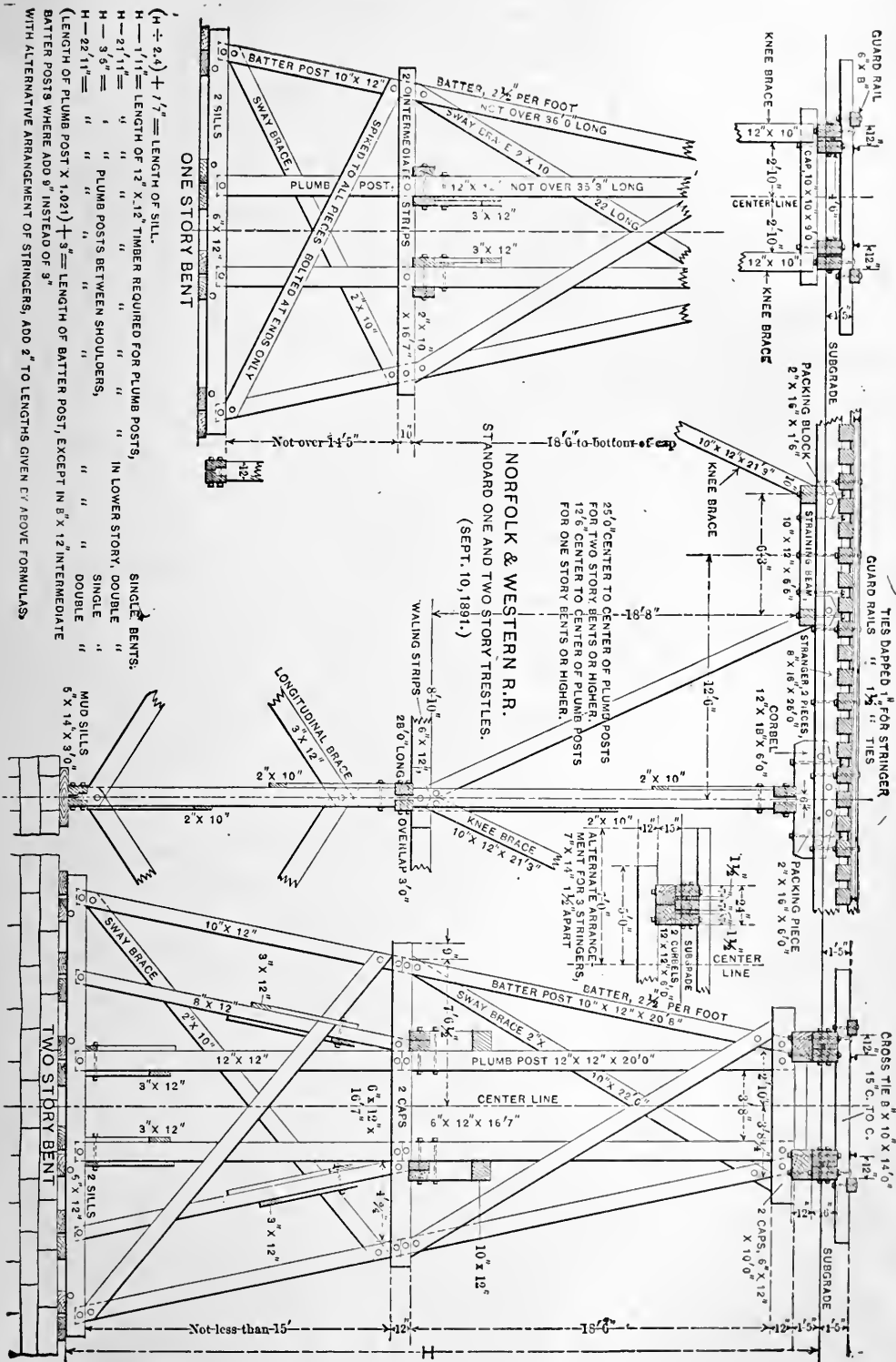
**149. Timber.** Any strong durable timber may be used when the choice is limited, but oak, pine, or cypress are preferred when obtainable. When all of these are readily obtainable, the various parts of the trestle will be constructed of different kinds of wood—the stringers of long-leaf pine, the posts and braces of pine or red cypress, and the caps, sills, and corbels (if used) of white oak. The use of oak (or a similar hard wood) for caps, sills, and corbels is desirable because of its greater strength in resisting crushing across the grain, which is the critical test for these parts. There is no physiological basis to

the objection, sometimes made, that different species of timber, in contact with each other, will rot quicker than if only one kind of timber is used. When a very extensive trestle is to be built at a place where suitable growing timber is at hand but there is no convenient sawmill, it will pay to transport a portable sawmill and engine and cut up the timber as desired.

**150. Cost of framed timber trestles.** The cost varies widely on account of the great variation in the cost of timber. When a railroad is first penetrating a new and undeveloped region, the cost of timber is frequently small, and when it is obtainable from the company's right-of-way the only expense is felling and sawing. The work per M., B. M., is small, considering that a single stick  $12'' \times 12'' \times 25'$  contains 300 feet, B. M., and that sometimes a few hours' work, worth less than \$1, will finish all the work required on it. Smaller pieces will of course require more work per foot, B. M. Long-leaf pine can be purchased from the mills at from \$8 to \$12 per M. feet, B. M., according to the dimensions. To this must be added the freight and labor of erection. The cartage from the nearest railroad to the trestle may often be a considerable item. Wrought iron will cost about 3 c. per pound and cast iron 2 c., although the prices are often lower than these. The amount of iron used depends on the detailed design, but, as an average, will amount to \$1.50 to \$2 per 1000 feet, B. M., of timber. A large part of the trestling of the country has been built at a contract price of about \$30 per 1000 feet, B. M., erected. While the cost will frequently rise to \$40 and even \$50 when timber is scarce, it will drop to \$13 (cost quoted) when timber is cheap.

#### DESIGN OF WOODEN TRESTLES.

**151. Common practice.** A great deal of trestling has been constructed without any rational design except that custom and experience have shown that certain sizes and designs are *probably* safe. This method has resulted occasionally in failures but more frequently in a very large waste of timber. Many railroads





employ a uniform size for all posts, caps, and sills, and a uniform size for stringers, all regardless of the height or span of the trestle. For repair work there are practical reasons favoring this. "To attempt to run a large lot of sizes would be more wasteful in the end than to maintain a few stock sizes only. Lumber can be bought more cheaply by giving a general order for 'the run of the mill for the season,' or 'a cargo lot,' specifying approximate percentages of standard stringer size, of  $12 \times 12$ -inch stuff,  $10 \times 10$ -inch stuff, etc., and a liberal proportion of 3- or 4-inch plank, all lengths thrown in. The  $12 \times 12$ -inch stuff, etc., is ordered all lengths, from a certain specified length up. In case of a wreck, washout, burn-out, or sudden call for a trestle to be completed in a stated time, it is much more economical and practical to order a certain number of carloads of 'trestle stuff' to the ground and there to select piece after piece as fast as needed, dependent only upon the length of stick required. When there is time to make the necessary surveys of the ground and calculations of strength, and to wait for a special bill of timber to be cut and delivered, the use of different sizes for posts in a structure would be warranted to a certain extent." \* For new construction, when there is generally sufficient time to design and order the proper sizes, such wastefulness is less excusable, and under any conditions it is both safer and more economical to prepare *standard designs* which can be made applicable to varying conditions and which will at the same time utilize as much of the strength of the timber as can be depended on. In the following sections will be given the elements of the preparation of such standard designs, which will utilize uniform sizes with as little waste of timber as possible. It is *not* to be understood that special designs should be made for each individual trestle.

**152. Required elements of strength.** The *stringers* of trestles are subject to transverse strains, to crushing across the grain at the ends, and to shearing along the neutral axis. The

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\* From "Economical Designing of Timber Trestle Bridges."

strength of the timber must therefore be computed for all these kinds of stress. *Caps* and *sills* will fail, if at all, by crushing across the grain; although subject to other forms of stress, these could hardly cause failure in the sizes usually employed. There is an apparent exception to this: if piles are improperly driven and an uneven settlement subsequently occurs, it may have the effect of transferring practically all of the weight to two or three piles, while the *cap* is subjected to a severe transverse strain which may cause its failure. Since such action is caused generally by avoidable errors of construction it may be considered as abnormal, and since such a failure will generally occur by a *gradual* settlement, all danger may be avoided by reasonable care in inspection. *Posts* must be tested for their columnar strength. These parts form the bulk of the trestle and are the parts which can be definitely designed from known stresses. The stresses in the bracing are more indefinite, depending on indeterminate forces, since the inclined posts take up an unknown proportion of the lateral stresses, and the design of the bracing may be left to what experience has shown to be safe, without involving any large waste of timber.

**153. Strength of timber.** Until recently tests of the strength of timber have generally been made by testing small, selected, well-seasoned sticks of "clear stuff," free from knots or imperfections. Such tests would give results so much higher than the vaguely known strength of large unseasoned "commercial" timber that very large factors of safety were recommended—factors so large as to detract from any confidence in the whole theoretical design. Recently the U. S. Government has been making a thoroughly scientific test of the strength of full-size timber under various conditions as to seasoning, etc. The work has been so extensive and thorough as to render possible the economical designing of timber structures.

One important result of the investigation is the determination of the great influence of the moisture in the timber and the law of its effect on the strength. It has been also shown that timber soaked with water has substantially the same

strength as green timber, even though the timber had once been thoroughly seasoned. Since trestles are exposed to the weather they should be designed on the basis of using green timber. It has been shown that the strength of green timber is very regularly about 55 to 60% of the strength of timber in which the moisture is 12% of the dry weight, 12% being the proportion of moisture usually found in timber that is protected from the weather but not heated, as, e.g., the timber in a barn. Since the moduli of rupture have all been reduced to this standard of moisture (12%), if we take *one-eighth* of the rupture values, it still allows a factor of safety of about five, even on green timber.

**Moduli of rupture for various timbers. [12% moisture.]**

(Condensed from U. S. Forestry Circular, No. 15.)

| No. | Species.            | Weight<br>per<br>cubic<br>foot. | Cross-bending.        |                           | Crush-<br>ing end<br>wise. | Crush-<br>ing<br>across<br>grain. | Shear-<br>ing<br>along<br>grain. |
|-----|---------------------|---------------------------------|-----------------------|---------------------------|----------------------------|-----------------------------------|----------------------------------|
|     |                     |                                 | Ultimate<br>Strength. | Modulus of<br>Elasticity. |                            |                                   |                                  |
| 1   | Long-leaf pine..... | 38                              | 12 600                | 2 070 000                 | 8000                       | 1180                              | 700                              |
| 2   | Cuban " .....       | 39                              | 13 600                | 2 370 000                 | 8700                       | 1220                              | 700                              |
| 3   | Short-leaf " .....  | 32                              | 10 100                | 1 680 000                 | 6500                       | 960                               | 700                              |
| 4   | Loblolly " .....    | 33                              | 11 300                | 2 050 000                 | 7400                       | 1150                              | 700                              |
| 5   | White " .....       | 24                              | 7 900                 | 1 390 000                 | 5400                       | 700                               | 400                              |
| 6   | Red " .....         | 31                              | 9 100                 | 1 620 000                 | 6700                       | 1000                              | 500                              |
| 7   | Spruce " .....      | 39                              | 10 000                | 1 640 000                 | 7300                       | 1200                              | 800                              |
| 8   | Bald cypress.....   | 29                              | 7 900                 | 1 290 000                 | 6000                       | 800                               | 500                              |
| 9   | White cedar.....    | 23                              | 6 300                 | 910 000                   | 5200                       | 700                               | 400                              |
| 10  | Douglas spruce....  | 32                              | 7 900                 | 1 680 000                 | 5700                       | 800                               | 500                              |
| 11  | White oak.....      | 50                              | 13 100                | 2 090 000                 | 8500                       | 2200                              | 1000                             |
| 12  | Overcup " .....     | 46                              | 11 300                | 1 620 000                 | 7300                       | 1900                              | 1000                             |
| 13  | Post " .....        | 50                              | 12 300                | 2 030 000                 | 7100                       | 3000                              | 1100                             |
| 14  | Cow " .....         | 46                              | 11 500                | 1 610 000                 | 7400                       | 1900                              | 900                              |
| 15  | Red " .....         | 45                              | 11 400                | 1 970 000                 | 7200                       | 2300                              | 1100                             |
| 16  | Texan " .....       | 46                              | 13 100                | 1 860 000                 | 8100                       | 2000                              | 900                              |
| 19  | Willow " .....      | 45                              | 10 400                | 1 750 000                 | 7200                       | 1600                              | 900                              |
| 20  | Spanish " .....     | 46                              | 12 000                | 1 930 000                 | 7700                       | 1800                              | 900                              |
| 21  | Shagbark hickory..  | 51                              | 16 000                | 2 390 000                 | 9500                       | 2700                              | 1100                             |
| 27  | Pignut " ..         | 56                              | 18 700                | 2 730 000                 | 10900                      | 3200                              | 1200                             |
| 28  | White elm.....      | 34                              | 10 300                | 1 540 000                 | 6500                       | 1200                              | 800                              |
| 29  | Cedar " .....       | 46                              | 13 500                | 1 700 000                 | 8000                       | 2100                              | 1300                             |
| 30  | White ash.....      | 39                              | 10 800                | 1 640 000                 | 7200                       | 1900                              | 1100                             |

AVERAGE SAFE ALLOWABLE WORKING UNIT STRESSES, IN POUNDS, PER SQUARE INCH. RECOMMENDED BY THE COMMITTEE ON "STRENGTH OF BRIDGE AND TRESTLE TIMBERS." (ASSOCIATION OF RAILWAY SUPERINTENDENTS OF BRIDGES AND BUILDINGS: FIFTH ANNUAL CONVENTION, NEW ORLEANS, OCTOBER, 1895.)

| Kind of timber.                                   | Tension.    |               | Compression. |                            |               | Transverse.           |                        | Shearing.   |               |
|---------------------------------------------------|-------------|---------------|--------------|----------------------------|---------------|-----------------------|------------------------|-------------|---------------|
|                                                   | With grain. | Across grain. | With grain.  |                            | Across grain. | Extreme fibre stress. | Modulus of elasticity. | With grain. | Across grain. |
|                                                   |             |               | End bearing. | Column under 15 diameters. |               |                       |                        |             |               |
|                                                   | Ten.        | Ten.          | Five.        | Five.                      | Four.         | Six.                  | Two.                   | Four.       | Four.         |
| Factor of safety .....                            |             |               |              |                            |               |                       |                        |             |               |
| White oak .....                                   | 1000        | 200           | 1400         | 900                        | 500           | 1000                  | 550 000                | 200         | 1000          |
| White pine .....                                  | 700         | 50            | 1100         | 700                        | 200           | 700                   | 500 000                | 100         | 500           |
| Southern, long-leaf, or Georgia yellow pine ..... | 1200        | 60            | 1600         | 1000                       | 350           | 1200                  | 850 000                | 150         | 1250          |
| Douglas, Oregon, and Wash- } Yellow fir.          | 1200        | ...           | 1600         | 1200                       | 300           | 1100                  | 700 000                | 150         | ....          |
| ington fir or pine : } Red fir.....               | 1000        | ...           | ....         | ....                       | ...           | 800                   | ....                   | ...         | ....          |
| Northern or short-leaf yellow pine.....           | 900         | 50            | 1200         | 800                        | 250           | 1000                  | 600 000                | 100         | 1000          |
| Red pine.....                                     | 900         | 50            | 1200         | 800                        | 200           | 800                   | 600 000                | ...         | ....          |
| Norway pine.....                                  | 800         | ...           | 1200         | 800                        | 200           | 700                   | 600 000                | ...         | ....          |
| Canadian (Ottawa) white pine.....                 | 1000        | ...           | ....         | 1000                       | ...           | ....                  | ....                   | 100         | ....          |
| Canadian (Ontario) red pine .....                 | 1000        | ...           | ....         | 1000                       | ...           | 800                   | 700 000                | 100         | ....          |
| Spruce and Eastern fir.....                       | 800         | 50            | 1200         | 800                        | 200           | 700                   | 600 000                | 100         | 750           |
| Hemlock.....                                      | 600         | ...           | ....         | 800                        | 150           | 600                   | 450 000                | 100         | 600           |
| Cypress .....                                     | 600         | ...           | 1200         | 800                        | 200           | 800                   | 450 000                | ...         | ....          |
| Cedar.....                                        | 800         | ...           | 1200         | 800                        | 200           | 800                   | 350 000                | ...         | 400           |
| Chestnut.....                                     | 900         | ...           | ....         | 1000                       | 250           | 800                   | 500 000                | 150         | 400           |
| California redwood. ....                          | 700         | ...           | ....         | 800                        | 200           | 750                   | 350 000                | 100         | ....          |
| California spruce.....                            | ....        | ...           | ....         | 800                        | ...           | 800                   | 600 000                | ...         | ....          |



On page 177 there are quoted the values taken from the U. S. Government reports on the strength of timber, the tests probably being the most thorough and reliable that were ever made.

On page 178 are given the "average safe allowable working unit stresses in pounds per square inch," as recommended by the committee on "Strength of Bridge and Trestle Timbers," the work being done under the auspices of the Association of Railway Superintendents of Bridges and Buildings. The report was presented at their fifth annual convention, held in New Orleans, in October, 1895.

**154. Loading.** As shown in § 138, the span of trestles is always small, is generally 14 feet, and is never greater than 18' except when supported by knee-braces. The greatest load that will ever come on any one span will be the concentrated loading of the drivers of a consolidation locomotive. With spans of 14 feet or less it is impossible for even the four pairs of drivers to be on the same span at once. The weight of the rails, ties, and guard-rails should be added to obtain the total load on the stringers, and the weight of these, plus the weight of the stringers, should be added to obtain the pressure on the caps or corbels. This dead load is almost insignificant compared with the live load and may be included with it. The weight of rails, ties, etc., may be estimated at 200 pounds per foot. To obtain the weight on the caps the weight of the stringers must be added, which depends on the design and on the weight per cubic foot of the wood employed. But as the weight of the stringers is comparatively small, a considerable percentage of variation in weight will have but an insignificant effect on the result. Disregarding all refinements as to actual dimensions, the ordinary maximum loading for standard gauge railroads may be taken as that due to four pairs of driving-axles, spaced 5' 0" apart and giving a pressure of 25,000 pounds per axle. This should be increased to 40,000 pounds per axle (same spacing) for the heaviest traffic. On the basis of 25,000 pounds per axle the following results have been computed:

STRESSES ON VARIOUS SPANS DUE TO MOVING LOADS OF 25,000 POUNDS,  
SPACED 5' 0" APART.

| Span in feet. | Max. mom.—<br>ft. lbs. | Max. shear. | Max load on<br>one cap. |
|---------------|------------------------|-------------|-------------------------|
| 10            | 65 000                 | 38 500      | 52 100                  |
| 12            | 103 600                | 45 000      | 62 700                  |
| 14            | 142 400                | 49 600      | 74 200                  |
| 16            | 181 400                | 54 725      | 85 700                  |
| 18            | 220 600                | 60 100      | 97 900                  |

Although the dead load does not vary in proportion to the live load, yet, considering the very small influence of the dead load, there will be no appreciable error in assuming the corresponding values, for a load of 40,000 lbs. per axle, to be  $\frac{4}{3}$  of those given in the above tabulation.

**155. Factors of safety.**—The most valuable result of the government tests is the knowledge that under given moisture conditions the strength of various species of sound timber is not the variable uncertain quantity it was once supposed to be, but that its strength can be relied on to a comparatively close percentage. This confidence in values permits the employment of lower factors of safety than have heretofore been permissible. Stresses, which when excessive would result in immediate destruction, such as cross-breaking and columnar stresses, should be allowed a higher factor of safety—say 6 or 8 for green timber. Other stresses, such as crushing across the grain and shearing along the neutral axis, which will be apparent to inspection before it is dangerous, may be allowed lower factors—say 3 to 5.

**156. Design of stringers.**—The strength of rectangular beams of equal width varies as the square of the depth; therefore deep beams are the strongest. On the other hand, when any cross-sectional dimension of timber much exceeds 12" the cost is much higher per M., B.M., and it is correspondingly difficult to obtain thoroughly sound sticks, free from wind-shakes, etc. Wind-shakes especially affect the shearing strength. Also, if the required transverse strength is obtained by using high narrow stringers, the area of pressure between the stringers and the

cap may become so small as to induce crushing across the grain. This is a very common defect in trestle design. As already indicated in § 138, the span should vary roughly with the average height of the trestle, the longer spans being employed when the trestle bents are very high, although it is usual to employ the same span throughout any one trestle.

To illustrate, if we select a span of 14 feet, the load on one cap will be 74,200 lbs. If the stringers and cap are made of long-leaf yellow pine, which require the closely determined value of 1180 lbs. per square inch to produce a crushing amounting to 3% of the height on timber with 12% moisture, we may use 200 lbs. per square inch as a safe pressure even for green timber; this will require 371 square inches of surface. If the cap is 12" wide, this will require a width of 31 inches, or say 2 stringers under each rail, each 8 inches wide. For rectangular beams

$$\text{Moment} = \frac{1}{6} R' b h^2.$$

Using for  $R'$  the safe value 1575 lbs. per square inch, we have

$$142400 \times 12 = \frac{1}{6} \times 1575 \times 32 \times h^2,$$

from which  $h = 15''.9$ . If desired, the width may be increased to 9" and the depth correspondingly reduced, which will give similarly  $h = 14''.8$ , or say 15". This shows that two beams,  $9'' \times 15''$ , under each rail will stand the transverse bending and have more than enough area for crushing.

The shear per square inch will equal

$$\frac{3}{2} \frac{\text{total shear}}{\text{cross section}} = \frac{3}{2} \frac{49600}{4 \times 9 \times 15} = 138 \text{ lbs. per sq. inch,}$$

which is a safe value, although it should preferably be less. Hence the above combination of dimensions will answer.

The deflection should be computed to see if it exceeds the

somewhat arbitrary standard of  $\frac{1}{200}$  of the span. The deflection for *uniform loading* is

$$\Delta = \frac{5 W l^3}{32 b h^3 E},$$

in which  $l$  = length in inches;

$W$  = total load, assumed as uniform;

$E$  = modulus of elasticity, given as 2,070,000 lbs.

per sq. in. for long-leaf pine, 12% dry, and assumed to be 1,200,000 for green timber. Then

$$\Delta = \frac{5 \times 72800 \times 168^3}{32 \times 36 \times 15^3 \times 1200000} = 0''.37$$

$$\frac{1}{200} \times 168'' = 0''.84,$$

so that the calculated deflection is well within the limit. Of course the loading is not strictly uniform, but even with a liberal allowance the deflection is still safe.

For the heaviest practice (40000 lbs. per axle) these stringer dimensions must be correspondingly increased.

**157. Design of posts.** Four posts are generally used for single-track work. The inner posts are usually braced by the cross-braces, so that their columnar strength is largely increased; but as they are apt to get more than their share of work, the advantage is compensated and they should be treated as unsupported columns for the total distance between cap and sill in simple bents, or for the height of stories in multiple-story construction. The caps and sills are assumed to have a width of 12''. It facilitates the application of bracing to have the columns of the same width and vary the other dimension as required.

Unfortunately the experimental work of the U. S. Government on timber testing has not yet progressed far enough to establish unquestionably a general relation between the strength of long columns and the crushing strength of short blocks. The



but it is certainly a safe dimension.  $12'' \times 6''$  would possibly prove amply safe in practice. One method of allowing for weathering is to disregard the outer half-inch on all sides of the post, i.e., to calculate the strength of a post one inch smaller in each dimension than the post actually employed. On this basis an  $8'' \times 12'' \times 20'$  post, computed as a  $7'' \times 11'$  post, would have a *safe* columnar strength of 706 lbs. per square inch. With an area of 77 square inches, this gives a working load of 54362 lbs. for *each post*, or 217448 lbs. for the four posts. Considering that 74200 lbs. is the maximum load on one cap (14 feet span), the great excess of strength is apparent.

**158. Design of caps and sills.** The stresses in caps and sills are very indefinite, except as to crushing across the grain. As the stringers are placed almost directly over the inner posts, and as the sills are supported just under the posts, the transverse stresses are almost insignificant. In the above case four posts have an area of  $4 \times 12'' \times 8'' = 384$  sq. in. The total load, 74200 lbs., will then give a pressure of 193 pounds per square inch, which is within the allowable limit. This one feature might require the use of  $8'' \times 12''$  posts rather than  $6'' \times 12''$  posts, for the smaller posts, although probably strong enough as posts, would produce an objectionably high pressure.

**159. Bracing.** Although some idea of the stresses in the bracing could be found from certain assumptions as to wind-pressure, etc., yet it would probably not be found wise to decrease, for the sake of economy, the dimensions which practice has shown to be sufficient for the work. The economy that would be possible would be too insignificant to justify any risk. Therefore the usual dimensions, given in §§ 139 and 140, should be employed.

## CHAPTER V.

### TUNNELS.

#### SURVEYING.

**160. Surface surveys.** As tunnels are always dug from each end and frequently from one or more intermediate shafts, it is necessary that an accurate surface survey should be made between the two ends. As the natural surface in a locality where a tunnel is necessary is almost invariably very steep and rough, it requires the employment of unusually refined methods of work to avoid inaccuracies. It is usual to run a line on the surface that will be at every point vertically over the center line of the tunnel. Tunnels are generally made straight unless curves are absolutely necessary, as curves add greatly to the cost. Fig. 85 represents roughly a longitudinal section of the

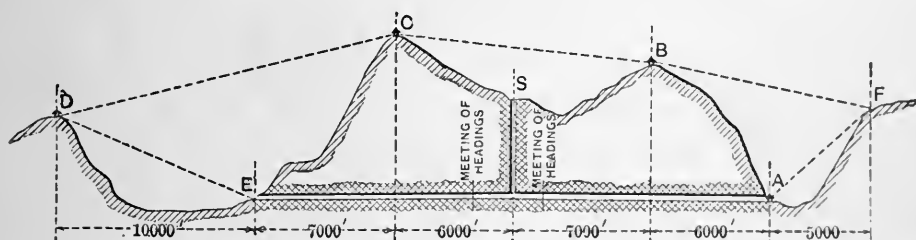


FIG. 85.—SKETCH OF SECTION OF THE HOOSAC TUNNEL.

Hoosac Tunnel. Permanent stations were located at *A*, *B*, *C*, *D*, *E*, and *F*, and stone houses were built at *A*, *B*, *C*, and *D*. These were located with ordinary field transits at first, and then all the points were placed as nearly as possible in one vertical plane by repeated trials and minute corrections, using a very large specially constructed transit. The stations *D* and *F* were necessary because *E* and *A* were invisible from *C* and *B*.

The alignment at *A* and *E* having been determined with great accuracy, the true alignment was easily carried into the tunnel.

The relative elevations of *A* and *E* were determined with great accuracy. Steep slopes render necessary many settings of the level per unit of horizontal distance and require that the work be unusually accurate to obtain even fair accuracy per unit of distance. The levels are usually re-run many times until the probable error is a very small quantity.

The exact horizontal distance between the two ends of the tunnel must also be known, especially if the tunnel is on a grade. The usual steep slopes and rough topography likewise render accurate horizontal measurements very difficult. Frequently when the slope is steep the measurement is best obtained by measuring along the slope and allowing for grade. This may be very accurately done by employing two tripods (level or transit tripods serve the purpose very well), setting them up slightly less than one tape-length apart and measuring between horizontal needles set in wooden blocks inserted in the top of each tripod. The elevation of each needle is also observed. The true horizontal distance between two successive positions of the needles then equals the square root of the difference of the squares of the inclined distance and the difference of elevation. Such measurements will probably be more accurate than those made by attempting to hold the tape horizontal and plumbing down with plumb-bobs, because (1) it is practically difficult to hold both ends of the tape truly horizontal; (2) on steep slopes it is impossible to hold the downhill end of a 100-foot tape (or even a 25-foot length) on a level with the other end, and the great increase in the number of applications of the unit of measurement very greatly increases the probable error of the whole measurement; (3) the vibrations of a plumb-bob introduce a large probability of error in transferring the measurement from the elevated end of the tape to the ground, and the increased number of such applications of the unit of measurement still further increases the probable error.



**161. Surveying down a shaft.** If a shaft is sunk, as at *S*, Fig. 85, and it is desired to dig out the tunnel in both directions from the foot of the shaft so as to meet the headings from the outside, it is necessary to know, when at the bottom of the shaft, the elevation, alignment, and horizontal distance from each end of the tunnel.

The *elevation* is generally carried down a shaft by means of a steel tape. This method involves the least number of applications of the unit of measurement and greatly increases the accuracy of the final result.

The *horizontal distance from each end* may be easily transferred down the shaft by means of a plumb-bob, using some of the precautions described in the next paragraph.

To transfer the *alignment* from the surface to the bottom of a shaft requires the highest skill because the shaft is always small, and to produce a line perhaps several thousand feet long in a direction given by two points 6 or 8 feet apart requires that the two points must be determined with extreme accuracy. The eminently successful method adopted in the Hoosac Tunnel will be briefly described: Two beams were securely fastened across the top of the shaft (1030 feet deep), the beams being placed transversely to the direction of the tunnel and as far apart as possible and yet allow plumb-lines, hung from the intersection of each beam with the tunnel center line, to swing freely at the bottom of the shaft. These intersections of the beams with the center line were determined by averaging the results of a large number of careful observations for alignment. Two fine parallel wires, spaced about  $\frac{1}{16}$ " apart, were then stretched between the beams so that the center line of the tunnel bisected at all points the space between the wires. Plumb-bobs, weighing 15 pounds, were suspended by fine wires beside each cross-beam, the wires passing between the two parallel alignment wires and bisecting the space. The plumb-bobs were allowed to swing in pails of water at the bottom. Drafts of air up the shaft required the construction of boxes surrounding the wires. Even these precautions did not suffice

to absolutely prevent vibration of the wire at the bottom through a very small arc. The mean point of these vibrations in each case was then located on a rigid cross-beam suitably placed at the bottom of the shaft and at about the level of the roof of the tunnel. Short plumb-lines were then suspended from these points whenever desired; a transit was set (by trial) so that its line of collimation passed through both plumb lines and the line at the bottom could thus be prolonged.

**162. Underground surveys.** Survey marks are frequently placed on the timbering, but they are apt to prove unreliable on account of the shifting of the timbering due to settlement of the surrounding material. They should never be placed at the bottom of the tunnel on account of the danger of being disturbed or covered up. Frequently holes are drilled in the roof and filled with wooden plugs in which a hook is screwed exactly on line. Although this is probably the safest method, even these plugs are not always undisturbed, as the material, unless very hard, will often settle slightly as the excavation proceeds. When a tunnel is perfectly straight and not too long, alignment-points may be given as frequently as desired from permanent stations located *outside* the tunnel where they are not liable to disturbance. This has been accomplished by running the alignment through

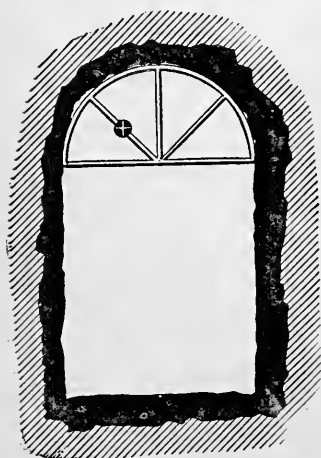


FIG. 86.

the upper part of the cross-section, at one side of the center, where it is out of the way of the piles of masonry material, débris, etc., which are so apt to choke up the lower part of the cross-section. The position of this line relative to the cross-section being fixed, the alignment of any required point of the cross-section is readily found by means of a light frame or template with a fixed target located where this line would intersect the frame when properly placed. A level-bubble

on the frame will assist in setting the frame in its proper position.

In all tunnel surveying the cross-wires must be illuminated

by a lantern, and the object sighted at must also be illuminated. A powerful dark-lantern with the opening covered with *ground glass* has been found useful. This may be used to illuminate a plumb-bob string or a very fine rod, or to place behind a brass plate having a narrow slit in it, the axis of the slit and plate being coincident with the plumb-bob string by which it is hung.

On account of the interference to the surveying caused by the work of construction and also by the smoke and dust in the air resulting from the blasting, it is generally necessary to make the surveys at times when construction is temporarily suspended.

**163. Accuracy of tunnel surveying.** Apart from the very natural desire to do surveying which shall check well, there is an important financial side to accurate tunnel surveying. If the survey lines do not meet as desired when the headings come together, it may be found necessary, if the error is of appreciable size, to introduce a slight curve, perhaps even a reversed curve, into the alignment, and it is even conceivable that the tunnel section would need to be enlarged somewhat to allow for these curves. The cost of these changes and the perpetual annoyance due to an enforced and undesirable alteration of the original design will justify a considerable increase in the expenses of the survey. Considering that the cost of surveys is usually but a small fraction of the total cost of the work, an increase of 10 or even 20% in the cost of the surveys will mean an insignificant addition to the total cost and frequently, if not generally, it will result in a saving of many times the increased cost. The accuracy actually attained in two noted American tunnels is given as follows: The Musconetcong tunnel is about 5000 feet long, bored through a mountain 400 feet high. The error of alignment at the meeting of the headings was 0'.04, error of levels 0'.015, error of distance 0'.52. The Hoosac tunnel is over 25,000 feet long. The heading from the east end met the heading from the central shaft at a point 11274 feet from the east end and 1563 feet from the shaft. The error in alignment was  $\frac{5}{16}$  of an inch, that of levels "a few hundredths,"

error of distance "trifling." The alignment, corrected at the shaft, was carried on through and met the heading from the west end at a point 10138 feet from the west end and 2056 feet from the shaft. Here the error of alignment was  $\frac{9}{16}$ " and that of levels 0.134 ft.

#### DESIGN.

**164. Cross-sections.** Nearly all tunnels have cross-sections peculiar to themselves—all varying at least in the details. The *general* form of a great many tunnels is that of a rectangle surmounted by a semi-circle or semi-ellipse. In very soft material an inverted arch is necessary along the bottom. In such cases the sides will generally be arched instead of vertical. The sides are frequently battered. With very long tunnels, several forms of cross-section will often be used in the same tunnel, owing to differences in the material encountered. In solid rock, which will not disintegrate upon exposure, no lining is required, and the cross-section will be the irregular section left by the blasting, the only requirement being that no rock shall be left within the required cross-sectional figure. Farther on, in the same tunnel, when passing through some very soft treacherous material, it may be necessary to put in a full arch lining—top, sides, and bottom—which will be nearly circular in cross-section. For an illustration of this see Figs. 87 and 88.

The width of tunnels varies as greatly as the designs. Single-track tunnels generally have a width of 15 to 16 feet. Occasionally they have been built 14 feet wide, and even less, and also up to 18 feet, especially when on curves. 24 to 26 feet is the most common width for double track. Many double-track tunnels are only 22 feet wide, and some are 28 feet wide. The heights are generally 19 feet for single track and 20 to 22 feet for double track. The variations from these figures are considerable. The lower limits depend on the cross-section of the rolling stock, with an indefinite allowance for clearance and ventilation. Cross-sections which coincide too closely with what is

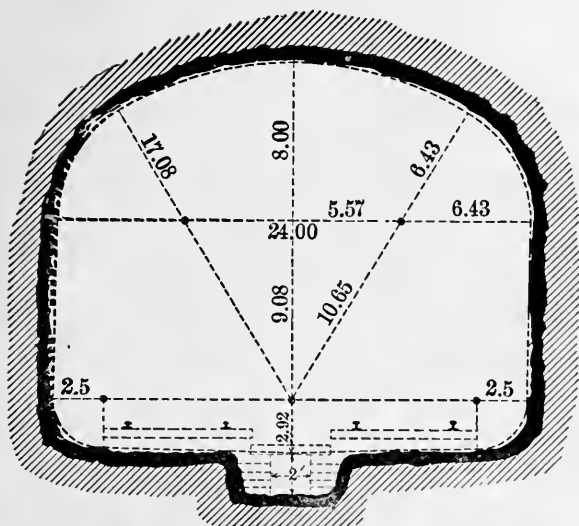


FIG. 87.—HOOSAC TUNNEL. SECTION THROUGH SOLID ROCK.

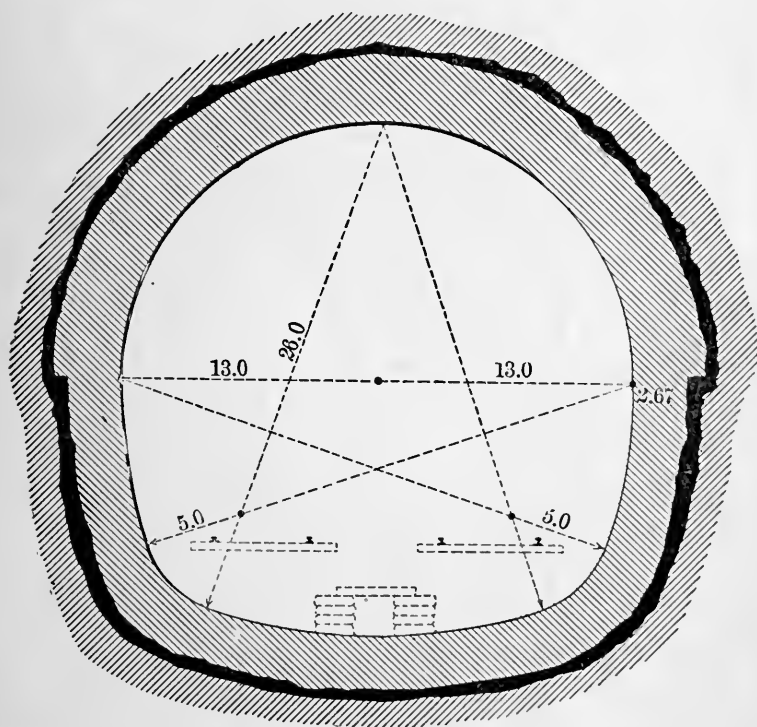


FIG. 88.—HOOSAC TUNNEL. SECTION THROUGH SOFT GROUND.

absolutely required for clearance are objectionable, because any slight settlement of the lining which would otherwise be harmless would then become troublesome and even dangerous. Figs. 87, 88, and 89 \* show some typical cross-sections.

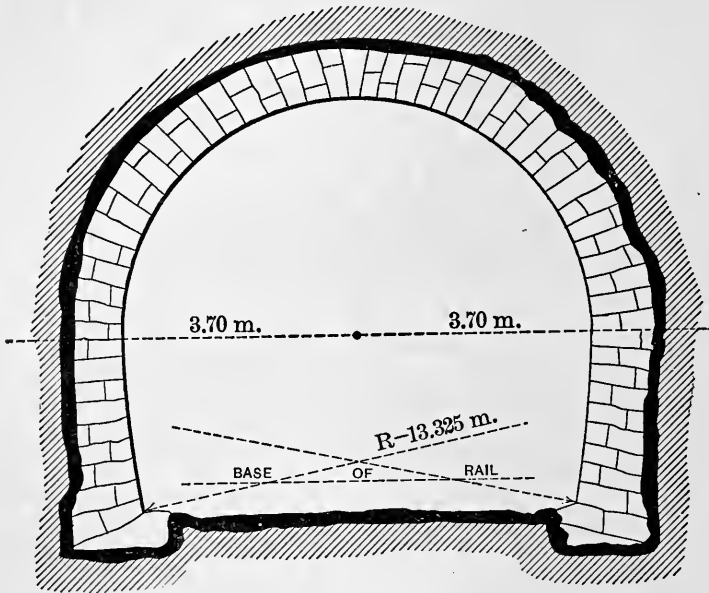
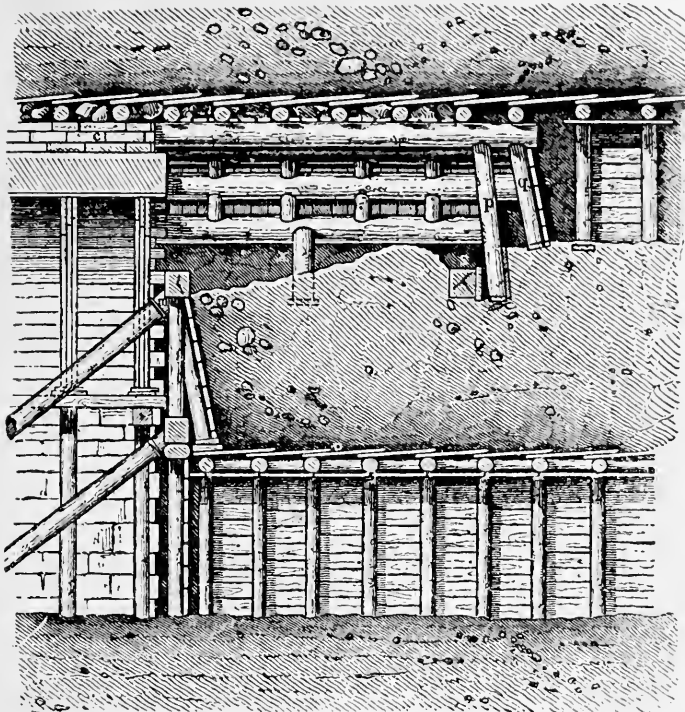


FIG. 89.—ST. CLOUD TUNNEL.

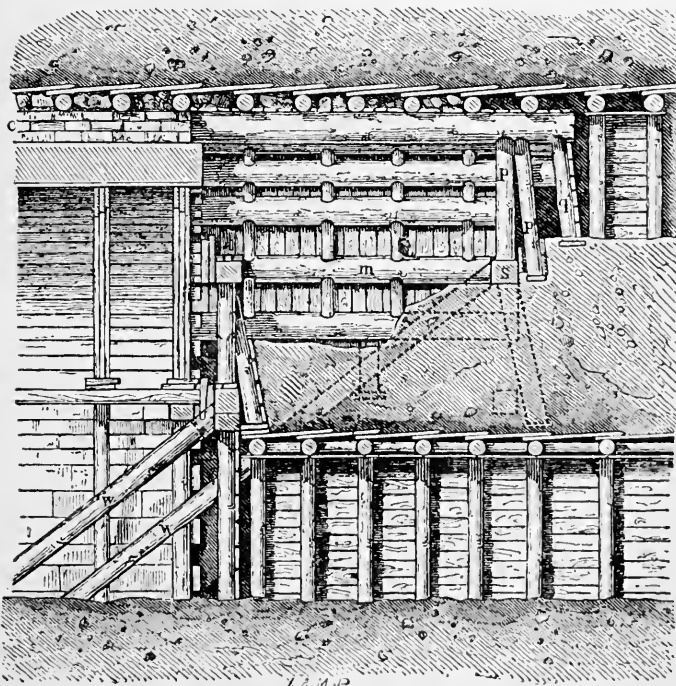
**165. Grade.** A grade of at least 0.2% is needed for drainage. If the tunnel is at the summit of two grades, the tunnel grade should be practically level, with an allowance for drainage, the actual summit being perhaps in the center so as to drain both ways. When the tunnel forms part of a long ascending grade, it is advisable to reduce the grade through the tunnel unless the tunnel is very short. The additional atmospheric resistance and the decreased adhesion of the driver wheels on the damp rails in a tunnel will cause an engine to work very hard and still more rapidly vitiate the atmosphere until the accumulation of poisonous gases becomes a source of actual danger to the engineer and fireman of the locomotive and of extreme discomfort to the passengers. If the nominal ruling grade of the road were maintained through a tunnel, the maximum resistance would be

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\* Drinker's "Tunneling."



TUNNEL-TIMBERING—ENGLISH SYSTEM (*a*).

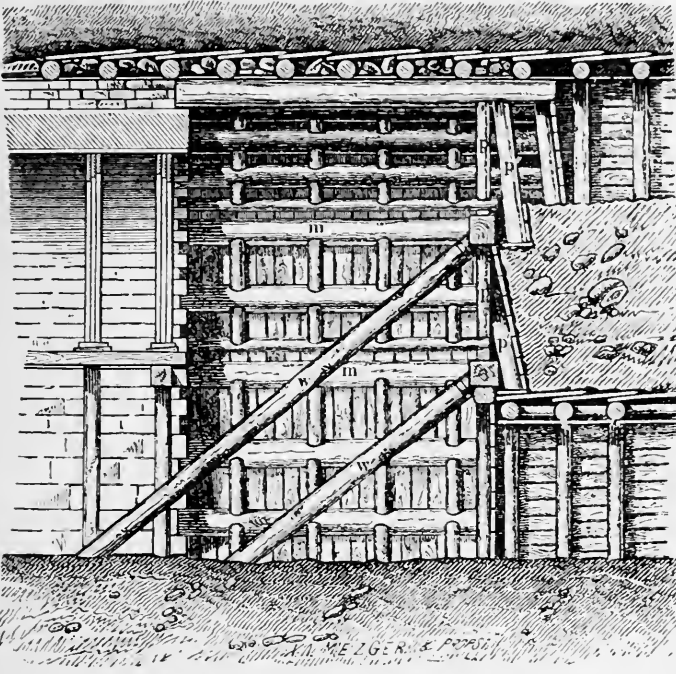


TUNNEL-TIMBERING—ENGLISH SYSTEM (*b*).

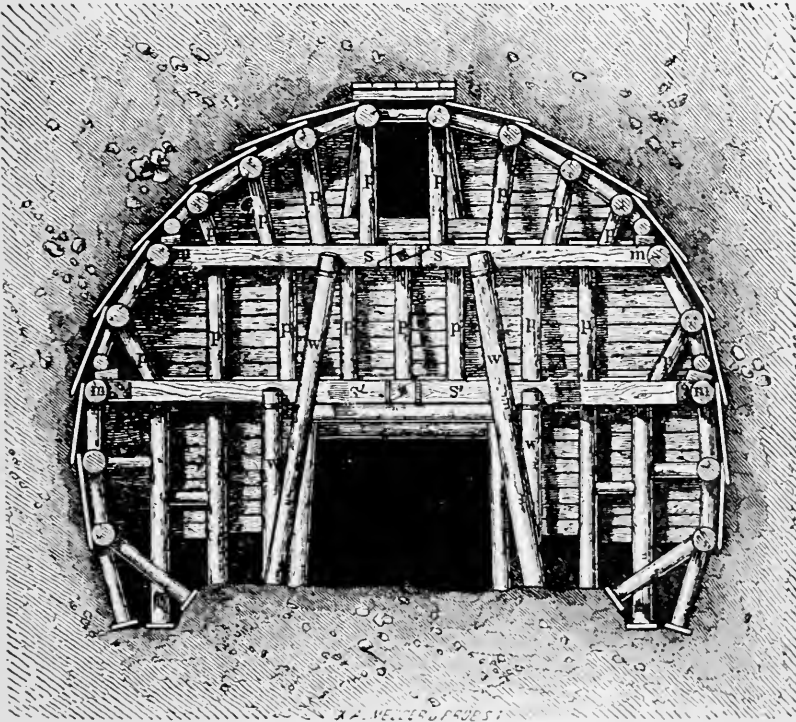
(To face page 192.)







TUNNEL-TIMBERING—ENGLISH SYSTEM (c).



TUNNEL-TIMBERING—ENGLISH SYSTEM (d).

(To face page 192.)



found in the tunnel. This would probably cause trains to stall there, which would be objectionable and perhaps dangerous.

**166. Lining.** It is a characteristic of many kinds of rock and of all earthy material that, although they may be self-sustaining when first exposed to the atmosphere, they rapidly disintegrate and require that the top and perhaps the sides and even the bottom shall be lined to prevent caving in. In this country, when timber is cheap, it is occasionally framed as an arch and used as the *permanent* lining, but masonry is always to be preferred. Frequently the cross-section is made extra large so that a masonry lining may subsequently be placed inside the wooden lining and thus postpone a large expense until the road is better able to pay for the work. In very soft unstable material, like quicksand, an arch of cut stone voussoirs may be necessary to withstand the pressure. A good quality of brick is occasionally used for lining, as they are easily handled and make good masonry if the pressure is not excessive. Only the best of cement mortar should be used, economy in this feature being the worst of folly. Of course the excavation must include the outside line of the lining. Any excavation which is made outside of this line (by the fall of earth or loose rock or by excessive blasting) must be refilled with stone well packed in. Occasionally it is necessary to fill these spaces with concrete. Of course it is not necessary that the lining be uniform throughout the tunnel.

**167. Shafts.** Shafts are variously made with square, rectangular, elliptical, and circular cross-sections. The rectangular cross-section, with the longer axis parallel with the tunnel, is most usually employed. Generally the shaft is directly over the center of the tunnel, but that always implies a complicated connection between the linings of the tunnel and shaft, provided such linings are necessary. It is easier to sink a shaft near to one side of the tunnel and make an opening through the nearly vertical side of the tunnel. Such a method was employed in the Church Hill Tunnel, illustrated in Fig. 90.\* Fig. 91 † shows

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\* Drinker's "Tunneling."

† Ržiha, "Lehrbuch der Gesamten Tunnelbaukunst."

a cross-section for a large main shaft. Many shafts have been built with the idea of being left open permanently for ventilation and have therefore been elaborately lined with masonry.

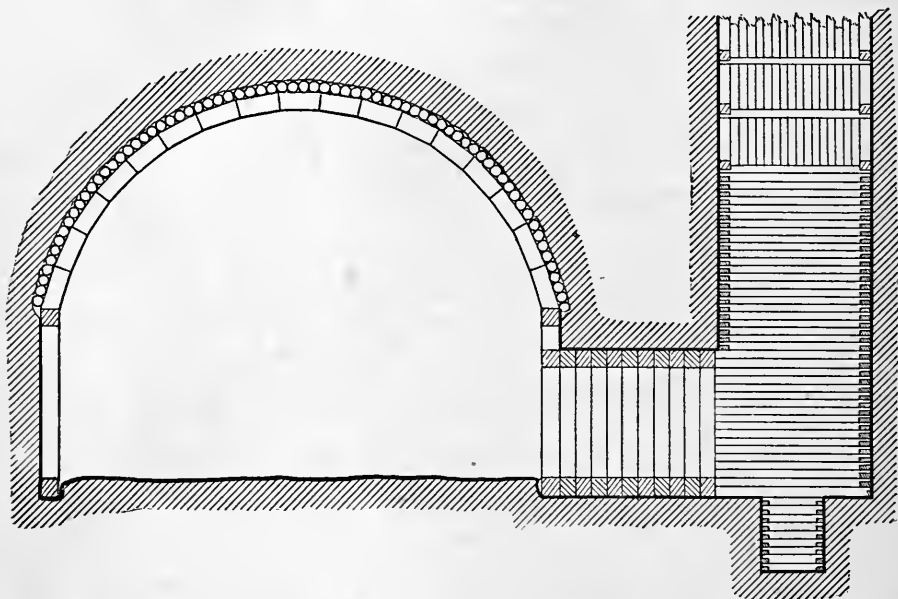


FIG. 90.—CONNECTION WITH SHAFT, CHURCH HILL TUNNEL.

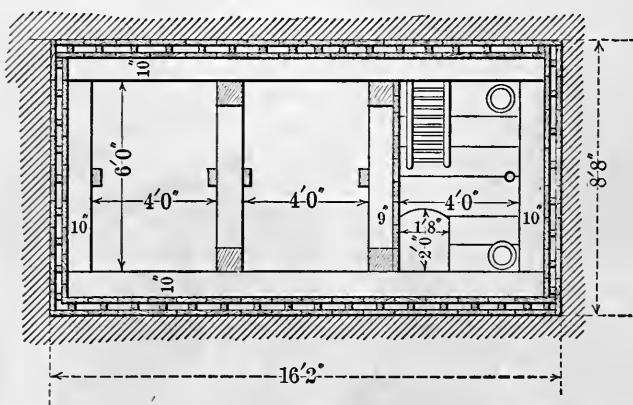


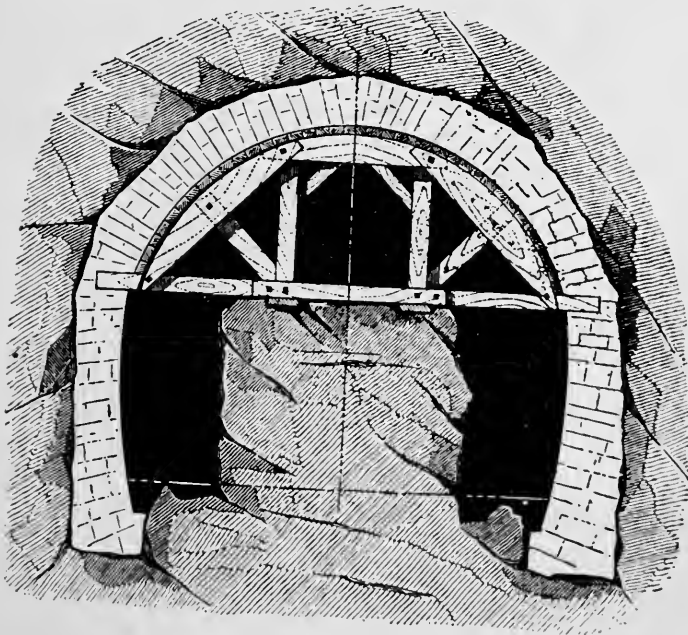
FIG. 91.—CROSS-SECTION, LARGE MAIN SHAFT.

The general consensus of opinion now appears to be that shafts are worse than useless for ventilation; that the quick passage of a train through the tunnel is the most effective ventilator; and that shafts only tend to produce cross-currents and are ineffective to clear the air. In consequence, many of these elaborately lined shafts have been permanently closed, and the more recent

PLATE IV.



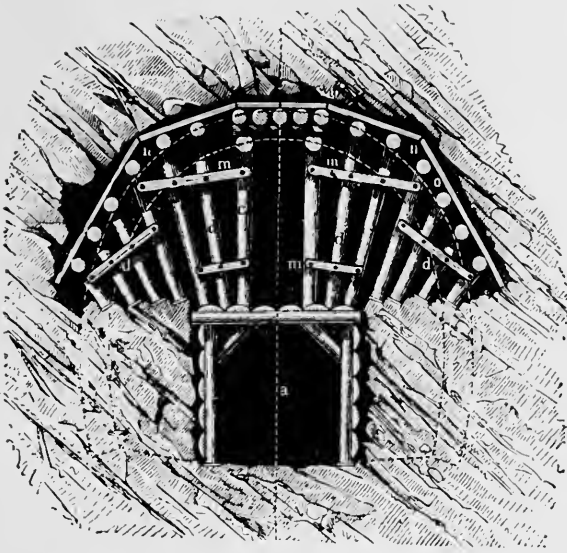
TUNNEL-TIMBERING—FRENCH SYSTEM (a).



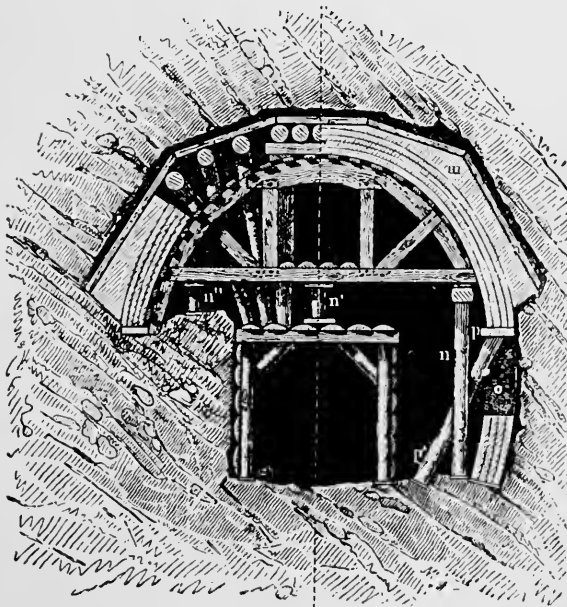
TUNNEL-TIMBERING—FRENCH SYSTEM (b).  
(To face page 194.)



PLATE V.



TUNNEL TIMBERING—BELGIAN SYSTEM (a).



TUNNEL-TIMBERING—BELGIAN SYSTEM (b).  
(To face page 194.)





practice is to close up a shaft as soon as the tunnel is completed. Shafts always form drainage-wells for the material they pass through, and sometimes to such an extent that it is a serious matter to dispose of the water that collects at the bottom, requiring the construction of large and expensive drains.

**168. Drains.** A tunnel will almost invariably strike veins of water which will promptly begin to drain into the tunnel and not only cause considerable trouble and expense during construction, but necessitate the provision of permanent drains for its perpetual disposal. These drains must frequently be so large as to appreciably increase the required cross-section of the tunnel. Generally a small open gutter on each side will suffice for this purpose, but in double-track tunnels a large covered drain is often built between the tracks. It is sometimes necessary to thoroughly grout the outside of the lining so that water will not force its way through the masonry and perhaps injure it, but may freely drain down the sides and pass through openings in the side walls near their base into the gutters.

#### CONSTRUCTION.

**169. Headings.** The methods of all tunnel excavation depend on the general principle that all earthy material, except the softest of liquid mud and quicksand, will be self-sustaining over a greater or less area and for a greater or less time after excavation is made, and the work consists in excavating some material and immediately propping up the exposed surface by timbering and poling-boards. The excavation of the cross-section begins with cutting out a "heading," which is a small horizontal drift whose breast is constantly kept 15 feet or more in advance of the full cross-sectional excavation. In solid self-sustaining rock, which will not decompose upon exposure to air, it becomes simply a matter of excavating the rock with the least possible expenditure of time and energy. In soft ground the heading must be heavily timbered, and as the heading is gradually enlarged the timbering must be gradually extended

and perhaps replaced, according to some regular system, so that when the full cross-section has been excavated it is supported by such timbering as is intended for it. The heading is sometimes made on the center line near the top; with other plans,

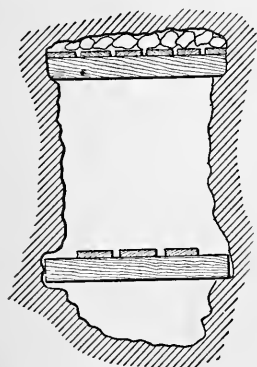


FIG. 92.

on the center line near the bottom; and sometimes two simultaneous headings are run in the two lower corners. Headings near the bottom serve the purpose of draining the material above it and facilitating the excavation. The simplest case of heading timbering is that shown in Fig. 92, in which cross-timbers are placed at intervals just under the roof, set in notches cut in the side walls and supporting poling-boards which sustain what-

ever pressure may come on them. Cross-timbers near the bottom support a flooring on which vehicles for transporting material may be run and under which the drainage may freely escape. As the necessity for timbering becomes greater, side timbers and even bottom timbers must be added, these timbers supporting poling-boards, and even the breast of the heading must be protected by boards suitably braced, as shown in Fig. 93. The

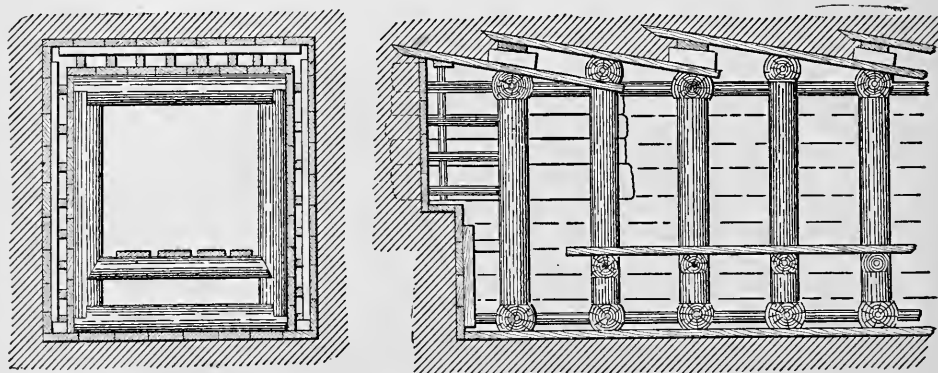
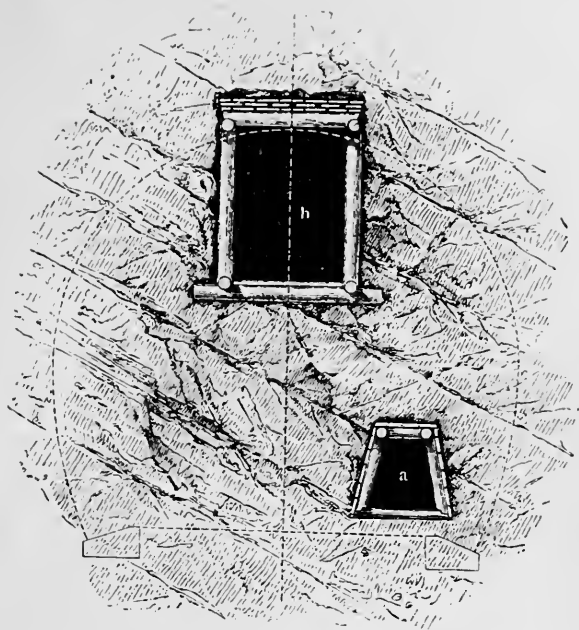


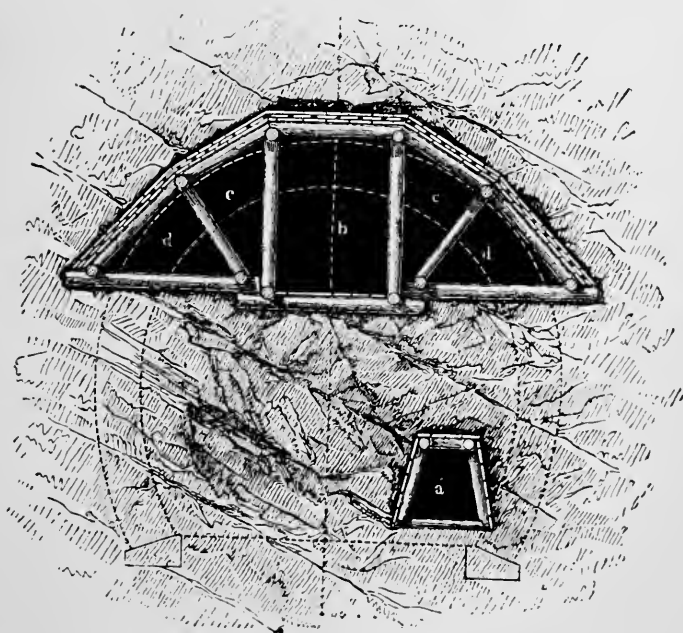
FIG 93 —TIMBERING FOR TUNNEL HEADING.

supporting timbers are framed into collars in such a manner that added pressure only increases their rigidity.

**170. Enlargement.** Enlargement is accomplished by removing the poling-boards, one at a time, excavating a greater or less

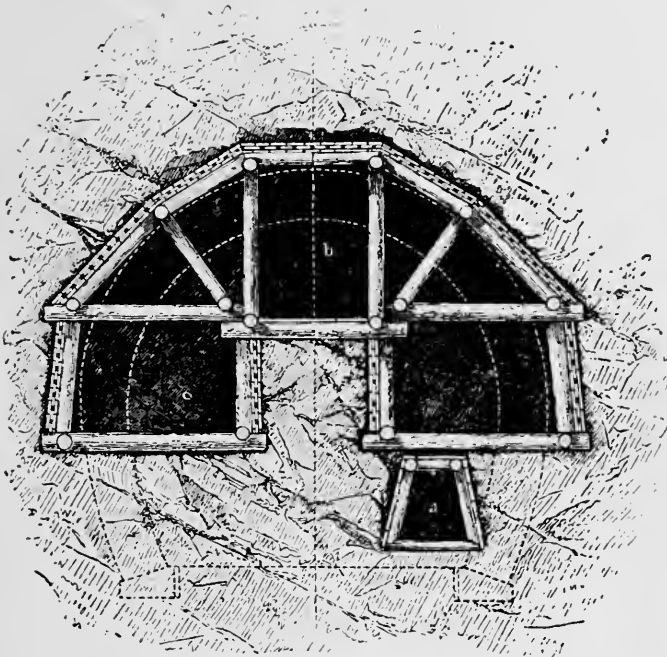


TUNNEL TIMBERING—GERMAN SYSTEM (*a*).

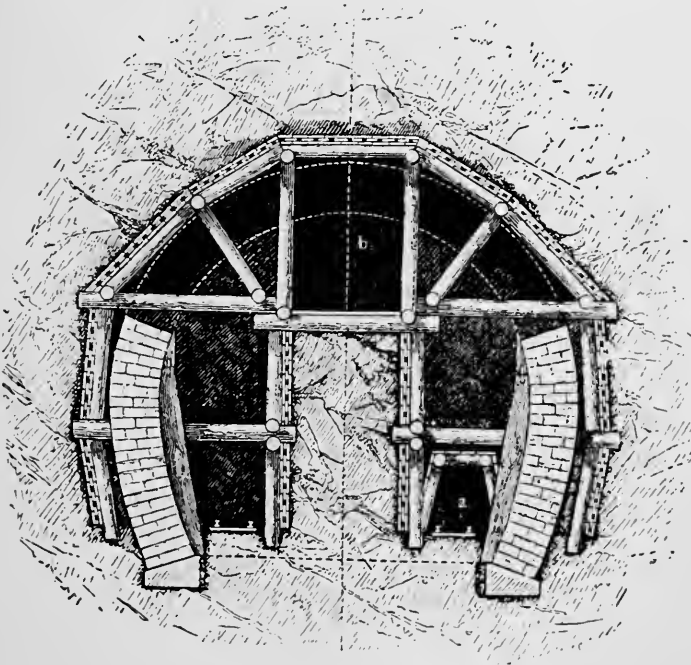


TUNNEL-TIMBERING—GERMAN SYSTEM (*b*).  
(*To face page 196.*)





TUNNEL-TIMBERING—GERMAN SYSTEM (c).



TUNNEL-TIMBERING—GERMAN SYSTEM (d)

(To face page 196.)



amount of material, and immediately supporting the exposed material with poling-boards suitably braced. (See Figs. 93 and 94.) This work being systematically done, space is thereby

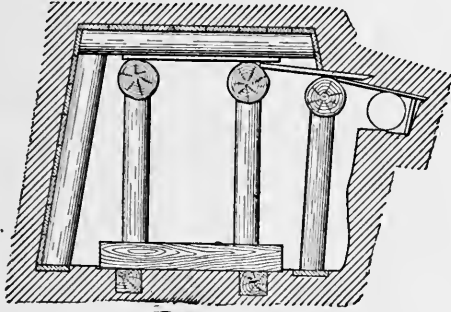


FIG. 94.

obtained in which the framing for the full cross-section may be gradually introduced. The framing is constructed with a cross-section so large that the masonry lining may be constructed within it.

#### 171. Distinctive features of various methods of construction.

There are six general systems, known as the English, German, Belgian, French, Austrian, and American. They are so named from the origin of the methods, although their use is not confined to the countries named. Fig. 95 shows by numbers (1 to 5) the order of the excavation within the cross-sections. The English, Austrian, and American systems are alike in excavating the entire cross-section before beginning the construction of the masonry lining. The German method leaves a solid core (5) until practically the whole of the lining is complete. This has the disadvantage of extremely cramped quarters for work, poor ventilation, etc. The Belgian and French methods agree in excavating the upper part of the section, building the arch at once, and supporting it temporarily until the side walls are built. The Belgian method then takes out the core (3), removes very short sections of the sides (4), immediately underpinning the arch with short sections of the side walls and thus gradually constructing the whole side wall. The French method digs out the sides (3), supporting the arch temporarily with timbers and

then replacing the timbers with masonry; the core (4) is taken out last. The French method has the same disadvantage as the German—working in a cramped space. The Belgian and French systems have the disadvantage that the arch, supported temporarily on timber, is very apt to be strained and cracked by the slight settlement that so frequently occurs in soft material. The English, Austrian, and American methods differ mainly in the

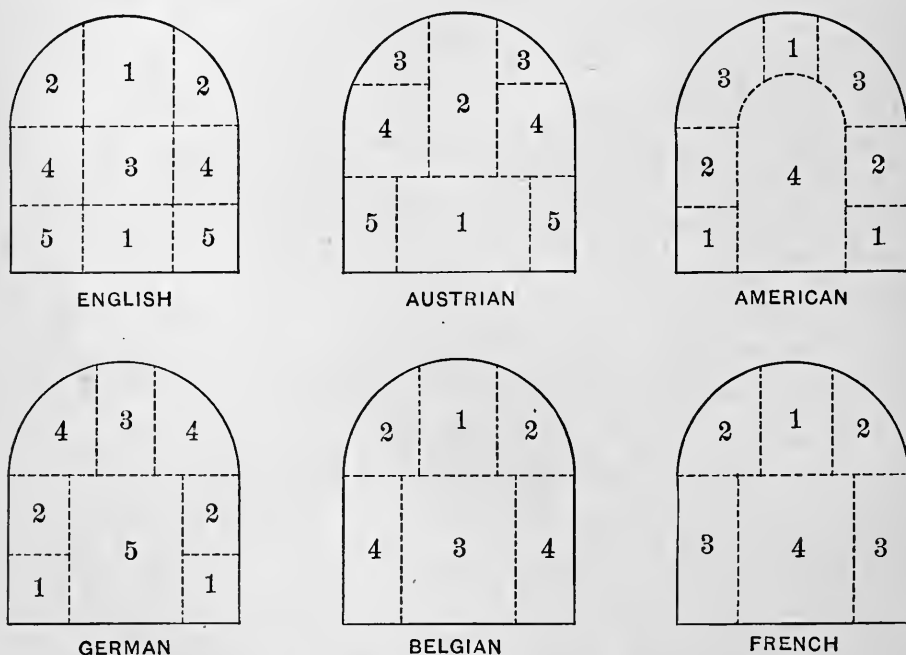
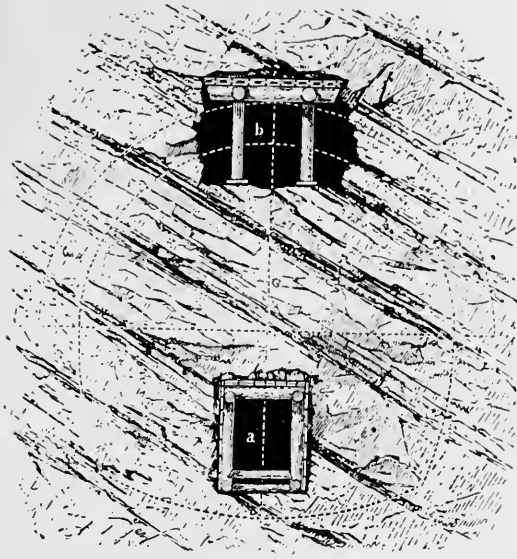


FIG. 95.—ORDER OF WORKING BY THE VARIOUS SYSTEMS.

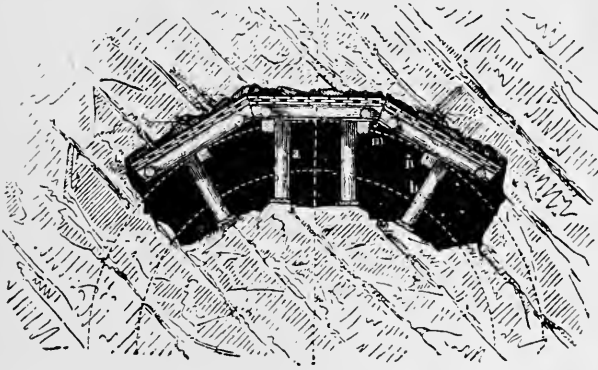
design of the timbering. The English support the roof by lines of very heavy *longitudinal* timbers which are supported at comparatively wide intervals by a heavy framework occupying the whole cross-section. The Austrian system uses such frequent cross-frames of timber-work that poling-boards will suffice to support the material between the frames. The American system agrees with the Austrian in using frequent cross-frames supporting poling-boards, but differs from it in that the “cross-frames” consist simply of arches of 3 to 15 wooden voussoirs, the voussoirs being blocks of 12"  $\times$  12" timber about 2 to 8 feet long and cut with joints normal to the arch. These arches are put together on a centering which is removed as soon as the arch



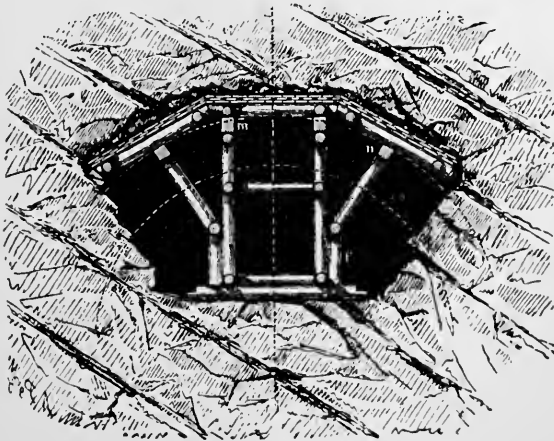
PLATE VIII.



TUNNEL-TIMBERING—AUSTRIAN SYSTEM (a).



TUNNEL-TIMBERING—AUSTRIAN SYSTEM (b).

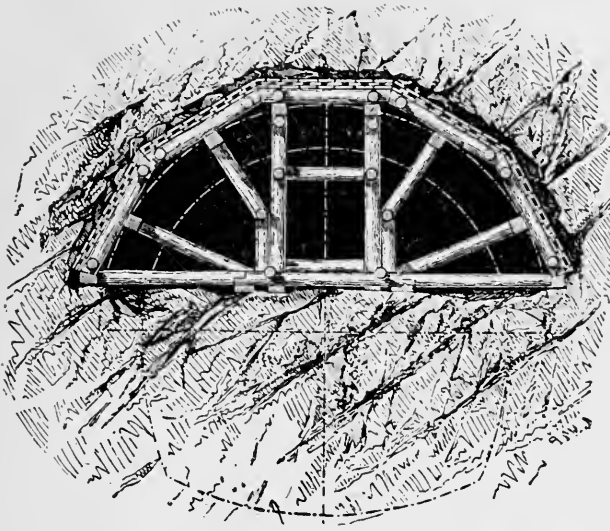


TUNNEL-TIMBERING—AUSTRIAN SYSTEM (c).

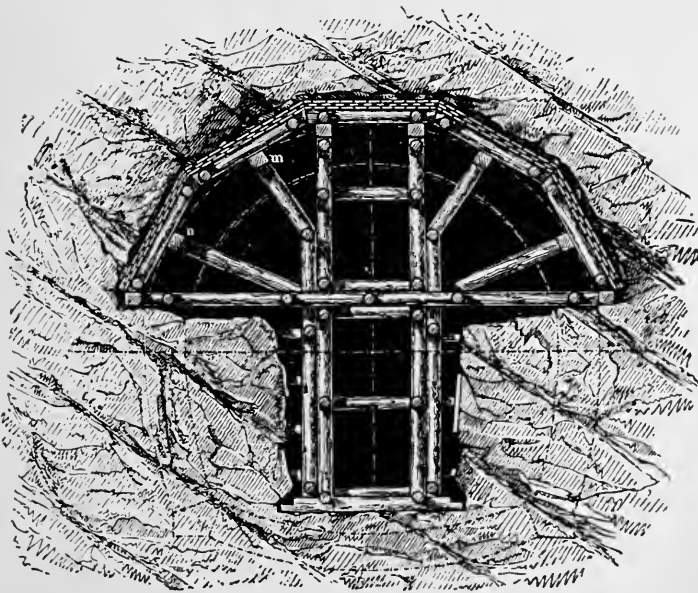
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PLATE IX.



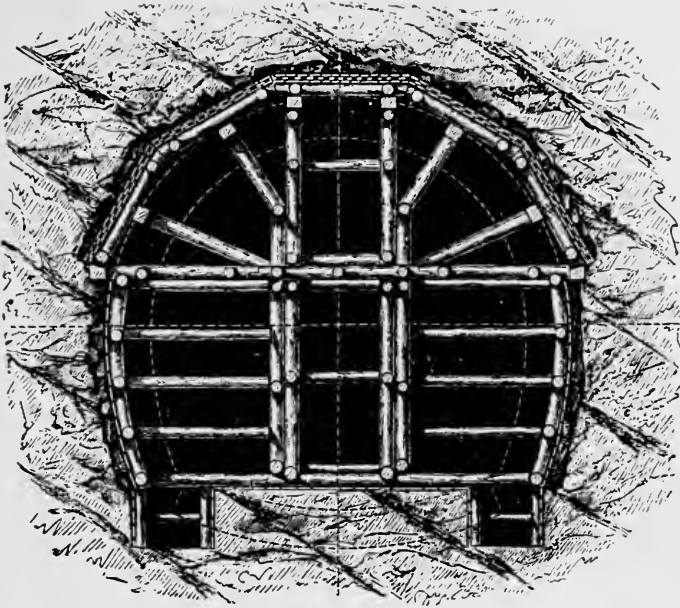
TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*d*).



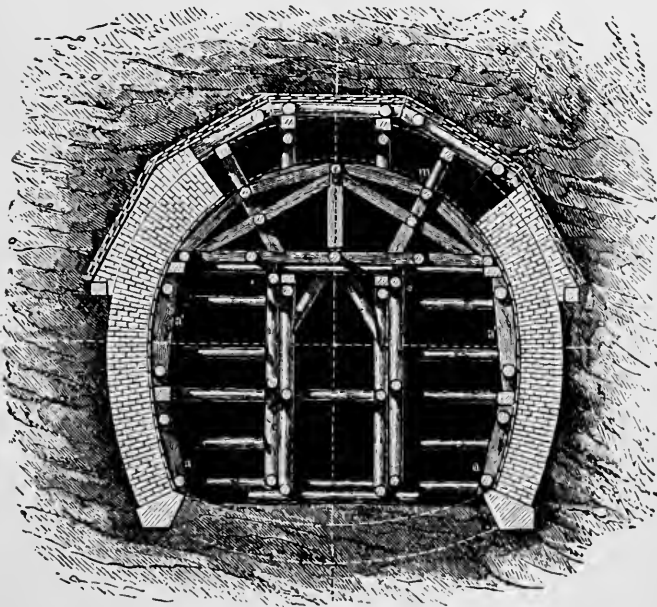
TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*e*).

(*To face page 198.*)





TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*f*).



TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*g*).

(*To face page 198* )



is keyed up and thus immediately opens up the full cross-section, so that the center core (4) may be immediately dug out and the masonry constructed in a large open space. The American system has been used successfully in very soft ground, but its advantages are greater in loose rock, when it is much cheaper than the other methods which employ more timber. Fig. 90 illustrates the use of the American system. The figure shows the wooden arch in place. The masonry arch may be placed when convenient, since it is *possible* to lay the track and commence traffic as soon as the wooden arch is in place. Plates II to XIV illustrate the methods of excavating and timbering by these various systems.

**172. Ventilation during construction.** Tunnels of any great length must be artificially ventilated during construction. If the excavated material is rock so that blasting is necessary, the need for ventilation becomes still more imperative. The invention of compressed-air drills simultaneously solved two difficulties. It introduced a motive power which is unobjectionable in its application (as gas would be), and it also furnished at the same time a supply of just what is needed—pure air. If no blasting is done (and sometimes even when there is blasting), air must be supplied by direct pumping. The cooling effect of the sudden expansion of compressed air only reduces the otherwise objectionably high temperature sometimes found in tunnels. Since pure air is being continually pumped in, the foul air is thereby forced out.

**173. Excavation for the portals.** Under normal conditions there is always a greater or less amount of open cut preceding and following a tunnel. Since all tunnel methods depend (to some slight degree at least) on the capacity of the exposed material to act as an arch, there is implied a considerable thickness of material above the tunnel. This thickness is reduced to nearly zero over the tunnel portals and therefore requires special treatment, particularly when the material is very soft. Fig. 96 \*

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\* Ržiha, "Lehrbuch der Gersamnten Tunnelbaukunst."

illustrates one method of breaking into the ground at a portal. The loose stones are piled on the framing to give stability to the framing by their weight and also to retain the earth on the slope above. Another method is to sink a temporary shaft to the tunnel near the portal; immediately enlarge to the full size and build the masonry lining; then work back to the portal.

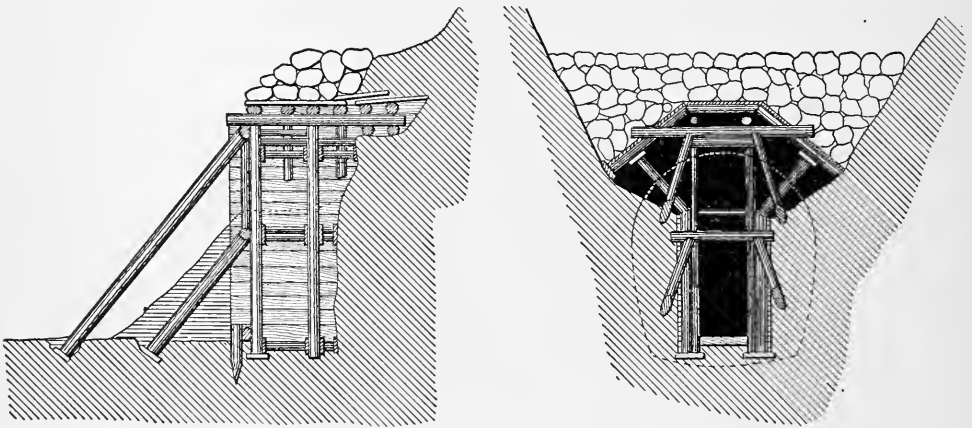


FIG. 96.—TIMBERING FOR TUNNEL PORTAL.

This method is more costly, but is preferable in very treacherous ground, it being less liable to cause landslides of the surface material.

**174. Tunnels vs. open cuts.** In cases in which an open cut rather than a tunnel is a possibility the ultimate consideration is generally that of first cost combined with other financial considerations and annual maintenance charges directly or indirectly connected with it. Even when an open cut may be constructed at the same cost as a tunnel (or perhaps a little cheaper) the tunnel may be preferable under the following conditions:

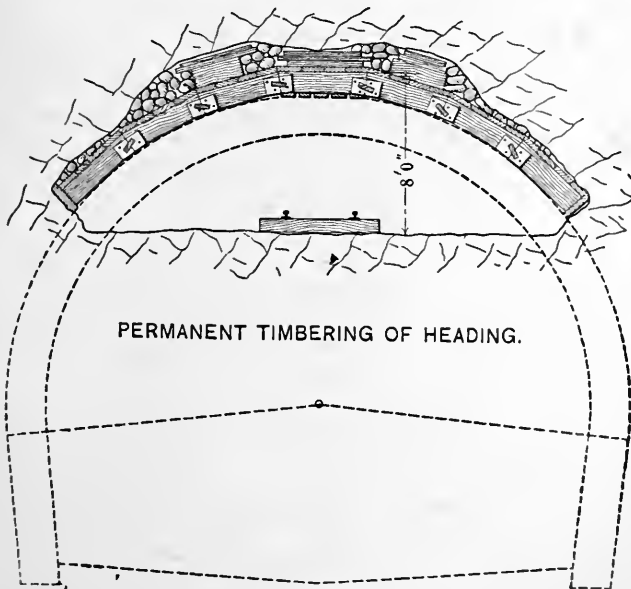
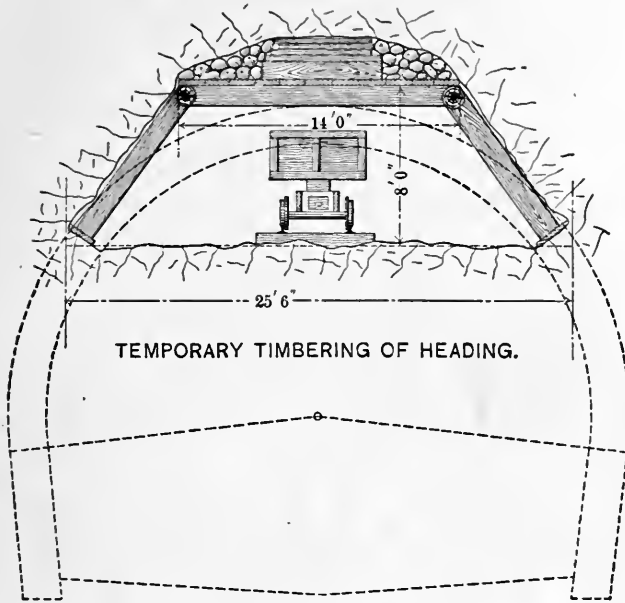
1. When the soil indicates that the open cut would be liable to landslides.

2. When the open cut would be subject to excessive snow-drifts or avalanches.

3. When land is especially costly or it is desired to run under existing costly or valuable buildings or monuments. When running through cities, tunnels are sometimes constructed as open cuts and then arched over.



PLATE XI.

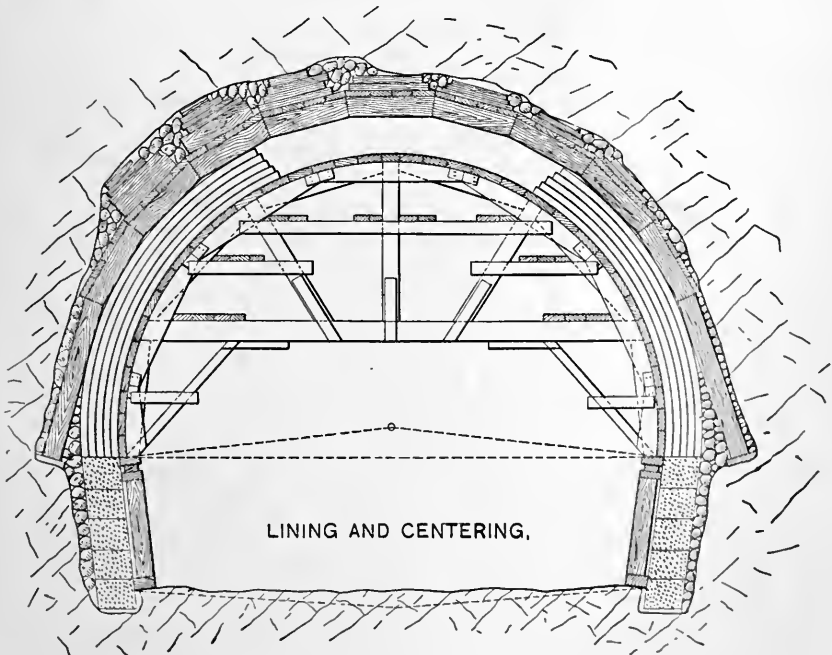
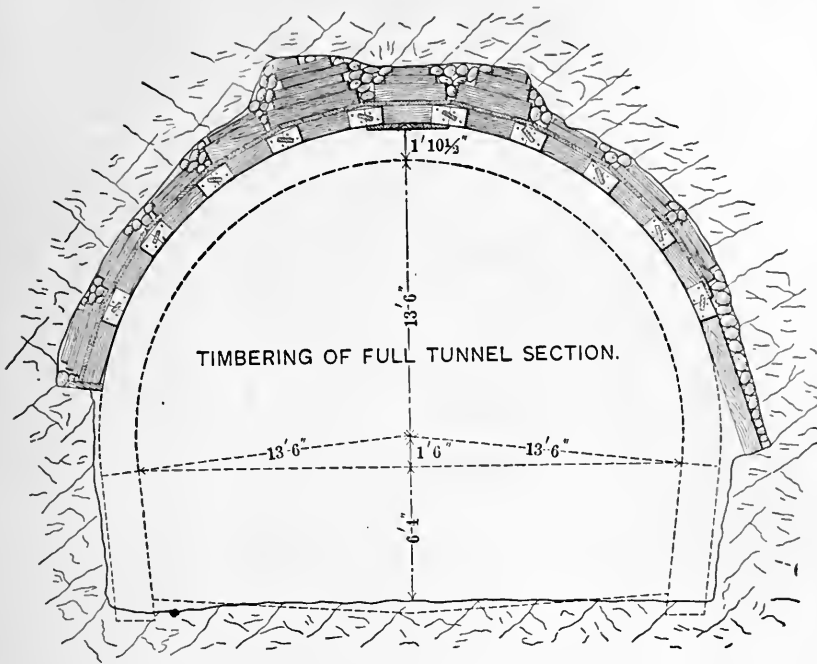


PHOENIXVILLE TUNNEL. P. S. V. R.R.

(To face page 200.)



PLATE XII.

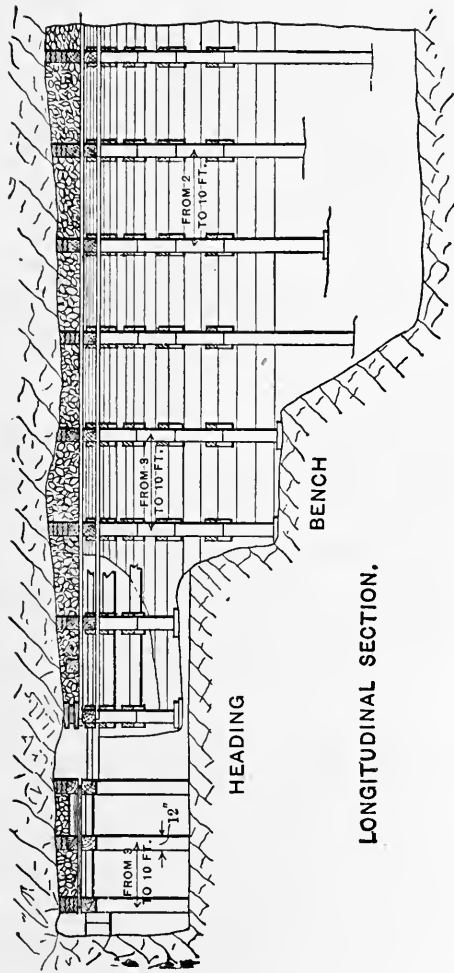


PHOENIXVILLE TUNNEL. P. S. V. R.R.

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# PLATE XIII.

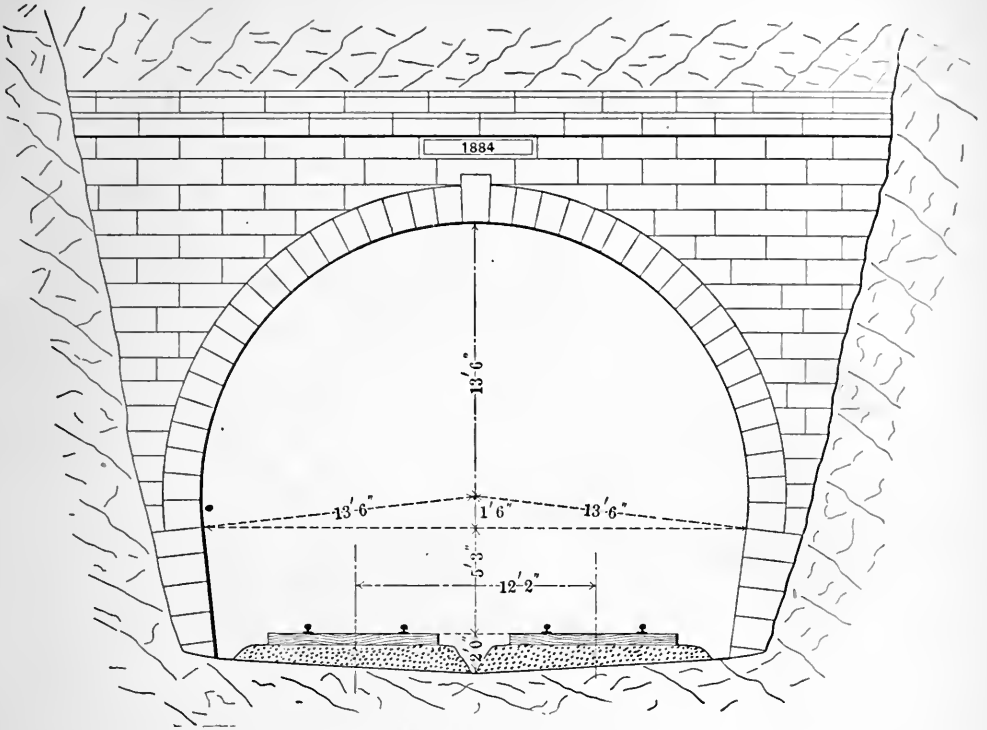


PHOENIXVILLE TUNNEL. P. S. V. R.R.

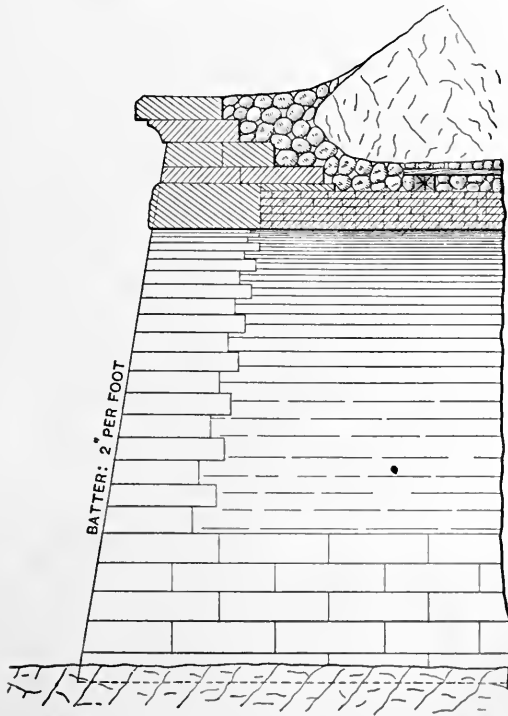
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PLATE XIV.



ELEVATION OF PORTAL.



LONGITUDINAL SECTION OF PORTAL.  
PHENIXVILLE TUNNEL. P. S. V. R.R.

(To face page 200.)





These cases apply to tunnels *vs.* open cuts when the alignment is fixed by other considerations than the mere topography. The broader question of excavating tunnels to avoid excessive grades or to save distance or curvature, and similar problems, are hardly susceptible of general analysis except as questions of railway economics and must be treated individually.

**175. Cost of tunneling.** The cost of any construction which involves such uncertainties as tunneling is very variable. It depends on the material encountered, the amount and kind of timbering required, on the size of the cross-section, on the price of labor, and especially on the *reconstruction* that *may* be necessary on account of mishaps.

Headings generally cost \$4 to \$5 per cubic yard for excavation, while the remainder of the cross-section in the same tunnel may cost about half as much. The average cost of a large number of tunnels in this country may be seen from the following table: \*

| Material.          | Cost per cubic yard. |         |          |         | Cost per lineal foot. |          |
|--------------------|----------------------|---------|----------|---------|-----------------------|----------|
|                    | Excavation.          |         | Masonry. |         | Single.               | Double.  |
|                    | Single.              | Double. | Single.  | Double. |                       |          |
| Hard rock. . . . . | \$5.89               | \$5.45  | \$12.00  | \$ 8.25 | \$ 69.76              | \$142.82 |
| Loose rock. . . .  | 3.12                 | 3.48    | 9.07     | 10.41   | 80.61                 | 119.26   |
| Soft ground. . .   | 3.62                 | 4.64    | 15.00    | 10.50   | 135.31                | 174.42   |

A considerable variation from these figures may be found in individual cases, due sometimes to unusual skill (or the lack of it) in prosecuting the work, but the figures will generally be sufficiently accurate for preliminary estimates or for the comparison of two proposed routes.

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\* Figures derived from Drinker's "Tunneling."

## CHAPTER VI.

### CULVERTS AND MINOR BRIDGES.

**176. Definition and object.** Although a variable percentage of the rain falling on any section of country soaks into the ground and does not immediately reappear, yet a very large percentage flows over the surface, always seeking and following the lowest channels. The roadbed of a railroad is constantly intersecting these channels, which frequently are normally dry. In order to prevent injury to railroad embankments by the impounding of such rainfall, it is necessary to construct waterways through the embankment through which such rainflow may freely pass. Such waterways, called culverts, are also applicable for the bridging of very small although perennial streams, and therefore in this work the term culvert will be applied to all water-channels passing through a railroad embankment which are not of sufficient magnitude to require a special structural design, such as is necessary for a large masonry arch or a truss bridge.

**177. Elements of the design.** A well-designed culvert must afford such free passage to the water that it will not "back up" over the adjoining land nor cause any injury to the embankment or culvert. The ability of the culvert to discharge freely all the water that comes to it evidently depends chiefly on the area of the waterway, but also on the form, length, slope, and materials of construction of the culvert and the nature of the approach and outfall. When the embankment is very low and the amount of water to be discharged very great, it sometimes becomes necessary to allow the water to discharge "under a head," i.e.,

with the surface of the water above the top of the culvert. Safety then requires a much stronger construction than would otherwise be necessary to avoid injury to the culvert or embankment by washing. The necessity for such construction should be avoided if possible.

#### AREA OF THE WATERWAY.

**178. Elements involved.** The determination of the required area of the waterway involves such a multiplicity of indeterminate elements that any close determination of its value from purely theoretical considerations is a practical impossibility. The principal elements involved are:

**a. Rainfall.** The real test of the culvert is its capacity to discharge without injury the flow resulting from the extraordinary rainfalls and "cloud bursts" that may occur once in many years. Therefore, while a knowledge of the average annual rainfall is of very little value, a record of the maximum rainfall during heavy storms for a long term of years may give a relative idea of the maximum demand on the culvert.

**b. Area of watershed.** This signifies the total area of country draining into the channel considered. When the drainage area is very small it is sometimes included within the area surveyed by the preliminary survey. When larger it is frequently possible to obtain its area from other maps with a percentage of accuracy sufficient for the purpose. Sometimes a special survey for the purpose is considered justifiable.

**c. Character of soil and vegetation.** This has a large influence on the rapidity with which the rainflow from a given area will reach the culvert. If the soil is hard and impermeable and the vegetation scant, a heavy rain will run off suddenly, taxing the capacity of the culvert for a short time, while a spongy soil and dense vegetation will retard the flow, making it more nearly uniform and the maximum flow at any one time much less.

**d. Shape and slope of watershed.** If the watershed is very long and narrow (other things being equal), the water from the

remoter parts will require so much longer time to reach the culvert that the flow will be comparatively uniform, especially when the slope of the whole watershed is very low. When the slope of the remoter portions is quite steep it may result in the nearly simultaneous arrival of a storm-flow from all parts of the watershed, thus taxing the capacity of the culvert.

**e. Effect of design of culvert.** The principles of hydraulics show that the slope of the culvert, its length, the form of the cross-section, the nature of the surface, and the form of the approach and discharge all have a considerable influence on the area of cross-section required to discharge a given volume of water in a given time, but unfortunately the combined hydraulic effect of these various details is still a very uncertain quantity.

**179. Methods of computation of area.** There are three possible methods of computation.

**(a) Theoretical.** As shown above it is a practical impossibility to estimate correctly the combined effect of the great multiplicity of elements which influence the final result. The nearest approach to it is to estimate by the use of empirical formulæ the amount of water which will be presented at the upper end of the culvert in a given time and then to compute, from the principles of hydraulics, the rate of flow through a culvert of given construction, but (as shown in § 178, *e*) such methods are still very unreliable, owing to lack of experimental knowledge. This method has *apparently* greater scientific accuracy than other methods, but a little study will show that the elements of uncertainty are as great and the final result no more reliable. The method is most reliable for streams of uniform flow, but it is under these conditions that method (*c*) is most useful. The theoretical method will not therefore be considered further.

**(b) Empirical.** As illustrated in § 180, some formulæ make the area of waterway a function of the drainage area, the formula being affected by a coefficient the value of which is estimated between limits according to the judgment. Assuming that the formulæ are sound, their use only narrows the limits of

error, the final determination depending on experience and judgment.

(c) **From observation.** This method, considered by far the best for permanent work, consists in observing the high-water marks on contracted channel-openings which are on the same stream and as near as possible to the proposed culvert. If the country is new and there are no such openings, the wisest plan is to bridge the opening by a temporary structure in wood which has an ample waterway (see § 126, *b*, 4) and carefully observe all high-water marks on that opening during the 6 to 10 years which is ordinarily the minimum life of such a structure. As shown later, such observations may be utilized for a close computation of the required waterway. Method (*b*) may be utilized for an approximate calculation for the required area for the temporary structure, using a value which is intentionally excessive, so that a permanent structure of sufficient capacity may subsequently be constructed *within* the temporary structure.

**180. Empirical formulæ.** Two of the best known empirical formulæ for area of the waterway are the following:

(a) **Myer's formula:**

Area of waterway in square feet =  $C \times \sqrt{\text{drainage area in acres}}$ , where  $C$  is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground. As an illustration, if the drainage area is 100 acres, the waterway area should be from 10 to 40 square feet, according to the value of the coefficient chosen. It should be noted that this formula does not regard the great variations in rainfall in various parts of the world nor the design of the culvert, and also that the final result depends largely on the choice of the coefficient.

(b) **Talbot's formula:**

Area of waterway in square feet =  $C \times \sqrt[4]{(\text{drainage area in acres})^3}$ . "For steep and rocky ground  $C$  varies from  $\frac{2}{3}$  to 1. For rolling agricultural country subject to floods at times of melting snow, and with the length of the valley three or four times its width,  $C$  is about  $\frac{1}{3}$ ; and if the stream is longer in proportion to the area, decrease  $C$ . In districts not affected by accumulated snow, and

where the length of the valley is several times the width,  $\frac{1}{5}$  or  $\frac{1}{6}$ , or even less, may be used.  $C$  should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert." \* As an illustration, if the drainage area is 100 acres the area of waterway should be  $C \times 31.6$ . The area should then vary from 5 to 31 square feet, according to the character of the country. Like the previous estimate, the result depends on the choice of a coefficient and disregards local variations in rainfall, except as they may be arbitrarily allowed for in choosing the coefficient.

**181. Value of empirical formulæ.** The fact that these formulæ, as well as many others of similar nature that have been suggested, depend so largely upon the choice of the coefficient shows that they are valuable "more as a guide to the judgment than as a working rule," as Prof. Talbot explicitly declares in commenting on his own formula. In short, they are chiefly valuable in indicating a probable maximum and minimum between which the true result probably lies.

**182. Results based on Observation.** As already indicated in § 179, observation of the stream in question gives the most reliable results. If the country is new and no records of the flow of the stream during heavy storms has been taken, even the life of a temporary wooden structure may not be long enough to include one of the unusually severe storms which must be allowed for, but there will usually be some high-water mark which will indicate how much opening will be required. The following quotation illustrates this: "A tidal estuary may generally be safely narrowed considerably from the extreme water lines if stone revetments are used to protect the bank from wash. Above the true estuary, where the stream cuts through the marsh, we generally find nearly vertical banks, and we are safe if the faces of abutments are placed even with the banks. In level sections of the country, where the current is sluggish, it is usually safe to encroach somewhat on the

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\* Prof. A. N. Talbot, "Selected Papers of the Civil Engineers' Club of the Univ. of Illinois."

general width of the stream, but in rapid streams among the hills the width that the stream has cut for itself through the soil should not be lessened, and in ravines carrying mountain torrents the openings must be left very much larger than the ordinary appearance of the banks of the stream would seem to make necessary.” \*

As an illustration of an observation of a storm-flow through a temporary trestle, the following is quoted: “Having the flood height and velocity, it is an easy matter to determine the volume of water to be taken care of. I have one ten-bent pile trestle 135 feet long and 24 feet high over a spring branch that ordinarily runs about six cubic inches per second. Last summer during one of our heavy rainstorms (four inches in less than three hours) I visited this place and found by float observations the surface velocity at the highest stage to be 1.9 feet per second. I made a high-water mark, and after the flood-water receded found the width of stream to be 12 feet and an average depth of  $2\frac{3}{4}$  feet. This, with a surface velocity of 1.9 feet per second, would give approximately a discharge of 50 cubic feet, or 375 gallons, per second. Having this information it is easy to determine size of opening required.” †

**183. Degree of accuracy required.** The advantages resulting from the use of standard designs for culverts (as well as other structures) have led to the adoption of a comparatively small number of designs. The practical use made of a computation of required waterway area is to determine which one of several standard designs will most nearly fulfill the requirements. For example, if a 24-inch iron pipe, having an area of 3.14 square feet, is considered to be a little small, the next size (30-inch) would be adopted; but a 30-inch pipe has an area of 4.92 square feet, which is 56% larger. A similar result, except that the percentage of difference might not be quite so marked,

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\* J. P. Snow, Boston & Maine Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

† A. J. Kelley, Kansas City Belt Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

will be found by comparing the areas of consecutive standard designs for stone box culverts.

The advisability of designing a culvert to withstand any storm-flow that may *ever* occur is considered doubtful. Several years ago a record-breaking storm in New England carried away a very large number of bridges, etc., hitherto supposed to be safe. It was not afterward considered that the design of those bridges was faulty, because the extra cost of constructing bridges capable of withstanding such a flood, added to interest for a long period of years, would be enormously greater than the cost of repairing the damages of such a storm once or twice in a century. Of course the element of danger has some weight, but not enough to justify a great additional expenditure, for common prudence would prompt unusual precautions during or immediately after such an extraordinary storm.

#### PIPE CULVERTS.

**184. Advantages.** Pipe culverts, made of cast iron or earthenware, are very durable, readily constructed, moderately cheap, will pass a larger volume of water in proportion to the area than many other designs on account of the smoothness of the surface, and (when using iron pipe) may be used very close to the track when a low opening of large capacity is required. Another advantage lies in the ease with which they may be inserted through a somewhat larger opening that has been temporarily lined with wood, without disturbing the roadbed or track.

**185. Construction.** Permanency requires that the foundation shall be firm and secure against being washed out. To accomplish this, the soil of the trench should be hollowed out to fit the lower half of the pipe, making suitable recesses for the bells. In very soft treacherous soil a foundation-block of concrete is sometimes placed under each joint, or even throughout the whole length. When pipes are laid through a slightly larger timber culvert great care should be taken that the pipes are properly supported, so that there will be no settling nor



development of unusual strains when the timber finally decays and gives way. To prevent the washing away of material around the pipe the ends should be protected by a bulkhead. This is best constructed of masonry (see Fig. 97), although wood is sometimes used for cheap and minor constructions. The joints should be calked, especially when the culvert is liable to run full or when the outflow is impeded and the culvert is liable to be partly or wholly filled during freezing weather. The cost of a calking of clay or even hydraulic cement is insignificant compared with the value of the additional safety afforded. When the grade of the pipe is perfectly uniform, a very low rate of grade will suffice to drain a pipe culvert, but since some unevenness of grade is inevitable through uneven settlement or imperfect construction, a grade of 1 in 20 should preferably be required, although much less is often used. The length of a pipe culvert is approximately determined as follows:

$$\text{Length} = 2s \text{ (depth of embankment to top of pipe) + (width of roadbed),}$$

in which  $s$  is the slope ratio (horizontal to vertical) of the banks. In practice an even number of lengths will be used which will most nearly agree with this formula.

**186. Iron-pipe culverts.** Simple cast-iron pipes are used in sizes from 12" to 48" diameter. These are usually made in lengths of 12 feet with a few lengths of 6 feet, so that any required length may be more nearly obtained. The lightest pipes made are sufficiently strong for the purpose, and even those which would be rejected because of incapacity to withstand pressure may be utilized for this work. In Fig. 97 are shown the standard plans used on the C. C. C. & St. L. Ry., which may be considered as typical plans.

Pipes formed of cast-iron segments have been used up to 12 feet diameter. The shell is then made comparatively thin, but is stiffened by ribs and flanges on the outside. The segments break joints and are bolted together through the flanges. The joints are made tight by the use of a tarred rope, together with neat cement.



**187. Tile-pipe culverts.** The pipes used for this purpose vary from 12" to 24" in diameter. When a larger capacity is required two or more pipes may be laid side by side, but in such a case another design might be preferable. It is frequently specified that "double-strength" or "extra-heavy" pipe shall be used, evidently with the idea that the stresses on a culvert-pipe are greater than on a sewer-pipe. But it has been conclusively demonstrated that, no matter how deep the embankment, the pressure cannot exceed a somewhat uncertain maximum, also that the greatest danger consists in placing the pipe so near the ties that shocks may be directly transferred to the pipe without the cushioning effect of the earth and ballast. When the pipes are well bedded in *clear* earth and there is a sufficient depth of earth over them to avoid direct impact (at least three feet) the ordinary sewer-pipe will be sufficiently strong. "Double-strength" pipe is frequently less perfectly burned, and

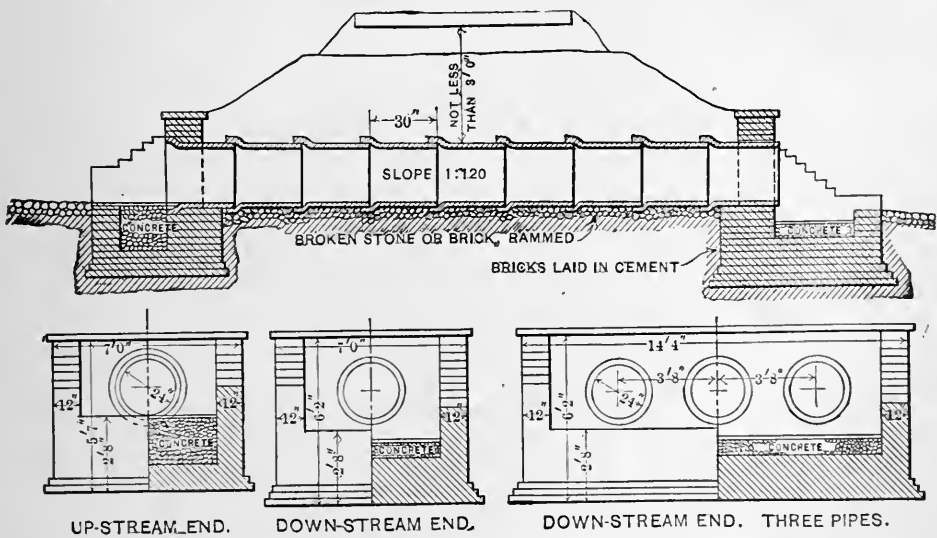


FIG. 98.—STANDARD VITRIFIED-PIPE CULVERT. PLANT SYSTEM. (1891.)

the supposed extra strength is not therefore obtained. In Fig. 98 are shown the standard plans for vitrified-pipe culverts as used on the "Plant system." Tile pipe is much cheaper than iron pipe, but is made in much shorter lengths and requires much more work in laying and especially to obtain a uniform grade.

## BOX CULVERTS.

**188. Wooden box culverts.** This form serves the purpose of a cheap temporary construction which allows the use of a ballasted roadbed. As in all temporary constructions, the area should be made considerably larger than the calculated area (§§ 179–182), not only for safety but also in order that, if the smaller area is demonstrated to be sufficiently large, the permanent construction (probably pipe) may be placed inside without disturbing the embankment. All designs agree in using heavy timbers ( $12'' \times 12''$ ,  $10'' \times 12''$ , or  $8'' \times 12''$ ) for the side walls, cross-timbers for the roof, every fifth or sixth timber being notched down so as to take up the thrust of the side walls, and planks for the flooring. Fig. 99 shows some of the standard designs as used by the C., M. & St. P. Ry.

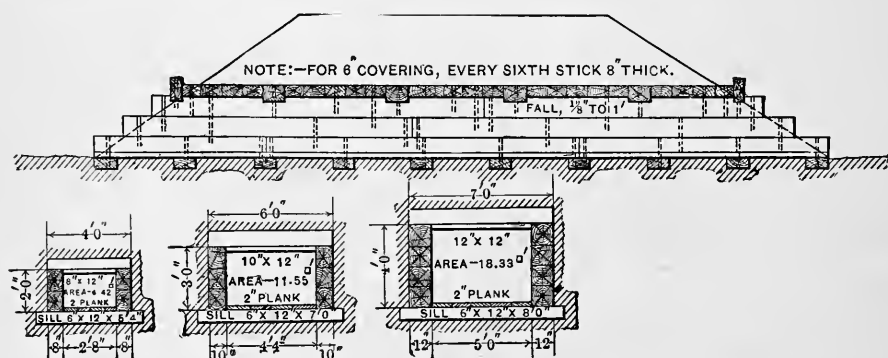


FIG. 99.—STANDARD TIMBER BOX CULVERT. C., M. & ST. P. RY. (Feb. 1889.)

**189. Stone box culverts.** In localities where a *good* quality of stone is cheap, stone box culverts are the cheapest form of permanent construction for culverts of medium capacity, but their use is decreasing owing to the frequent difficulty in obtaining really suitable stone within a reasonable distance of the culvert. The clear span of the cover-stones varies from 2 to 4 feet. The required thickness of the cover-stones is sometimes calculated by the theory of transverse strains on the basis of certain assumptions of loading—as a function of the height of the embankment and the unit strength of the stone used. Such a method is simply another illustration of a class of calculations.

which look very precise and beautiful, but which are worse than useless (because misleading) on account of the hopeless uncertainty as to the true value of certain quantities which must be used in the computations. In the first place the true value of the unit tensile strength of stone is such an uncertain and variable quantity that calculations based on any assumed value for it are of small reliability. In the second place the weight of the prism of earth lying directly above the stone, plus an allowance for live load, is by no means a measure of the load on the stone nor of the forces that tend to fracture it. All earthwork will tend to form an arch above any cavity and thus relieve an uncertain and probably variable proportion of the pressure that might otherwise exist. The higher the embankment the *less* the *proportionate* loading, until at some uncertain height an increase in height will not increase the load on the cover-stones. The effect of frost is likewise large, but uncertain and not computable. The usual practice is therefore to make the thickness such as experience has shown to be safe with a good quality of stone, i.e., about 10 or 12 inches for 2 feet span and up to 16 or 18 inches for 4 feet span. The side walls should be carried down deep enough to prevent their being undermined by scour or heaved by frost. The use of cement mortar is also an important feature of first-class work, especially when there is a rapid scouring current or a liability that the culvert will run under a head. In Fig. 100 are shown standard plans for single and double stone box culverts as used on the Norfolk and Western R.R.

**190. Old-rail culverts.** It sometimes happens (although very rarely) that it is necessary to bring the grade line within 3 or 4 feet of the bottom of a stream and yet allow an area of 10 or 12 square feet. A single large pipe of sufficient area could not be used in this case. The use of several smaller pipes side by side would be both expensive and inefficient. For similar reasons neither wooden nor stone box culverts could be used. In such cases, as well as in many others where the head-room is not so limited, the plan illustrated in Fig. 101 is a very satisfactory solution of the problem. The old rails, having a length of 8 or



9 feet, are laid close together across a 6-foot opening. Sometimes the rails are held together by long bolts passing through the webs of the rails. In the plan shown the rails are confined

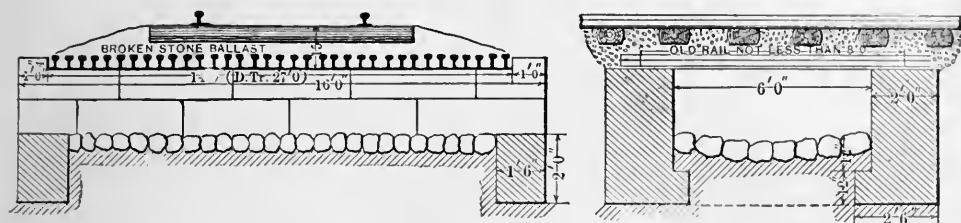


FIG. 101.—STANDARD OLD-RAIL CULVERT. N. & W. R.R. (1895.)

by low end walls on each abutment. This plan requires only 15 inches between the base of the rail and the top of the culvert channel. It also gives a continuous ballasted roadbed.

#### ARCH CULVERTS.

**191. Influence of design on flow.** The variations in the design of arch culverts have a very marked influence on the cost and efficiency. To combine the least cost with the greatest efficiency, due weight should be given to the following elements: (*a*) the amount of masonry, (*b*) the simplicity of the constructive work, (*c*) the design of the wing walls, (*d*) the design of the junction of the wing walls with the barrel

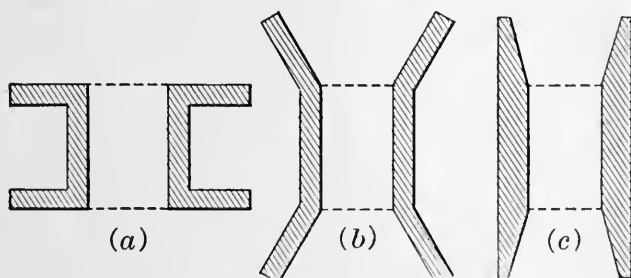


FIG. 102.—TYPES OF CULVERTS.

and faces of the arch, and (*e*) the safety and permanency of the construction. These elements are more or less antagonistic to each other, and the defects of most designs are due to a lack of proper proportion in the design of these opposing interests. The simplest construction (satisfying elements *b* and *c*) is the straight

barrel arch between two parallel vertical head walls, as sketched in Fig. 102, *a*. From a hydraulic standpoint the design is poor, as the water eddies around the corners, causing a great resistance which decreases the flow. Fig. 102, *b*, shows a much better design in many respects, but much depends on the details of the design as indicated in elements (*b*) and (*d*). As a general thing a good hydraulic design requires complicated and expensive masonry construction, i.e., elements (*b*) and (*d*) are opposed. Design 102, *c*, is sometimes inapplicable because the water is liable to work in behind the masonry during floods and perhaps cause scour. This design uses less masonry than (*a*) or (*b*).

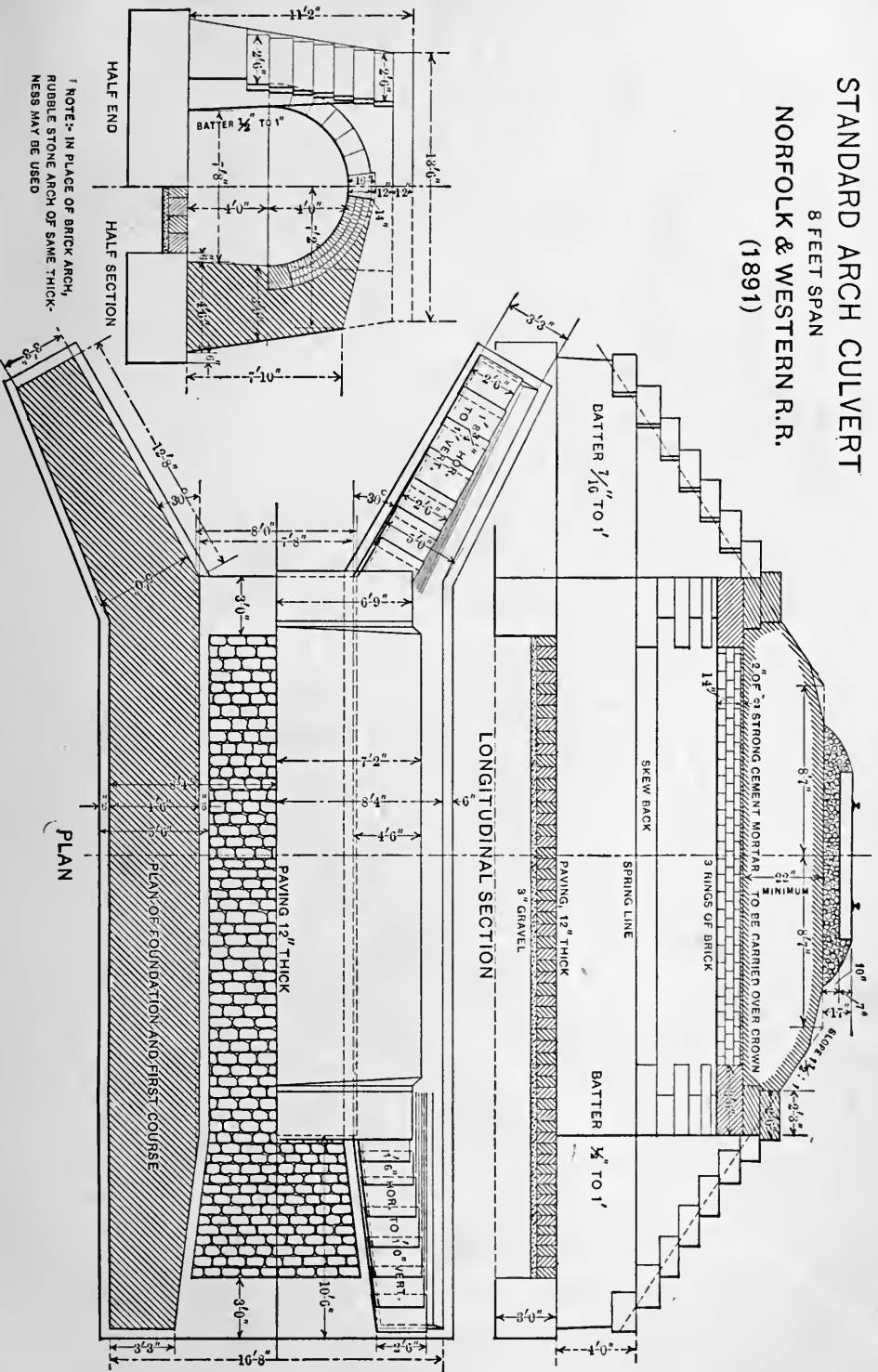
**192. Example of arch culvert design.** In Plate XV is shown the design for an 8-foot arch culvert according to the standard of the Norfolk and Western R.R. Note that the plan uses the flaring wing walls (Fig. 102, *b*) on the up-stream side (thus protecting the abutments from scour) and straight wing walls (similar to Fig. 102, *c*) on the down-stream end. This economizes masonry and also simplifies the constructive work. Note also the simplicity of the junction of the wing walls with the barrel of the arch, there being no re-entrant angles below the springing line of the arch. The design here shown is but one of a set of designs for arches varying in span from 6' to 30'.

#### MINOR OPENINGS.

**193. Cattle-guards. (a) Pit guards.** Cattle-guards will be considered under the head of minor openings, since the old-fashioned plan of pit guards, which are even now defended and preferred by some railroad men, requires a break in the continuity of the roadbed. A pit about three feet deep, five feet long, and as wide as the width of the roadbed, is walled up with stone (sometimes with wood), and the rails are supported on heavy timbers laid longitudinally with the rails. The break in the continuity of the roadbed produces a disturbance in the elastic wave running through the rails, the effect of which is noticeable at high velocities. The greatest objection, however, lies in the



STANDARD ARCH CULVERT  
8 FEET SPAN  
NORFOLK & WESTERN R.R.  
(1891)



NOTE: IN PLACE OF BRICK ARCH,  
RUBBLE STONE ARCH OF SAME THICK-  
NESS MAY BE USED

(To face page 216.)



dangerous consequences of a derailment or a failure of the timbers owing to unobserved decay or destruction by fire—caused perhaps by sparks and cinders from passing locomotives. The very insignificance of the structure often leads to careless in-

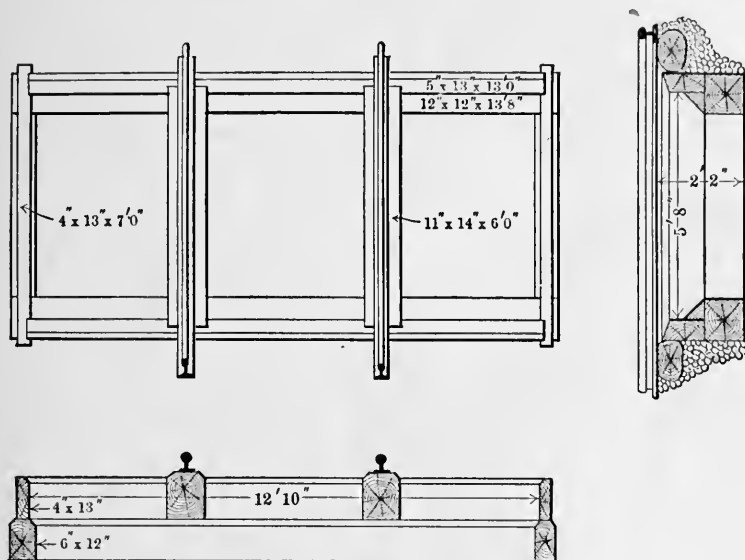


FIG. 103.—PIT CATTLE-GUARDS. P. R.R.

spection. But if a single pair of wheels gets off the rails and drops into the pit, a costly wreck is inevitable. The (once) standard design for such a structure on the Pennsylvania R.R. is shown in Fig. 103.

(b) **Surface cattle-guards.** These are fastened on top of the ties; the continuity of the roadbed is absolutely unbroken and thus is avoided much of the danger of a bad wreck owing to a possible derailment. The device consists essentially of overlaying the ties (both inside and outside the rails) with a surface on which cattle will not walk. The multitudinous designs for such a surface are variously effective in this respect. An objection, which is often urged indiscriminately against all such designs, is the liability that a brake-chain which may happen to be dragging *may* catch in the rough bars which are used. The bars are sometimes “home-made,” of wood, as shown in Fig. 104. Iron or steel bars are made as shown in Fig. 105. The general construction is the same as for the wooden bars. The

metal bars have far greater durability, and it is claimed that they are more effective in discouraging cattle from attempting to cross.

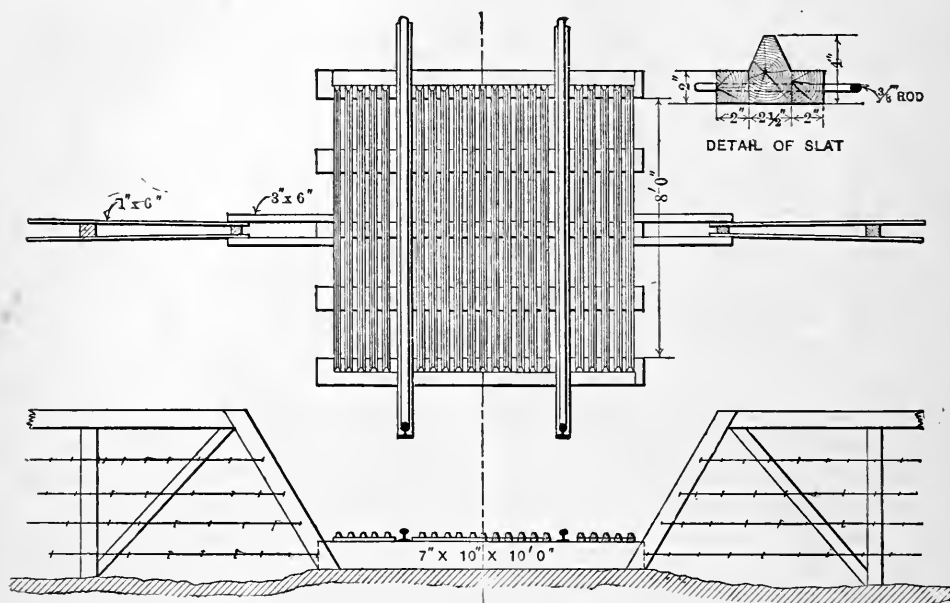


FIG. 104.—CATTLE-GUARD WITH WOODEN SLATS.

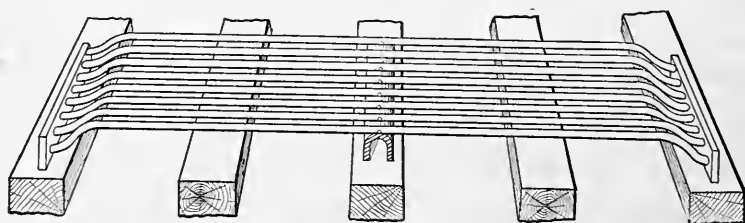
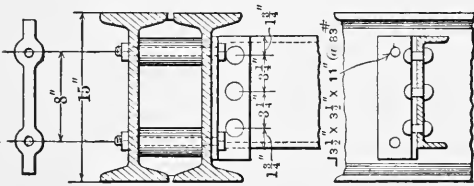
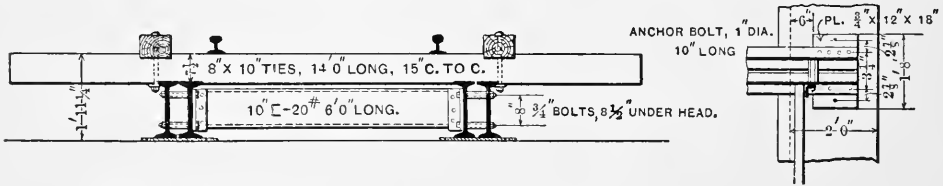
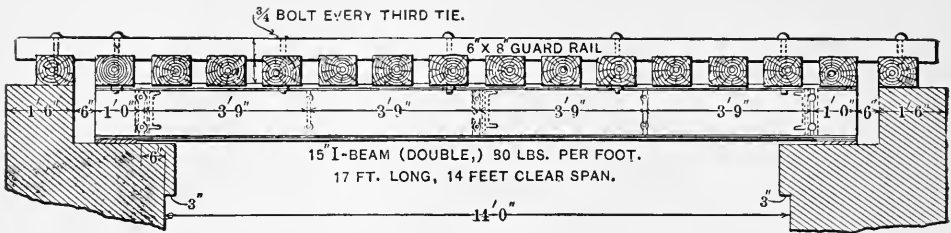


FIG. 105.—MERRILL-STEVENS STEEL CATTLE-GUARD.

**194. Cattle-passes.** Frequently when a railroad crosses a farm on an embankment, cutting the farm into two parts, the railroad company is obliged to agree to make a passageway through the embankment sufficient for the passage of cattle and perhaps even farm-wagons. If the embankment is high enough so that a stone arch is practicable, the initial cost is the only great objection to such a construction; but if an open wooden structure is necessary, all the objections against the old-fashioned cattle-guards apply with equal force here. The avoidance of a grade crossing which would otherwise be necessary is one of the



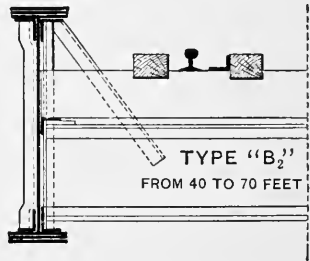
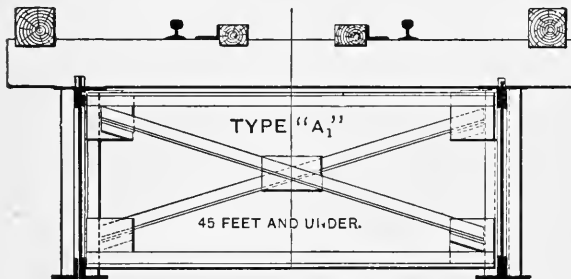
PLATE XVI.



STANDARD I-BRIDGES—14-FT. SPAN.

NORFOLK AND WESTERN R.R.

(1891.)



## TYPES OF PLATE GIRDER BRIDGES.

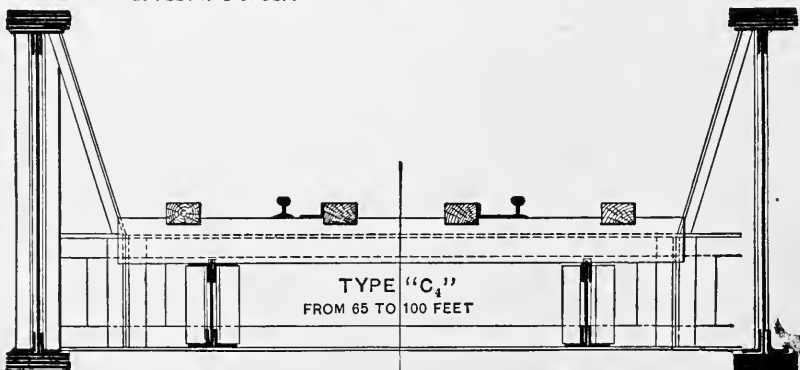


### TYPE "E" GIRDER

25 FEET AND UNDER.

C. M. & ST.P. RY.

(DEC. 1895.)



*(To face page 219.)*

great compensations for the expense of the construction and maintenance of these structures. The construction is sometimes made by placing two pile trestle bents about 6 to 8 feet apart, supporting the rails by stringers in the usual way, the special feature of this construction being that the embankments are filled in behind the trestle bents, and the thrust of the embankments is mutually taken up through the stringers, which are notched at the ends or otherwise constructed so that they may take up such a thrust. The designs for old-rail culverts and arch culverts are also utilized for cattle-passes when suitable and convenient, as well as the designs illustrated in the following section.

**195. Standard stringer and I-beam bridges.** The advantages of standard designs apply even to the covering of short spans with wooden stringers or with I beams—especially since the methods do not require much vertical space between the rails and the upper side of the clear opening, a feature which is often of prime importance. These designs are chiefly used for culverts or cattle-passes and for crossing *over* highways—providing such a narrow opening would be tolerated. The plans all imply stone abutments, or at least abutments of sufficient stability to withstand all thrust of the embankments. Some of the designs are illustrated in Plate XVI. The preparation of these standard designs should be attacked by the same general methods as already illustrated in § 156. When computing the required transverse strength, due allowance should be made for lateral bracing, which should be amply provided for. Note particularly the methods of bracing illustrated in Plate XVI. The designs calling for iron (or steel) stringers may be classed as permanent constructions, which are cheap, safe, easily inspected and maintained and therefore a desirable method of construction.

## CHAPTER VII.

### BALLAST.

**196. Purpose and requirements.** “The object of the ballast is to transfer the applied load over a large surface; to hold the timber work in place horizontally; to carry off the rain-water from the superstructure and to prevent freezing up in winter; to afford means of keeping the ties truly up to the grade line; and to give elasticity to the roadbed.” This extremely condensed statement is a description of an ideally perfect ballast. The value of any given kind of ballast is proportional to the extent to which it fulfills these requirements. The ideally perfect ballast is not necessarily the most economical ballast for all roads. Light traffic generally justifies something cheaper, but a very common error is to use a very cheap ballast when a small additional expenditure would procure a much better ballast which would be much more economical in the long run.

**197. Materials.** The materials most commonly employed are gravel and broken stone. Burnt clay, cinders, shells, and small coal are occasionally used as ballast when they are especially cheap and convenient or when better kinds are especially expensive. Although it is hardly correct to speak of the natural soil as ballast, yet many miles of cheap railways are “ballasted” with the natural soil, which is then called “mud ballast.”

**Mud ballast.** When the natural soil is gravelly so that rain will drain through it quickly, it will make a fair roadbed for light traffic, but for heavy traffic, and for the greater part of the length of most roads, the natural soil is a very poor material for ballast; for, no matter how suitable the soil might be along



limited sections of the road, it would practically never happen that the soil would be uniformly good throughout the whole length of the road. Considering that a heavy rain will in one day spoil the results of weeks of patient "surfacing" with mud ballast, it is seldom economical to use "mud" if there is a gravel-bed or other source of ballast anywhere on the line of the road.

**Cinders.** The advantages consist in the excellent facilities for drainage, ease of handling, and cheapness—after the road is in operation. One disadvantage is excessive dust in dry weather. Cinders are considered preferable to gravel in yards.

**Slag.** When slag is readily obtainable it furnishes an excellent ballast, free from dust and perfect in drainage qualities. Some kinds of slag are objectionable on account of their deleterious chemical effect on the ties and spikes—especially on metallic ties.

**Shells, small coal, etc.** These comparatively inferior kinds of ballast are used for light traffic when they are especially cheap and convenient. They are extremely dusty in dry weather, break up into very fine dust, and are but little better than mud.

**Gravel.** This is the most common form of ballast which may be called good ballast. In 1885, the Roadmasters Association of America voted in favor of gravel ballast as against rock ballast. Although not so stated, this action was perhaps due to a conviction of its real economy for the *average* railroad of this country, which may be called a "light traffic" road. Gravel should preferably be screened over a screen having a  $\frac{1}{2}$ " mesh, so as to screen out all dirt and the finest stones. Generally a railroad will be able to find at some point along its line a "gravel-pit" affording a suitable supply. This may be dug out with a steam-shovel, screened if necessary, and sent out over the line by the train-load at a comparatively small cost.

**Rock or broken stone.** Rock ballast is generally specified to be such as will pass through a  $1\frac{1}{2}$ " (or 2") ring. Although preferably broken by hand, machine-broken stone is much cheaper. It is most easily handled with forks. This also has the effect of

screening out the dirt and fine chips which would interfere with effectual drainage. Rock ballast is more expensive in first cost, and also more troublesome to handle, than any other kind, but under heavy traffic will keep in surface better and will require less work for maintenance after the ties have become thoroughly bedded. For roads with very light traffic, running few trains, at comparatively low velocities, the advantages of rock ballast over other kinds are not so pronounced. For such roads rock ballast is an expensive luxury. The amount of traffic which will justify the use of rock ballast will depend on the cost of obtaining ballast of the various kinds.

**198. Cross-sections.** A depth of 12'' under the tie is generally required on the best roads, but for light traffic this is sometimes reduced to 6'' and even less. The width is generally 1 to 2 feet less than the width of the roadbed proper—excluding ditches. If the ballast has an average width of 10 feet (12 feet at bottom and 8 feet at top) and an average depth of 15 inches (including that placed between the ties), it will require 2444 cubic yards per mile of track. The P. R.R. estimates 2500 cubic yards of gravel and 2800 cubic yards of stone ballast per mile of single track. On account of the requirements of drainage the best form of cross-section depends on the kind of ballast used.

**Mud ballast.** Since the great objection to mud ballast lies in its liability to become soft by soaking up the rain that falls, it becomes necessary that it should be drained as quickly and readily as its nature will permit. Fig. 106 shows a typical

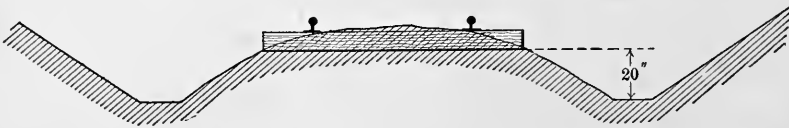


FIG. 106.—“MUD” BALLAST.

cross-section for mud ballast. It should be crowned 2'' above the top of the tie at the center, thence sloped so as to leave a slight clearance under the rail between the ties, thence sloping down to the bottom of the tie at each end and continuing to

slope down to the ditch (in cut), which should be 18'' or 20'' below the bottom of the tie.

**Gravel, cinders, slag, etc.** The subgrade is crowned 6'' or 8'' in the center, as shown in Fig. 107. The ballast is crowned

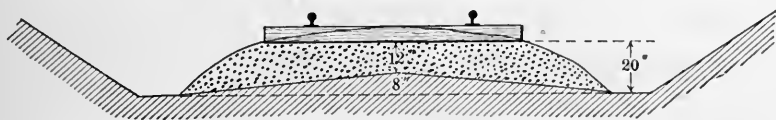


FIG. 107.—GRAVEL BALLAST.

to the top of the tie in the center, but is sloped down to the bottom of the tie at each end. This is necessary (and more especially so with mud ballast) to prevent a possible accumulation and settlement of water at the ends of the tie, which would readily soak into the end fibers and produce decay.

**Broken stone.** Stone ballast is shouldered out beyond the ends of the ties so as to afford greater lateral binding. The space between the ties is filled up level with the tops. The



FIG. 108.—BROKEN STONE BALLAST.

perfect drainage of stone ballast permits this to be done without any danger of causing decay of the ties by the accumulation and retention of water.

**199. Methods of laying ballast.** The cheapest method of laying ballast on new roads is to lay ties and rails directly on the prepared subgrade and run a construction train over the track to distribute the ballast. Then the track is lifted up until sufficient ballast is worked under the ties and the track is properly surfaced. This method, although cheap, is apt to injure the rails by causing bends and kinks, due to the passage of loaded construction trains when the ties are very unevenly and roughly supported, and the method is therefore condemned and prohibited in some specifications. The best method is to draw

in carts (or on a contractor's temporary track) the ballast that is required under the level of the *bottom* of the ties. Spread this ballast carefully to the required surface. Then lay the ties and rails, which will then have a very fair surface and uniform support. A construction train can then be run on the rails and distribute sufficient additional ballast to pack around and between the ties and make the required cross-section.

The necessity for constructing some lines at an absolute minimum of cost and of opening them for traffic as soon as possible has often led to the policy of starting traffic when there is little or no ballast—perhaps nothing more than a mere tamping of the natural soil under the ties. When this is done ballast may subsequently be drawn where required by the train-load on flat cars and unloaded at a minimum of cost by means of a “plough.” The plough has the same width as the cars and is guided either by a ridge along the center of each car or by short posts set up at the sides of the cars. It is drawn from one end of the train to the other by means of a cable. The cable is sometimes operated by means of a small hoisting-engine carried on a car at one end of the train. Sometimes the locomotive is detached temporarily from the train and is run ahead with the cable attached to it.

**200. Cost.** The cost of ballast *in the track* is quite a variable item for different roads, since it depends (*a*) on the first cost of the material as it comes to the road, (*b*) on the distance from the source of supply to the place where it is used, and (*c*) on the method of handling. The first cost of cinder or slag is frequently insignificant. A gravel-pit may cost nothing except the price of a little additional land beyond the usual limits of the right of way. Broken stone will usually cost \$1 or more per cubic yard. If suitable stone is obtainable on the company's land, the cost of blasting and breaking should be somewhat less than this. The cost of loading the ballast on to trains will be small (per cubic yard) if it is handled with steam-shovels—as in the case of gravel taken from a gravel-pit. Hand-shovelling will cost more. The cost of hauling will depend on the distance

hauled, and also, to a considerable extent, on the limitations on the operation of the train due to the necessity of keeping out of the way of regular trains. There is often a needless waste in this way. The "mud train" is considered a pariah and entitled to no rights whatever, regardless of the large daily cost of such a train and of the necessary gang of men. The cost of broken stone ballast *in the track* is estimated at \$1.25 per cubic yard. The cost of gravel ballast is estimated at 60 c. per cubic yard in the track. The cost of placing and tamping gravel ballast is estimated at 20 c. to 24 c. per cubic yard, for cinders 12 c. to 15 c. per cubic yard. The cost of loading gravel on cars, using a steam-shovel, is estimated at 6 c. to 10 c. per cubic yard.\*

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\* Report Roadmasters Association, 1885.

## CHAPTER VIII.

### TIES,

#### AND OTHER FORMS OF RAIL SUPPORT.

**201. Various methods of supporting rails.** It is necessary that the rails shall be sufficiently supported and braced, so that the gauge shall be kept constant and that the rails shall not be subjected to excessive transverse stress. It is also preferable that the rail support shall be neither rigid (as if on solid rock) nor too yielding, but shall have a *uniform* elasticity throughout. These requirements are more or less fulfilled by the following methods.

(a) **Longitudinals.** Supporting the rails throughout their entire length. This method is very seldom used in this country except occasionally on bridges and in terminals when the longitudinals are supported on cross-ties. In § 224 will be described a system of rails, used to some extent in Europe, having such broad bases that they are self-supporting on the ballast and are only connected by tie-rods to maintain the gauge.

(b) **Cast-iron "bowls" or "pots."** These are castings resembling large inverted bowls or pots, having suitable chairs on top for holding and supporting the rails, and tied together with tie-rods. They will be described more fully later (§ 223).

(b) **Cross-ties of metal or wood.** These will be discussed in the following sections.

**202. Economics of ties.** The true cost of ties depends on the relative total cost of maintenance for long periods of time. The first cost of the ties delivered to the road is but one item in the

economics of the question. Cheap ties require frequent renewals, which cost for the *labor* of each renewal practically the same whether the tie is of oak or hemlock. Cheap ties make a poor roadbed which will require more track labor to keep even in tolerable condition. The roadbed will require to be disturbed so frequently on account of renewals that the ties never get an opportunity to get settled and to form a smooth roadbed for any length of time. Irregularity in width, thickness, or length of ties is especially detrimental in causing the ballast to act and wear unevenly. The life of ties has thus a more or less direct influence on the life of the rails, on the wear of rolling stock, and on the speed of trains. } These last items are not so readily reducible to dollars and cents, but when it can be shown that the total cost, for a long period of time, of several renewals of cheap ties, with all the extra track labor involved, is as great as or greater than that of a few renewals of durable ties, then there is no question as to the real economy. In the following discussions of the merits of untreated ties (either cheap or costly), chemically treated ties, or metal ties, the true question is therefore of the ultimate cost of maintaining any particular kind of ties for an indefinite period, the cost including the first cost of the ties, the labor of placing them and maintaining them to surface, and the somewhat uncertain (but not therefore nonexistent) effect of frequent renewals on repairs of rolling stock, on possible speed, etc.

#### WOODEN TIES.

**203. Choice of wood.** This naturally depends, for any particular section of country, on the supply of wood which is most readily available. The woods most commonly used, especially in this country, are oak and pine, oak being the most durable and generally the most expensive. Redwood is used very extensively in California and proves to be extremely durable, so far as decay is concerned, but it is very soft and is much injured by "rail-cutting." This defect is being partly remedied by the

use of tie-plates, as will be explained later. [Cedar, chestnut, hemlock, and tamarack are frequently used in this country. In tropical countries very durable ties are frequently obtained from the hard woods peculiar to those countries. According to a recent bulletin of the U. S. Department of Agriculture the proportions of the various kinds used in the United States are about as follows :

|            |     |                   |    |              |      |
|------------|-----|-------------------|----|--------------|------|
| Oak.....   | 60% | Chestnut.....     | 5% | Cypress..... | 2%   |
| Pine.....  | 20  | Hemlock and Tama- |    | Various..... | 1    |
| Cedar..... | 6   | rack.....         | 3  | Total.....   | 100% |
|            |     | Redwood.....      | 3  |              |      |

**204. Durability.** The durability of ties depends on the climate; the drainage of the ballast ; the volume, weight, and speed of the traffic; the curvature, if any; the use of tie-plates; the time of year of cutting the timber; the age of the timber and the degree of its seasoning before placing in the track; the nature of the soil in which the timber was grown; and, chiefly, on the species of wood employed. The variability in these items will account for the discrepancies in the reports on the life of various woods used for ties.

*White oak* is credited with a life of 5 to 12 years, depending principally on the traffic. Is is both hard and durable, the hardness enabling it to withstand the cutting tendency of the rail-flanges, and the durability enabling it to resist decay. *Pine* and *redwood* resist decay very well, but are so soft that they are badly cut by the rail-flanges and do not hold the spikes very well, necessitating frequent respiking. Since the spikes must be driven within certain very limited areas on the face of each tie, it does not require many spike-holes to "spike-kill" the tie. On sharp curves, especially with heavy traffic, the wheel-flange pressure produces a side pressure on the rail tending to overturn it, which tendency is resisted by the spike, aided sometimes by rail-braces. Whenever the pressure becomes too great the spike will yield somewhat and will be slightly withdrawn. The resistance is then somewhat less and the spike is soon so loose that it must be redriven in a new hole. If this occurs very



often, the tie may need to be replaced long before any decay has set in. When the traffic is very light, the wood very durable, and the climate favorable ties have been known to last 25 years.

**205. Dimensions.** The usual dimensions for the best roads (standard gauge) are 8' to 8' 6" long, 6" to 7" thick, and 8" to 10" wide on top and bottom (if they are hewed) or 8" to 9" wide if they are sawed. For cheap roads and light traffic the length is shortened sometimes to 7' and the cross-section also reduced. On the other hand a very few roads use ties 9' long.

Two objections are urged against sawed ties: first, that the grain is torn by the saw, leaving a woolly surface which induces decay; and secondly, that, since timber is not perfectly straight-grained, some of the fibers are cut obliquely, exposing their ends, which are thus liable to decay. The use of a "planer-saw" obviates the first difficulty. Chemical treatment of ties obviates both of these difficulties. Sawed ties are more convenient to handle, are a necessity on bridges and trestles, and it is even claimed, although against commonly received opinion, that actual trial has demonstrated that they are more durable than hewed ties.

**206. Spacing.** The spacing is usually 14 to 16 ties to a 30-foot rail. This number is sometimes reduced to 12 and even 10, and on the other hand occasionally increased to 18 or 20 by employing narrower ties. There is no economy in reducing the number of ties very much, since for any required stiffness of track it is more economical to increase the number of supports than to increase the weight of the rail. The decreasing cost of rails and the increasing cost of ties have materially changed the relation between number of ties and weight of rail to produce a given stiffness at minimum cost, but many roads have found it economical to employ a large number of ties rather than increase the weight of the rail. On the other hand there is a practical limit to the number that may be employed, on account of the necessary space between the ties that is required for proper tamping. This width is ordinarily about twice the width of the tie. At this rate, with light ties 6" wide and with 12" clear

space, there would be 20 ties per 30-foot rail, or 3520 per mile. The smaller ties can generally be bought much cheaper (proportionately) than the larger sizes, and hence the economy.

Track instructions to foremen generally require that the spacing of ties shall *not* be uniform along the length of any rail. Since the joint is generally the weakest part of the rail structure, the joint requires more support than the center of the rail. Therefore the ties are placed with but 8" or 10" clear space between them at the joints, this applying to 3 or 4 ties at each joint; the remaining ties, required for each rail length, are equally spaced along the remaining distance.

**207. Specifications.** The specifications for ties are apt to include the items of size, kind of wood, and method of construction, besides other minor directions about time of cutting, seasoning, delivery, quality of timber, etc.

(a) **Size.** The particular size or sizes required will be somewhat as indicated in § 205.

(b) **Kind of wood.** When the kind or kinds of wood are specified, the most suitable kinds that are available in that section of country are usually required.

(c) **Method of construction.** It is generally specified that the ties shall be hewed on two sides; that the two faces thus made shall be parallel planes and that the bark shall be removed. It is sometimes required that the ends shall be sawed off square; that the timber shall be cut in the winter (when the sap is down); and that the ties shall be seasoned for six months. These last specifications are not required or lived up to as much as their importance deserves. It is sometimes required that the ties shall be delivered on the right of way, neatly piled in rows, the alternate rows at right angles, piled if possible on ground not lower than the rails and at least seven feet away from them, the lower row of ties resting on two ties which are themselves supported so as to be clear of the ground.

(d) **Quality of timber.** The usual specifications for sound timber are required, except that they are not so rigid as for a better class of timber work. The ties must be sound, reason-

ably straight-grained, and not very crooked—one test being that a line joining the center of one end with the center of the middle shall not pass outside of the other end. Splits or shakes, especially if severe, should cause rejection.

Specifications sometimes require that the ties shall be cut from single trees, making what is known as “pole ties” and definitely condemning those which are cut or split from larger trunks, giving two “slab ties” or four “quarter ties” for each cross-



FIG. 109.—METHODS OF CUTTING TIES.

109. Even if pole ties are better, their exclusive use means the rapid destruction of forests of young trees.

**208. Regulations for laying and renewing ties.** The regulations issued by railroad companies to their track foremen will generally include the following, in addition to directions regarding dimensions, spacing, and specifications given in §§ 204–207. When hewn ties of somewhat variable size are used, as is frequently the case, the largest and best are to be selected for use as joint ties. If the upper surface of a tie is found to be warped (contrary to the usual specifications) so that one or both rails do not get a full bearing across the whole width of the tie, it must be adzed to a true surface along its whole length and not merely notched for a rail-seat. When respiking is necessary and spikes have been pulled out, the holes should be immediately plugged with “wooden spikes,” which are supplied to the foremen for that express purpose, so as to fill up the holes and prevent the decay which would otherwise take place when the hole becomes filled with rain-water. Ties should always be laid at right angles to the rails and never obliquely. Minute regulations to prevent premature rejection and renewal of ties are frequently made. It is generally required that the requisitions for renewals shall be made by the actual count of the individual ties to be renewed instead of by any wholesale estimates. It is unwise to have ties of widely variable size, hardness, or durability adjacent to each

other in the track, for the uniform elasticity, so necessary for smooth riding, will be unobtainable under those circumstances.

**209. Cost of ties.** When railroads can obtain ties cut by farmers from woodlands in the immediate neighborhood, the price will frequently be as low as 20 c. for the smaller sizes, running up to 50 c. for the larger sizes and better qualities, especially when the timber is not very plentiful. Sometimes if a railroad cannot procure suitable ties from its immediate neighborhood, it will find that adjacent railroads control all adjacent sources of supply for their own use and that ties can only be procured from a considerable distance, with a considerable added cost for transportation. First-class oak ties cost about 75 to 80 c. and frequently much more. Hemlock ties can generally be obtained for 35 c. or less.

#### PRESERVATIVE PROCESSES FOR WOODEN TIES.

**210. General principle.** Wood has a fibrous cellular structure, the cells being filled with sap or air. The woody fiber is but little subject to decay unless the sap undergoes fermentation. Preservative processes generally aim at removing as much of the water and sap as possible and filling up the pores of the wood with an antiseptic compound. The most common methods (except one) all agree in this general process and only differ in the method employed to get rid of the sap and in the antiseptic chemical with which the fibers are filled. One valuable feature of these processes lies in the fact that the softer cheaper woods (such as hemlock and pine) are more readily treated than are the harder woods and yet will produce practically as good a tie as a treated hard-wood tie and a very much better tie than an untreated hard-wood tie. The various processes will be briefly described, taking up first the process which is fundamentally different from the others, viz., vulcanizing.

**211. Vulcanizing.** The process consists in heating the timber to a temperature of 300° to 500° F. in a cylinder, the air being under a pressure of 100 to 175 lbs. per square inch. By this process the albumen in the sap is coagulated, the water evap-

orated, and the pores are partially closed by the coagulation of the albumen. It is claimed that the heat sterilizes the wood and produces chemical changes in the wood which give it an antiseptic character. It has been very extensively used on the elevated lines of New York City, and it is claimed to give perfect satisfaction. The treatment has cost that road 25 c. per tie.

**212. Creosoting.** This process consists in impregnating the wood with *wood-creosote* or with *dead oil of coal-tar*. *Wood-creosote* is one of the products of the destructive distillation of wood—usually long-leaf pine. *Dead oil of coal-tar* is a product of the distillation of coal-tar at a temperature between 480° and 760° F. It would require about 35 to 50 pounds of creosote to completely fill the pores of a cubic foot of wood. But it would be impossible to force such an amount into the wood, nor is it necessary or desirable. About 10 pounds per cubic foot, or about 35 pounds per tie, is all that is necessary. For piling placed in salt water about 18 to 20 pounds per cubic foot is used, and the timber is then perfectly protected against the ravages of the *teredo navalis*. To do the work, long cylinders, which may be opened at the ends, are necessary. Usually the timbers are run in and out on iron carriages running on rails fastened to braces on the inside of the cylinder. When the load has been run in, the ends of the cylinder are fastened on. The water and air in the pores of the wood are first drawn out by subjecting the wood alternately to steam-pressure and to the action of a vacuum-pump. This is continued for several hours. Then, after one of the vacuum periods, the cylinder is filled with creosote oil at a temperature of about 170° F. The pumps are kept at work until the pressure is about 80 to 100 pounds per square inch, and is maintained at this pressure from one to two hours according to the size of the timber. The oil is then withdrawn, the cylinders opened, the train pulled out and another load made up in 40 to 60 minutes. The average time required for treating a load is about 18 or 20 hours, the absorption about 10 or 11 pounds of oil per cubic foot, and the cost (1894) from \$12.50 to \$14.50 per thousand feet B. M.

**213. Burnettizing (chloride-of-zinc process).** This process is very similar to the creosoting process except that the chemical is chloride of zinc, and that the chemical is not heated before use. The preliminary treatment of the wood to alternate vacuum and pressure is not continued for quite so long a period as in the creosoting process. Care must be taken, in using this process, that the ties are of as uniform quality as possible, for seasoned ties will absorb much more zinc chloride than unseasoned (in the same time), and the product will lack uniformity unless the seasoning is uniform. The A., T. & S. F<sup>e</sup> R.R. has works of its own at which ties are treated by this process at a cost of about 25 c. per tie. The Southern Pacific R.R. also has works for burnettizing ties at a cost of 9.5 to 12 c. per tie. The zinc-chloride solution used in these works contains only 1.7% of zinc chloride instead of over 3% as used in the Santa F<sup>e</sup> works, which perhaps accounts partially for the great difference in cost per tie. One great objection to burnettized ties is the fact that the chemical is somewhat easily washed out, when the wood again becomes subject to decay. Another objection, which is more forcible with respect to timber subject to great stresses, as in trestles, than to ties, is the fact that when the solution of zinc chloride is made strong (over 3%) the timber is made very brittle and its strength is reduced. The reduction in strength has been shown by tests to amount to  $\frac{1}{4}$  to  $\frac{1}{10}$  of the ultimate strength, and that the elastic limit has been reduced by about  $\frac{1}{7}$ .

**214. Kyanizing (bichloride-of-mercury or corrosive-sublimate process).** This is a process of "steeping." It requires a much longer time than the previously described processes, but does not require such an expensive plant. Wooden tanks of sufficient size for the timber are all that is necessary. The corrosive sublimate is first made into a concentrated solution of one part of chemical to six parts of *hot* water. When used in the tanks this solution is weakened to 1 part in 100 or 150. The wood will absorb about 5 to 6.5 pounds of the bichloride per 100 cubic feet, or about one pound for each 4 to 6 ties. The timber is allowed to soak in the tanks for several days, the general rule

being about one day for each inch of least thickness and one day over—which means seven days for six-inch ties, or thirteen (to fifteen) days for 12" timber (least dimension). The process is somewhat objectionable on account of the chemical being such a virulent poison, workmen sometimes being sickened by the fumes arising from the tanks. On the Baden railway (Germany) kyanized ties last 20 to 30 years. On this railway the wood is always air-dried for two weeks after impregnation and before being used, which is thought to have an important effect on its durability. The solubility of the chemical and the liability of the chemical washing out and leaving the wood unprotected is an element of weakness in the method.

**215. Wellhouse (or zinc-tannin) process.** The last two methods described (as well as some others employing similar chemicals) are open to the objection that since the wood is impregnated with an aqueous solution, it is liable to be washed out very rapidly if the wood is placed under water, and will even disappear, although more slowly, under the action of moisture and rain. Several processes have been proposed or patented to prevent this. Many of them belong to one class, of which the Wellhouse process is a sample. By these processes the timber is successively subjected to the action of two chemicals, each individually soluble in water, and hence readily impregnating the timber, but the chemicals when brought in contact form insoluble compounds which cannot be washed out of the wood-cells. By the Wellhouse process, the wood is first impregnated with a solution of chloride of zinc and glue, and is then subjected to a bath of tannin under pressure. The glue and tannin combine to form an insoluble leathery compound in the cells, which will prevent the zinc chloride from being washed out. It is being used by the A., T. & S. Fé R.R., their works being located at Las Vegas, New Mexico, and also by the Union Pacific R.R. at their works at Laramie, Wyo. In 1897 Mr. J. M. Meade, a resident engineer on the A., T. & S. Fé, exhibited to the Roadmasters Association of America a piece of a tie treated by this process which had been taken from the tracks after

nearly 13 years' service. The tie was selected at random, was taken out for the sole purpose of having a specimen, and was still in sound condition and capable of serving many years longer. The cost of the treatment was then quoted as 13 c. per tie. It was claimed that the treatment trebled the life of the tie besides adding to its spike-holding power.

**216. Cost of treating.** The cost of treating ties by the various methods has been estimated as follows\*—assuming that the plant was of sufficient capacity to do the work economically: creosoting, 25 c. per tie; vulcanizing, 25 c. per tie; burnettizing (chloride of zinc), 8.25 c. per tie; kyanizing (steeping in corrosive sublimate), 14.6 c. per tie; Wellhouse process (chloride of zinc and tannin), 11.25 c. per tie. These estimates are only for the net cost at the works and do not include the cost of hauling the ties to and from the works, which may mean 5 to 10 c. per tie. Some of these processes have been installed on cars which are transported over the road and operated where most convenient.

**217. Economics of treated ties.** The fact that treated ties are not universally adopted is due to the argument that the added life of the tie is not worth the extra cost. If ties can be bought for 25 c., and cost 25 c. for treatment, and the treatment only doubles their life, there is apparently but little gained except the work of placing the extra tie in the track, which is more or less offset by the interest on 25 c. for the life of the untreated tie, and the larger initial outlay makes a stronger impression on the mind than the computed ultimate economy. But when ties cost 75 c. and treatment costs only 25 c., or perhaps less, then the economy is more apparent and unquestionable. But this analysis may be made more closely. As shown in § 202, the disturbance of the roadbed on account of frequent renewals of untreated ties is a disadvantage which would justify an appreciable expenditure to avoid, although it is

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\* Bull. No. 9, U. S. Dept. of Agric., Div. of Forestry. App. No. 1, by Henry Flad.



very difficult to closely estimate its true value. The annual cost of a system of ties may be considered as the sum of (*a*) the interest on the first cost, (*b*) the annual sinking fund that would buy a new tie at the end of its life, and (*c*) the average annual cost of maintenance for the life of the tie, which includes the cost of laying and the considerable amount of subsequent tamping that must be done until the tie is fairly settled in the roadbed, beside the regular trackwork on the tie, which is practically constant. This last item is difficult to compute, but it is easy to see that, since the cost of laying the tie and the subsequent tamping to obtain proper settlement is the same for all ties (of similar form), the *average* annual charge on the longer-lived tie would be much less. In the following comparison item (*c*) is disregarded, simply remembering that the advantage is with the longer-lived tie.

|                                                          | Untreated tie. | Treated tie. |
|----------------------------------------------------------|----------------|--------------|
| Original cost . . . . .                                  | 40 cents       | 65 cents     |
| Life (assumed at). . . . .                               | 7 years        | 14 years     |
| Item ( <i>a</i> )—interest on first cost @ 4% . . . . .  | 1.6 cents      | 2.6 cents    |
| “ ( <i>b</i> )—sinking fund @ 4% . . . . .               | 5.1 “          | 3.6 “        |
| “ ( <i>c</i> )—(considered here as offsetted) . . . . .  | —              | —            |
| Average annual cost (except item ( <i>c</i> )) . . . . . | 6.7 cents      | 6.2 cents    |

On this basis treated ties will cost 0.5 cent *less* per annum *besides* the advantage of item (*c*) and the still more indefinite advantages resulting from smoother running of trains, less wear and tear on rolling stock, etc., due to less disturbance of the roadbed.

In Europe, where wood is expensive, untreated ties are seldom used, as the treatment is always considered to be worth more than it costs. The rapid destruction of the forests of timber in this country is having the effect of increasing the price, so that it will not be long before treated ties (or metal ties) will be economical for a large majority of the railroads of the country.

## METAL TIES.

**218. Extent of use.** In 1894\* there were nearly 35000 miles of "metal track" in various parts of the world. Of this total, there were 3645 miles of "longitudinals" (see § 224), found exclusively in Europe, nearly all of it being in Germany. There were over 12000 miles of "bowls and plates" (see § 223), found almost entirely in British India and in the Argentine Republic. The remainder, over 18000 miles, was laid with metal cross-ties of various designs. There were over 8000 miles of metal cross-ties in Germany alone, about 1500 miles in the rest of Europe, over 6000 miles in British India, nearly 1000 miles in the rest of Asia, and about 1500 miles more in various other parts of the world. Several railroads in this country have tried various designs of these ties, but their use has never passed the experimental stage. These 35000 miles represent about 9% of the total railroad mileage of the world—nearly 400000 miles. They represent about 17.6% of the total railroad mileage, exclusive of the United States and Canada, where they are not used at all, except experimentally. In the four years from 1890 to 1894 the use of metal track increased from less than 25000 miles to nearly 35000 miles. This increase was practically equal to the total increase in railroad mileage during that time, exclusive of the increase in the United States and Canada. This indicates a large growth in the percentage of metal track to total mileage, and therefore an increased appreciation of the advantages to be derived from their use.

**219. Durability.** The durability of metal track is still far from being a settled question, due largely to the fact that the best form for such track is not yet determined, and that a large part of the apparent failures in metal track have been evidently due to defective design. Those in favor of them estimate the life as from 30 to 50 years. The opponents place it as not more than 20 years, or perhaps as long as the best of wooden ties.

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\* Bulletin No. 9, U. S. Dept. of Agriculture, Div. of Forestry.

Unlike the wooden tie, however, which deteriorates as much with time as with usage, the life of a metal tie is more largely a function of the traffic. The life of a well-designed metal tie has been estimated at 150000 to 200000 trains; for 20 trains per day, or say 6000 per year, this would mean from 25 to 33 years. 20 trains per day on a *single* track is a much larger number than will be found on the majority of railroads. Metal ties are found to be subject to rust, especially when in damp localities, such as tunnels; but on the other hand it is in such confined localities, where renewals are troublesome, that it is especially desirable to employ the best and longest-lived ties. Paint, tar, etc., have been tried as a protection against rust, but the efficacy of such protection is as yet uncertain, the conditions preventing any renewal of the protection—such as may be done by repainting a bridge, for example. Failures in metal cross-ties have been largely due to cracks which begin at a corner of one of the *square* holes which are generally *punched* through the tie, the holes being made for the bolts by which the rails are fastened to the tie. The holes are generally *punched* because it is cheaper. Reaming the holes after punching is thought to be a safeguard against this frequent cause of failure. Another method is to round the corners of the square punch with a radius of about  $\frac{1}{8}$ ". If a crack has already started, the spread of the crack may be prevented by drilling a small hole at the end of it.

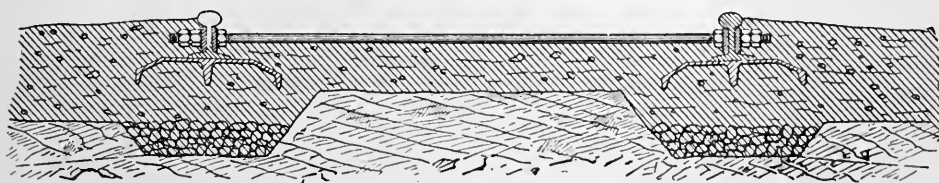
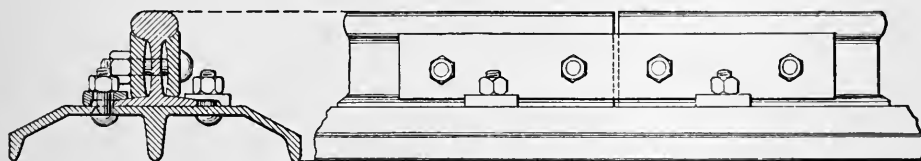
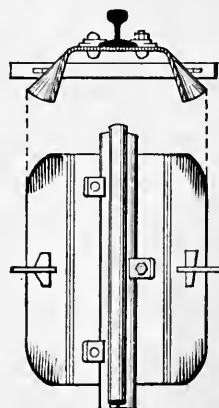
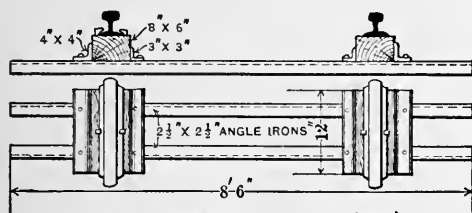
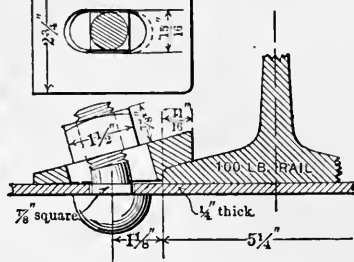
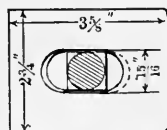
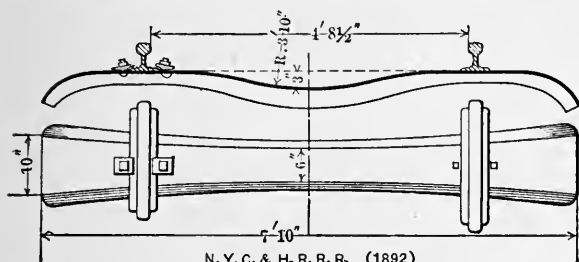
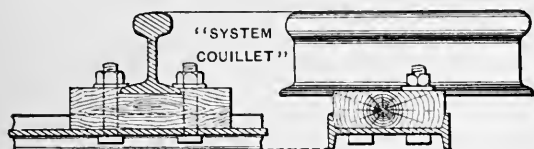
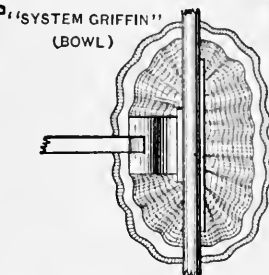
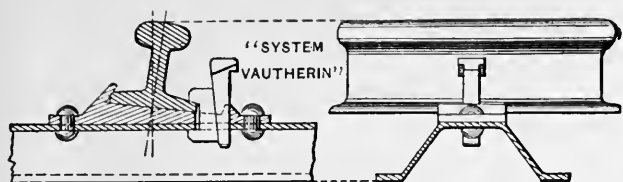
**220. Form and dimensions of metal cross-ties.** Since stability in the ballast is an essential quality for a tie, this must be accomplished either by turning down the end of the tie or by having some form of lug extending downward from one or more points of the tie. The ties are sometimes depressed in the center (see Plate XVII, N. Y. C. & H. R. R.R. tie) to allow for a thick covering of ballast on top in order to increase its stability in the ballast. This form requires that the ties should be sufficiently well tamped to prevent a tendency to bend out straight, thus widening the gauge. Many designs of ties are objectionable because they cannot be placed in the track without disturbing adjacent ties. The failure of many metal cross-

ties, otherwise of good design, may be ascribed to too light weight. Those weighing much less than 100 pounds have proved too light. From 100 to 130 pounds weight is being used satisfactorily on German railroads. The general outside dimensions are about the same as for wooden ties, except as to thickness. The metal is generally from  $\frac{1}{4}$ " to  $\frac{3}{8}$ " thick. They are, of course, only made of wrought iron or steel, cast iron being used only for "bowls" or "plates" (see § 213). The details of construction of some of the most commonly used ties may be seen by a study of Plate XVII.

**221. Fastenings.** The devices for fastening the rails to the ties should be such that the gauge may be widened if desired on curves, also that the gauge can be made true regardless of slight inaccuracies in the manufacture of the ties, and also that shims may be placed under the rail if necessary during cold weather when the tie is frozen into the ballast and cannot be easily disturbed. Some methods of fastening require that the base of the rail be placed against a lug which is riveted to the tie or which forms a part of it. This has the advantage of reducing the number of pieces, but is apt to have one or more of the disadvantages named above. Metal keys or wooden wedges are sometimes used, but the majority of designs employ some form of bolted clamp. The form adopted for the experimental ties used by the N. Y. C. & H. R. R.R. (see Plate XVII) is especially ingenious in the method used to vary the gauge or allow for inaccuracies of manufacture. Plate XVII shows some of the methods of fastening adopted on the principal types of ties.

**222. Cost.** The cost of metal cross-ties in Germany averages about 1.6 c. per pound or about \$1.60 for a 100-lb. tie. The ties manufactured for the N. Y. C. & H. R. R.R. in 1892 weighed about 100 lbs. and cost \$2.50 per tie, but if they had been made in larger quantities and with the present price of steel the cost would possibly have been much lower. The item of freight from the place of manufacture to the place where used is no inconsiderable item of cost with some roads. Metal cross-ties have been used by some street railroads in this country.

PLATE XVII.



## METAL TIES.

(To face page 240 )



Those used on the Terre Haute Street Railway weigh 60 pounds and cost about 66 c. for the tie, or 74 c. per tie with the fastenings.

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**223. Bowls or plates.** As mentioned before, over 12000 miles of railway, chiefly in British India and in the Argentine Republic, are laid with this form of track. It consists essentially of large cast-iron inverted "bowls" laid at intervals under each rail and opposite each other, the opposite bowls being tied together with tie-rods. A suitable chair is riveted or bolted on to the top of each bowl so as to properly hold the rail. Being made of cast iron, they are not so subject to corrosion as steel or wrought iron. They have the advantage that when old and worn out their scrap value is from 60 to 80% of their initial cost, while the scrap value of a steel or wrought-iron tie is practically nothing. Failure generally occurs from breakage, the failures from this cause in India being about 0.4 per cent per annum. They weigh about 250 lbs. apiece and are therefore quite expensive in first cost and transportation charges. There are miles of them in India which have already lasted 25 years and are still in a serviceable condition. Some illustrations of this form of tie are shown in Plate XVII.

**224. Longitudinals.\*** This form, the use of which is confined almost exclusively to Germany, is being gradually replaced on many lines by metal cross-ties. The system generally consists of a compound rail of several parts, the upper bearing rail being very light and supported throughout its length by other rails, which are suitably tied together with tie-rods so as to maintain the proper gauge, and which have a sufficiently broad

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\* Although the discussion of longitudinals might be considered to belong more properly to the subject of RAILS, yet the essential idea of all designs must necessarily be the *support* of a rail-head on which the rolling stock may run, and therefore this form, unused in this country, will be briefly described here.

base to be properly supported in the ballast. One great objection to this method of construction is the difficulty of obtaining proper drainage especially on grades, the drainage having a tendency to follow along the lines of the rails.

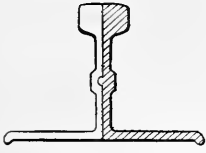


FIG. 110.

The construction is much more complicated on sharp curves and at frogs and switches. Another fundamentally different form of longitudinal is the Haarman compound "self-bearing" rail, having a base 12" wide and a height of 8", the alternate sections breaking joints so as to form a practically continuous rail.

Some of the other forms of longitudinals are illustrated in Plate XVII.

For a very complete discussion of the subject of metal ties, see the "Report on the Substitution of Metal for Wood in Railroad Ties" by E. E. Russell Tratman, it being Bulletin No. 4, Forestry Division of the U. S. Dept. of Agriculture.



## CHAPTER IX.

### RAILS.

**225. Early forms.** The first rails ever laid were wooden stringers which were used on very short tram-roads around coal-mines. As the necessity for a more durable rail increased, owing chiefly to the invention of the locomotive as a motive power, there were invented successively the cast-iron "fish-belly" rail and various forms of wrought-iron strap rails which finally developed into the T rail used in this country and the double-headed rail, supported by chairs, used so extensively in England. The cast-iron rails were cast in lengths of about 3 feet and were supported in iron chairs which were sometimes set upon stone piers. A great deal of the first railroad track of this country was laid with longitudinal stringers of wood placed upon cross-ties, the inner edge of the stringers being

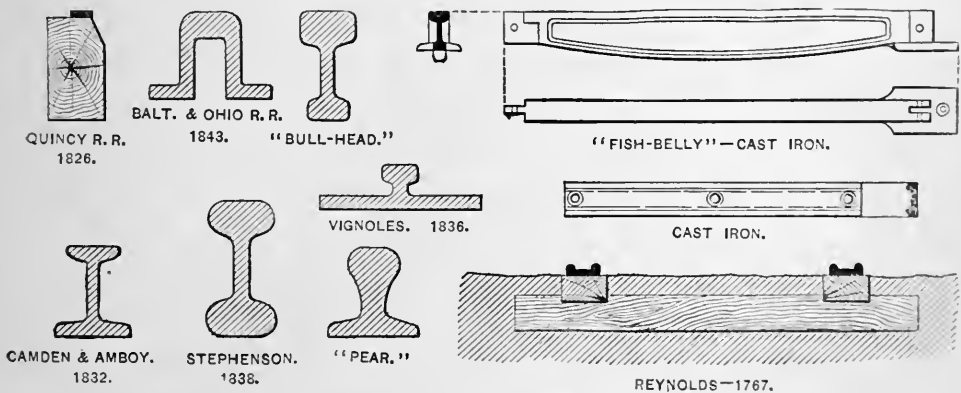
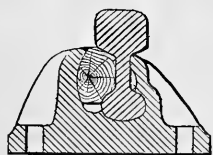


FIG. 111.—EARLY FORMS OF RAILS.

protected by wrought-iron straps. The "bridge" rails were first rolled in this country in 1844. The "pear" section was

an approach to the present form, but was very defective on account of the difficulty of designing a good form of joint. The "Stevens" section was designed in 1830 by Col. Robert L. Stevens, Chief Engineer of the Camden and Amboy Railroad; although quite defective in its proportions, according to the present knowledge of the requirements, it is essentially the present form. In 1836, Charles Vignoles invented essentially the same form in England; this form is therefore known throughout England and Europe as the Vignoles rail.

**226. Present standard forms.** The larger part of modern railroad track is laid with rails which are either "T" rails or the double-headed or "bull-headed" rails which are carried in chairs. The double-headed rail was designed with a symmetrical form with the idea that after one head had been worn out by traffic the rail could be reversed, and that its life would be practically doubled. Experience has shown that the wear of the rail in the chairs is very great; so much so that when one head has been worn out by traffic the whole rail is generally useless. If the rail is turned over, the worn places, caused by the chairs, make a rough track and the rail appears to be more brittle and subject to fracture, possibly due to the crystallization that may have occurred during the previous usage and to the reversal of stresses in the fibers. Whatever the explanation, experience has demonstrated the *fact*. The "bull-headed" rail has the lower



**FIG. 112.—BULL-HEADED RAIL AND CHAIR.** head only large enough to properly hold the wooden keys with which the rail is secured to the chairs (see Fig. 112) and furnish the necessary strength. The use of these rails requires the use of two cast-iron chairs for each tie. It is claimed that such track is better for heavy and fast traffic, but it is more expensive to build and maintain. It is the standard form of track in England and some parts of Europe.

Until a few years ago there was a very great multiplicity in the designs of "T" rails as used in this country, nearly every prominent railroad having its own special design, which

perhaps differed from that of some other road by only a very minute and insignificant detail, but which nevertheless would require a complete new set of rolls for rolling. This certainly must have had a very appreciable effect on the cost of rails. In 1893, the American Society of Civil Engineers, after a very exhaustive investigation of the subject, extending over several years, having obtained the opinions of the best experts of the country, adopted a series of sections which have been very extensively adopted by the railroads of this country. Instead of having the rail sections for various weights to be geometrically similar figures, certain dimensions are made constant, regardless of the weight. It was decided that the metal should be distributed through the section in the proportions of—head 42%, web 21%, and flange 37%. The top of the head should have a radius of  $12''$ ; the top corner radius of head should be  $\frac{5}{16}''$ ; the

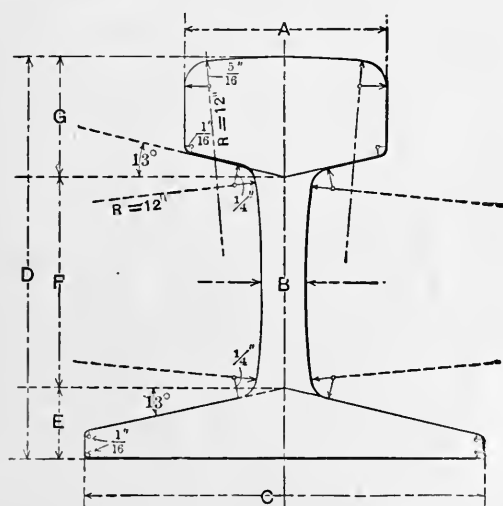


FIG. 113.—AM. SOC. C. E. STANDARD RAIL SECTION.

lower corner radius of head should be  $\frac{1}{16}''$ ; the corners of the flanges,  $\frac{1}{16}''$  radius; side radius of web,  $12''$ ; top and bottom radii of web corners,  $\frac{1}{4}''$ ; and angles with the horizontal of the under side of the head and the top of the flange,  $13^\circ$ . The sides of the head are vertical.

The height of the rail ( $D$ ) and the width of the base ( $C$ ) are always made equal to each other.

|       | Weight per Yard.                 |                                 |                                |                                |                                 |                                 |                                 |                                 |                                |                                 |                                 |                                 |                                 |
|-------|----------------------------------|---------------------------------|--------------------------------|--------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|--------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|       | 40                               | 45                              | 50                             | 55                             | 60                              | 65                              | 70                              | 75                              | 80                             | 85                              | 90                              | 95                              | 100                             |
| A     | 17 <sup>11</sup> / <sub>16</sub> | 2 <sup>11</sup> / <sub>16</sub> | 2 <sup>11</sup> / <sub>8</sub> | 2 <sup>11</sup> / <sub>4</sub> | 2 <sup>3</sup> / <sub>2</sub>   | 2 <sup>13</sup> / <sub>16</sub> | 2 <sup>7</sup> / <sub>8</sub>   | 2 <sup>15</sup> / <sub>16</sub> | 2 <sup>11</sup> / <sub>8</sub> | 2 <sup>9</sup> / <sub>16</sub>  | 2 <sup>5</sup> / <sub>8</sub>   | 2 <sup>11</sup> / <sub>16</sub> | 2 <sup>3</sup> / <sub>4</sub>   |
| B     | 2 <sup>5</sup> / <sub>8</sub>    | 2 <sup>7</sup> / <sub>16</sub>  | 7 <sup>1</sup> / <sub>16</sub> | 1 <sup>5</sup> / <sub>2</sub>  | 3 <sup>1</sup> / <sub>4</sub>   | 3 <sup>1</sup> / <sub>2</sub>   | 3 <sup>3</sup> / <sub>4</sub>   | 17 <sup>1</sup> / <sub>16</sub> | 3 <sup>5</sup> / <sub>16</sub> | 9 <sup>1</sup> / <sub>16</sub>  | 9 <sup>1</sup> / <sub>16</sub>  | 9 <sup>1</sup> / <sub>16</sub>  | 9 <sup>1</sup> / <sub>16</sub>  |
| C & D | 3 <sup>1</sup> / <sub>2</sub>    | 3 <sup>11</sup> / <sub>16</sub> | 3 <sup>7</sup> / <sub>8</sub>  | 4 <sup>1</sup> / <sub>16</sub> | 4 <sup>1</sup> / <sub>4</sub>   | 4 <sup>7</sup> / <sub>16</sub>  | 4 <sup>5</sup> / <sub>8</sub>   | 4 <sup>13</sup> / <sub>16</sub> | 5                              | 5 <sup>3</sup> / <sub>16</sub>  | 5 <sup>3</sup> / <sub>8</sub>   | 5 <sup>9</sup> / <sub>16</sub>  | 5 <sup>3</sup> / <sub>4</sub>   |
| E     | 5 <sup>5</sup> / <sub>8</sub>    | 3 <sup>1</sup> / <sub>2</sub>   | 1 <sup>1</sup> / <sub>16</sub> | 3 <sup>3</sup> / <sub>2</sub>  | 4 <sup>9</sup> / <sub>16</sub>  | 2 <sup>5</sup> / <sub>2</sub>   | 1 <sup>3</sup> / <sub>2</sub>   | 3 <sup>7</sup> / <sub>16</sub>  | 7 <sup>5</sup> / <sub>16</sub> | 5 <sup>7</sup> / <sub>16</sub>  | 5 <sup>9</sup> / <sub>16</sub>  | 1 <sup>3</sup> / <sub>8</sub>   | 3 <sup>1</sup> / <sub>2</sub>   |
| F     | 1 <sup>55</sup> / <sub>64</sub>  | 1 <sup>33</sup> / <sub>32</sub> | 2 <sup>1</sup> / <sub>16</sub> | 2 <sup>1</sup> / <sub>4</sub>  | 2 <sup>17</sup> / <sub>64</sub> | 2 <sup>3</sup> / <sub>8</sub>   | 2 <sup>15</sup> / <sub>32</sub> | 2 <sup>35</sup> / <sub>64</sub> | 2 <sup>5</sup> / <sub>8</sub>  | 2 <sup>4</sup> / <sub>16</sub>  | 2 <sup>55</sup> / <sub>64</sub> | 2 <sup>63</sup> / <sub>64</sub> | 3 <sup>5</sup> / <sub>64</sub>  |
| G     | 1 <sup>1</sup> / <sub>64</sub>   | 1 <sup>1</sup> / <sub>16</sub>  | 1 <sup>1</sup> / <sub>8</sub>  | 1 <sup>1</sup> / <sub>4</sub>  | 1 <sup>7</sup> / <sub>32</sub>  | 1 <sup>3</sup> / <sub>16</sub>  | 1 <sup>11</sup> / <sub>32</sub> | 1 <sup>27</sup> / <sub>64</sub> | 1 <sup>1</sup> / <sub>2</sub>  | 1 <sup>25</sup> / <sub>64</sub> | 1 <sup>19</sup> / <sub>32</sub> | 1 <sup>41</sup> / <sub>64</sub> | 1 <sup>45</sup> / <sub>64</sub> |

The chief features of disagreement among railroad men relate to the radius of the upper corner of the head and the slope of the side of the head. The radius ( $\frac{5}{16}$ " ) adopted for the upper corner (constant for all weights) is a little more than is advocated by those in favor of "sharp corners" who often use a radius of  $\frac{1}{4}$ ". On the other hand it is much less than is advocated by those who consider that it should be nearly equal to (or even greater than) the larger

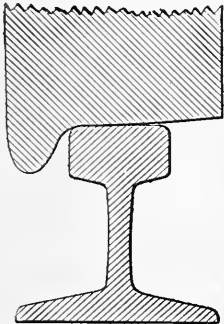


FIG. 114. — RELATION OF RAIL TO WHEEL-TREAD.

radius universally adopted for the corner of the wheel-flange. The discussion turns on the relative rapidity of rail wear and the wear of the wheel-flanges as affected by the relation of the form of the wheel-tread to that of the rail. It is argued that sharp rail corners wear the wheel-flanges so as to produce sharp flanges, which are liable to cause derailment at switches and also to require that the tires of engine-drivers must be more frequently turned down to their true form. On the other hand it is generally believed that rail wear is much less rapid while the area of contact between the rail and wheel-flange is small, and that when the rail has worn down, as it invariably does, to nearly the same form as the wheel-flange, the rail wears away very quickly.

**227. Weight for various kinds of traffic.** The heaviest rails in regular use weigh 100 lbs. per yard, and even these are only used on some of the heaviest traffic sections of such roads as the N. Y. Central, the Pennsylvania, the N. Y., N. H. & H., and

a few others. Probably the larger part of the mileage of the country is laid with 60- to 75-lb. rails—considering the fact that “the larger part of the mileage” consists of comparatively light-traffic roads and may exclude all the heavy trunk lines. Very light-traffic roads are sometimes laid with 56-lb. rails. Roads with fairly heavy traffic generally use 75- to 85-lb. rails, especially when grades are heavy and there is much and sharp curvature. The tendency on all roads is toward an increase in the weight, rendered necessary on account of the increase in the weight and capacity of rolling stock, and due also to the fact that the price of rails has been so reduced that it is both better and cheaper to obtain a more solid and durable track by increasing the weight of the rail rather than by attempting to support a weak rail by an excessive number of ties or by excessive track labor in tamping. It should be remembered that in buying rails the mere weight is, in one sense, of no importance. The important thing to consider is the *STRENGTH* and the *STIFFNESS*. If we assume that all weights of rails have *similar* cross-sections (which is nearly although not exactly true), then, since for beams of similar cross-sections the *strength* varies as the *cube* of the homologous dimensions and the *stiffness* as the *fourth power*, while the area (and therefore the weight per unit of length) only varies as the *square*, it follows that the stiffness varies as the square of the weight, and the strength as the  $\frac{2}{3}$  power of the weight. Since for ordinary variations of weight the price per ton is the same, adding (say) 10% to the weight (and cost) adds 21% to the stiffness and over 15% to the strength. As another illustration, using an 80-lb. rail instead of a 75-lb. rail adds only  $6\frac{2}{3}\%$  to the cost, but adds about 14% to the stiffness and nearly 11% to the strength. This shows why heavier rails are more economical and are being adopted even when they are not absolutely needed on account of heavier rolling stock. The stiffness, strength, and consequent durability are increased in a much greater ratio than the cost.

**228. Effect of stiffness on traction.** A very important but generally unconsidered feature of a stiff rail is its effect on trac-

tive force. An extreme illustration of this principle is seen when a vehicle is drawn over a soft sandy road. The constant compression of the sand in front of the wheel has virtually the same effect on traction as drawing the wheel up a grade whose steepness depends on the radius of the wheel and the depth of the rut. On the other hand, if a wheel, made of perfectly elastic material, is rolled over a surface which, while supported with absolute rigidity, is also perfectly elastic, there would be a forward component, caused by the expanding of the compressed metal just behind the center of contact, which would just balance the backward component. If the rail was supported throughout its length by an absolutely rigid support, the high elasticity of the wheel-tires and rails would reduce this form of resistance to an insignificant quantity, but the ballast and even the ties are comparatively inelastic. When a weak rail yields, the ballast is more or less compressed or displaced, and even though the elasticity of the rail brings it back to nearly its former place, the work done in compressing an inelastic material is wholly lost. The effect of this on the fuel account is certainly very considerable and yet is frequently entirely overlooked. It is practically impossible to compute the saving in tractive power, and therefore in cost of fuel, resulting from a given increase in the weight and stiffness of the rail, since the yielding of the rail is so dependent on the spacing of the ties, the tamping, etc. But it is not difficult to perceive in a general way that such an economy is possible and that it should not be neglected in considering the value of stiffness in rails.

**229. Length of rails.** The standard length of rails with most railroads is 30 feet. In recent years many roads have been trying 45-foot and even 60-foot rails. The argument in favor of longer rails is chiefly that of the reduction in track-joints, which are costly to construct and to maintain and are a fruitful source of accidents. Mr. Morrison of the Lehigh Valley R.R.\* declares that, as a result of extensive experience with 45-foot rails

on that road, he finds that they are much less expensive to handle, and that, being so long, they can be laid around sharp curves without being curved in a machine, as is necessary with the shorter rails. The great objection to longer rails lies in the difficulty in allowing for the expansion, which will require, in the coldest weather, an opening at the joint of nearly  $\frac{3}{4}$ " for a 60-foot rail. The Pennsylvania R.R. and the Norfolk and Western R.R. each have a considerable mileage laid with 60-foot rails.

**230. Expansion of rails.** Steel expands at the rate of .0000065 of its length per degree Fahrenheit. The extreme range of temperature to which any rail will be subjected will be about  $160^{\circ}$ , or say from  $-20^{\circ}$  F. to  $+140^{\circ}$  F. With the above coefficient and a rail length of 60 feet the expansion would be 0.0624 foot, or about  $\frac{3}{4}$  inch. But it is doubtful whether there would ever be such a range of motion even if there were such a range of temperature. Mr. A. Torrey, chief engineer of the Mich. Cent. R.R., experimented with a section over 500 feet long, which, although not a single rail, was made "continuous" by rigid splicing, and he found that there was no appreciable additional contraction of the rail at any temperature below  $+20$  F. The reason is not clear, but the *fact* is undeniable.

The heavy girder rails, used by the street railroads of the country, are bonded together with perfectly tight rigid joints which do not permit expansion. If the rails are laid at a temperature of  $60^{\circ}$  F. and the temperature sinks to  $0^{\circ}$ , the rails have a *tendency* to contract .00039 of their length. If this tendency is resisted by the friction of the pavement in which the rails are buried, it only results in a tension amounting to .00039 of the modulus of elasticity, or say 10920 pounds per square inch, assuming 28 000 000 as the modulus of elasticity. This stress is not dangerous and may be permitted. If the temperature rises to  $120^{\circ}$  F., a tendency to expansion and buckling will take place, which will be resisted as before by the pavement, and a compression of 10920 pounds per square inch will be induced, which will likewise be harmless. The range of tempera-

ture of rails which are buried in pavement is much less than when they are entirely above the ground and will probably never reach the above extremes. Rails supported on ties which are only held in place by ballast must be allowed to expand and contract almost freely, as the ballast cannot be depended on to resist the distortion induced by any considerable range of temperature, especially on curves.

**231. Rules for allowing for temperature.** Track regulations generally require that the track foremen shall use iron (*not* wooden) shims for placing between the ends of the rails while splicing them. The thickness of these shims should vary with the temperature. Some roads use such approximate rules as the following: "The proper thickness for coldest weather is  $\frac{5}{16}$  of an inch; during spring and fall use  $\frac{1}{8}$  of an inch, and in the very hottest weather  $\frac{1}{16}$  of an inch should be allowed." This is on the basis of a 30-foot rail. When a more accurate adjustment than this is desired, it may be done by assuming some very high temperature (120° to 150° F.) as a maximum, when the joints should be *tight*; then compute in tabular form the spacing for each temperature, varying by 20°, allowing 0".0468 (almost exactly  $\frac{3}{64}$ ") for each 20° change. Such a tabular form would be about as follows (rail length 30 feet):

| Temperature. .... | 150° | 130°             | 110°             | 90°              | 70°              | 50°               | 30°              | 10°               | - 10°           | - 30°             |
|-------------------|------|------------------|------------------|------------------|------------------|-------------------|------------------|-------------------|-----------------|-------------------|
| Rail opening...   | 0    | $\frac{3}{64}$ " | $\frac{3}{32}$ " | $\frac{9}{64}$ " | $\frac{3}{16}$ " | $\frac{15}{64}$ " | $\frac{9}{32}$ " | $\frac{21}{64}$ " | $\frac{3}{8}$ " | $\frac{27}{64}$ " |

One practical difficulty in the way of great refinement in this work is the determination of the real temperature of the rail when it is laid. A rail lying in the hot sun has a very much higher temperature than the air. The temperature of the rail cannot be obtained even by exposing a thermometer directly to the sun, although such a result might be the best that is easily obtainable. On a cloudy or rainy day the rail has practically the same temperature as the air; therefore on such days there need be no such trouble.



**232. Chemical composition.** About 98 to 99.5% of the composition of steel rails is iron, but the value of the rail, as a rail, is almost wholly dependent upon the large number of other chemical elements which are, or may be, present in very small amounts. The manager of a steel-rail mill once declared that their aim was to produce rails having in them—

|                 |                |
|-----------------|----------------|
| Carbon.....     | 0.32 to 0.40%  |
| Silicon.....    | 0.04 to 0.06%  |
| Phosphorus..... | 0.09 to 0.105% |
| Manganese.....  | 1.00 to 1.50%  |

The analysis of 32 specimens of rails on the Chic., Mil. & St. Paul R.R. showed variations as follows:

|                 |                 |
|-----------------|-----------------|
| Carbon.....     | 0.211 to 0.52%  |
| Silicon.....    | 0.013 to 0.256% |
| Phosphorus..... | 0.055 to 0.181% |
| Manganese.....  | 0.35 to 1.63%   |

These quantities have the same general relative proportions as the rail-mill standard given above, the differences lying mainly in the broadening of the limits. Increasing the percentage of carbon by even a few hundredths of one per cent makes the rail harder, but likewise more brittle. If a track is well ballasted and not subject to heaving by frost, a harder and more brittle rail may be used without excessive danger of breakage, and such a rail will wear much longer than a softer tougher rail, although the softer tougher rail may be the better rail for a road having a less perfect roadbed.

A small but objectionable percentage of sulphur is sometimes found in rails, and very delicate analysis will often show the presence, in very minute quantities, of several other chemical elements. The use of a very small quantity of nickel or aluminum has often been suggested as a means of producing a more durable rail. The added cost and the uncertainty of

the amount of advantage to be gained has hitherto prevented the practical use or manufacture of such rails.

**233. Testing.** Chemical and mechanical testing are both necessary for a thorough determination of the value of a rail. The chemical testing has for its main object the determination of those minute quantities of chemical elements which have such a marked influence on the rail for good or bad. The mechanical testing consists of the usual tests for elastic limit, ultimate strength, and elongation at rupture, determined from pieces cut out of the rail, besides a "drop test." The drop test consists in dropping a weight of 2000 lbs. from a height of 16 to 20 feet on to the center of a rail which is supported on abutments placed three or four feet apart. The number of blows required to produce rupture or to produce a permanent set of specified magnitude gives a measure of the strength and toughness of the rail.

**234. Rail wear on tangents.** When the wheel loads on a rail are abnormally heavy, and particularly when the rail has but little carbon and is unusually soft, the concentrated pressure on



FIG. 115.

the rail is frequently greater than the elastic limit, and the metal "flows" so that the head, although greatly abraded, will spread somewhat outside of its original lines, as shown in Fig.

115. The rail wear that occurs on tangents is almost exclusively on top. Statistics show that the rate of rail wear on tangents decreases as the rails are more worn. Tests of a large number of rails on tangents have shown a rail wear averaging nearly one pound per yard per 10 000 000 tons of traffic. There is about 33 pounds of metal in one yard of the head of an 80-lb. rail. As an extreme value this may be worn down one-half, thus giving a tonnage of 165 000 000 tons for the life of the rail. Other estimates bring the tonnage down to 125 000 000 tons. Since the locomotive is considered to be responsible for one half (and possibly more) of the damage done to the rail, it is found that the rate of wear on roads with shorter trains is more rapid in proportion to the tonnage, and it

is therefore thought that the life of a rail should be expressed in terms of the number of trains. This has been estimated at 300 000 to 500 000 trains.

**235. Rail wear on curves.** On curves the maximum rail wear occurs on the inner side of the head of the outer rail, giving a worn form somewhat as shown in Fig. 116. The dotted line shows the nature and progress of the rail wear on the inner rail of a curve. Since the pressure on the outer rail is somewhat lateral rather than vertical, the “flow” does not take place to the same extent, if at all, on the outside, and whatever flow would take place on the inside is immediately worn off by the wheel-flange. Unlike the wear on tangents, the wear on curves is at a greater rate as the rail becomes more worn.

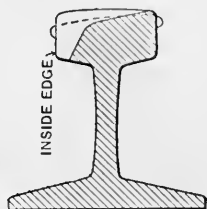


FIG. 116.

The inside rail on curves wears chiefly on top, the same as on a tangent, except that the wear is much greater owing to the longitudinal slipping of the wheels on the rail, and the lateral slipping that must occur when a rigid four-wheeled truck is guided around a curve. The outside rail is subjected to a greater or less proportion of the longitudinal slipping, likewise to the lateral slipping, and, worst of all, to the grinding action of the flange of the wheel, which grinds off the side of the head.

The results of some very elaborate tests, made by Mr. A. M. Wellington, on the Atlantic and Great Western R.R., on the wear of rails, seem to show that the rail wear on curves may be expressed by the formula: “Total wear of rails on a  $d$  degree curve in pounds per yard per 10 000 000 tons duty  $= 1 + 0.03d^2$ .” “It is not pretended that this formula is strictly correct even in theory, but several theoretical considerations indicate that it may be nearly so.” According to this formula the average rail wear on a  $6^\circ$  curve will be about twice the rail wear on a tangent. While this is approximately true, the various causes modifying the rate of rail wear (length of trains, age and quality of rails, etc.) will result in numerous and

large variations from the above formula, which should only be taken as indicating an approximate law.

**236. Cost of rails.** In 1873 the cost of steel rails was about \$120 per ton, and the cost of iron rails about \$70 per ton. Although the steel rails were at once recognized as superior to iron rails on account of more uniform wear, they were an expensive luxury. The manufacture of steel rails by the Bessemer process created a revolution in prices, and they have steadily dropped in price until, during the last few years, steel rails have been manufactured and sold for \$22 per ton. At such prices there is no longer any demand for iron rails, since the cost of manufacturing them is substantially the same as that of steel rails, while their durability is unquestionably inferior to that of steel rails.

## CHAPTER X.

### RAIL-FASTENINGS.

#### RAIL-JOINTS.

**237. Theoretical requirements for a perfect joint.** A perfect rail-joint is one that has the *same strength* and *stiffness*—no more and no less—as the rails which it joins, and which will not interfere with the regular and uniform spacing of ties. It should also be reasonably cheap both in first cost and in cost of maintenance. Since the action of heavy loads on an elastic rail is to cause a wave of translation in front of each wheel, any change in the stiffness or elasticity of the rail structure will cause more or less of a shock, which must be taken up and resisted by the joint. The greater the change in stiffness the greater the shock, and the greater the destructive action of the shock. The perfect rail-joint must keep both rail ends truly in line both laterally and vertically, so that the flange or tread of the wheel need not jump or change its direction of motion suddenly in passing from one rail to the other. A consideration of all the above requirements will show that only a perfect welding of rail-ends would produce a joint of uniform strength and stiffness which would give a uniform elastic wave ahead of each wheel. As welding is impracticable for ordinary railroad work (see § 230), some other contrivance is necessary which will approach this ideal as closely as may be.

**238. Efficiency of the ordinary angle-bar.** Throughout the middle portion of a rail the rail acts as a continuous girder. If we consider for simplicity that the ties are unyielding, the deflection of such a continuous girder between the ties will be but

one-fourth of the deflection that would be found if the rail were cut half-way between the ties and an equal concentrated load were divided equally between the two unconnected ends. The maximum stress for the continuous girder would be but one-half of that in the cantilevers. Joining these ends with rail-joints will give the ordinary "suspended" joint. In order to maintain uniform strength and stiffness the angle-bars must supply the deficiency. These theoretical relations are modified to an unknown extent by the unknown and variable yielding of the ties. From some experiments made by the Association of Engineers of Maintenance of Way of the P. R.R.\* the following deductions were made:

1. The capacity of a "suspended" joint is greater than that of a "supported" joint—whether supported on one or three ties. (See § 240.)

2. That (with the particular patterns tested) the angle-bars alone can carry only 53 to 56% of a concentrated load placed on a joint.

3. That the capacity of the whole joint (angle-bars and rail) is only 52.4% of the strength of the unbroken rail.

4. That the ineffectiveness of the angle-bar is due chiefly to a deficiency in compressive resistance.

Although it has been universally recognized that the angle-bar is not a perfect form of joint, its simplicity, cheapness, and reliability have caused its almost universal adoption. Within a very few years other forms (to be described later) have been adopted on trial sections and have been more and more extended, until their present use is very large. The present time (1900) is evidently a transition period, and it is quite probable that within a very few years the now common angle-plate will be as unknown in standard practice as the old-fashioned "fish-plate" is at the present time.

**239. Effect of rail gap at joints.** It has been found that the jar at a joint is due almost entirely to the *deflection* of the joint

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\* Roadmasters Association of America—Reports for 1897.

and scarcely at all to the small gap required for expansion. This gap causes a drop equal to the versed sine of the arc having a chord equal to the gap and a radius equal to the radius of the wheel. Taking the extreme case (for a 30-foot rail) of a  $\frac{3}{8}$ " gap and a 33" freight-car wheel, the drop is about  $\frac{1}{1000}$ ". In order to test how much the jarring at a joint is due to a gap between the rails, the experiment was tried of cutting shallow notches in the top of an otherwise solid rail and running a locomotive and an inspection car over them. The resulting jarring was practically imperceptible and not comparable to the jar produced at joints. Notwithstanding this fact, many plans have been tried for avoiding this gap. The most of these plans consist essentially of some form of compound rail, the sections breaking joints. (Of course the design of the compound rail has also several other objects in view.) In Fig. 117 are shown a

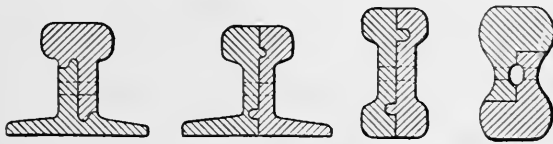


FIG. 117.—COMPOUND RAIL SECTIONS.

few of the very many designs which have been proposed. These designs have invariably been abandoned after trial. Another plan, which has been extensively tried on the Lehigh Valley R.R., is the use of mitered joints. The advantages gained by their use are as yet doubtful, while the added expense is unquestionable. The "Roadmasters Association of America" in 1895 adopted a resolution recommending mitered joints for double track, but their use does not seem to be growing.

**240. "Supported," "suspended," and "bridge" joints.** In a supported joint the ends of the rails are on a tie. If the angle-plates are short, the joint is entirely supported on one tie; if very long, it may be possible to place three ties under one angle-bar and thus the joint is virtually supported on three ties rather than one. In a suspended joint the ends of the rails are midway between two ties and the joint is supported by the two. There

have always been advocates of both methods, but suspended joints are more generally used than supported joints. The opponents of three-tie joints claim that either the middle tie will be too strongly tamped, thus making it a supported joint, or that, if the middle tie is weakest, the joint becomes a very long (and therefore weak) suspended joint between the outer joint-ties, or that possibly one of the outer joint-ties gives way, thus breaking the angle-plate at the joint. Another objection which is urged is that unless the bars are very long (say 44 inches, as used on the Mich. Cent. R.R.) the ties are too close for proper tamping. The best answer to these objections is the successful use of these joints on several heavy-traffic roads.

“Bridge”-joints are similar to suspended joints in that the joint is supported on two ties, but there is the important difference that the bridge-joint supports the rail from *underneath* and there is no transverse stress in the rail, whereas the supported joint requires the combined transverse strength of both angle-bars and rail. A serious objection to bridge-joints lies in the fact of their considerable thickness between the rail base and the tie. When joints are placed “staggered” rather than “opposite” (as is now the invariable standard practice), the ties supporting a bridge-joint must either be notched down, thus weakening the tie and promoting decay at the cut, or else the tie must be laid on a slope and the joint and the opposite rail do not get a fair bearing.

**241. Failures of rail-joints.** It has been observed on double-track roads that the maximum rail wear occurs a few inches beyond the rail gap at the joint in the direction of the traffic. On single-track roads the maximum rail wear is found a few inches *each* side of the joint rather than at the extreme ends of the rail, thus showing that the rail end deflects down under the wheel until (with fast trains especially) the wheel actually jumps the space and strikes the rail a few inches beyond the joint, the impact producing excessive wear. This action, which is called the “drop,” is apt to cause the first tie beyond the joint to become depressed, and unless this tie is carefully watched and main-



tained at its proper level, the stresses in the angle-bar may actually become reversed and the bar may break at the top. The angle-bars of a suspended joint are normally in compression at the top. The mere reversal of the stresses would cause the bars

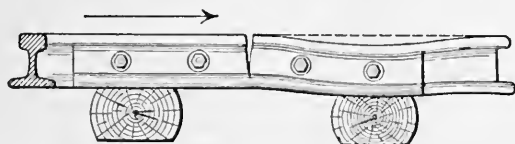


FIG. 118.—EFFECT OF “WHEEL DROP” (EXAGGERATED).

to give way with a less stress than if the stress were always the same in kind. A supported joint, and especially a three-tie joint (see § 240), is apt to be broken in the same manner.

**242. Standard angle-bars.**—An angle-bar must be so made as to closely fit the rails. The great multiplicity in the designs of rails (referred to in Chapter IX) results in nearly as great variety in the detailed dimensions of the angle-bars. The sections here illustrated must be considered only as types of the variable forms necessary for each different shape of rail. The absolutely essential features required for a fit are (1) the angles

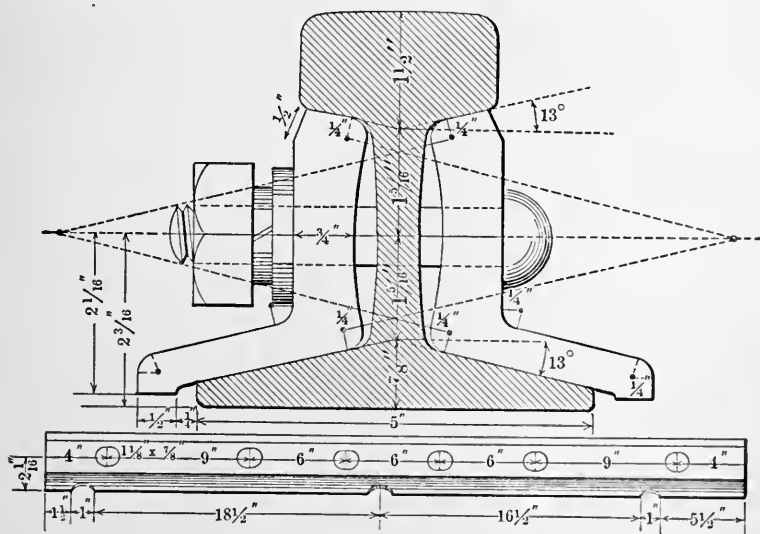


FIG. 119.—STANDARD ANGLE-BAR—80-LB. RAIL. M. C. R.R.

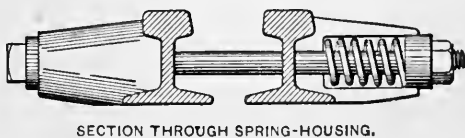
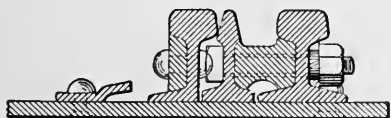
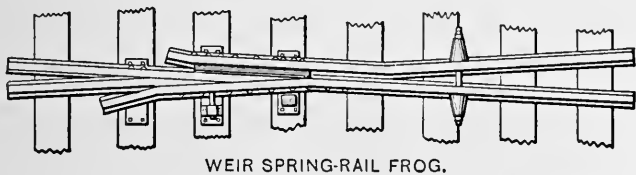
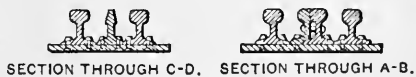
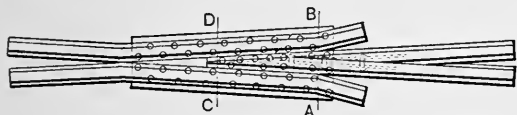
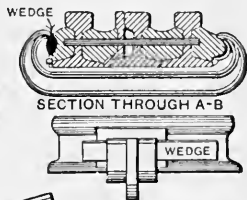
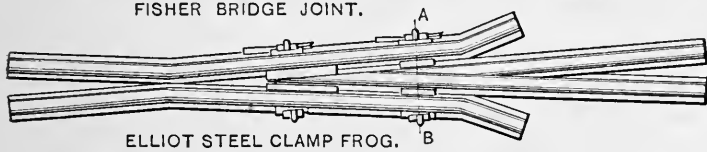
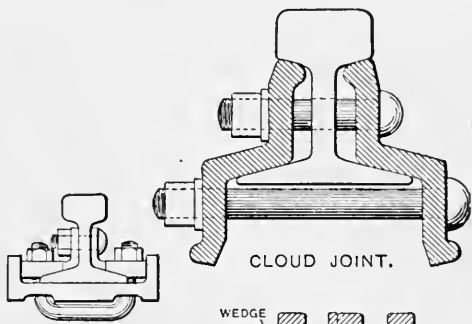
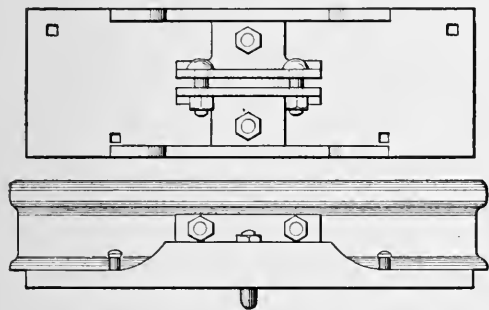
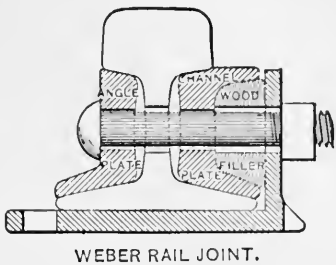
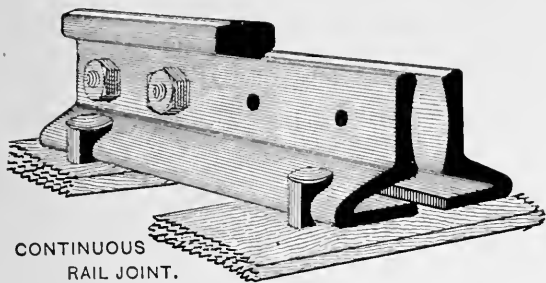
of the upper and lower surfaces of the bar where they fit against the rail, and (2) the height of the bar. The bolt-holes in the

bar and rail must also correspond. The holes in the angle-plates are elongated or made oval, so that the track-bolts, which are made of corresponding shape immediately under the head, will not be turned by jarring or vibration. The holes in the rails are made of larger diameter (by about  $\frac{1}{4}$ "') than the bolts, so as to allow the rail to expand with temperature.

**243. Later designs of rail-joints.** In Plate XVIII are shown various designs which are competing for adoption. The most prominent of these (judging from the discussion in the convention of the Roadmasters Association of America in 1897) are the "Continuous" and the "Weber." Each of them has been very extensively adopted, and where used are universally preferred to angle-plates. Nearly all the later designs embody more or less directly the principle of the bridge-joint, i.e., support the rail from underneath. An experience of several years will be required to demonstrate which form of joint best satisfies the somewhat opposed requirements of minimum cost (both initial and for maintenance) and minimum wear of rails and rolling stock.

#### TIE-PLATES.

**244. Advantages.** (a) As already indicated in § 204, the life of a soft-wood tie is very much reduced by "rail-cutting" and "spike-killing," such ties frequently requiring renewal long before any serious decay has set in. It has been practically demonstrated that the "rail-cutting" is not due to the mere pressure of the rail on the tie, even with a maximum load on the rail, but is due to the impact resulting from vibration and to the longitudinal working of the rail. It has been proved that this rail-cutting is practically prevented by the use of tie-plates. (b) On curves there is a tendency to overturn the outer rail due to the lateral pressure on the side of the head. This produces a concentrated pressure of the outer edge of the base on the tie which produces rail-cutting and also draws the inner spikes. Formerly the only method of guarding



RAIL JOINTS AND FROGS.

(To face page 260.)



against this was by the use of "rail-braces," one pattern of which is shown in Fig. 120. But it has been found that tie-

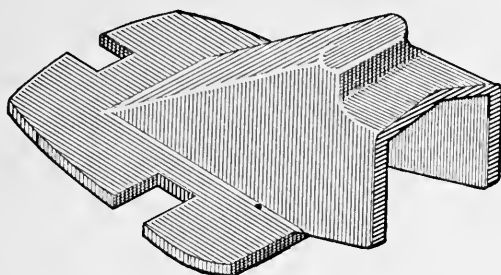


FIG. 120.

plates serve the purpose even better, and rail-braces have been abandoned where tie-plates are used. (c) Driving spikes through holes in the plate enables the spikes on *each* side of the rail to mutually support each other, no matter in which (lateral) direction the rail may tend to move, and this probably accounts in large measure for the added stability obtained by the use of tie-plates. (d) The wear in spikes, called "necking," caused by the vertical vibration of the rail against them, is very greatly reduced. (e) The cost is very small compared with the value of the added life of the tie, the large reduction in the work of track maintenance, and the smoother running on the better track which is obtained. It has been estimated that by the use of tie-plates the life of hard-wood ties is increased from one to three years, and the life of soft-wood ties is increased from three to six years. From the very nature of the case, the value of tie-plates is greater when they are used to protect soft ties.

**245. Elements of the design.** The earliest forms of tie-plates were flat on the bottom, but it was soon found that they would work loose, allow sand and dirt to get between the rail and the plate and also between the plate and the tie, which would cause excessive wear. Such plates are also apt to produce an objectionable rattle. Another fault of the earlier designs was the use of plates so thin that they would buckle. The latest designs have flanges or "teeth" formed on the lower surface which penetrate the tie about  $\frac{3}{4}$ " to  $1\frac{3}{8}$ ". Opinion is still divided on the question of whether these teeth should run with the grain

or across the grain. If the flanges run with the grain, they generally extend the whole length of the tie-plate—as in the Wollhaupter design. If the grain is to be cut crosswise, several teeth about 1" wide will be used—as in the Goldie design.

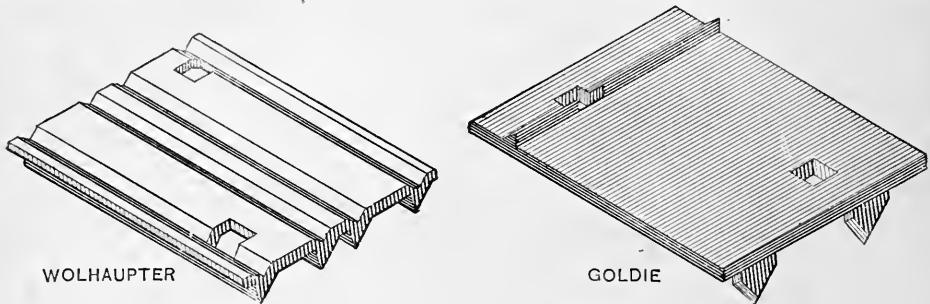


FIG. 121.—TIE-PLATES.

It is a very important feature that the spike-holes should be so punched that the spikes will fit closely to the base of the rail. Otherwise a lateral motion of the rail will be permitted which will defeat one of the main objects of the use of the plate.

Another unsettled detail is the use of “shoulders” on the upper surface. On the one hand it is claimed that the use of shoulders relieves the spikes of side pressure from the rail and prevents “necking.” On the other hand it is claimed that if the plain plate is once properly set with new spikes (at least with spikes not already necked) the spikes will not neck appreciably, and that, as the shouldered plates cost more, the additional expenditure is unnecessary.

The above designs should be studied with reference to the manner in which they fulfill the requirements which have been already stated. As in the case of rail-joints, the best forms of tie-plates are of comparatively recent design, and experience with them is still insufficient to determine beyond all question which designs are the best.

**246. Methods of setting.** A very important detail in the process of setting the tie-plates on the ties is that the flanges or teeth should penetrate the tie as far as desired when the plates are first put in position. It requires considerable force to press the teeth into a tie. In a few cases trackmen have depended on the easy process of waiting for passing trains to force the teeth

down. Until the teeth are down the spikes cannot be driven home, and this apparently cheap and easy process results in loose spikes and rails. If the trackmen neglect even temporarily to tighten these spikes, it will become impossible to make them tight ultimately. The plates are generally pounded into place with a 10- to 16-pound sledge-hammer. A very good method was adopted once during the construction of a bridge when a pile-driver was at hand. The bridge-ties were placed under the pile-hammer. The plates, accurately set to gauge, were then forced in by a blow from the 3000-lb. hammer falling 2 or 3 feet.

## SPIKES.

**247. Requirements.** The rails must be held to the ties by a fastening which will not only give sufficient resistance, but which will retain its capacity for resistance. It must also be cheap and easily applied. The ordinary track-spike fulfills the last requirements, but has comparatively small resisting power, compared with screws or bolts. Worse than all, the tendency to vertical vibration in the rail produces a series of upward pulls on the spike that soon loosens it. When motion has once begun the capacity for resistance is greatly reduced, and but little more vibration is required to pull the spike out so much that redriving is necessary. Driving the spike to place again in the same hole is of small value except as a very temporary expedient, as its holding power is then very small. Redriving the spikes in new holes very soon "spike-kills" the tie. Many plans have been devised to increase the holding power of spikes, such as making them jagged, twisting the spike, swelling the spike at about the center of its length, etc. But it has been easily demonstrated that the fibers of the wood are generally so crushed and torn by driving such spikes that their holding power is less than that of the plain spike.

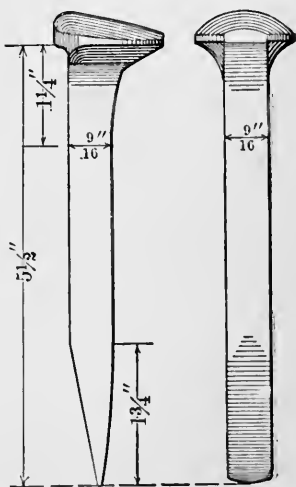


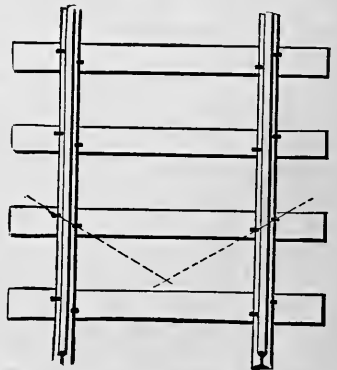
FIG. 122.



The ordinary spike (see Fig. 122) is made with a square cross-section which is uniform through the middle of its length, the lower  $1\frac{3}{4}$ " tapering down to a chisel edge, the upper part swelling out to the head. The Goldie spike (see Fig. 123) aims to improve this form by reducing to a minimum the destruction of the fibers. To this end, the sides are made smooth, the edges are clean-cut, and the point, instead of being chisel-shaped, is ground down to a pyramidal form. Such fiber-cutting as occurs is thus accomplished without much crushing, and the fibers are thus pressed away from the spike and slightly downward. Any tendency to draw the spike will therefore cause

the fibers to press still harder on the spike and thus increase the resistance.

**248. Driving.** The holding power of a spike depends largely on how it is driven. If the blows are eccentric and irregular in direction, the hole will be somewhat enlarged and the holding power largely decreased. The spikes on each side of the rail in any one tie should not be directly opposite, but should be staggered. Placing them directly opposite will tend to split the tie, or at least decrease the holding power of the spikes. The direction of staggering should be reversed in the two pairs of spikes in any one tie



(see Fig. 124). This will tend to prevent any twisting of the tie in the ballast, which would otherwise loosen the rail from the tie.

**249. Screws and bolts.** The use of these abroad is very extensive, but their use in this country has not passed the experimental stage. The screws are "wood"-screws (see Fig. 125), having large square heads, which are screwed down with a track-wrench. Holes, having the same diameter as the *base* of the screw-threads, should first be bored into the tie, at exactly the right position and at the proper angle with the vertical.



A light wooden frame is sometimes used to guide the auger at the proper angle. Sometimes the large head of the screw bears directly against the base of the rail, as with the ordinary spike. Other designs employ a plate, made to fit the rail on one side, bearing on the tie on the other side, and through which the screw passes. These screws cost much more than spikes and require more work to put in place, but their holding power is much greater and the work of track maintenance is very much less. Screw-bolts, passing entirely through the tie, having the head at the bottom of the tie and the nut on the upper side, are also used abroad. These are quite difficult to replace, requiring that the ballast be dug out beneath the tie, but on the other hand the occasions for replacing such a bolt are comparatively rare, as their durability is very great. The



FIG. 125.

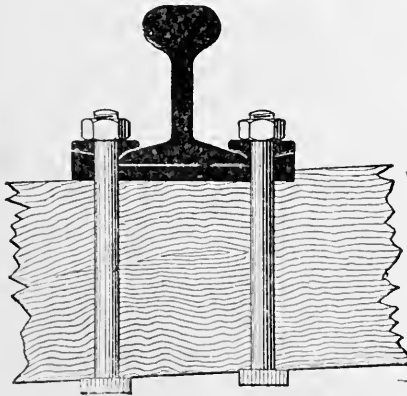


FIG. 126.

use of screws or bolts increases the life of the tie by the avoidance of "spike-killing." It is capable of demonstration that the reduced cost of maintenance and the resulting improvement in track would much more than repay the added cost of screws and bolts, but it seems impossible to induce railroad directors to authorize a large and immediate additional expenditure to make an annual saving whose value, although unquestionably considerable, cannot be exactly computed.

**250. "Wooden spikes."** Among the regulations for track-laying given in § 208, mention was made of wooden "spikes," or plugs, which are used to fill up the holes when spikes are withdrawn. The value of the policy of filling up these holes is unquestionable, since the expense is insignificant compared with the loss due to the quick and certain decay of the tie if these holes are allowed to fill with water and remain so. But the method of making these plugs is variable. On some roads they are "hand-made" by the trackmen out of otherwise useless



FIG. 127.

scraps of lumber, the work being done at odd moments. This policy, while apparently cheap, is not necessarily so, for the hand-made plugs are irregular in size and therefore more or less inefficient. It is also quite probable that if a track gang is required to make their own plugs, they may spend time on these very cheap articles which could be more profitably employed otherwise. Since the holes made by the spikes are larger at the top than they are near the bottom, the plugs should *not* be of uniform cross section but should be slightly wedge-shaped. The "Goldie tie-plug" (see Fig. 127) has been designed to fill these requirements. Being machine-

made, they are uniform in size; they are of a shape which will best fit the hole; they can be furnished of any desired wood, and at a cost which makes it a wasteful economy to attempt to cut them by hand.

#### TRACK-BOLTS AND NUT-LOCKS.

**251. Essential requirements.** The track-bolts must have sufficient strength and must be screwed up tight enough to hold the angle-plates against the rail with sufficient force to develop the full transverse strength of the angle-bars. On the other hand the bolts should not be screwed so tight that slipping may not take place when the rail expands or contracts with temperature. It would be impossible to screw the bolts tight enough to prevent

slipping during the contraction due to a considerable fall of temperature on a straight track, but when the track is curved, or when expansion takes place, it is conceivable that the resistance of the ties in the ballast to lateral motion may be less than the resistance at the joint. A test to determine this resistance was made by Mr. A. Torrey, chief engineer of the Mich. Cent. R.R., using 80-lb. rails and ordinary angle-bars, the bolts being screwed up as usual. It required a force of about 31000 to 35000 lbs. to start the joint, which would be equivalent to the stress induced by a change of temperature of about  $22^{\circ}$ . But if the central angle of any given curve is small, a comparatively small lateral component will be sufficient to resist a compression of even 35000 lbs. in the rails. Therefore there will ordinarily be no trouble about having the joints screwed too tight. The vibration caused by the passage of a train reduces the resistance to slipping. This vibration also facilitates an objectionable feature, viz., loosening of the nuts of the track-bolts. The bolt is readily prevented from turning by giving it a form which is *not* circular immediately under the head and making corresponding holes in the angle-plate. Square holes would answer the purpose, except that the square corners in the holes in the angle-plates would increase the danger of fracture of the plates. Therefore the holes (and also the bolts, under the head) are made of an oval form, or perhaps a square form with rounded corners, avoiding angles in the outline.

The nut-locks should be simple and cheap, should have a life at least as long as the bolt, should be effective, and should not lose their effectiveness with age. Many of the designs that have been tried have been failures in one or more of these particulars, as will be described in detail below.

**252. Design of track-bolts.** In Fig. 128 is shown a common design of track-bolt. In its general form this represents the bolt used on nearly all roads, being used not only with the common angle-plates, but also with many of the improved designs of rail-joints. The variations are chiefly a general increase in size to correspond with the increased

weight of rails, besides variations in detail dimensions which are frequently unimportant. The diameter is usually  $\frac{3}{4}$ " to

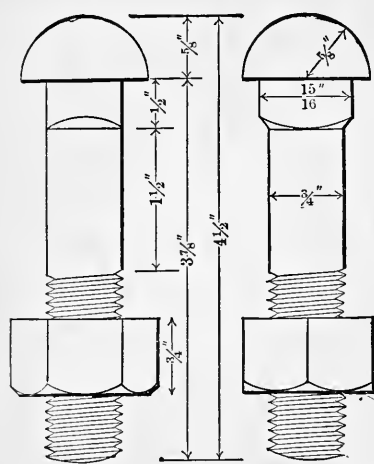


FIG. 128.—TRACK-BOLT.

$\frac{7}{8}$ "; 1" bolts are sometimes used for the heaviest sections of rails. As to length, the bolts should not extend more than  $\frac{1}{2}$ " outside of the nut when it is screwed up. If it extends farther than this, it is liable to be broken off by a possible derailment at that point. The lengths used vary from  $3\frac{1}{4}$ ", which may be used with 60 lbs. rails, to 5", which is required with 100-lb. rails. The length required depends somewhat on the type of nut-lock used.

**253. Design of nut-locks.** The designs for nut-locks may be divided into three classes: (a) those depending entirely on an elastic washer which absorbs the vibration which might otherwise induce turning; (b) those which jam the threads of the bolt and nut so that, when screwed up, the frictional resistance is too great to be overcome by vibration; (c) the "positive" nut-locks—those which mechanically hold the nut from turning. Some of the designs combine these principles to some extent. The "vulcanized fiber" nut-lock is an example of the first class. It consists essentially of a rubber washer which is protected by an iron ring. When first placed this lock is effective, but the rubber soon hardens and loses its elasticity and it is then ineffective and worthless. Another illustration of class (a) is the use of wooden blocks, generally of 1" to 2" oak, which extend the entire length of the angle-bar, a single piece forming the washer for the four or six bolts of a joint. This form is cheap, but the wood soon shrinks, loses its elasticity, or decays so that it soon becomes worthless, and it requires constant adjustment to keep it in even tolerable condition. The "Verona" nut-lock is another illustration of class (a) which also combines some of the positive elements of class (c). It is made of

tempered steel and, as shown in Fig. 129, is warped and has sharp edges or points. The warped form furnishes the element of elastic pressure when the nut is screwed up. The steel being harder than the iron of the angle-bar or of the nut, it bites into them, owing to the great pressure that must exist

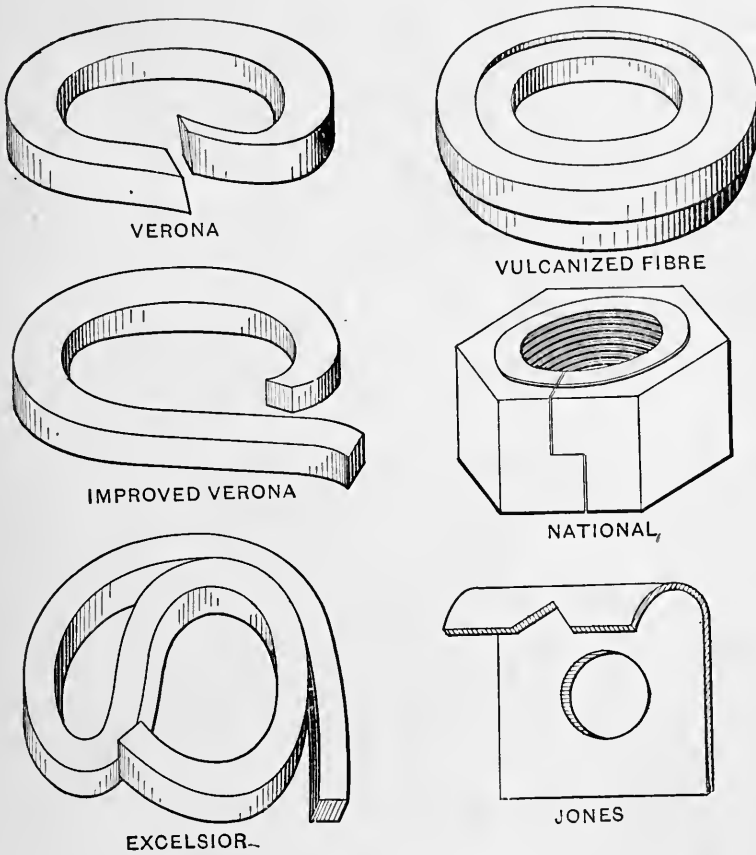


FIG. 129.—TYPES OF NUT-LOCKS.

when the washer is squeezed nearly flat, and thus prevents any *backward* movement, although forward movement (or tightening the bolt) is not interfered with. The “National” nut-lock is a type of the second class (*b*), in which, like the “Harvey” nut-lock, the nut and lock are combined in one piece. With six-bolt angle-bars and 30-foot rails, this means a saving of 2112 pieces on each mile of single track. The “National” nuts are open on one side. The hole is drilled and the thread is cut slightly smaller than the bolt, so that when the nut is screwed

up it is forced slightly open and therefore presses on the threads of the bolt with such force that vibration cannot jar it loose. Unlike the "National" nut, the "Harvey" nut is solid, but the form of the thread is progressively varied so that the thread pinches the thread of the bolt and the frictional resistance to turning is too great to be affected by vibration.

The "Jones" nut-lock, belonging to class (c), is a type of a nut-lock that does not depend on elasticity or jamming of screw-threads. It is made of a thin flexible plate, the square part of which is so large that it will not turn after being placed on the bolt. After the nut is screwed up, the thin plate is bent over so that the re-entrant angle of the plate engages the corner of the nut and thus mechanically prevents any turning. The metal is supposed to be sufficiently tough to endure without fracture as many bendings of the plate as will ever be desired. Nut-locks of class (c) are not in common use.

## CHAPTER XI.

### SWITCHES AND CROSSINGS.

#### SWITCH CONSTRUCTION.

**254. Essential elements of a switch.** Flanges of some sort are a necessity to prevent car-wheels from running off from the rails on which they may be moving. But the flanges, although a necessity, are also a source of complication in that they require some special mechanism which will, when desired, guide the wheels out from the controlling influence of the main-line rails. This must either be done by raising the wheels high enough so that the flanges may pass *over* the rails, or by breaking the continuity of the rails in such a way that channels or "flange spaces" are formed *through* the rails. An ordinary stub switch breaks the continuity of the main-line rails in three places, two of them at the switch-block and one at the frog. The Wharton switch avoids two of these breaks by so placing inclined planes that the wheels, rolling on their flanges, will surmount these inclines until they are a little higher than the rails. Then the wheels on the side toward which the switch runs are guided over and across the main rail on that side. This rise being accomplished in a short distance, it becomes impracticable to operate these switches except at slow speeds, as any sudden change in the path of the center of gravity of a car causes very destructive jars both to the switch and to the rolling stock. The other general method makes a break in one main rail (or both) at the switch-block. In both methods the wheels are led to one side by means of the "lead rails," and finally one line of wheels passes *through* the main rail on that side by means of a "frog." There are some designs by which even this break in the main rail is avoided, the wheels being led *over* the main rail by means





assembled by riveting them to a flat plate, but this method is now but little used, except for very light work. The usual practice is now chiefly divided between "bolted" and "keyed" frogs. In each case the space between the rails, except a sufficient flange-way, is filled with a cast-iron filler and the whole assemblage of parts is suitably bolted or clamped together, as is illustrated in Plate XVIII. The operation of a spring-rail frog is evident from the figure. Since a siding is usually operated at slow speed, while the main track may be operated at fast speed, a spring-rail frog will be so set that the tread is continuous for the main track and broken for the siding. This also means that the spring rail will only be moved by trains moving at a (presumably) slow speed on to the siding. For the fast trains on the main line such a frog is substantially a "fixed" frog and has a tread which is practically continuous.

**256. To find the frog number.** The frog number ( $n$ ) equals the ratio of the distance of any point on the tongue of the frog from the theoretical point of the frog divided by the width of the tongue at that point, i.e.  $= hc \div ab$  (Fig. 130). This value may be directly measured by applying any convenient unit of measure (even a knife, a short pencil, etc.) to some point of the tongue where the width just equals the unit of measure, and then noting how many times the unit of measure is contained in the distance from that place to the theoretical point. But since  $c$ , the theoretical point, is not so readily determinable with exactitude, it being the imaginary intersection of the gauge lines, it may be more accurate to measure  $de$ ,  $ab$ , and  $hs$ ; then  $n$ , the frog number,  $= hs \div (ab + de)$ . If the frog angle be called  $F$ , then

$$n = hc \div ab = hs \div (ab + de) = \frac{1}{2} \cot \frac{1}{2}F;$$

$$\text{i.e.} \quad \cot \frac{1}{2}F = 2n.$$

**257. Stub switches.** The use of these, although once nearly universal, has been practically abandoned as turnouts from *main track* except for the poorest and cheapest roads. In some States, their use on main track is prohibited by law. They

have the sole merit of cheapness with adaptability to the circumstances of very light traffic operated at slow speed when a considerable element of danger may be tolerated for the sake of economy. The rails from *A* to *B* (see Fig. 131\*) are not fastened

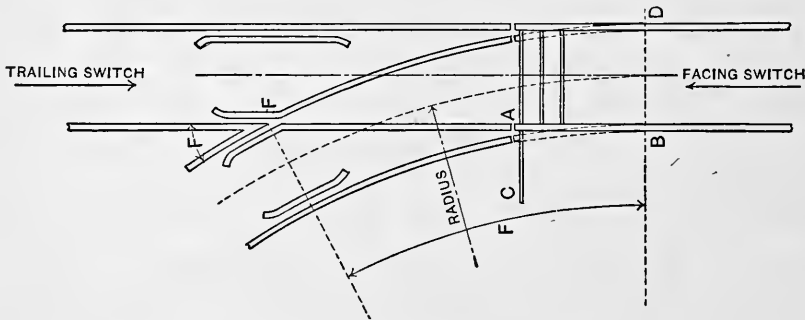


FIG. 131.—STUB SWITCH.

to the ties; they are fastened to each other by tie-rods which keep them at the proper gauge; at and back of *B* they are securely spiked to the ties, and at *A* they are kept in place by the connecting bar (*C*) fastened to the switch-stand. One great objection to the switch is that, in its usual form, when operated as a trailing switch, a derailment is inevitable if the switch is misplaced. The very least damage resulting from such a derailment must include the bending or breaking of the tie-rods of the switch-rail. Several devices have been invented to obviate this objection, some of which succeed very well mechanically, although their added cost precludes any economy in the total cost of the switch. Another objection to the switch is the looseness of construction which makes the switches objectionable at high speeds. The gap of the rails at the head-block is always considerable, and is sometimes as much as two inches. A

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\* The student should at once appreciate that in Fig. 131, as well as in nearly all the remaining figures in this chapter, it becomes necessary to use excessively large frog angles, short radii, and a very wide gauge in order to illustrate the desired principles with figures which are sufficiently small for the page. In fact, the proportions used in the figures are such that serious mechanical difficulties would be encountered if they were used. These difficulties are here ignored because they can be neglected in the proportions used in practice.

driving-wheel with a load of 12000 to 20000 pounds, jumping this gap with any considerable velocity, will do immense damage to the farther rail end, besides producing such a stress in the construction that a breakage is rendered quite likely, and such a breakage might have very serious consequences.

**258. Point switches.** The essential principle of a point switch is illustrated in Fig. 132. As is shown, one main rail and also one of the switch-rails is unbroken and immovable.

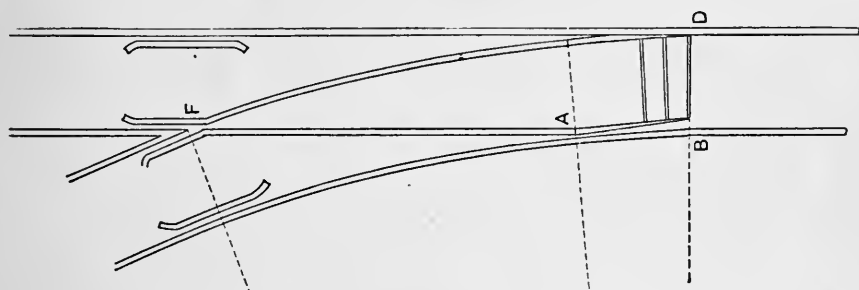


FIG. 132.—POINT SWITCH.

The other main rail (from *A* to *F*) and the corresponding portion of the other lead rail are substantially the same as in a stub switch. A portion of the main rail (*AB*) and an equal length of the opposite lead rail (usually 15 to 24 feet long) are fastened together by tie-rods. The end at *A* is jointed as usual and the other end is pointed, both sides being trimmed down so that the feather edge at *B* includes the web of the rail. In order to retain in it as much strength as possible, the point-rail is raised so that it rests on the base of the stock-rail, one side of the base of the point-rail being entirely cut away. As may be seen in Fig. 133, although the influence of the point of the rail in moving the wheel-flange away from the stock-rail is really zero at that point, yet the rail has all the strength of the web and about one-half that of the base—a very fair angle-iron.

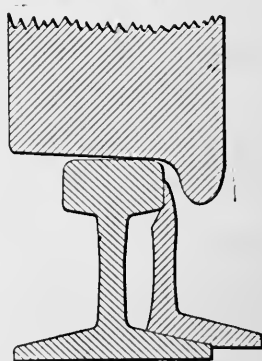


FIG. 133.

The planing runs back in *straight* lines, until at about six or seven feet back from the point the full width of the head is

obtained. The full width of the base will only be obtained at about 13 feet from the point. An 80-lb. rail is 5 inches

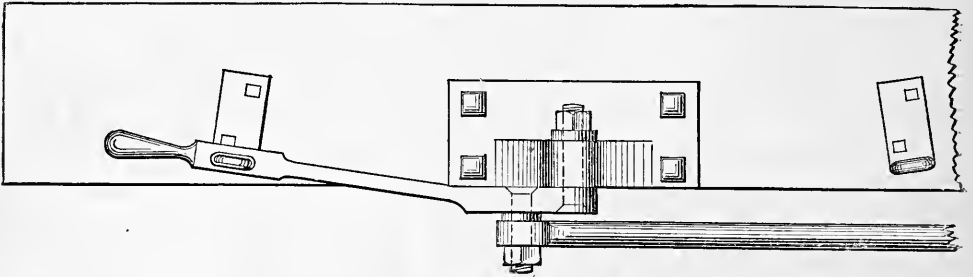


FIG. 134.—GROUND LEVER FOR THROWING A SWITCH.

wide at the base. Allowing  $\frac{3}{4}$ " more for a spike between the rails, this gives  $5\frac{3}{4}$ " as the minimum width between rail centers at the joint. The minimum angle of the switch-point (using a 15-foot point rail) is therefore the angle whose tangent is

$$\frac{5.75}{15 \times 12} = .03914, \text{ which is the tangent of } 1^\circ 50'.$$

Switch-rails are sometimes used with a length of 24 feet, which reduces the angle of the switch-point to  $1^\circ 09'$ .

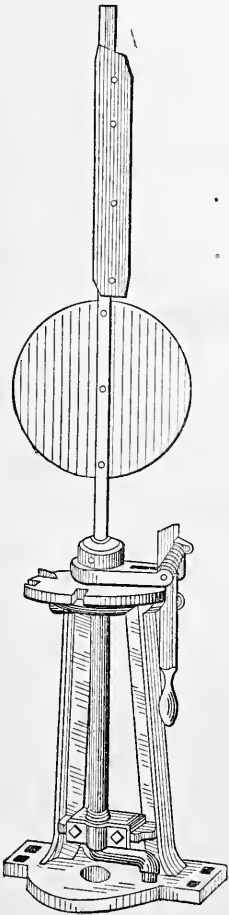


FIG. 135.

**259. Switch-stands.** The simplest and cheapest form is the "ground lever," which has no target. The radius of the circle described by the connecting-rod pin is precisely one-half the throw. From the nature of the motion the device is practically self-locking in either position, padlocks being only used to prevent malicious tampering. The numerous designs of upright stands are always combined with targets, one design of which is illustrated in Fig. 135. When the road is equipped with interlocking signals, the switch-throw mechanism forms a part of the design.

**260. Tie-rods.** These are fastened to the webs of the rails by means of lugs which are bolted on, there

being usually a hinge-joint between the rod and the lug. Four such tie-rods are generally necessary. The first rod is sometimes made without hinges, which gives additional stiffness to the comparatively weak rail-points. The old fashioned tie-rod, having jaws fitting the base of the rail, was almost universally used in the days of stub switches. One great inconvenience in their use lies in the fact that they must be slipped on, one by one, over the *free* ends of the switch-rails. Sometimes the lugs are fastened to the rail-webs by rivets instead of bolts.

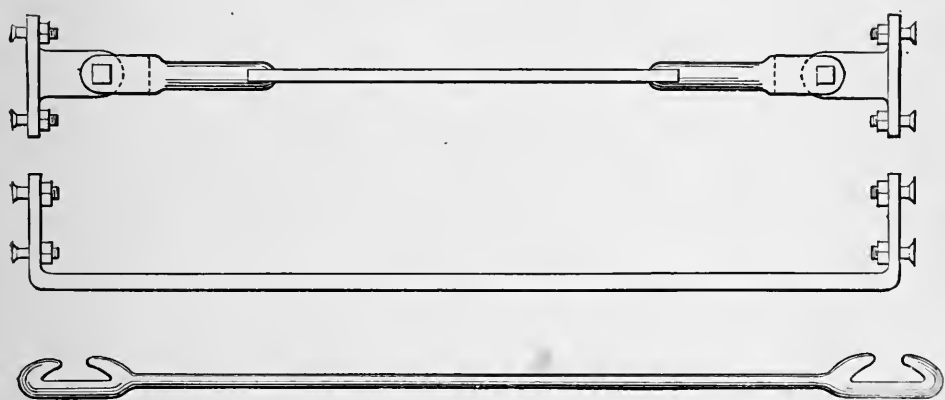


FIG. 136.—FORMS OF TIE-RODS.

**261. Guard-rails.** As shown in Figs. 131 and 132, guard-rails are used on both the main and switch tracks opposite the frog-point. Their function is not only to prevent the possibility of the wheel-flanges passing on the wrong side of the frog-point, but also to save the side of the frog-tongue from excessive wear. The necessity for their use may be realized by noting the very apparent wear usually found on the side of the head of the guard-rail. The flange-way space between the heads of the guard-rail and wheel-rail therefore becomes a definite quantity and should equal about two inches. Since this is less than the space between the heads of ordinary (say 80-pound) rails when placed base to base, to say nothing of the  $\frac{3}{4}$ " necessary for spikes, it becomes necessary to cut the flange of the guard-rail. The length of the rail is made from 10 to 15 feet, the ends being bent as shown in Fig. 132, so as to



$$\begin{aligned}
 \text{then} \quad r &= \frac{1}{2}L (\cot F + \operatorname{cosec} F) \\
 &= \frac{1}{2}g \cot \frac{1}{2}F (\cot F + \operatorname{cosec} F) \\
 &= \frac{1}{2}g \cot^2 \frac{1}{2}F, \text{ since } (\cot \alpha + \operatorname{cosec} \alpha) = \cot \frac{1}{2}\alpha \\
 &= 2gn^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)
 \end{aligned}$$

$$\text{Also} \quad L = 2gn, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

$$\text{from which} \quad r = n \times L. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

These extremely simple relations may obviate altogether the necessity for tables, since they involve only the frog-number and the gauge. On account of the great simplicity of these rules, they are frequently used as they are, regardless of the fact that the curve is never a uniform simple curve from switch-block to frog. In the first place there is a considerable length of the gauge-line within the frog, which is straight unless it is purposely curved to the proper curve while being manufactured, which is seldom if ever done—except for the very large-angled frogs used for street-railway work, etc. It is also doubtful whether the switch-rails (*BA*, Fig. 131) are bent to the computed curve when the rails are set for the switch. The switch-rails of point switches are *straight*, thus introducing a stretch of straight track which is about one-fifth of the total length of the lead-rails. The effect of these modifications on the length and radius of the lead-rails will be developed and discussed in the next four sections.

The throw (*t*) of a stub switch depends on the weight of the rail, or rather on the width of its base. The throw must be at least  $\frac{3}{4}$ " more than that width. The head-block should therefore be placed at such a distance from the heel of the switch (*B*) that the versed sine of the arc equals the throw. These points *must* be opposite on the two rails, but the points on the two rails where these relations are exactly true will not be opposite. Therefore, instead of considering either of the two radii ( $r + \frac{1}{2}g$ ) and ( $r - \frac{1}{2}g$ ), the mean radius *r* is used. Then (see Fig. 137)

$$\text{vers } KOQ = t \div r,$$

and the length of the switch-rails is

$$QK = r \sin KOQ. \quad . \quad . \quad . \quad . \quad (81)$$

These relations develop another disadvantage in the use of a stub switch. The required value of  $BG$ , using a No. 10 frog and 80-pound rail, is 30.1 feet—slightly more than a full rail length. It would be unsafe to leave so much of the track unspiked from the ties. Whether this is obviated by spiking down a portion of the switch-rails (virtually shortening the lead) or by moving the switch-block nearer the heel of the switch (shortening the switch-rails), but still maintaining the required throw, the theoretical accuracy of the curve is hopelessly lost.

**263. Effect of straight frog-rails.** A portion of the ends of the rails of a frog are free and *may* be bent to conform to the switch-rail curve, but there is a considerable portion which is fitted to the cast-iron filler, and this portion is always straight. Call the length of this straight portion back from the frog-point  $f$  ( $= FH$ , Fig. 138). Then we have

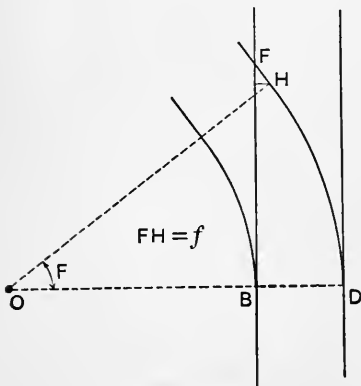


FIG. 138.

$$\begin{aligned} r + \frac{1}{2}g &= (g - f \sin F) \div \text{vers } F \\ &= \frac{g}{\text{vers } F} - f \cot \frac{1}{2}F \\ &= \frac{g}{\text{vers } F} - 2fn. \quad . \quad . \quad (82) \end{aligned}$$

$$\begin{aligned} BF = L &= (g - f \sin F) \cot \frac{1}{2}F + f \cos F \\ &= 2gn - f \sin F \cot \frac{1}{2}F + f \cos F \\ &= 2gn - f(1 + \cos F) + f \cos F \\ &= 2gn - f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83) \end{aligned}$$

Since  $r - \frac{1}{2}g = (L - f \sec F) \cot F$ , and

$$r + \frac{1}{2}g = (L - f \cos F) \text{cosec } F,$$



$$r = \frac{1}{2}L (\cot F + \operatorname{cosec} F) - \frac{1}{2}f \sec F \cot F - \frac{1}{2}f \cos F \operatorname{cosec} F$$

$$= Ln - \frac{1}{2}f \left( \frac{1 + \cos F}{\sin F} \right).$$

$$r = Ln - \frac{1}{2}f \cot \frac{1}{2}F$$

$$= Ln - fn. \quad \text{Then from (83)}$$

$$r = 2gn^2 - 2fn. \quad . \quad . \quad . \quad . \quad . \quad (84)$$

**264. Effect of straight point-rails.** The “point switches,” now so generally used, have *straight* switch-rails. This requires an *angle* in the alignment rather than turning off by a tangential curve. The angle is, however, very small (between  $1^\circ$  and  $2^\circ$ ), and the disadvantages of this angle are small compared with the very great advantages of the device.

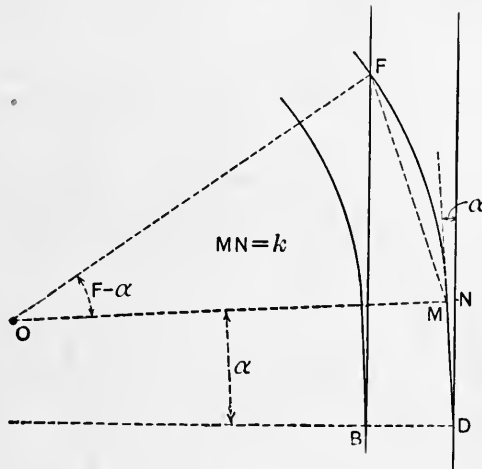


FIG. 139.

$$FM = \frac{g - k}{\sin \frac{1}{2}(F + \alpha)};$$

$$r + \frac{1}{2}g = \frac{FM}{2 \sin \frac{1}{2}(F - \alpha)}$$

$$= \frac{g - k}{2 \sin \frac{1}{2}(F + \alpha) \sin \frac{1}{2}(F - \alpha)}$$

$$= \frac{g - k}{\cos \alpha - \cos F} \quad . \quad . \quad . \quad . \quad . \quad (85)$$

$$\begin{aligned} BF = L &= FM \cos \tfrac{1}{2}(F + \alpha) + DN \\ &= (g - k) \cot \tfrac{1}{2}(F + \alpha) + DN. \end{aligned} \quad (86)$$

**265. Combined effect of straight frog-rails and straight point-rails.** It becomes necessary in this case to find a curve which shall be tangent to both the point-rail and the frog-rail. The curve therefore begins at *M*, its tangent making an angle of  $\alpha$  (usually  $1^\circ 50'$ ) with the main rail, and runs to *H*. The central

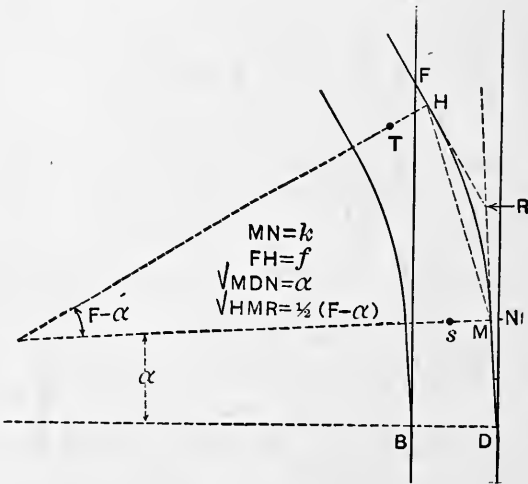


FIG. 140.

angle of the curve is therefore  $(F - \alpha)$ . The angle of the chord *HM* with the main rails is therefore

$$\begin{aligned} \tfrac{1}{2}(F - \alpha) + \alpha &= \tfrac{1}{2}(F + \alpha); \\ HM &= \frac{g - f \sin F - k}{\sin \tfrac{1}{2}(F + \alpha)}; \\ r + \tfrac{1}{2}g &= \frac{HM}{2 \sin \tfrac{1}{2}(F - \alpha)} \\ &= \frac{g - f \sin F - k}{2 \sin \tfrac{1}{2}(F + \alpha) \sin \tfrac{1}{2}(F - \alpha)} \\ &= \frac{g - f \sin F - k}{\cos \alpha - \cos F}; \end{aligned} \quad (87)$$

$$ST = 2r \sin \tfrac{1}{2}(F - \alpha). \quad (88)$$

$$BF = L = HM \cos \frac{1}{2}(F + \alpha) + f \cos F + DN$$

$$= (g - f \sin F - k) \cot \frac{1}{2}(F + \alpha) + f \cos F + DN. \quad (89)$$

It may be more simple, if  $(r + \frac{1}{2}g)$  has already been computed, to write

$$L = 2(r + \frac{1}{2}g) \sin \frac{1}{2}(F - \alpha) \cos \frac{1}{2}(F + \alpha) + f \cos F + DN$$

$$= (r + \frac{1}{2}g)(\sin F - \sin \alpha) + f \cos F + DN. \quad . \quad . \quad (90)$$

**266. Comparison of the above methods.** Computing values for  $r$  and  $L$  by the various methods, on the uniform basis of a No. 9 frog, standard gauge  $4' 8\frac{1}{2}''$ ,  $f = 3'.37$ ,  $k = 5\frac{3}{4}'' = 0'.479$ ,  $DN = 15' 0''$ , and  $\alpha = 1^\circ 50'$ , we may tabulate the comparative results:

|               | § 262.<br>Simple circle<br>Curved frog r.<br>Curved switch-r. | § 263.<br>Straight frog-r.<br>Curved switch-r. | § 264.<br>Curved frog-r.<br>Straight switch-r. | § 265.<br>Straight frog-r.<br>Straight switch r. |
|---------------|---------------------------------------------------------------|------------------------------------------------|------------------------------------------------|--------------------------------------------------|
| $r$           | 762.75                                                        | 702.00                                         | 747.48                                         | 681.16                                           |
| Deg. of curve | 7° 31'                                                        | 8° 10'                                         | 7° 40'                                         | 8° 25'                                           |
| $L$           | 84.75                                                         | 81.37                                          | 74.00                                          | 72.13                                            |

This shows that the effect of using straight frog-rails and straight switch-rails is to sharpen the curve and shorten the lead in each case separately, and that the combined effect is still greater. The effect of the straight switch-rails is especially marked in reducing the length of lead, and therefore Eq. 78 to 80, although having the advantage of extreme simplicity, cannot be used for point-switches without material error. The effect of the straight frog-rail is less, and since it can be materially reduced by bending the free end of the frog-rails, the influence of this feature is frequently ignored, the frog-rails are assumed to be curved and Eq. 85 and 86 are used. (See § 276 for a further discussion of this point.)

**267. Dimensions for a turnout from the OUTER side of a curved track.** In this demonstration the switch-rails will be considered as uniformly circular from the switch-points to the frog-point.

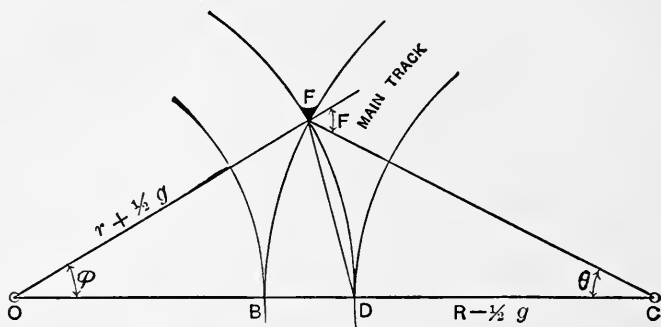


FIG. 141.

In the triangle  $FCD$  (Fig. 141) we have

$$(FC + CD) : (FC - CD) :: \tan \frac{1}{2}(FDC + DFC) : \tan \frac{1}{2}(FDC - DFC);$$

but  $\frac{1}{2}(FDC + DFC) = 90^\circ - \frac{1}{2}\theta$

and  $\frac{1}{2}(FDC - DFC) = \frac{1}{2}F$ .

Also  $FC + CD = 2R$  and  $FC - CD = g$ ;

$$\therefore 2R : g :: \cot \frac{1}{2}\theta : \tan \frac{1}{2}F$$

$$:: \cot \frac{1}{2}F : \tan \frac{1}{2}\theta;$$

$$\therefore \tan \frac{1}{2}\theta = \frac{g \cot \frac{1}{2}F}{2R} \quad \text{. . . . . (91)}$$

Also  $OF : FC :: \sin \theta : \sin \phi$ ; but  $\phi = (F - \theta)$ ;

then  $r + \frac{1}{2}g = (R + \frac{1}{2}g) \frac{\sin \theta}{\sin (F - \theta)} \quad \text{. . . . . (92)}$

$$BF = L = 2(R + \frac{1}{2}g) \sin \frac{1}{2}\theta. \quad \text{. . . . . (93)}$$

If the curvature of the main track is very sharp or the frog angle unusually small,  $F$  may be less than  $\theta$ ; in which case the center  $O$  will be on the same side of the main track as  $C$ . Eq. 92 will become (by calling  $r = -r$  and changing the signs)

$$(r - \frac{1}{2}g) = (R + \frac{1}{2}g) \frac{\sin \theta}{\sin (\theta - F)}. \quad \text{. . . . . (94)}$$

If we call  $d$  the degree of curve corresponding to the radius  $r$ , and  $D$  the degree of curve corresponding to the radius  $R$ , also  $d'$  the degree of curve of a turnout from a straight track (the frog angle  $F$  being the same), it may be shown that  $d = d' - D$  (very nearly). To illustrate we will take three cases, a number 6 frog (very blunt), a number 9 frog (very commonly used), and a number 12 frog (unusually sharp). Suppose  $D = 4^\circ 0'$ ; also  $D = 10^\circ 0'$ ;  $g = 4' 8\frac{1}{2}'' = 4'.708$ .

| Frog number. | $D = 4^\circ$ . |             |            |        | "L" for straight track. |
|--------------|-----------------|-------------|------------|--------|-------------------------|
|              | $d$             | $d' - D$    | Error.     | $L$    |                         |
| 6            | 12° 54' 20"     | 12° 57' 52" | 0° 03' 32" | 56.57  | 56.50                   |
| 9            | 3 30 27         | 3 31 04     | 0 0 37     | 84.85  | 84.75                   |
| 12           | 0 13 33         | 0 13 36     | 0 0 03     | 112.72 | 113.00                  |

| Frog number. | $D = 10^\circ$ |            |            |        | "L" for straight track. |
|--------------|----------------|------------|------------|--------|-------------------------|
|              | $d$            | $d' - D$   | Error.     | $L$    |                         |
| 6            | 6° 53' 24"     | 6° 57' 52" | 0° 04' 28" | 56.66  | 56.50                   |
| 9            | 2 27 54        | 2 28 56    | 0 01 02    | 84.86  | 84.75                   |
| 12           | 5 44 26        | 5 46 24    | 0 01 58    | 112.91 | 113.00                  |

A brief study of the above tabular form will show that the error involved in the use of the approximate rule for ordinary curves ( $4^\circ$  or less) and for the usual frogs (about No. 9) is really insignificant, and that, even for sharper curves ( $10^\circ$  or more), or for very blunt frogs, the error would never cause damage, considering the lower probable speed. In the most unfavorable case noted above the change in radius is about 1%. On account of the closeness of the approximation the method is frequently used. The remarkable agreement of the computed values of  $L$  with the corresponding values for a straight main track (the lead



somewhat greater than in the other, especially when the curvature of the main track is sharp. If the frog angle is also large, the curvature of the turnout is excessively sharp. If the frog angle is very small, the liability to derailment is great. Turnouts to the inside of a curved track should therefore be avoided, unless the curvature of the main track is small.

**269. Double turnout from a straight track.** In Fig. 143 the frogs  $F_l$  and  $F_r$  are generally made equal. Then, if there are

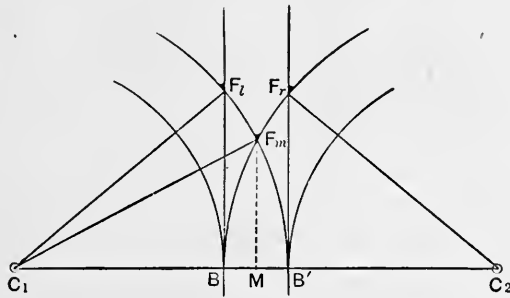


FIG. 143.

uniform curves from  $B'$  to  $F_l$  and from  $B$  to  $F_r$ , the required value of  $F_m$  is obtained from

$$\text{vers } \frac{1}{2}F_m = \frac{g}{2(r + \frac{1}{2}g)}, \quad \cdot \cdot \cdot \cdot (98)$$

$r$  being found from Eq. 78, in which  $n$  is the frog number of  $F_l$  or  $F_r$ .

$$MF_m = r \tan \frac{1}{2}F_m;$$

but since  $n_m = \frac{1}{2} \cot \frac{1}{2}F_m$ ,

$$MF_m = \frac{r}{2n_m} \cdot \cdot \cdot \cdot (99)$$

Since  $\text{vers } F_l = \frac{g}{(r + \frac{1}{2}g)},$

$$\text{vers } \frac{1}{2}F_m = \frac{1}{2} \text{vers } F_l, \cdot \cdot \cdot \cdot (100)$$

Also, since  $(C_1 F_m)^2 = (M F_m)^2 + (C_1 M)^2$ , we have

$$(r + \frac{1}{2}g)^2 = \left(\frac{r}{2n_m}\right)^2 + r^2;$$

$$r^2 + rg + \frac{1}{4}g^2 = \frac{r^2}{4n_m^2} + r^2.$$

Simplifying and substituting  $r = 2gn^2$ , we have

$$2g^2n^2 + \frac{1}{4}g^2 = \frac{4g^2n^4}{4n_m^2};$$

$$n_m^2 = \frac{n^4}{2n^2 + \frac{1}{4}}.$$

Dropping the  $\frac{1}{4}$ , which is always insignificant in comparison with  $2n^2$ , we have

$$n_m = \frac{n}{\sqrt{2}} = n \times .707 \text{ (approx.)}. \quad . \quad . \quad (101)$$

Frogs are usually made with angles corresponding to integral values of  $n$ , or sometimes in "half" sizes, e.g. 6,  $6\frac{1}{2}$ , 7,  $7\frac{1}{2}$ , etc. If No.  $8\frac{1}{2}$  frogs are used for  $F_l$  and  $F_r$ , the exact frog number for  $F_m$  is 6.01. This is so nearly 6 that a No. 6 frog may be used without sensible inaccuracy. Numbers 7 and 10 are a less perfect combination. If sharp frogs must be used,  $8\frac{1}{2}$  and 12 form a very good combination.

If it becomes necessary to use other frogs because the right combination is unobtainable, it may be done by compounding the curve at the middle frog.  $F_l$  and  $F_r$  should be greater than  $\frac{1}{2}F_m$ . If equal to  $\frac{1}{2}F_m$ , the rails would be straight from the middle frog to the outer frogs. In Fig. 144,  $\theta_1 = F_l - \frac{1}{2}F_m$ . Drawing the chord  $\overline{F_l F_m}$ ,

$$\overline{KF_l F_m} = F_l - \frac{1}{2}\theta_1 = F_l - \frac{1}{2}F_l + \frac{1}{4}F_m = \frac{1}{2}(F_l + \frac{1}{2}F_m);$$



$$\overline{F_l F_m} = \frac{\overline{K F_m}}{\sin \overline{K F_l F_m}} = \frac{g}{2 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m)}; \quad . \quad . \quad . \quad (102)$$

$$\overline{K F_l} = \overline{K F_m} \cot \overline{K F_l F_m} = \frac{1}{2}g \cot \frac{1}{2}(F_l + \frac{1}{2}F_m); \quad (103)$$

$$\begin{aligned} (r_1 + \frac{1}{2}g) &= \frac{\overline{F_l F_m}}{2 \sin \frac{1}{2}\theta} = \frac{g}{4 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m) \sin \frac{1}{2}(F_l - \frac{1}{2}F_m)} \\ &= \frac{\frac{1}{2}g}{\cos \frac{1}{2}F_m - \cos F_l'} \quad . \quad . \quad . \quad (104) \end{aligned}$$

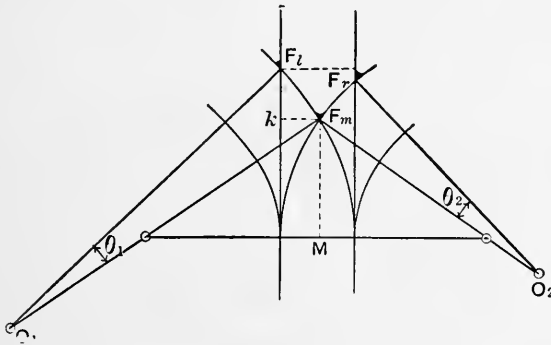


FIG. 144.

If three frogs, all different, *must* be used, the largest may be selected as  $F_m$ ; the radius of the lead rails may be found by an inversion of Eq. 98;  $F_m$  may be located in the center of the tracks by Eq. 99; then each of the smaller frogs may be located by separate applications of Eq. 102 or 103, the radius being determined by Eq. 104.

**270. Two turnouts on the same side.** In Fig. 145, let  $O_1$  bisect  $O_2D$ . Then  $(r_1 + \frac{1}{2}g) = \frac{1}{2}(r_2 + \frac{1}{2}g)$ ; also,  $O_1O_2 = O_1F_l$  and  $F_r = F_l$ .

$$\text{vers } F_m = \frac{g}{r_1 + \frac{1}{2}g} = \frac{2g}{r_2 + \frac{1}{2}g}; \quad . \quad . \quad . \quad (105)$$

$$BF_m = (r_1 + \frac{1}{2}g) \sin F_m. \quad . \quad . \quad . \quad (106)$$

It may readily be shown that the relative values of  $F_r$ ,  $F_l$ , and  $F_m$  are almost identical with those given in § 269; as may

be apparent when it is considered that the middle switch may be regarded simply as a curved main track, and that, as

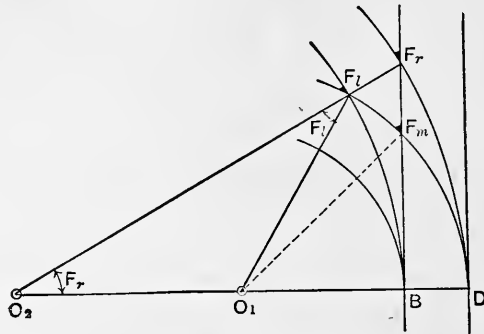


FIG. 145.

developed in § 267, the dimensions of turnouts are nearly the same whether the main track is straight or slightly curved.

**271. Connecting curve from a straight track.** The “connecting curve” is the track lying

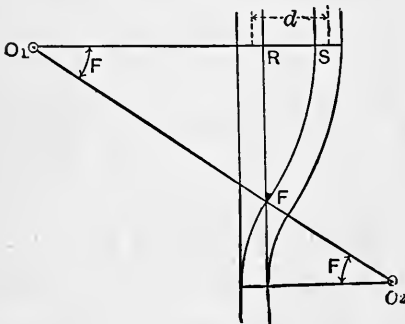


FIG. 146.

between the frog and the side track where it becomes parallel to the main track ( $FS$  in Fig. 146 or 147). Call  $d$  the distance between track centers. The angle  $FO_1R = F$  (see Fig. 146). Call  $r'$  the radius of the connecting curve. Then

$$(r' - \frac{1}{2}g) = \frac{d - g}{\text{vers } F}; \quad . \quad . \quad . \quad . \quad . \quad (107)$$

$$FR = (r' - \frac{1}{2}g) \sin F. \quad . \quad . \quad . \quad (108)$$

If it is considered that the distance  $FR$  consumes too much track room, it may be shortened by the method indicated in Fig. 151.

**272. Connecting curve from a curved track to the OUTSIDE.**

When the main track is curved, the required quantities are the radius  $r$  of the connecting curve from  $F$  to  $S$ , Fig. 147, and its length or central angle. In the triangle  $CSF$

$$CS + CF : CS - CF :: \tan \frac{1}{2}(CFS + CSF) : \tan \frac{1}{2}(CFS - CSF);$$

but  $\frac{1}{2}(CFS + CSF) = 90 - \frac{1}{2}\psi$ ; and, since the triangle  $O_1SF$  is isosceles,  $\frac{1}{2}(CFS - CSF) = \frac{1}{2}F$ ;

$$\begin{aligned} \therefore 2R + d : d - g &:: \cot \frac{1}{2}\psi : \tan \frac{1}{2}F \\ &:: \cot \frac{1}{2}F : \tan \frac{1}{2}\psi; \end{aligned}$$

$$\therefore \tan \frac{1}{2}\psi = \frac{2n(d - g)}{2R + d}. \quad \dots \quad (109)$$

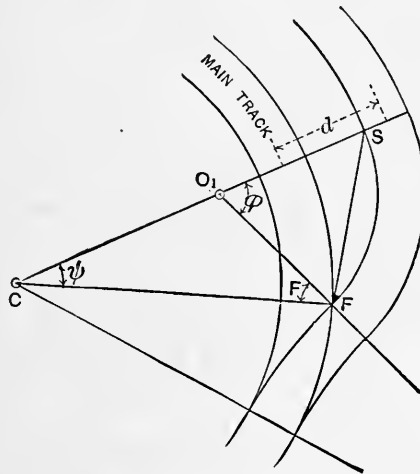


FIG. 147.

From the triangle  $CO_1F$  we may derive

$$\begin{aligned} r - \frac{1}{2}g : R + \frac{1}{2}g &:: \sin \psi : \sin (F + \psi); \\ r - \frac{1}{2}g &= (R + \frac{1}{2}g) \frac{\sin \psi}{\sin (F + \psi)} \quad \dots \quad (110) \end{aligned}$$

Also  $FS = 2(r - \frac{1}{2}g) \sin \frac{1}{2}(F + \psi). \quad \dots \quad (111)$

**273. Connecting curve from a curved track to the INSIDE.**  
As above, it may readily be deduced from the triangle  $CFS$  (see Fig. 148) that

$$(2R - d) : (d - g) :: \cot \frac{1}{2}\psi : \tan \frac{1}{2}F,$$

and finally that

$$\tan \frac{1}{2}\psi = \frac{2n(d - g)}{2R - d}. \quad \dots \quad (112)$$



the same as Eq. 112, but

$$r + \frac{1}{2}g = (R - \frac{1}{2}g) \frac{\sin \psi}{\sin (\psi - F')}. \quad \dots \quad (116)$$

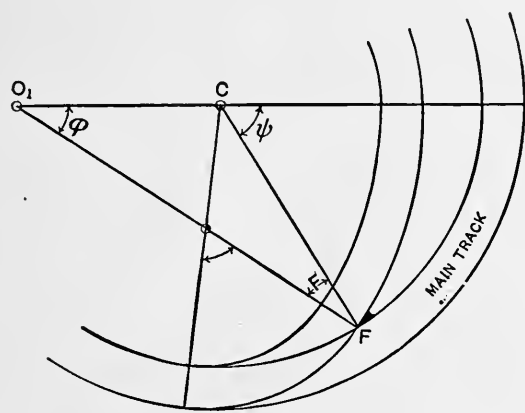


FIG. 150.

**274. Crossover between two parallel straight tracks.** (See Fig. 151.) The turnouts are as usual. The crossover track may be straight, as shown by the full lines, or it may be a reversed curve, as shown by the dotted lines. The reversed curve shortens the total length of track required, but is somewhat objectionable. The first method requires that both frogs must be equal. The second method permits unequal frogs, although equal frogs are preferable. The length of straight crossover track is  $F_1T$ .

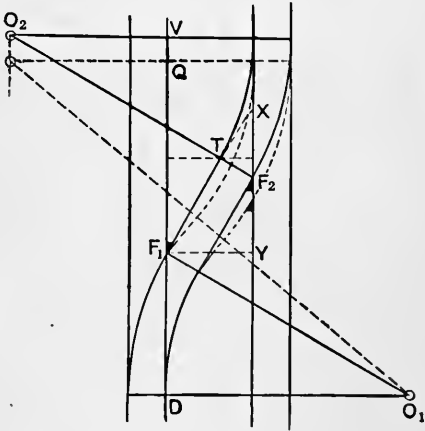


FIG. 151.

$$F_1T \sin F_1 + g \cos F_1 = d - g;$$

$$F_1T = \frac{d - g}{\sin F_1} - g \cot F_1. \quad \dots \quad (117)$$

The total distance along the track may be derived as follows:

$$DV = 2DF_1 + F_2Y = 2DF_1 + XY - XF_2;$$

$$XY = (d - g) \cot F_1; \quad XF_2 = g \div \sin F_2;$$

$$\therefore DV = 2DF_1 + (d - g) \cot F_1 - \frac{g}{\sin F_2}. \quad (118)$$

If a reversed curve with equal frogs is used, we have

$$\text{vers } \theta = \frac{d}{2r}; \quad (119)$$

also

$$DQ = 2r \sin \theta. \quad (120)$$

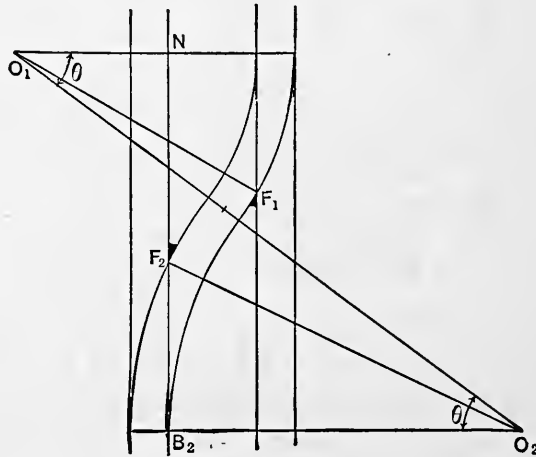


FIG. 152.

If the frogs are unequal, we will have (see Fig. 152)

$$r_2 \text{ vers } \theta + r_1 \text{ vers } \theta = d;$$

$$\therefore \text{vers } \theta = \frac{d}{r_1 + r_2}; \quad (121)$$

also the distance along the track

$$B_2N = (r_1 + r_2) \sin \theta. \quad (122)$$



Knowing  $F_1$ ,  $\theta_1$  is determinable from Eq. 91. Fig. 153 shows the case where  $\theta_2$  is greater than  $F_2$ . Fig. 154 shows the case where it is less. The demonstration of Eq. 123 is applicable to

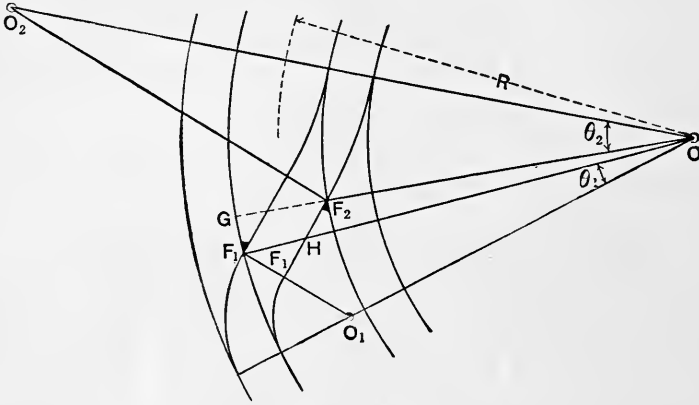


FIG. 154.

both figures. The relative position of the frogs  $F_1$  and  $F_2$  may be determined as follows, the solution being applicable to both Figs. 153 and 154:

$$HOF_2 = 180^\circ - (90^\circ - F_1) - (90^\circ + F_2) = F_1 - F_2.$$

Then

$$GF_1 = 2(R + \frac{1}{2}d - \frac{1}{2}g) \sin \frac{1}{2}(F_1 - F_2). \quad (124)$$

Since  $F_2$  comes out *any* angle, its value will not be in general that of an even frog number, and it will therefore need to be made to order.

(b) **Continuing the switch-rail curves until they meet as a reversed curve.** In this case  $F_1$  and  $F_2$  may be chosen at pleasure (within limitations), and they will of course be of regular sizes and equal or unequal as desired.  $F_1$  and  $F_2$  being known,  $\theta_1$  and  $\theta_2$  are computed by Eq. 95 and 91. In the triangle  $OO_1O_2$  (see Fig. 155)

$$\text{vers } \psi = \frac{2(S - OO_2)(S - OO_1)}{OO_2 - OO_1},$$

in which

$$S = \frac{1}{2}(OO_1 + OO_2 + O_1O_2);$$







In accordance with this plan Table III has been computed from Eq. 87, 88, and 90. The *leads* there given may be used for all main tracks straight or curved. The table gives the degree of curve of the lead rails for *straight* main track; for a turnout to the *inside*, *add* the degree of curve of the main track; for a turnout to the *outside*, *subtract* it.

If the position of the switch-block is definitely determined, then the rails must be cut accordingly; but when some freedom is allowable (which never need exceed 15 feet and may require but a few inches), one rail-cutting may be avoided. Mark on the rails at *B*, *F*, and *D*; measure off the length of the switch-rails *DN*; offset  $\frac{1}{2}g + k$  from *N* for the point *S*.

The point *H* may be located (temporarily) by measuring along the rail a distance *FH* ( $=f$ ) and then swinging out a distance of  $f \div n$  ( $n$  being the frog number).  $HT = \frac{1}{2}g$  and is measured at right angles to *FH*. Points for track centers between *S* and *T* may be laid off by a transit or by the use of a string and tape. Substituting in Eq. 31 the value of *R* and of *chord* ( $=ST$ ), we may compute  $x$  ( $=db$ ). Locate the middle point *d* and the quarter points *a''* and *c''*. Then *a''a* and *c''c* each equal

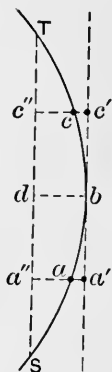


FIG. 156.

three-fourths of *db*. Theoretically this gives a parabola rather than a circle, but the difference for all practical cases is too small for measurement.

*Example.* Given a main track on a  $4^\circ$  curve; a turnout to the outside, using a number 9 frog; gauge  $4' 8\frac{1}{2}''$ ;  $f = 3'.37$ ;  $k = 5\frac{3}{4}''$ ;  $DN = 15' 0''$  and  $\alpha = 1^\circ 50'$ . Then for a *straight* track  $r$  would equal 681.16 [ $d = 8^\circ 25'$ ]. For this curved track  $d$  will be nearly  $(8^\circ 25' - 4^\circ) = 4^\circ 25'$ , or  $r$  will be 1297.6.  $L$  for the *straight* track would be 72.20; but since the lead is slightly increased (see § 267) when the turnout is on the outside of a curve,  $L$  may here be called 72.5.  $FH = f = 3'.37$ ;  $f \div n = 3.37 \div 9 = 0'.375 = 4''.5$ . *H*, *T*, and *S* may be located as described above. *ST* may be measured on the ground, or it may be computed from Eq. 88, giving the value

of 53.80 feet for straight track. Since it is slightly more for a turnout to the outside of a curve, it may be called 54.0. Then  $x = db = \frac{(54.0)^2}{8 \times 1297.6} = 0.281$  feet, and  $aa''$  and  $cc'' = 0.21$  foot.

## CROSSINGS.

**277. Two straight tracks.** When two straight tracks cross each other, four frogs are necessary, the angles of two of them being supplementary to the angles of the other. Since such crossings are sometimes operated at high speeds, they should be

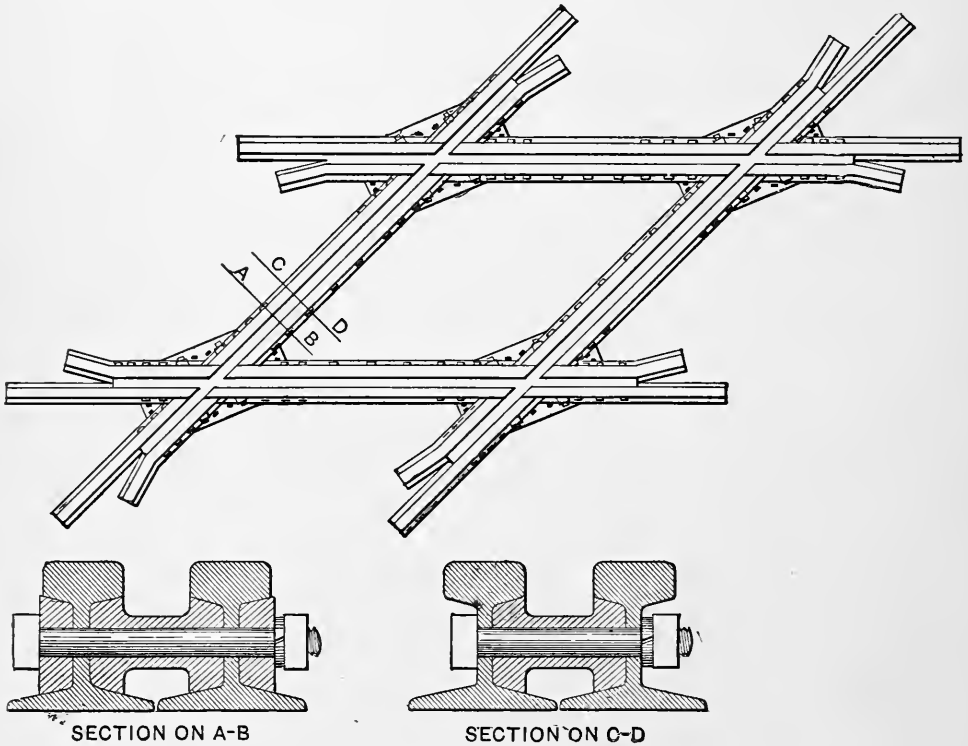


FIG. 157.—CROSSING.

very strongly constructed, and the angles should preferably be  $90^\circ$  or as near that as possible. The frogs will not in general be “stock” frogs of an even number, especially if the angles are large, but must be made to order with the required angles as measured. In Fig. 157 are shown the details of such a crossing. Note the fillers, bolts, and guard-rails.

**278. One straight and one curved track.** Structurally the crossing is about the same as above, but the frog angles are all unequal. In Fig. 158,  $R$  is known, and the angle  $M$ , made by the center lines of the tracks at their point of intersection, is also known.

$$M = NCM. \quad NC = R \cos M.$$

$$\left. \begin{aligned} (R - \tfrac{1}{2}g) \cos F_1 &= NC + \tfrac{1}{2}g; \\ \therefore \cos F_1 &= \frac{R \cos M + \tfrac{1}{2}g}{R - \tfrac{1}{2}g}. \\ \text{Similarly } \cos F_2 &= \frac{R \cos M + \tfrac{1}{2}g}{R + \tfrac{1}{2}g}, \\ \cos F_3 &= \frac{R \cos M - \tfrac{1}{2}g}{R + \tfrac{1}{2}g}, \\ \cos F_4 &= \frac{R \cos M - \tfrac{1}{2}g}{R - \tfrac{1}{2}g}. \end{aligned} \right\} (129)$$

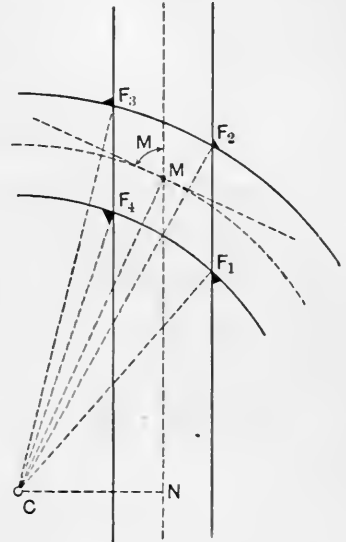


FIG. 158.

**279. Two curved tracks.** The four frogs are unequal, and the angle of each must be computed. The radii  $R_1$  and  $R_2$  are

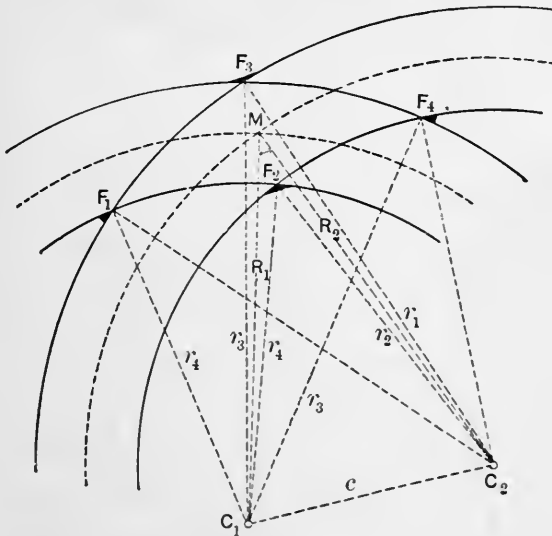


FIG. 159.

known; also the angle  $M$ .  $r_1$ ,  $r_2$ ,  $r_3$ , and  $r_4$  are therefore known by adding or subtracting  $\tfrac{1}{2}g$ , but the lines are so indi-

cated for brevity. Call the angle  $MC_1C_2 = C_1$ , the angle  $MC_2C_1 = C_2$ , and the line  $C_1C_2 = c$ . Then

$$\frac{1}{2}(C_1 + C_2) = 90^\circ - \frac{1}{2}M$$

and

$$\tan \frac{1}{2}(C_1 - C_2) = \cot \frac{1}{2}M \frac{R_2 - R_1}{R_2 + R_1}.$$

$C_1$  and  $C_2$  then become known and

$$c = C_1C_2 = R_2 \frac{\sin M}{\sin C_1}.$$

In the triangle  $F_1C_1C_2$ , call  $\frac{1}{2}(c + r_1 + r_4) = s_1$ ; then

$$\left. \begin{aligned} \text{vers } F_1 &= \frac{2(s_1 - r_1)(s_1 - r_4)}{r_1 r_4}, \\ \text{vers } F_2 &= \frac{2(s_2 - r_2)(s_2 - r_4)}{r_2 r_4}, \\ \text{vers } F_3 &= \frac{2(s_3 - r_1)(s_3 - r_3)}{r_1 r_3}, \\ \text{vers } F_4 &= \frac{2(s_4 - r_2)(s_4 - r_3)}{r_2 r_3}. \end{aligned} \right\} \quad \cdot \quad \cdot \quad (130)$$

Similarly

In the above equations

$$s_2 = \frac{1}{2}(c + r_2 + r_4),$$

$$s_3 = \frac{1}{2}(c + r_1 + r_3),$$

$$s_4 = \frac{1}{2}(c + r_2 + r_3).$$

## APPENDIX.

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### THE ADJUSTMENTS OF INSTRUMENTS.

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THE accuracy of instrumental work may be vitiated by any one of a large number of inaccuracies in the geometrical relations of the parts of the instruments. Some of these relations are so apt to be altered by ordinary usage of the instrument that the makers have provided adjusting-screws so that the inaccuracies may be readily corrected. There are other possible defects, which, however, will seldom be found to exist, provided the instrument was properly made and has never been subjected to treatment sufficiently rough to distort it. Such defects, when found, can only be corrected by a competent instrument maker or repairer.

A WARNING is necessary to those who would test the accuracy of instruments, and especially to those whose experience in such work is small. Lack of skill in handling an instrument will often indicate an apparent error of adjustment when the real error is very different or perhaps non-existent. It is always a safe plan when testing an adjustment to note the amount of the apparent error; then, beginning anew, make another independent determination of the amount of the error. When two or more *perfectly independent* determinations of such an error are made it will generally be found that they differ by an appreciable amount. The *differences* may be due in variable measure to careless inaccurate manipulation and to instrumental *defects* which are wholly independent of the particular test being made. Such careful determinations of the amounts of the errors are generally advisable in view of the next paragraph.

DO NOT DISTURB THE ADJUSTING-SCREWS ANY MORE THAN NECESSARY. Although metals are apparently rigid, they are really elastic and yielding. If some parts of a complicated mechanism, which is held together largely by friction, are subjected to greater internal stresses than other parts of the mechanism, the jarring resulting from handling will frequently cause a slight readjustment in the parts which will tend to more nearly equalize the internal stresses. Such action frequently occurs with the adjusting mechanism of instruments. One screw may be strained more than others. The friction of parts may prevent the opposing screw from *immediately* taking up an equal stress. Perhaps the adjustment appears perfect under these conditions. Jarring diminishes the friction between the parts, and the unequal stresses tend to equalize. A motion takes place which, although microscopically minute, is sufficient to indicate an error of adjustment. A readjustment, made by unskillful hands, may not make the final adjustment any more perfect. The frequent shifting of adjusting-screws wears them badly, and when the screws are worn it is still more difficult to keep them from moving enough to vitiate the adjustments. It is therefore preferable in many cases to refrain from disturbing the adjusting-screws, especially as the accuracy of the work done is not *necessarily* affected by errors of adjustment, as may be illustrated:

(a) Certain operations are *absolutely* unaffected by certain errors of adjustment.

(b) Certain operations are so slightly affected by certain *small* errors of adjustment that their effect may properly be neglected.

(c) Certain errors of adjustment may be readily allowed for and neutralized so that no error results from the use of the unadjusted instrument. Illustrations of all these cases will be given under their proper heads.

#### ADJUSTMENTS OF THE TRANSIT.

1. *To have the plate-bubbles in the center of the tubes when the axis is vertical.* Clamp the upper plate and, with the lower



clamp loose, swing the instrument so that the plate-bubbles are parallel to the lines of opposite leveling-screws. Level up until both bubbles are central. Swing the instrument  $180^\circ$ . If the bubbles again settle at the center, the adjustment is perfect. If either bubble does not settle in the center, move the leveling-screws until the bubble is *half-way* back to the center. Then, before touching the adjusting-screws, note carefully the position of the bubbles and observe whether the bubbles always settle at the *same* place in the tube, no matter to what position the instrument may be rotated. When the instrument is so leveled, the axis is truly vertical and the discrepancies between this constant position of the bubbles and the centers of the tubes measure the errors of adjustment. By means of the adjusting-screws bring each bubble to the center of the tube. If this is done so skillfully that the true level of the instrument is not disturbed, the bubbles should settle in the center for all positions of the instrument. Under unskillful hands, two or more such trials may be necessary.

When the plates are not horizontal, the measured angle is greater than the true horizontal angle by the difference between the measured angle and its projection on a horizontal plane. When this angle of inclination is small, the difference is insignificant. Therefore when the plate-bubbles are *very nearly* in adjustment, the error of measurement of horizontal angles may be far within the lowest unit of measurement used. A *small* error of adjustment of the plate-bubble *perpendicular* to the telescope will affect the horizontal angles by only a small proportion of the error, which will be perhaps imperceptible. Vertical angles will be affected by the same insignificant amount. A *small* error of adjustment of the plate-bubble *parallel* to the telescope will affect horizontal angles *very slightly*, but will affect vertical angles by the full amount of the error.

All error due to unadjusted plate-bubbles may be avoided by noting in what positions in the tubes the bubbles will remain fixed for all positions of azimuth and then keeping the bubbles adjusted to these positions, for the axis is then truly vertical. It will often save time to work in this way temporarily rather than to stop to make the adjustments. This should especially be done when accurate vertical angles are required.

When the bubbles are truly adjusted, they should remain stationary, regardless of whether the telescope is revolved with the upper plate loose and the lower plate clamped or whether the whole instrument is revolved, the plates being clamped together. If there is any appreciable difference,

it shows that the two vertical axes or "centers" of the plates are not concentric. This may be due to cheap and faulty construction or to the excessive wear that may be sometimes observed in an old instrument originally well made. In either case it can only be corrected by a maker.

2. *To make the revolving axis of the telescope perpendicular to the vertical axis of the instrument.* This is best tested by using a long plumb-line, so placed that the telescope must be pointed upward at an angle of about  $45^\circ$  to sight at the top of the plumb-line and downward about the same amount, if possible, to sight at the lower end. The vertical axis of the transit must be made truly vertical. Sight at the upper part of the line, clamping the horizontal plates. Swing the telescope down and see if the cross-wire again bisects the cord. If so, the adjustment is *probably* perfect (a conceivable exception will be noted later); if not, raise or lower one end of the axis by means of the adjusting-screws, placed at the top of one of the standards, until the cross-wire will bisect the cord both at top and bottom. The plumb-bob may be steadied, if necessary, by hanging it in a pail of water. As many telescopes cannot be focused on an object nearer than 6 or 8 feet from the telescope, this method requires a long plumb-line swung from a high point, which may be inconvenient.

Another method is to set up the instrument about 10 feet from a high wall. After leveling, sight at some convenient mark high up on the wall. Swing the telescope down and make a mark (when working alone some convenient natural mark may generally be found) low down on the wall. Plunge the telescope and revolve the instrument about its vertical axis and again sight at the upper mark. Swing down to the lower mark. If the wire again bisects it, the adjustment is perfect. If not, fix a point *half-way* between the two positions of the lower mark. The plane of this point, the upper point, and the center of the instrument is truly vertical. Adjust the axis to these upper and lower points as when using the plumb-line.

3. *To make the line of collimation perpendicular to the revolving axis of the telescope.* With the instrument level and

the telescope nearly horizontal point at some well-defined point at a distance of 200 feet or more. Plunge the telescope and establish a point in the opposite direction. Turn the whole instrument about the vertical axis until it again points at the first mark. Again plunge to "direct position" (i.e., with the level-tube *under* the telescope). If the vertical cross-wire again points at the second mark, the adjustment is perfect. If not, the error is *one-fourth* of the distance between the two positions of the second mark. Loosen the capstan-screw on one side of the telescope and tighten it on the other side until the vertical wire is set at the one-fourth mark. Turn the whole instrument by means of the tangent screw until the vertical wire is *midway* between the two positions of the second mark. Plunge the telescope. If the adjusting has been skillfully done, the cross-wire should come exactly to the first mark. As an "erecting eyepiece" reinverts an image already inverted, the ring carrying the cross-wires must be moved in the *same* direction as the *apparent* error in order to correct that error.

The necessity for the third adjustment lies principally in the practice of producing a line by plunging the telescope, but when this is required to be done with great accuracy it is always better to obtain the forward point by reversion (as described above for making the test) and take the *mean* of the two forward points. Horizontal and vertical angles are practically unaffected by *small* errors of this adjustment, unless, in the case of horizontal angles, the vertical angles to the points observed are very different.

Unnecessary motion of the adjusting-screws may sometimes be avoided by carefully establishing the forward point on line by repeated reversions of the instrument, and thus determining by repeated trials the exact amount of the error. *Differences* in the amount of error determined would be evidence of inaccuracy in manipulating the instrument, and would show that an adjustment based on the first trial would *probably* prove unsatisfactory.

The 2d and 3d adjustments are mutually dependent. If either adjustment is badly out, the other adjustment cannot be made except as follows :

(a) The second adjustment can be made regardless of the third when the lines to the high point and the low point make *equal* angles with the horizontal.

(b) The third adjustment can be made regardless of the second when the front and rear points are *on a level* with the instrument.

When both of these requirements are *nearly* fulfilled, and especially when the error of either adjustment is small, no trouble will be found in perfecting either adjustment on account of a small error in the other adjustment.

If the test for the second adjustment is made by means of the plumb-line and the vertical cross-wire intersects the line at all points as the telescope is raised or lowered, it not only demonstrates at once the accuracy of that adjustment, but also shows that the third adjustment is either perfect or has so small an error that it does not affect the second.

4. *To have the bubble of the telescope-level in the center of the tube when the line of collimation is horizontal.* The line of collimation should coincide with the optical axis of the telescope. If the object-glass and eyepiece have been properly centered, the previous adjustment will have brought the vertical cross-wire to the center of the field of view. The horizontal cross-wire should also be brought to the center of the field of view, and the bubble should be adjusted to it.

*a. Peg method.* Set up the transit at one end of a nearly level stretch of about 300 feet. Clamp the telescope with its bubble in the center. Drive a stake vertically under the eyepiece of the transit, and another about 300 feet away. Observe the height of the center of the eyepiece (the telescope being level) above the stake (calling it  $a$ ); observe the reading of the rod when held on the other stake (calling it  $b$ ); take the instrument to the other stake and set it up so that the eyepiece is vertically over the stake, observing the height,  $c$ ; take a reading on the first stake, calling it  $d$ . If this adjustment is perfect, then

$$a - d = b - c,$$

$$\text{or} \quad (a - d) - (b - c) = 0.$$

$$\text{Call} \quad (a - d) - (b - c) = 2m.$$

When  $m$  is positive, the line points downward;

“  $m$  “ negative, “ “ “ upward.

To adjust: if the line points *up*, sight the horizontal cross-wire (by moving the vertical tangent screw) at a point which is  $m$  lower, then adjust the bubble so that it is in the center.

By taking several independent values for  $a$ ,  $b$ ,  $c$ , and  $d$ , a mean value for  $m$  is obtained, which is more reliable and which may save much unnecessary working of the adjusting-screws.

*b. Using an auxiliary level.* When a carefully adjusted level is at hand, this adjustment may sometimes be more easily made by setting up the transit and level, so that their lines of collimation are as nearly as possible at the same height. If a point may be found which is half a mile or more away and which is on the horizontal cross-wire of the level, the horizontal cross-wire of the transit may be pointed directly at it, and the bubble adjusted accordingly. Any slight difference in the heights of the lines of collimation of the transit and level (say  $\frac{1}{4}''$ ) may almost be disregarded at a distance of  $\frac{1}{2}$  mile or more, or, if the difference of level would have an appreciable effect, even this may be practically eliminated by making an estimated allowance when sighting at the distant point. Or, if a distant point is not available, a level-rod with target may be used at a distance of (say) 300 feet, making allowance for the carefully determined difference of elevation of the two lines of collimation.

*5. Zero of vertical circle.* When the line of collimation is truly horizontal and the vertical axis is truly vertical, the reading of the vertical circle should be  $0^\circ$ . If the arc is adjustable, it should be brought to  $0^\circ$ . If it is not adjustable, the *index error* should be observed, so that it may be applied to all readings of vertical angles.

#### ADJUSTMENTS OF THE WYE LEVEL.

*1. To make the line of collimation coincide with the center of the rings.* Point the intersection of the cross-wires at some well-defined point which is at a considerable distance. The instrument need not be level, which allows much greater liberty in choosing a convenient point. The vertical axis should be

clamped, and the clips over the wyes should be loosened and raised. Rotate the telescope in the wyes. The intersection of the cross-wires should be continually on the point. If it is not, it requires adjustment. Rotate the telescope  $180^\circ$  and adjust *one-half* of the error by means of the capstan-headed screws that move the cross-wire ring. It should be remembered that, with an erecting telescope, on account of the inversion of the image, the ring should be moved in the direction of the *apparent* error. Adjust the other half of the error with the leveling-screws. Then rotate the telescope  $90^\circ$  from its usual position, sight accurately at the point, and then rotate  $180^\circ$  from that position and adjust any error as before. It may require several trials, but it is necessary to adjust the ring until the intersection of the cross-wires will remain on the point for any position of rotation.

If such a test is made on a very distant point and again on a point only 10 or 15 feet from the instrument, the adjustment may be found correct for one point and incorrect for the other. This indicates that the object-slide is improperly centered. Usually this defect can only be corrected by an instrument-maker. If the difference is very small it may be ignored, but the adjustment should then be made on a point which is at about the mean distance for usual practice—say 150 feet.

If the whole image appears to shift as the telescope is rotated, it indicates that the eyepiece is improperly adjusted. This defect is likewise usually corrected only by the maker. It does not interfere with instrumental accuracy, but it usually causes the intersection of the cross-wires to be eccentric with the field of view.

2. *To make the axis of the level tube parallel to the line of collimation.* Raise the clips as far as possible. Swing the level so that it is parallel to a pair of opposite leveling-screws and clamp it. Bring the bubble to the middle of the tube by means of the leveling-screws. Take the telescope out of the wyes and replace it end for end, using *extreme care* that the wyes are not jarred by the action. If the bubble does not come to the center, correct *one-half* of the error by the vertical adjusting-screws at one end of the bubble. Correct the other half by the leveling-screws. Test the work by again changing the telescope end for end in the wyes.

Care should be taken while making this adjustment to see that the level-tube is vertically under the telescope. With the bubble in the center of the tube, rotate the telescope in the wyes for a considerable angle each side of the vertical. If the first half of the adjustment has been made and the bubble moves, it shows that the axis of the wyes and the axis of the level-tube are not in the same vertical plane although both have been made horizontal. By moving one end of the level-tube *sidewise* by means of the horizontal screws at one end of the tube, the two axes may be brought into the same plane. As this adjustment is liable to disturb the other, both should be alternately tested until both requirements are complied with.

By these methods the axis of the bubble is made parallel to the axis of the wyes; and as this has been made parallel to the lines of collimation by means of the previous adjustment, the axis of the bubble is therefore parallel to the line of collimation.

3. *To make the line of collimation perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope  $180^\circ$ . If it is not level, adjust half of the error by means of the capstan-headed screw under one of the wyes, and the other half by the leveling-screws. Reverse again as a test.

When the first two adjustments have been accurately made, good leveling may always be done by bringing the bubble to the center by means of the leveling-screws, at every sight if necessary, even if the third adjustment is not made. Of course this third adjustment should be made as a matter of convenience, so that the line of collimation may be always level no matter in what direction it may be pointed, but it is not *necessary* to stop work to make this adjustment every time it is found to be defective.

#### ADJUSTMENTS OF THE DUMPY LEVEL.

1. *To make the axis of the level-tube perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope  $180^\circ$ . If

it is not level, adjust *one-half* of the error by means of the adjusting-screws at one end of the bubble, and the other half by means of the leveling-screws. Reverse again as a test.

2. *To make the line of collimation perpendicular to the vertical axis.* The method of adjustment is identical with that for the transit (No. 4, p. 308) except that the cross-wire must be adjusted to agree with the level-bubble rather than *vice versa*, as is the case with the corresponding adjustment of the transit; i.e., with the level-bubble in the center, raise or lower the horizontal cross-wire until it points at the mark known to be on a level with the center of the instrument.

If the instrument has been well made and has not been distorted by rough usage, the cross-wires will intersect at the center of the field of view when adjusted as described. If they do not, it indicates an error which ordinarily can only be corrected by an instrument-maker. The error may be due to any one of several causes, which are

(a) faulty centering of object-slide;

(b) faulty centering of eyepiece;

(c) distortion of instrument so that the geometric axis of the telescope is not perpendicular to the vertical axis. If the error is only just perceptible, it will not probably cause any error in the work.



# EXPLANATORY NOTE ON THE USE OF THE TABLES.

The logarithms here given are “five-place,” but the last figure sometimes has a special mark over it (e.g.,  $\hat{6}$ ) which indicates that one-half a unit in the last place should be *added*. For example :

| the value       | includes all values between |                   |
|-----------------|-----------------------------|-------------------|
| .69586          | .6958575000 + and           | .6958624999 . . . |
| .6958 $\hat{6}$ | .6958625000 + and           | .6958674999 . . . |

The maximum error in any one value therefore does not exceed one-quarter of a fifth-place unit.

When adding or subtracting such logarithms allow a half-unit for such a sign. For example :

|                   |                   |                   |
|-------------------|-------------------|-------------------|
| .69586            | .69586            | .6958 $\hat{6}$   |
| .10841            | .1084 $\hat{1}$   | .1084 $\hat{1}$   |
| .1294 $\hat{7}$   | .1294 $\hat{7}$   | .1294 $\hat{7}$   |
| <u>          </u> | <u>          </u> | <u>          </u> |
| .9337 $\hat{4}$   | .93375            | .9337 $\hat{5}$   |

All other logarithmic operations are performed as usual and are supposed to be understood by the student.

TABLE I.—RADI OF CURVES.

| Deg. | 0°      |         | 1°      |         | 2°      |         | 3°      |         | Deg. |
|------|---------|---------|---------|---------|---------|---------|---------|---------|------|
| Min. | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Min. |
| 0    | ∞       | ∞       | 5729.6  | 3.75813 | 2864.9  | 3.45711 | 1910.1  | 3.28105 | 0    |
| 1    | 343775  | 5.53627 | 5635.7  | .75095  | 2841.3  | .45351  | 1899.5  | .27864  | 1    |
| 2    | 171887  | 5.23524 | 5544.8  | .74389  | 2818.0  | .44993  | 1889.1  | .27625  | 2    |
| 3    | 114592  | 5.05915 | 5456.8  | .73694  | 2795.1  | .44639  | 1878.8  | .27387  | 3    |
| 4    | 85944   | 4.93421 | 5371.6  | .73010  | 2772.5  | .44287  | 1868.6  | .27151  | 4    |
| 5    | 68755   | 4.83730 | 5288.9  | .72336  | 2750.4  | .43939  | 1858.5  | .26913  | 5    |
| 6    | 57296   | 4.75812 | 5208.8  | 3.71673 | 2728.5  | 3.43593 | 1848.5  | 3.26681 | 6    |
| 7    | 49111   | .69117  | 5131.0  | .71020  | 2707.0  | .43249  | 1838.6  | .26448  | 7    |
| 8    | 42972   | .63318  | 5055.6  | .70377  | 2685.9  | .42909  | 1828.8  | .26217  | 8    |
| 9    | 38197   | .58203  | 4982.3  | .69743  | 2665.1  | .42571  | 1819.1  | .25986  | 9    |
| 10   | 34377   | .53627  | 4911.2  | .69118  | 2644.6  | .42235  | 1809.6  | .25757  | 10   |
| 11   | 31252   | 4.49488 | 4842.0  | 3.68502 | 2624.4  | 3.41903 | 1800.1  | 3.25529 | 11   |
| 12   | 28648   | .45709  | 4774.7  | .67895  | 2604.5  | .41572  | 1790.7  | .25303  | 12   |
| 13   | 26444   | .42233  | 4709.3  | .67296  | 2584.9  | .41245  | 1781.5  | .25077  | 13   |
| 14   | 24555   | .39014  | 4645.7  | .66705  | 2565.6  | .40919  | 1772.3  | .24853  | 14   |
| 15   | 22918   | .36018  | 4583.8  | .66122  | 2546.6  | .40597  | 1763.2  | .24629  | 15   |
| 16   | 21486   | 4.33213 | 4523.4  | 3.65547 | 2527.9  | 3.40276 | 1754.2  | 3.24407 | 16   |
| 17   | 20222   | .30582  | 4464.7  | .64979  | 2509.5  | .39958  | 1745.3  | .24186  | 17   |
| 18   | 19099   | .28100  | 4407.5  | .64419  | 2491.3  | .39642  | 1736.5  | .23967  | 18   |
| 19   | 18093   | .25752  | 4351.7  | .63865  | 2473.4  | .39329  | 1727.8  | .23748  | 19   |
| 20   | 17189   | .23524  | 4297.3  | .63319  | 2455.7  | .39017  | 1719.1  | .23530  | 20   |
| 21   | 16370   | 4.21405 | 4244.2  | 3.62780 | 2438.3  | 3.38708 | 1710.6  | 3.23314 | 21   |
| 22   | 15626   | .19385  | 4192.5  | .62247  | 2421.1  | .38401  | 1702.1  | .23098  | 22   |
| 23   | 14947   | .17454  | 4142.0  | .61720  | 2404.2  | .38097  | 1693.7  | .22884  | 23   |
| 24   | 14324   | .15606  | 4092.7  | .61200  | 2387.5  | .37794  | 1685.4  | .22670  | 24   |
| 25   | 13751   | .13833  | 4044.5  | .60686  | 2371.0  | .37494  | 1677.2  | .22458  | 25   |
| 26   | 13222   | 4.12130 | 3997.5  | 3.60178 | 2354.8  | 3.37195 | 1669.1  | 3.22247 | 26   |
| 27   | 12732   | .10491  | 3951.5  | .59676  | 2338.8  | .36899  | 1661.0  | .22037  | 27   |
| 28   | 12278   | .08911  | 3906.6  | .59180  | 2323.0  | .36604  | 1653.0  | .21827  | 28   |
| 29   | 11854   | .07387  | 3862.7  | .58689  | 2307.4  | .36312  | 1645.1  | .21619  | 29   |
| 30   | 11459   | .05915  | 3819.8  | .58204  | 2292.0  | .36021  | 1637.3  | .21412  | 30   |
| 31   | 11090   | 4.04491 | 3777.9  | 3.57724 | 2276.8  | 3.35733 | 1629.5  | 3.21206 | 31   |
| 32   | 10743   | .03112  | 3736.8  | .57250  | 2261.9  | .35446  | 1621.8  | .21000  | 32   |
| 33   | 10417   | .01776  | 3696.6  | .56780  | 2247.1  | .35162  | 1614.2  | .20796  | 33   |
| 34   | 10111   | 4.00479 | 3657.3  | .56316  | 2232.5  | .34879  | 1606.7  | .20593  | 34   |
| 35   | 9822.2  | 3.99221 | 3618.8  | .55856  | 2218.1  | .34598  | 1599.2  | .20390  | 35   |
| 36   | 9549.3  | 3.97997 | 3581.1  | 3.55401 | 2203.9  | 3.34318 | 1591.8  | 3.20189 | 36   |
| 37   | 9291.3  | .96807  | 3544.2  | .54951  | 2189.8  | .34041  | 1584.5  | .19988  | 37   |
| 38   | 9046.7  | .95649  | 3508.0  | .54506  | 2176.0  | .33765  | 1577.2  | .19789  | 38   |
| 39   | 8814.8  | .94521  | 3472.6  | .54065  | 2162.3  | .33491  | 1570.0  | .19590  | 39   |
| 40   | 8594.4  | .93421  | 3437.9  | .53629  | 2148.8  | .33219  | 1562.9  | .19392  | 40   |
| 41   | 8384.8  | 3.92349 | 3403.8  | 3.53197 | 2135.4  | 3.32949 | 1555.8  | 3.19195 | 41   |
| 42   | 8185.2  | .91302  | 3370.5  | .52769  | 2122.3  | .32680  | 1548.8  | .18999  | 42   |
| 43   | 7994.8  | .90281  | 3337.7  | .52345  | 2109.2  | .32412  | 1541.9  | .18804  | 43   |
| 44   | 7813.1  | .89282  | 3305.7  | .51925  | 2096.4  | .32147  | 1535.0  | .18610  | 44   |
| 45   | 7639.5  | .88306  | 3274.2  | .51510  | 2083.7  | .31883  | 1528.2  | .18417  | 45   |
| 46   | 7473.4  | 3.87352 | 3243.3  | 3.51098 | 2071.1  | 3.31621 | 1521.4  | 3.18224 | 46   |
| 47   | 7314.4  | .86418  | 3213.0  | .50691  | 2058.7  | .31360  | 1514.7  | .18032  | 47   |
| 48   | 7162.0  | .85503  | 3183.2  | .50287  | 2046.5  | .31101  | 1508.1  | .17842  | 48   |
| 49   | 7015.9  | .84608  | 3154.0  | .49886  | 2034.4  | .30843  | 1501.5  | .17652  | 49   |
| 50   | 6875.6  | .83731  | 3125.4  | .49490  | 2022.4  | .30587  | 1495.0  | .17462  | 50   |
| 51   | 6740.7  | 3.82871 | 3097.2  | 3.49097 | 2010.6  | 3.30332 | 1488.5  | 3.17274 | 51   |
| 52   | 6611.1  | .82027  | 3069.6  | .48707  | 1998.9  | .30079  | 1482.1  | .17087  | 52   |
| 53   | 6486.4  | .81200  | 3042.4  | .48321  | 1987.3  | .29827  | 1475.7  | .16900  | 53   |
| 54   | 6366.3  | .80388  | 3015.7  | .47939  | 1975.9  | .29577  | 1469.4  | .16714  | 54   |
| 55   | 6250.5  | .79591  | 2989.5  | .47559  | 1964.6  | .29328  | 1463.2  | .16529  | 55   |
| 56   | 6138.9  | 3.78809 | 2963.7  | 3.47183 | 1953.5  | 3.29081 | 1457.0  | 3.16344 | 56   |
| 57   | 6031.2  | .78040  | 2938.4  | .46811  | 1942.4  | .28835  | 1450.8  | .16161  | 57   |
| 58   | 5927.2  | .77285  | 2913.5  | .46441  | 1931.5  | .28590  | 1444.7  | .15978  | 58   |
| 59   | 5826.8  | .76542  | 2889.0  | .46075  | 1920.7  | .28347  | 1438.7  | .15796  | 59   |
| 60   | 5729.6  | .75813  | 2864.9  | .45711  | 1910.1  | .28105  | 1432.7  | .15615  | 60   |

TABLE I.—RADII OF CURVES.

| Deg. | 4°      |         | 5°      |         | 6°      |         | 7°      |         | Deg. |
|------|---------|---------|---------|---------|---------|---------|---------|---------|------|
| Min. | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Min. |
| 0    | 1432.7  | 3.15615 | 1146.3  | 3.05929 | 955.37  | 2.98017 | 819.02  | 2.91329 | 0    |
| 1    | 1426.7  | .15434  | 1142.5  | .05784  | 952.72  | .97896  | 817.08  | .91226  | 1    |
| 2    | 1420.8  | .15255  | 1138.7  | .05640  | 950.09  | .97776  | 815.14  | .91123  | 2    |
| 3    | 1415.0  | .15076  | 1134.9  | .05497  | 947.48  | .97657  | 813.22  | .91021  | 3    |
| 4    | 1409.2  | .14897  | 1131.2  | .05354  | 944.88  | .97537  | 811.30  | .90918  | 4    |
| 5    | 1403.5  | .14720  | 1127.5  | .05211  | 942.29  | .97418  | 809.40  | .90816  | 5    |
| 6    | 1397.8  | 3.14543 | 1123.8  | 3.05069 | 939.72  | 2.97300 | 807.50  | 2.90714 | 6    |
| 7    | 1392.1  | .14367  | 1120.2  | .04928  | 937.16  | .97181  | 805.61  | .90612  | 7    |
| 8    | 1386.5  | .14191  | 1116.5  | .04787  | 934.62  | .97063  | 803.73  | .90511  | 8    |
| 9    | 1380.9  | .14017  | 1112.9  | .04646  | 932.09  | .96945  | 801.86  | .90410  | 9    |
| 10   | 1375.4  | .13843  | 1109.3  | .04506  | 929.57  | .96828  | 800.00  | .90309  | 10   |
| 11   | 1369.9  | 3.13669 | 1105.8  | 3.04366 | 927.07  | 2.96711 | 798.14  | 2.90208 | 11   |
| 12   | 1364.5  | .13497  | 1102.2  | .04227  | 924.58  | .96594  | 796.30  | .90107  | 12   |
| 13   | 1359.1  | .13325  | 1098.7  | .04088  | 922.10  | .96478  | 794.46  | .90007  | 13   |
| 14   | 1353.8  | .13154  | 1095.2  | .03949  | 919.64  | .96361  | 792.63  | .89907  | 14   |
| 15   | 1348.4  | .12983  | 1091.7  | .03811  | 917.19  | .96246  | 790.81  | .89807  | 15   |
| 16   | 1343.2  | 3.12813 | 1088.3  | 3.03674 | 914.75  | 2.96130 | 789.00  | 2.89708 | 16   |
| 17   | 1338.0  | .12644  | 1084.8  | .03537  | 912.33  | .96015  | 787.20  | .89608  | 17   |
| 18   | 1332.8  | .12475  | 1081.4  | .03400  | 909.92  | .95900  | 785.41  | .89509  | 18   |
| 19   | 1327.6  | .12307  | 1078.1  | .03264  | 907.52  | .95785  | 783.62  | .89410  | 19   |
| 20   | 1322.5  | .12140  | 1074.7  | .03128  | 905.13  | .95671  | 781.84  | .89312  | 20   |
| 21   | 1317.5  | 3.11974 | 1071.3  | 3.02992 | 902.76  | 2.95557 | 780.07  | 2.89213 | 21   |
| 22   | 1312.4  | .11808  | 1068.0  | .02857  | 900.40  | .95443  | 778.31  | .89115  | 22   |
| 23   | 1307.4  | .11642  | 1064.7  | .02723  | 898.05  | .95330  | 776.55  | .89017  | 23   |
| 24   | 1302.5  | .11477  | 1061.4  | .02589  | 895.71  | .95217  | 774.81  | .88919  | 24   |
| 25   | 1297.6  | .11313  | 1058.2  | .02455  | 893.39  | .95104  | 773.07  | .88821  | 25   |
| 26   | 1292.7  | 3.11150 | 1054.9  | 3.02322 | 891.08  | 2.94991 | 771.34  | 2.88724 | 26   |
| 27   | 1287.9  | .10987  | 1051.7  | .02189  | 888.78  | .94879  | 769.61  | .88627  | 27   |
| 28   | 1283.1  | .10825  | 1048.5  | .02056  | 886.49  | .94767  | 767.90  | .88530  | 28   |
| 29   | 1278.3  | .10663  | 1045.3  | .01924  | 884.21  | .94655  | 766.19  | .88433  | 29   |
| 30   | 1273.6  | .10502  | 1042.1  | .01792  | 881.95  | .94544  | 764.49  | .88337  | 30   |
| 31   | 1268.9  | 3.10341 | 1039.0  | 3.01661 | 879.69  | 2.94433 | 762.80  | 2.88241 | 31   |
| 32   | 1264.2  | .10182  | 1035.9  | .01530  | 877.45  | .94322  | 761.11  | .88145  | 32   |
| 33   | 1259.6  | .10022  | 1032.8  | .01400  | 875.22  | .94212  | 759.43  | .88049  | 33   |
| 34   | 1255.0  | .09864  | 1029.7  | .01270  | 873.00  | .94101  | 757.76  | .87953  | 34   |
| 35   | 1250.4  | .09705  | 1026.6  | .01140  | 870.80  | .93991  | 756.10  | .87858  | 35   |
| 36   | 1245.9  | 3.09548 | 1023.5  | 3.01010 | 868.60  | 2.93882 | 754.44  | 2.87762 | 36   |
| 37   | 1241.4  | .09391  | 1020.5  | .00882  | 866.41  | .93772  | 752.80  | .87668  | 37   |
| 38   | 1236.9  | .09234  | 1017.5  | .00753  | 864.24  | .93663  | 751.16  | .87573  | 38   |
| 39   | 1232.5  | .09079  | 1014.5  | .00625  | 862.07  | .93554  | 749.52  | .87478  | 39   |
| 40   | 1228.1  | .08923  | 1011.5  | .00497  | 859.92  | .93446  | 747.89  | .87384  | 40   |
| 41   | 1223.7  | 3.08769 | 1008.6  | 3.00370 | 857.78  | 2.93337 | 746.27  | 2.87290 | 41   |
| 42   | 1219.4  | .08614  | 1005.6  | .00242  | 855.65  | .93229  | 744.66  | .87196  | 42   |
| 43   | 1215.1  | .08461  | 1002.7  | .00116  | 853.53  | .93122  | 743.06  | .87102  | 43   |
| 44   | 1210.8  | .08308  | 999.76  | 2.99989 | 851.42  | .93014  | 741.46  | .87008  | 44   |
| 45   | 1206.6  | .08155  | 996.87  | .99863  | 849.32  | .92907  | 739.86  | .86915  | 45   |
| 46   | 1202.4  | 3.08003 | 993.99  | 2.99738 | 847.23  | 2.92800 | 738.28  | 2.86822 | 46   |
| 47   | 1198.2  | .07852  | 991.13  | .99613  | 845.15  | .92693  | 736.70  | .86729  | 47   |
| 48   | 1194.0  | .07701  | 988.28  | .99488  | 843.08  | .92587  | 735.13  | .86636  | 48   |
| 49   | 1189.9  | .07550  | 985.45  | .99363  | 841.02  | .92480  | 733.56  | .86544  | 49   |
| 50   | 1185.8  | .07400  | 982.64  | .99239  | 838.97  | .92374  | 732.01  | .86451  | 50   |
| 51   | 1181.7  | 3.07251 | 979.84  | 2.99115 | 836.93  | 2.92269 | 730.45  | 2.86359 | 51   |
| 52   | 1177.7  | .07102  | 977.06  | .98992  | 834.90  | .92163  | 728.91  | .86267  | 52   |
| 53   | 1173.6  | .06954  | 974.29  | .98869  | 832.89  | .92058  | 727.37  | .86175  | 53   |
| 54   | 1169.7  | .06806  | 971.54  | .98746  | 830.88  | .91953  | 725.84  | .86084  | 54   |
| 55   | 1165.7  | .06658  | 968.81  | .98624  | 828.88  | .91849  | 724.31  | .85992  | 55   |
| 56   | 1161.8  | 3.06511 | 966.09  | 2.98501 | 826.89  | 2.91744 | 722.79  | 2.85901 | 56   |
| 57   | 1157.9  | .06365  | 963.39  | .98380  | 824.91  | .91640  | 721.28  | .85810  | 57   |
| 58   | 1154.0  | .06219  | 960.70  | .98258  | 822.93  | .91536  | 719.77  | .85719  | 58   |
| 59   | 1150.1  | .06074  | 958.03  | .98137  | 820.97  | .91433  | 718.27  | .85629  | 59   |
| 60   | 1146.3  | .05929  | 955.37  | .98017  | 819.02  | .91329  | 716.78  | .85538  | 60   |

TABLE I.—RADII OF CURVES.

| Deg. | 8°      |         | 9°      |         | 10°     |         | 11°     |         | Deg. |
|------|---------|---------|---------|---------|---------|---------|---------|---------|------|
| Min. | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Min. |
| 0    | 716.78  | 2.85538 | 637.27  | 2.80432 | 573.69  | 2.75867 | 521.67  | 2.71739 | 0    |
| 1    | 715.29  | .85448  | 636.10  | .80352  | 572.73  | .75795  | 520.88  | .71674  | 1    |
| 2    | 713.81  | .85358  | 634.93  | .80272  | 571.78  | .75723  | 520.10  | .71608  | 2    |
| 3    | 712.34  | .85268  | 633.76  | .80192  | 570.84  | .75651  | 519.32  | .71543  | 3    |
| 4    | 710.87  | .85178  | 632.60  | .80113  | 569.90  | .75579  | 518.54  | .71478  | 4    |
| 5    | 709.40  | .85089  | 631.44  | .80033  | 568.96  | .75508  | 517.76  | .71413  | 5    |
| 6    | 707.95  | 2.85000 | 630.29  | 2.79954 | 568.02  | 2.75436 | 516.99  | 2.71348 | 6    |
| 7    | 706.49  | .84911  | 629.14  | .79874  | 567.09  | .75365  | 516.21  | .71283  | 7    |
| 8    | 705.05  | .84822  | 627.99  | .79795  | 566.16  | .75293  | 515.44  | .71218  | 8    |
| 9    | 703.61  | .84733  | 626.85  | .79716  | 565.23  | .75222  | 514.68  | .71153  | 9    |
| 10   | 702.17  | .84644  | 625.71  | .79637  | 564.31  | .75151  | 513.91  | .71088  | 10   |
| 11   | 700.75  | 2.84556 | 624.58  | 2.79558 | 563.38  | 2.75080 | 513.15  | 2.71024 | 11   |
| 12   | 699.33  | .84468  | 623.45  | .79480  | 562.47  | .75009  | 512.38  | .70959  | 12   |
| 13   | 697.91  | .84380  | 622.32  | .79401  | 561.55  | .74939  | 511.63  | .70895  | 13   |
| 14   | 696.50  | .84292  | 621.20  | .79323  | 560.64  | .74868  | 510.87  | .70831  | 14   |
| 15   | 695.09  | .84204  | 620.09  | .79245  | 559.73  | .74798  | 510.11  | .70767  | 15   |
| 16   | 693.70  | 2.84117 | 618.97  | 2.79167 | 558.82  | 2.74727 | 509.36  | 2.70702 | 16   |
| 17   | 692.30  | .84029  | 617.87  | .79089  | 557.92  | .74657  | 508.61  | .70638  | 17   |
| 18   | 690.91  | .83942  | 616.76  | .79011  | 557.02  | .74587  | 507.86  | .70575  | 18   |
| 19   | 689.53  | .83855  | 615.66  | .78934  | 556.12  | .74517  | 507.12  | .70511  | 19   |
| 20   | 688.16  | .83768  | 614.56  | .78856  | 555.23  | .74447  | 506.38  | .70447  | 20   |
| 21   | 686.78  | 2.83682 | 613.47  | 2.78779 | 554.34  | 2.74377 | 505.64  | 2.70383 | 21   |
| 22   | 685.42  | .83595  | 612.38  | .78702  | 553.45  | .74307  | 504.90  | .70320  | 22   |
| 23   | 684.06  | .83509  | 611.30  | .78625  | 552.56  | .74238  | 504.16  | .70257  | 23   |
| 24   | 682.70  | .83423  | 610.21  | .78548  | 551.68  | .74168  | 503.42  | .70193  | 24   |
| 25   | 681.35  | .83337  | 609.14  | .78471  | 550.80  | .74099  | 502.69  | .70130  | 25   |
| 26   | 680.01  | 2.83251 | 608.06  | 2.78395 | 549.92  | 2.74030 | 501.96  | 2.70067 | 26   |
| 27   | 678.67  | .83166  | 606.99  | .78318  | 549.05  | .73961  | 501.23  | .70004  | 27   |
| 28   | 677.34  | .83080  | 605.93  | .78242  | 548.17  | .73892  | 500.51  | .69941  | 28   |
| 29   | 676.01  | .82995  | 604.86  | .78165  | 547.30  | .73823  | 499.78  | .69878  | 29   |
| 30   | 674.69  | .82910  | 603.80  | .78089  | 546.44  | .73754  | 499.06  | .69815  | 30   |
| 31   | 673.37  | 2.82825 | 602.75  | 2.78013 | 545.57  | 2.73685 | 498.34  | 2.69752 | 31   |
| 32   | 672.06  | .82740  | 601.70  | .77938  | 544.71  | .73617  | 497.62  | .69690  | 32   |
| 33   | 670.75  | .82656  | 600.65  | .77862  | 543.86  | .73548  | 496.91  | .69627  | 33   |
| 34   | 669.45  | .82571  | 599.61  | .77786  | 543.00  | .73480  | 496.19  | .69565  | 34   |
| 35   | 668.15  | .82487  | 598.57  | .77711  | 542.15  | .73412  | 495.48  | .69503  | 35   |
| 36   | 666.86  | 2.82403 | 597.53  | 2.77636 | 541.30  | 2.73343 | 494.77  | 2.69440 | 36   |
| 37   | 665.57  | .82319  | 596.50  | .77561  | 540.45  | .73275  | 494.07  | .69378  | 37   |
| 38   | 664.29  | .82235  | 595.47  | .77486  | 539.61  | .73207  | 493.36  | .69316  | 38   |
| 39   | 663.01  | .82152  | 594.44  | .77411  | 538.76  | .73140  | 492.66  | .69254  | 39   |
| 40   | 661.74  | .82068  | 593.42  | .77336  | 537.92  | .73072  | 491.96  | .69192  | 40   |
| 41   | 660.47  | 2.81985 | 592.40  | 2.77261 | 537.09  | 2.73004 | 491.26  | 2.69131 | 41   |
| 42   | 659.21  | .81902  | 591.38  | .77187  | 536.25  | .72937  | 490.56  | .69069  | 42   |
| 43   | 657.95  | .81819  | 590.37  | .77112  | 535.42  | .72869  | 489.86  | .69007  | 43   |
| 44   | 656.69  | .81736  | 589.36  | .77038  | 534.59  | .72802  | 489.17  | .68946  | 44   |
| 45   | 655.45  | .81653  | 588.36  | .76964  | 533.77  | .72735  | 488.48  | .68884  | 45   |
| 46   | 654.20  | 2.81571 | 587.36  | 2.76890 | 532.94  | 2.72668 | 487.79  | 2.68823 | 46   |
| 47   | 652.96  | .81489  | 586.36  | .76816  | 532.12  | .72601  | 487.10  | .68762  | 47   |
| 48   | 651.73  | .81406  | 585.37  | .76742  | 531.30  | .72534  | 486.42  | .68701  | 48   |
| 49   | 650.50  | .81324  | 584.37  | .76669  | 530.49  | .72467  | 485.73  | .68640  | 49   |
| 50   | 649.27  | .81243  | 583.38  | .76595  | 529.67  | .72401  | 485.05  | .68579  | 50   |
| 51   | 648.05  | 2.81161 | 582.40  | 2.76522 | 528.86  | 2.72334 | 484.37  | 2.68518 | 51   |
| 52   | 646.84  | .81079  | 581.42  | .76449  | 528.05  | .72267  | 483.69  | .68457  | 52   |
| 53   | 645.63  | .80998  | 580.44  | .76376  | 527.25  | .72201  | 483.02  | .68396  | 53   |
| 54   | 644.42  | .80917  | 579.47  | .76303  | 526.44  | .72135  | 482.34  | .68335  | 54   |
| 55   | 643.22  | .80836  | 578.49  | .76230  | 525.64  | .72069  | 481.67  | .68275  | 55   |
| 56   | 642.02  | 2.80755 | 577.53  | 2.76157 | 524.84  | 2.72003 | 481.00  | 2.68214 | 56   |
| 57   | 640.83  | .80674  | 576.56  | .76084  | 524.05  | .71937  | 480.33  | .68154  | 57   |
| 58   | 639.64  | .80593  | 575.60  | .76012  | 523.25  | .71871  | 479.67  | .68094  | 58   |
| 59   | 638.45  | .80513  | 574.64  | .75939  | 522.46  | .71805  | 479.00  | .68033  | 59   |
| 60   | 637.27  | .80432  | 573.69  | .75867  | 521.67  | .71739  | 478.34  | .67973  | 60   |

TABLE I.—RADIII OF CURVES.

| Deg. | Radius. | Log R   | Deg. | Radius. | Log R   | Deg. | Radius. | Log R   | Deg. | Radius. | Log R   |
|------|---------|---------|------|---------|---------|------|---------|---------|------|---------|---------|
| 12°  | 478.34  | 2.67973 | 14°  | 410.28  | 2.61307 | 16°  | 359.26  | 2.55541 | 21°  | 274.37  | 2.43833 |
| 2    | 477.02  | .67853  | 2    | 409.31  | .61205  | 5    | 357.42  | .55317  | 10   | 272.23  | .43494  |
| 4    | 475.71  | .67734  | 4    | 408.34  | .61102  | 10   | 355.59  | .55094  | 20   | 270.13  | .43157  |
| 6    | 474.40  | .67614  | 6    | 407.38  | .61000  | 15   | 353.77  | .54872  | 30   | 268.06  | .42823  |
| 8    | 473.10  | .67493  | 8    | 406.42  | .60898  | 20   | 351.98  | .54652  | 40   | 266.02  | .42492  |
| 10   | 471.81  | 2.67376 | 10   | 405.47  | 2.60796 | 25   | 350.21  | .54432  | 50   | 264.02  | .42163  |
| 12   | 470.53  | .67258  | 12   | 404.53  | .60694  | 30   | 348.45  | 2.54214 | 22°  | 262.04  | 2.41837 |
| 14   | 469.25  | .67140  | 14   | 403.58  | .60593  | 35   | 346.71  | .53997  | 10   | 260.10  | .41513  |
| 16   | 467.98  | .67022  | 16   | 402.65  | .60492  | 40   | 344.99  | .53780  | 20   | 258.18  | .41192  |
| 18   | 466.72  | .66905  | 18   | 401.71  | .60391  | 45   | 343.29  | .53563  | 30   | 256.29  | .40873  |
| 20   | 465.46  | 2.66788 | 20   | 400.78  | 2.60291 | 50   | 341.60  | .53351  | 40   | 254.43  | .40557  |
| 22   | 464.21  | .66671  | 22   | 399.86  | .60190  | 55   | 339.93  | .53138  | 50   | 252.60  | .40243  |
| 24   | 462.97  | .66555  | 24   | 398.94  | .60090  | 17°  | 338.27  | 2.52927 | 23°  | 250.79  | 2.39931 |
| 26   | 461.73  | .66439  | 26   | 398.02  | .59990  | 5    | 336.64  | .52716  | 10   | 249.01  | .39622  |
| 28   | 460.50  | .66323  | 28   | 397.11  | .59891  | 10   | 335.01  | .52506  | 20   | 247.26  | .39315  |
| 30   | 459.28  | 2.66207 | 30   | 396.20  | 2.59791 | 15   | 333.41  | .52297  | 30   | 245.53  | .39010  |
| 32   | 458.06  | .66092  | 32   | 395.30  | .59692  | 20   | 331.82  | .52090  | 40   | 243.82  | .38707  |
| 34   | 456.85  | .65977  | 34   | 394.40  | .59593  | 25   | 330.24  | .51883  | 50   | 242.14  | .38407  |
| 36   | 455.65  | .65863  | 36   | 393.50  | .59494  | 30   | 328.68  | 2.51677 | 24°  | 240.49  | 2.38109 |
| 38   | 454.45  | .65748  | 38   | 392.61  | .59396  | 35   | 327.13  | .51472  | 10   | 238.85  | .37813  |
| 40   | 453.26  | 2.65634 | 40   | 391.72  | 2.59298 | 40   | 325.60  | .51269  | 20   | 237.24  | .37519  |
| 42   | 452.07  | .65521  | 42   | 390.84  | .59199  | 45   | 324.09  | .51066  | 30   | 235.65  | .37227  |
| 44   | 450.89  | .65407  | 44   | 389.96  | .59102  | 50   | 322.59  | .50864  | 40   | 234.08  | .36937  |
| 46   | 449.72  | .65294  | 46   | 389.08  | .59004  | 55   | 321.10  | .50663  | 50   | 232.54  | .36649  |
| 48   | 448.56  | .65181  | 48   | 388.21  | .58907  | 18°  | 319.62  | 2.50464 | 25°  | 231.01  | 2.36363 |
| 50   | 447.40  | 2.65069 | 50   | 387.34  | 2.58809 | 5    | 318.16  | .50265  | 30   | 226.55  | .35517  |
| 52   | 446.24  | .64957  | 52   | 386.48  | .58713  | 10   | 316.71  | .50067  | 26°  | 222.27  | .34688  |
| 54   | 445.09  | .64845  | 54   | 385.62  | .58616  | 15   | 315.28  | .49869  | 30   | 218.15  | .33873  |
| 56   | 443.95  | .64733  | 56   | 384.77  | .58519  | 20   | 313.86  | .49673  | 27°  | 214.18  | 2.33078 |
| 58   | 442.81  | .64622  | 58   | 383.91  | .58423  | 25   | 312.45  | .49478  | 30   | 210.36  | .32296  |
| 13°  | 441.68  | 2.64511 | 15°  | 383.06  | 2.58327 | 30   | 311.06  | 2.49284 | 28°  | 206.68  | .31529  |
| 2    | 440.56  | .64400  | 2    | 382.22  | .58231  | 35   | 309.67  | .49090  | 30   | 203.13  | .30776  |
| 4    | 439.44  | .64290  | 4    | 381.38  | .58135  | 40   | 308.30  | .48898  | 29°  | 199.70  | 2.30037 |
| 6    | 438.33  | .64180  | 6    | 380.54  | .58040  | 45   | 306.95  | .48706  | 30   | 196.38  | .29310  |
| 8    | 437.22  | .64070  | 8    | 379.71  | .57945  | 50   | 305.60  | .48515  | 30°  | 193.19  | .28597  |
| 10   | 436.12  | 2.63960 | 10   | 378.88  | 2.57850 | 55   | 304.27  | .48325  | 30   | 190.09  | .27896  |
| 12   | 435.02  | .63851  | 12   | 378.05  | .57753  | 19°  | 302.94  | 2.48136 | 31°  | 187.10  | 2.27207 |
| 14   | 433.93  | .63742  | 14   | 377.23  | .57661  | 5    | 301.63  | .47948  | 32   | 181.40  | .25863  |
| 16   | 432.84  | .63633  | 16   | 376.41  | .57566  | 10   | 300.33  | .47760  | 33   | 176.05  | .24563  |
| 18   | 431.76  | .63524  | 18   | 375.60  | .57472  | 15   | 299.04  | .47573  | 34   | 171.02  | .23303  |
| 20   | 430.69  | 2.63416 | 20   | 374.79  | 2.57378 | 20   | 297.77  | .47388  | 35   | 166.28  | .22083  |
| 22   | 429.62  | .63308  | 22   | 373.98  | .57284  | 25   | 296.50  | .47203  | 36   | 161.80  | 2.20899 |
| 24   | 428.56  | .63201  | 24   | 373.17  | .57191  | 30   | 295.25  | 2.47018 | 37   | 157.58  | .19749  |
| 26   | 427.50  | .63093  | 26   | 372.37  | .57097  | 35   | 294.00  | .46835  | 38   | 153.58  | .18633  |
| 28   | 426.44  | .62986  | 28   | 371.57  | .57004  | 40   | 292.77  | .46652  | 39   | 149.79  | .17547  |
| 30   | 425.40  | 2.62879 | 30   | 370.78  | 2.56911 | 45   | 291.55  | .46471  | 40   | 146.19  | .16492  |
| 32   | 424.35  | .62773  | 32   | 369.99  | .56819  | 50   | 290.33  | .46289  | 41   | 142.77  | 2.15464 |
| 34   | 423.32  | .62666  | 34   | 369.20  | .56726  | 55   | 289.13  | .46109  | 42   | 139.52  | .14464  |
| 36   | 422.28  | .62560  | 36   | 368.42  | .56634  | 20   | 287.94  | 2.45930 | 43   | 136.43  | .13489  |
| 38   | 421.26  | .62454  | 38   | 367.64  | .56542  | 5    | 286.76  | .45751  | 44   | 133.47  | .12539  |
| 40   | 420.23  | 2.62349 | 40   | 366.86  | 2.56450 | 10   | 285.58  | .45573  | 45   | 130.66  | .11613  |
| 42   | 419.22  | .62243  | 42   | 366.09  | .56358  | 15   | 284.42  | .45396  | 46   | 127.97  | 2.10709 |
| 44   | 418.20  | .62138  | 44   | 365.31  | .56266  | 20   | 283.27  | .45219  | 47   | 125.39  | .09827  |
| 46   | 417.19  | .62034  | 46   | 364.55  | .56173  | 25   | 282.12  | .45044  | 48   | 122.93  | .08963  |
| 48   | 416.19  | .61929  | 48   | 363.78  | .56084  | 30   | 280.99  | 2.44869 | 49   | 120.57  | .08124  |
| 50   | 415.19  | 2.61825 | 50   | 363.02  | 2.55993 | 35   | 279.86  | .44694  | 50   | 118.31  | .07302  |
| 52   | 414.20  | .61721  | 52   | 362.26  | .55902  | 40   | 278.75  | .44521  | 52   | 114.06  | 2.05713 |
| 54   | 413.21  | .61617  | 54   | 361.51  | .55812  | 45   | 277.64  | .44348  | 54   | 110.13  | .04192  |
| 56   | 412.23  | .61514  | 56   | 360.76  | .55721  | 50   | 276.54  | .44176  | 56   | 106.50  | .02736  |
| 58   | 411.25  | .61410  | 58   | 360.01  | .55631  | 55   | 275.45  | .44004  | 58   | 103.13  | .01340  |
| 14°  | 410.28  | 2.61307 | 16°  | 359.26  | 2.55541 | 21°  | 274.37  | 2.43833 | 60   | 100.00  | 2.00000 |

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A  
1° CURVE.

| Δ   | Tangent<br><i>T</i> . | Ext. Dist.<br><i>E</i> . | Long Ch'd<br><i>LC</i> . | Δ   | Tangent<br><i>T</i> . | Ext. Dist.<br><i>E</i> . | Long Ch'd<br><i>LC</i> . | Δ   | Tangent<br><i>T</i> . | Ext. Dist.<br><i>E</i> . | Long Ch'd<br><i>LC</i> . |
|-----|-----------------------|--------------------------|--------------------------|-----|-----------------------|--------------------------|--------------------------|-----|-----------------------|--------------------------|--------------------------|
| 1°  | 50.00                 | 0.218                    | 100.00                   | 11° | 551.70                | 26.500                   | 1098.3                   | 21° | 1061.9                | 97.58                    | 2088.3                   |
| 10' | 58.34                 | 0.297                    | 116.67                   | 10  | 560.11                | 27.313                   | 1114.9                   | 10  | 1070.6                | 99.15                    | 2104.7                   |
| 20  | 66.67                 | 0.388                    | 133.33                   | 20  | 568.53                | 28.137                   | 1131.5                   | 20  | 1079.2                | 100.75                   | 2121.1                   |
| 30  | 75.01                 | 0.491                    | 150.00                   | 30  | 576.95                | 28.974                   | 1148.1                   | 30  | 1087.8                | 102.35                   | 2137.4                   |
| 40  | 83.34                 | 0.606                    | 166.66                   | 40  | 585.36                | 29.824                   | 1164.7                   | 40  | 1096.4                | 103.97                   | 2153.8                   |
| 50  | 91.68                 | 0.733                    | 183.33                   | 50  | 593.79                | 30.686                   | 1181.2                   | 50  | 1105.1                | 105.60                   | 2170.2                   |
| 2°  | 100.01                | 0.873                    | 199.99                   | 12° | 602.21                | 31.561                   | 1197.8                   | 22° | 1113.7                | 107.24                   | 2186.5                   |
| 10  | 108.35                | 1.024                    | 216.66                   | 10  | 610.64                | 32.447                   | 1214.4                   | 10  | 1122.4                | 108.90                   | 2202.9                   |
| 20  | 116.68                | 1.188                    | 233.32                   | 20  | 619.07                | 33.347                   | 1231.0                   | 20  | 1131.0                | 110.57                   | 2219.2                   |
| 30  | 125.02                | 1.364                    | 249.98                   | 30  | 627.50                | 34.259                   | 1247.5                   | 30  | 1139.7                | 112.25                   | 2235.6                   |
| 40  | 133.36                | 1.552                    | 266.65                   | 40  | 635.93                | 35.183                   | 1264.1                   | 40  | 1148.4                | 113.95                   | 2251.9                   |
| 50  | 141.70                | 1.752                    | 283.31                   | 50  | 644.37                | 36.120                   | 1280.7                   | 50  | 1157.0                | 115.66                   | 2268.3                   |
| 3°  | 150.04                | 1.964                    | 299.97                   | 13° | 652.81                | 37.069                   | 1297.2                   | 23° | 1165.7                | 117.38                   | 2284.6                   |
| 10  | 158.38                | 2.188                    | 316.63                   | 10  | 661.25                | 38.031                   | 1313.8                   | 10  | 1174.4                | 119.12                   | 2301.0                   |
| 20  | 166.72                | 2.425                    | 333.29                   | 20  | 669.70                | 39.006                   | 1330.3                   | 20  | 1183.1                | 120.87                   | 2317.3                   |
| 30  | 175.06                | 2.674                    | 349.95                   | 30  | 678.15                | 39.993                   | 1346.9                   | 30  | 1191.8                | 122.63                   | 2333.6                   |
| 40  | 183.40                | 2.934                    | 366.61                   | 40  | 686.60                | 40.992                   | 1363.4                   | 40  | 1200.5                | 124.41                   | 2349.9                   |
| 50  | 191.74                | 3.207                    | 383.27                   | 50  | 695.06                | 42.004                   | 1380.0                   | 50  | 1209.2                | 126.20                   | 2366.2                   |
| 4°  | 200.08                | 3.492                    | 399.92                   | 14° | 703.51                | 43.029                   | 1396.5                   | 24° | 1217.9                | 128.00                   | 2382.5                   |
| 10  | 208.43                | 3.790                    | 416.58                   | 10  | 711.97                | 44.066                   | 1413.1                   | 10  | 1226.6                | 129.82                   | 2398.8                   |
| 20  | 216.77                | 4.099                    | 433.24                   | 20  | 720.44                | 45.116                   | 1429.6                   | 20  | 1235.3                | 131.65                   | 2415.1                   |
| 30  | 225.12                | 4.421                    | 449.89                   | 30  | 728.90                | 46.178                   | 1446.2                   | 30  | 1244.0                | 133.50                   | 2431.4                   |
| 40  | 233.47                | 4.755                    | 466.54                   | 40  | 737.37                | 47.253                   | 1462.7                   | 40  | 1252.8                | 135.36                   | 2447.7                   |
| 50  | 241.81                | 5.100                    | 483.20                   | 50  | 745.85                | 48.341                   | 1479.2                   | 50  | 1261.5                | 137.23                   | 2464.0                   |
| 5°  | 250.16                | 5.459                    | 499.85                   | 15° | 754.32                | 49.441                   | 1495.7                   | 25° | 1270.2                | 139.11                   | 2480.2                   |
| 10  | 258.51                | 5.829                    | 516.50                   | 10  | 762.80                | 50.554                   | 1512.3                   | 10  | 1279.0                | 141.01                   | 2496.5                   |
| 20  | 266.86                | 6.211                    | 533.15                   | 20  | 771.29                | 51.679                   | 1528.8                   | 20  | 1287.7                | 142.93                   | 2512.8                   |
| 30  | 275.21                | 6.606                    | 549.80                   | 30  | 779.77                | 52.818                   | 1545.3                   | 30  | 1296.5                | 144.85                   | 2529.0                   |
| 40  | 283.57                | 7.013                    | 566.44                   | 40  | 788.26                | 53.969                   | 1561.8                   | 40  | 1305.3                | 146.79                   | 2545.3                   |
| 50  | 291.92                | 7.432                    | 583.09                   | 50  | 796.75                | 55.132                   | 1578.3                   | 50  | 1314.0                | 148.75                   | 2561.5                   |
| 6°  | 300.28                | 7.863                    | 599.73                   | 16° | 805.25                | 56.309                   | 1594.8                   | 26° | 1322.8                | 150.71                   | 2577.8                   |
| 10  | 308.64                | 8.307                    | 616.38                   | 10  | 813.75                | 57.498                   | 1611.3                   | 10  | 1331.6                | 152.69                   | 2594.0                   |
| 20  | 316.99                | 8.762                    | 633.02                   | 20  | 822.25                | 58.699                   | 1627.8                   | 20  | 1340.4                | 154.69                   | 2610.3                   |
| 30  | 325.35                | 9.230                    | 649.66                   | 30  | 830.76                | 59.914                   | 1644.3                   | 30  | 1349.2                | 156.70                   | 2626.5                   |
| 40  | 333.71                | 9.710                    | 666.30                   | 40  | 839.27                | 61.141                   | 1660.8                   | 40  | 1358.0                | 158.72                   | 2642.7                   |
| 50  | 342.08                | 10.202                   | 682.94                   | 50  | 847.78                | 62.381                   | 1677.3                   | 50  | 1366.8                | 160.76                   | 2658.9                   |
| 7°  | 350.44                | 10.707                   | 699.57                   | 17° | 856.30                | 63.634                   | 1693.8                   | 27° | 1375.6                | 162.81                   | 2675.1                   |
| 10  | 358.81                | 11.224                   | 716.21                   | 10  | 864.82                | 64.900                   | 1710.3                   | 10  | 1384.4                | 164.87                   | 2691.3                   |
| 20  | 367.17                | 11.753                   | 732.84                   | 20  | 873.35                | 66.178                   | 1726.8                   | 20  | 1393.2                | 166.95                   | 2707.5                   |
| 30  | 375.54                | 12.294                   | 749.47                   | 30  | 881.88                | 67.470                   | 1743.2                   | 30  | 1402.0                | 169.04                   | 2723.7                   |
| 40  | 383.91                | 12.847                   | 766.10                   | 40  | 890.41                | 68.774                   | 1759.7                   | 40  | 1410.9                | 171.15                   | 2739.9                   |
| 50  | 392.28                | 13.413                   | 782.73                   | 50  | 898.95                | 70.091                   | 1776.2                   | 50  | 1419.7                | 173.27                   | 2756.1                   |
| 8°  | 400.66                | 13.991                   | 799.36                   | 18° | 907.49                | 71.421                   | 1792.6                   | 28° | 1428.6                | 175.41                   | 2772.3                   |
| 10  | 409.03                | 14.582                   | 815.99                   | 10  | 916.03                | 72.764                   | 1809.1                   | 10  | 1437.4                | 177.55                   | 2788.4                   |
| 20  | 417.41                | 15.184                   | 832.61                   | 20  | 924.58                | 74.119                   | 1825.5                   | 20  | 1446.3                | 179.72                   | 2804.6                   |
| 30  | 425.79                | 15.799                   | 849.23                   | 30  | 933.13                | 75.488                   | 1842.0                   | 30  | 1455.1                | 181.89                   | 2820.7                   |
| 40  | 434.17                | 16.426                   | 865.85                   | 40  | 941.69                | 76.869                   | 1858.4                   | 40  | 1464.0                | 184.08                   | 2836.9                   |
| 50  | 442.55                | 17.066                   | 882.47                   | 50  | 950.25                | 78.264                   | 1874.9                   | 50  | 1472.9                | 186.29                   | 2853.0                   |
| 9°  | 450.93                | 17.717                   | 899.09                   | 19° | 958.81                | 79.671                   | 1891.3                   | 29° | 1481.8                | 188.51                   | 2869.2                   |
| 10  | 459.32                | 18.381                   | 915.70                   | 10  | 967.38                | 81.092                   | 1907.8                   | 10  | 1490.7                | 190.74                   | 2885.3                   |
| 20  | 467.71                | 19.058                   | 932.31                   | 20  | 975.96                | 82.525                   | 1924.2                   | 20  | 1499.6                | 192.99                   | 2901.4                   |
| 30  | 476.10                | 19.746                   | 948.92                   | 30  | 984.53                | 83.972                   | 1940.6                   | 30  | 1508.5                | 195.25                   | 2917.6                   |
| 40  | 484.49                | 20.447                   | 965.53                   | 40  | 993.12                | 85.431                   | 1957.1                   | 40  | 1517.4                | 197.53                   | 2933.7                   |
| 50  | 492.88                | 21.161                   | 982.14                   | 50  | 1001.70               | 86.904                   | 1973.5                   | 50  | 1526.3                | 199.82                   | 2949.8                   |
| 10° | 501.28                | 21.886                   | 998.74                   | 20° | 1010.29               | 88.389                   | 1989.9                   | 30° | 1535.3                | 202.12                   | 2965.9                   |
| 10  | 509.68                | 22.624                   | 1015.35                  | 10  | 1018.89               | 89.888                   | 2006.3                   | 10  | 1544.2                | 204.44                   | 2982.0                   |
| 20  | 518.08                | 23.375                   | 1031.95                  | 20  | 1027.49               | 91.399                   | 2022.7                   | 20  | 1553.1                | 206.77                   | 2998.1                   |
| 30  | 526.48                | 24.138                   | 1048.54                  | 30  | 1036.09               | 92.924                   | 2039.1                   | 30  | 1562.1                | 209.12                   | 3014.2                   |
| 40  | 534.89                | 24.913                   | 1065.14                  | 40  | 1044.70               | 94.462                   | 2055.5                   | 40  | 1571.0                | 211.48                   | 3030.2                   |
| 50  | 543.29                | 25.700                   | 1081.73                  | 50  | 1053.31               | 96.013                   | 2071.9                   | 50  | 1580.0                | 213.86                   | 3046.3                   |
| 11° | 551.70                | 26.500                   | 1098.33                  | 21° | 1061.93               | 97.577                   | 2088.3                   | 31° | 1589.0                | 216.25                   | 3062.4                   |



TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A  
1° CURVE.

| Δ   | Tangent<br><i>T</i> . | Ext. Dist.<br><i>E</i> . | Long Ch'd<br><i>LC</i> . | Δ   | Tangent<br><i>T</i> . | Ext. Dist.<br><i>E</i> . | Long Ch'd<br><i>LC</i> . | Δ   | Tangent<br><i>T</i> . | Ext. Dist.<br><i>E</i> . | Long Ch'd<br><i>LC</i> . |
|-----|-----------------------|--------------------------|--------------------------|-----|-----------------------|--------------------------|--------------------------|-----|-----------------------|--------------------------|--------------------------|
| 31° | 1589.0                | 216.25                   | 3062.4                   | 41° | 2142.2                | 387.38                   | 4013.1                   | 51° | 2732.9                | 618.39                   | 4933.4                   |
| 10' | 1598.0                | 218.66                   | 3078.4                   | 10  | 2151.7                | 390.71                   | 4028.7                   | 10  | 2743.1                | 622.81                   | 4948.4                   |
| 20  | 1606.9                | 221.08                   | 3094.5                   | 20  | 2161.2                | 394.06                   | 4044.3                   | 20  | 2753.4                | 627.24                   | 4963.4                   |
| 30  | 1615.9                | 223.51                   | 3110.5                   | 30  | 2170.8                | 397.43                   | 4059.9                   | 30  | 2763.7                | 631.69                   | 4978.4                   |
| 40  | 1624.9                | 225.96                   | 3126.6                   | 40  | 2180.3                | 400.82                   | 4075.5                   | 40  | 2773.9                | 636.16                   | 4993.4                   |
| 50  | 1633.9                | 228.42                   | 3142.6                   | 50  | 2189.9                | 404.22                   | 4091.1                   | 50  | 2784.2                | 640.66                   | 5008.4                   |
| 32° | 1643.0                | 230.90                   | 3158.6                   | 42° | 2199.4                | 407.64                   | 4106.6                   | 52° | 2794.5                | 645.17                   | 5023.4                   |
| 10  | 1652.0                | 233.39                   | 3174.6                   | 10  | 2209.0                | 411.07                   | 4122.2                   | 10  | 2804.9                | 649.70                   | 5038.4                   |
| 20  | 1661.0                | 235.90                   | 3190.6                   | 20  | 2218.6                | 414.52                   | 4137.7                   | 20  | 2815.2                | 654.25                   | 5053.4                   |
| 30  | 1670.0                | 238.43                   | 3206.6                   | 30  | 2228.1                | 417.99                   | 4153.3                   | 30  | 2825.6                | 658.83                   | 5068.3                   |
| 40  | 1679.1                | 240.96                   | 3222.6                   | 40  | 2237.7                | 421.48                   | 4168.8                   | 40  | 2835.9                | 663.42                   | 5083.3                   |
| 50  | 1688.1                | 243.52                   | 3238.6                   | 50  | 2247.3                | 424.98                   | 4184.3                   | 50  | 2846.3                | 668.03                   | 5098.2                   |
| 33° | 1697.2                | 246.08                   | 3254.6                   | 43° | 2257.0                | 428.50                   | 4199.8                   | 53° | 2856.7                | 672.66                   | 5113.1                   |
| 10  | 1706.3                | 248.66                   | 3270.6                   | 10  | 2266.6                | 432.04                   | 4215.3                   | 10  | 2867.1                | 677.32                   | 5128.0                   |
| 20  | 1715.3                | 251.26                   | 3286.6                   | 20  | 2276.2                | 435.59                   | 4230.8                   | 20  | 2877.5                | 681.99                   | 5142.9                   |
| 30  | 1724.4                | 253.87                   | 3302.5                   | 30  | 2285.9                | 439.16                   | 4246.3                   | 30  | 2888.0                | 686.68                   | 5157.8                   |
| 40  | 1733.5                | 256.50                   | 3318.5                   | 40  | 2295.6                | 442.75                   | 4261.8                   | 40  | 2898.4                | 691.40                   | 5172.7                   |
| 50  | 1742.6                | 259.14                   | 3334.4                   | 50  | 2305.2                | 446.35                   | 4277.3                   | 50  | 2908.9                | 696.13                   | 5187.6                   |
| 34° | 1751.7                | 261.80                   | 3350.4                   | 44° | 2314.9                | 449.98                   | 4292.7                   | 54° | 2919.4                | 700.89                   | 5202.4                   |
| 10  | 1760.8                | 264.47                   | 3366.3                   | 10  | 2324.6                | 453.62                   | 4308.2                   | 10  | 2929.9                | 705.66                   | 5217.3                   |
| 20  | 1770.0                | 267.16                   | 3382.2                   | 20  | 2334.3                | 457.27                   | 4323.6                   | 20  | 2940.4                | 710.46                   | 5232.1                   |
| 30  | 1779.1                | 269.86                   | 3398.2                   | 30  | 2344.1                | 460.95                   | 4339.0                   | 30  | 2951.0                | 715.28                   | 5246.9                   |
| 40  | 1788.2                | 272.58                   | 3414.1                   | 40  | 2353.8                | 464.64                   | 4354.5                   | 40  | 2961.5                | 720.11                   | 5261.7                   |
| 50  | 1797.4                | 275.31                   | 3430.0                   | 50  | 2363.5                | 468.35                   | 4369.9                   | 50  | 2972.1                | 724.97                   | 5276.5                   |
| 35° | 1806.6                | 278.05                   | 3445.9                   | 45° | 2373.3                | 472.08                   | 4385.3                   | 55° | 2982.7                | 729.85                   | 5291.3                   |
| 10  | 1815.7                | 280.82                   | 3461.8                   | 10  | 2383.1                | 475.82                   | 4400.7                   | 10  | 2993.3                | 734.76                   | 5306.1                   |
| 20  | 1824.9                | 283.60                   | 3477.7                   | 20  | 2392.8                | 479.59                   | 4416.1                   | 20  | 3003.9                | 739.68                   | 5320.9                   |
| 30  | 1834.1                | 286.39                   | 3493.5                   | 30  | 2402.6                | 483.37                   | 4431.4                   | 30  | 3014.5                | 744.62                   | 5335.6                   |
| 40  | 1843.3                | 289.20                   | 3509.4                   | 40  | 2412.4                | 487.16                   | 4446.8                   | 40  | 3025.2                | 749.59                   | 5350.4                   |
| 50  | 1852.5                | 292.02                   | 3525.3                   | 50  | 2422.3                | 490.98                   | 4462.2                   | 50  | 3035.8                | 754.57                   | 5365.1                   |
| 36° | 1861.7                | 294.86                   | 3541.1                   | 46° | 2432.1                | 494.82                   | 4477.5                   | 56° | 3046.5                | 759.58                   | 5379.8                   |
| 10  | 1870.9                | 297.72                   | 3557.0                   | 10  | 2441.9                | 498.67                   | 4492.8                   | 10  | 3057.2                | 764.61                   | 5394.5                   |
| 20  | 1880.1                | 300.59                   | 3572.8                   | 20  | 2451.8                | 502.54                   | 4508.2                   | 20  | 3067.9                | 769.66                   | 5409.2                   |
| 30  | 1889.4                | 303.47                   | 3588.6                   | 30  | 2461.7                | 506.42                   | 4523.5                   | 30  | 3078.7                | 774.73                   | 5423.9                   |
| 40  | 1898.6                | 306.37                   | 3604.5                   | 40  | 2471.5                | 510.33                   | 4538.8                   | 40  | 3089.4                | 779.83                   | 5438.6                   |
| 50  | 1907.9                | 309.29                   | 3620.3                   | 50  | 2481.4                | 514.25                   | 4554.1                   | 50  | 3100.2                | 784.94                   | 5453.3                   |
| 37° | 1917.1                | 312.22                   | 3636.1                   | 47° | 2491.3                | 518.20                   | 4569.4                   | 57° | 3110.9                | 790.08                   | 5467.9                   |
| 10  | 1926.4                | 315.17                   | 3651.9                   | 10  | 2501.2                | 522.16                   | 4584.7                   | 10  | 3121.7                | 795.24                   | 5482.5                   |
| 20  | 1935.7                | 318.13                   | 3667.7                   | 20  | 2511.2                | 526.13                   | 4599.9                   | 20  | 3132.6                | 800.42                   | 5497.2                   |
| 30  | 1945.0                | 321.11                   | 3683.5                   | 30  | 2521.1                | 530.13                   | 4615.2                   | 30  | 3143.4                | 805.62                   | 5511.8                   |
| 40  | 1954.3                | 324.11                   | 3699.3                   | 40  | 2531.1                | 534.15                   | 4630.4                   | 40  | 3154.2                | 810.85                   | 5526.4                   |
| 50  | 1963.6                | 327.12                   | 3715.0                   | 50  | 2541.0                | 538.18                   | 4645.7                   | 50  | 3165.1                | 816.10                   | 5541.0                   |
| 38° | 1972.9                | 330.15                   | 3730.8                   | 48° | 2551.0                | 542.23                   | 4660.9                   | 58° | 3176.0                | 821.37                   | 5555.6                   |
| 10  | 1982.2                | 333.19                   | 3746.5                   | 10  | 2561.0                | 546.30                   | 4676.1                   | 10  | 3186.9                | 826.66                   | 5570.2                   |
| 20  | 1991.5                | 336.25                   | 3762.3                   | 20  | 2571.0                | 550.39                   | 4691.3                   | 20  | 3197.8                | 831.98                   | 5584.7                   |
| 30  | 2000.9                | 339.32                   | 3778.0                   | 30  | 2581.0                | 554.50                   | 4706.5                   | 30  | 3208.8                | 837.31                   | 5599.3                   |
| 40  | 2010.2                | 342.41                   | 3793.8                   | 40  | 2591.1                | 558.63                   | 4721.7                   | 40  | 3219.7                | 842.67                   | 5613.8                   |
| 50  | 2019.6                | 345.52                   | 3809.5                   | 50  | 2601.1                | 562.77                   | 4736.9                   | 50  | 3230.7                | 848.06                   | 5628.3                   |
| 39° | 2029.0                | 348.64                   | 3825.2                   | 49° | 2611.2                | 566.94                   | 4752.1                   | 59° | 3241.7                | 853.46                   | 5642.8                   |
| 10  | 2038.4                | 351.78                   | 3840.9                   | 10  | 2621.2                | 571.12                   | 4767.3                   | 10  | 3252.7                | 858.89                   | 5657.3                   |
| 20  | 2047.8                | 354.94                   | 3856.6                   | 20  | 2631.3                | 575.32                   | 4782.4                   | 20  | 3263.7                | 864.34                   | 5671.8                   |
| 30  | 2057.2                | 358.11                   | 3872.3                   | 30  | 2641.4                | 579.54                   | 4797.5                   | 30  | 3274.8                | 869.82                   | 5686.3                   |
| 40  | 2066.6                | 361.29                   | 3888.0                   | 40  | 2651.5                | 583.78                   | 4812.7                   | 40  | 3285.8                | 875.32                   | 5700.8                   |
| 50  | 2076.0                | 364.50                   | 3903.6                   | 50  | 2661.6                | 588.04                   | 4827.8                   | 50  | 3296.9                | 880.84                   | 5715.2                   |
| 40° | 2085.4                | 367.72                   | 3919.3                   | 50° | 2671.8                | 592.32                   | 4842.9                   | 60° | 3308.0                | 886.38                   | 5729.7                   |
| 10  | 2094.9                | 370.95                   | 3935.0                   | 10  | 2681.9                | 596.62                   | 4858.0                   | 10  | 3319.1                | 891.95                   | 5744.1                   |
| 20  | 2104.3                | 374.20                   | 3950.6                   | 20  | 2692.1                | 600.93                   | 4873.1                   | 20  | 3330.3                | 897.54                   | 5758.5                   |
| 30  | 2113.8                | 377.47                   | 3966.3                   | 30  | 2702.3                | 605.27                   | 4888.2                   | 30  | 3341.4                | 903.15                   | 5772.9                   |
| 40  | 2123.3                | 380.76                   | 3981.9                   | 40  | 2712.5                | 609.62                   | 4903.2                   | 40  | 3352.6                | 908.79                   | 5787.3                   |
| 50  | 2132.7                | 384.06                   | 3997.5                   | 50  | 2722.7                | 614.00                   | 4918.3                   | 50  | 3363.8                | 914.45                   | 5801.7                   |
| 41° | 2142.2                | 387.38                   | 4013.1                   | 51° | 2732.9                | 618.39                   | 4933.4                   | 61° | 3375.0                | 920.14                   | 5816.0                   |

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A  
1° CURVE.

| Δ   | Tangent<br>T. | Ext. Dist.<br>E. | Long Ch'd<br>L.C. | Δ   | Tangent<br>T. | Ext. Dist.<br>E. | Long Ch'd<br>L.C. | Δ   | Tangent<br>T. | Ext. Dist.<br>E. | Long Ch'd<br>L.C. |
|-----|---------------|------------------|-------------------|-----|---------------|------------------|-------------------|-----|---------------|------------------|-------------------|
| 61° | 3375.0        | 920.14           | 5816.0            | 71° | 4086.9        | 1308.2           | 6654.4            | 81° | 4893.6        | 1805.3           | 7442.2            |
| 10' | 3386.3        | 925.85           | 5830.4            | 10  | 4099.5        | 1315.5           | 6668.0            | 10  | 4908.0        | 1814.7           | 7454.9            |
| 20  | 3397.5        | 931.58           | 5844.7            | 20  | 4112.1        | 1322.9           | 6681.6            | 20  | 4922.5        | 1824.1           | 7467.5            |
| 30  | 3408.8        | 937.34           | 5859.1            | 30  | 4124.8        | 1330.3           | 6695.1            | 30  | 4937.0        | 1833.6           | 7480.2            |
| 40  | 3420.1        | 943.12           | 5873.4            | 40  | 4137.4        | 1337.7           | 6708.6            | 40  | 4951.5        | 1843.1           | 7492.8            |
| 50  | 3431.4        | 948.92           | 5887.7            | 50  | 4150.1        | 1345.1           | 6722.1            | 50  | 4966.1        | 1852.6           | 7505.4            |
| 62° | 3442.7        | 954.75           | 5902.0            | 72° | 4162.8        | 1352.6           | 6735.6            | 82° | 4980.7        | 1862.2           | 7518.0            |
| 10  | 3454.1        | 960.60           | 5916.3            | 10  | 4175.6        | 1360.1           | 6749.1            | 10  | 4995.4        | 1871.8           | 7530.5            |
| 20  | 3465.4        | 966.48           | 5930.5            | 20  | 4188.4        | 1367.6           | 6762.5            | 20  | 5010.0        | 1881.5           | 7543.1            |
| 30  | 3476.8        | 972.39           | 5944.8            | 30  | 4201.2        | 1375.2           | 6776.0            | 30  | 5024.8        | 1891.2           | 7555.6            |
| 40  | 3488.2        | 978.31           | 5959.0            | 40  | 4214.0        | 1382.8           | 6789.4            | 40  | 5039.5        | 1900.9           | 7568.2            |
| 50  | 3499.7        | 984.27           | 5973.3            | 50  | 4226.8        | 1390.4           | 6802.8            | 50  | 5054.3        | 1910.7           | 7580.7            |
| 63° | 3511.1        | 990.24           | 5987.5            | 73° | 4239.7        | 1398.0           | 6816.3            | 83° | 5069.2        | 1920.5           | 7593.2            |
| 10  | 3522.6        | 996.24           | 6001.7            | 10  | 4252.6        | 1405.7           | 6829.6            | 10  | 5084.0        | 1930.4           | 7605.6            |
| 20  | 3534.1        | 1002.3           | 6015.9            | 20  | 4265.6        | 1413.5           | 6843.0            | 20  | 5099.0        | 1940.3           | 7618.1            |
| 30  | 3545.6        | 1008.3           | 6030.0            | 30  | 4278.5        | 1421.2           | 6856.4            | 30  | 5113.9        | 1950.3           | 7630.5            |
| 40  | 3557.2        | 1014.4           | 6044.2            | 40  | 4291.5        | 1429.0           | 6869.7            | 40  | 5128.9        | 1960.2           | 7643.0            |
| 50  | 3568.7        | 1020.5           | 6058.4            | 50  | 4304.6        | 1436.8           | 6883.1            | 50  | 5143.9        | 1970.3           | 7655.4            |
| 64° | 3580.3        | 1026.6           | 6072.5            | 74° | 4317.6        | 1444.6           | 6896.4            | 84° | 5159.0        | 1980.4           | 7667.8            |
| 10  | 3591.9        | 1032.8           | 6086.6            | 10  | 4330.7        | 1452.5           | 6909.7            | 10  | 5174.1        | 1990.5           | 7680.1            |
| 20  | 3603.5        | 1039.0           | 6100.7            | 20  | 4343.8        | 1460.4           | 6923.0            | 20  | 5189.3        | 2000.6           | 7692.5            |
| 30  | 3615.1        | 1045.2           | 6114.8            | 30  | 4356.9        | 1468.4           | 6936.2            | 30  | 5204.4        | 2010.8           | 7704.9            |
| 40  | 3626.8        | 1051.4           | 6128.9            | 40  | 4370.1        | 1476.4           | 6949.5            | 40  | 5219.7        | 2021.1           | 7717.2            |
| 50  | 3638.5        | 1057.7           | 6143.0            | 50  | 4383.3        | 1484.4           | 6962.8            | 50  | 5234.9        | 2031.4           | 7729.5            |
| 65° | 3650.2        | 1063.9           | 6157.1            | 75° | 4396.5        | 1492.4           | 6976.0            | 85° | 5250.3        | 2041.7           | 7741.8            |
| 10  | 3661.9        | 1070.2           | 6171.1            | 10  | 4409.8        | 1500.5           | 6989.2            | 10  | 5265.6        | 2052.1           | 7754.1            |
| 20  | 3673.7        | 1076.6           | 6185.2            | 20  | 4423.1        | 1508.6           | 7002.4            | 20  | 5281.0        | 2062.5           | 7766.3            |
| 30  | 3685.4        | 1082.9           | 6199.2            | 30  | 4436.4        | 1516.7           | 7015.6            | 30  | 5296.4        | 2073.0           | 7778.6            |
| 40  | 3697.2        | 1089.3           | 6213.2            | 40  | 4449.7        | 1524.9           | 7028.8            | 40  | 5311.9        | 2083.5           | 7790.8            |
| 50  | 3709.0        | 1095.7           | 6227.2            | 50  | 4463.1        | 1533.1           | 7041.9            | 50  | 5327.4        | 2094.1           | 7803.0            |
| 66° | 3720.9        | 1102.2           | 6241.2            | 76° | 4476.5        | 1541.4           | 7055.0            | 86° | 5343.0        | 2104.7           | 7815.2            |
| 10  | 3732.7        | 1108.6           | 6255.2            | 10  | 4489.9        | 1549.7           | 7068.2            | 10  | 5358.6        | 2115.3           | 7827.4            |
| 20  | 3744.6        | 1115.1           | 6269.1            | 20  | 4503.4        | 1558.0           | 7081.3            | 20  | 5374.2        | 2126.0           | 7839.6            |
| 30  | 3756.5        | 1121.7           | 6283.1            | 30  | 4516.9        | 1566.3           | 7094.4            | 30  | 5389.9        | 2136.7           | 7851.7            |
| 40  | 3768.5        | 1128.2           | 6297.0            | 40  | 4530.4        | 1574.7           | 7107.5            | 40  | 5405.6        | 2147.5           | 7863.8            |
| 50  | 3780.4        | 1134.8           | 6310.9            | 50  | 4544.0        | 1583.1           | 7120.5            | 50  | 5421.4        | 2158.4           | 7876.0            |
| 67° | 3792.4        | 1141.4           | 6324.8            | 77° | 4557.6        | 1591.6           | 7133.6            | 87° | 5437.2        | 2169.2           | 7888.1            |
| 10  | 3804.4        | 1148.0           | 6338.7            | 10  | 4571.2        | 1600.1           | 7146.6            | 10  | 5453.1        | 2180.2           | 7900.1            |
| 20  | 3816.4        | 1154.7           | 6352.6            | 20  | 4584.8        | 1608.6           | 7159.6            | 20  | 5469.0        | 2191.1           | 7912.2            |
| 30  | 3828.4        | 1161.3           | 6366.4            | 30  | 4598.5        | 1617.1           | 7172.6            | 30  | 5484.9        | 2202.2           | 7924.3            |
| 40  | 3840.5        | 1168.1           | 6380.3            | 40  | 4612.2        | 1625.7           | 7185.6            | 40  | 5500.9        | 2213.2           | 7936.3            |
| 50  | 3852.6        | 1174.8           | 6394.1            | 50  | 4626.0        | 1634.4           | 7198.6            | 50  | 5517.0        | 2224.3           | 7948.3            |
| 68° | 3864.7        | 1181.6           | 6408.0            | 78° | 4639.8        | 1643.0           | 7211.6            | 88° | 5533.1        | 2235.5           | 7960.3            |
| 10  | 3876.8        | 1188.4           | 6421.8            | 10  | 4653.6        | 1651.7           | 7224.5            | 10  | 5549.2        | 2246.7           | 7972.3            |
| 20  | 3889.0        | 1195.2           | 6435.6            | 20  | 4667.4        | 1660.5           | 7237.4            | 20  | 5565.4        | 2258.0           | 7984.2            |
| 30  | 3901.2        | 1202.0           | 6449.4            | 30  | 4681.3        | 1669.2           | 7250.4            | 30  | 5581.6        | 2269.3           | 7996.2            |
| 40  | 3913.4        | 1208.9           | 6463.1            | 40  | 4695.2        | 1678.1           | 7263.3            | 40  | 5597.8        | 2280.6           | 8008.1            |
| 50  | 3925.6        | 1215.8           | 6476.9            | 50  | 4709.2        | 1686.9           | 7276.1            | 50  | 5614.2        | 2292.0           | 8020.0            |
| 69° | 3937.9        | 1222.7           | 6490.6            | 79° | 4723.2        | 1695.8           | 7289.0            | 89° | 5630.5        | 2303.5           | 8031.9            |
| 10  | 3950.2        | 1229.7           | 6504.4            | 10  | 4737.2        | 1704.7           | 7301.9            | 10  | 5646.9        | 2315.0           | 8043.8            |
| 20  | 3962.5        | 1236.7           | 6518.1            | 20  | 4751.2        | 1713.7           | 7314.7            | 20  | 5663.4        | 2326.6           | 8055.7            |
| 30  | 3974.8        | 1243.7           | 6531.8            | 30  | 4765.3        | 1722.7           | 7327.5            | 30  | 5679.9        | 2338.2           | 8067.5            |
| 40  | 3987.2        | 1250.8           | 6545.5            | 40  | 4779.4        | 1731.7           | 7340.3            | 40  | 5696.4        | 2349.8           | 8079.3            |
| 50  | 3999.5        | 1257.9           | 6559.1            | 50  | 4793.6        | 1740.8           | 7353.1            | 50  | 5713.0        | 2361.5           | 8091.2            |
| 70° | 4011.9        | 1265.0           | 6572.8            | 80° | 4808.7        | 1749.9           | 7365.9            | 90° | 5729.7        | 2373.3           | 8103.0            |
| 10  | 4024.4        | 1272.1           | 6586.4            | 10  | 4822.0        | 1759.0           | 7378.7            | 10  | 5746.3        | 2385.1           | 8114.7            |
| 20  | 4036.8        | 1279.3           | 6600.1            | 20  | 4836.2        | 1768.2           | 7391.4            | 20  | 5763.1        | 2397.0           | 8126.5            |
| 30  | 4049.3        | 1286.5           | 6613.7            | 30  | 4850.5        | 1777.4           | 7404.1            | 30  | 5779.9        | 2408.9           | 8138.2            |
| 40  | 4061.8        | 1293.7           | 6627.3            | 40  | 4864.8        | 1786.7           | 7416.8            | 40  | 5796.7        | 2420.9           | 8150.0            |
| 50  | 4074.4        | 1300.9           | 6640.9            | 50  | 4879.2        | 1796.0           | 7429.5            | 50  | 5813.6        | 2432.9           | 8161.7            |
| 71° | 4086.9        | 1308.2           | 6654.4            | 81° | 4893.6        | 1805.3           | 7442.2            | 91° | 5830.5        | 2444.9           | 8173.4            |



TABLE III.—SWITCH LEADS AND DISTANCES.

LEAD-RAILS CIRCULAR THROUGHOUT; GAUGE 4' 8½". See § 262.

| Frog Number (n). | Frog Angle (F). | Lead (L) (Eq. 79). | Chord (QT) (Eq. 77). | Radius of Lead Rails (r, Eq. 78). | Log r.  | Degree of Curve (d'). | Frog Number (n). |
|------------------|-----------------|--------------------|----------------------|-----------------------------------|---------|-----------------------|------------------|
| 4                | 14° 15' 00"     | 37.67              | 37.38                | 150.67                            | 2.17801 | 38° 46'               | 4                |
| 4.5              | 12 40 59        | 42.37              | 42.12                | 190.69                            | .28032  | 30 24                 | 4.5              |
| 5                | 11 25 16        | 47.08              | 46.85                | 235.42                            | .37183  | 24 32                 | 5                |
| 5.5              | 10 23 20        | 51.79              | 51.58                | 284.85                            | .45462  | 20 13                 | 5.5              |
| 6                | 9 31 38         | 56.50              | 56.30                | 339.00                            | .53020  | 16 58                 | 6                |
| 6.5              | 8 47 51         | 61.21              | 61.03                | 397.85                            | .59972  | 14 26                 | 6.5              |
| 7                | 8 10 16         | 65.92              | 65.75                | 461.42                            | .66409  | 12 26                 | 7                |
| 7.5              | 7 37 41         | 70.62              | 70.47                | 529.69                            | .72402  | 10 50                 | 7.5              |
| 8                | 7 09 10         | 75.33              | 75.19                | 602.67                            | .78007  | 9 31                  | 8                |
| 8.5              | 6 43 59         | 80.04              | 79.90                | 680.36                            | .83273  | 8 26                  | 8.5              |
| 9                | 6 21 35         | 84.75              | 84.62                | 762.75                            | .88238  | 7 31                  | 9                |
| 9.5              | 6 01 32         | 89.46              | 89.33                | 849.85                            | .92934  | 6 45                  | 9.5              |
| 10               | 5 43 29         | 94.17              | 94.05                | 941.67                            | 2.97389 | 6 05                  | 10               |
| 10.5             | 5 27 09         | 98.87              | 98.76                | 1038.19                           | 3.01627 | 5 32                  | 10.5             |
| 11               | 5 12 18         | 103.58             | 103.47               | 1139.42                           | .05668  | 5 02                  | 11               |
| 11.5             | 4 58 45         | 108.29             | 108.19               | 1245.36                           | .09529  | 4 36                  | 11.5             |
| 12               | 4 46 19         | 113.00             | 112.90               | 1356.00                           | 3.13226 | 4 14                  | 12               |

TURNOUTS WITH STRAIGHT POINT-RAILS AND STRAIGHT FROG-RAILS; GAUGE 4' 8½". See § 265.

| Frog Number (n). | Switch Point Angle (a). | Length of Switch Point (D.V.). | Length of Straight Frog-rail (f). | Lead (L) (Eq. 90). | Chord (ST) (Eq. 88). | Radius of Lead-rails (r, Eq. 87). | Log r.  | Degree of Curve (d'). | Frog Number (n). |
|------------------|-------------------------|--------------------------------|-----------------------------------|--------------------|----------------------|-----------------------------------|---------|-----------------------|------------------|
| 4                | 3° 40'                  | 7.5                            | 1.50                              | 32.20              | 23.09                | 125.21                            | 2.09764 | 47° 05'               | 4                |
| 4.5              | 3 40                    | 7.5                            | 1.69                              | 34.29              | 25.03                | 159.25                            | .20208  | 36 36                 | 4.5              |
| 5                | 2 45                    | 10.0                           | 1.87                              | 41.85              | 29.88                | 197.65                            | .29589  | 29 22                 | 5                |
| 5.5              | 2 45                    | 10.0                           | 2.06                              | 44.16              | 32.03                | 240.44                            | .38100  | 24 00                 | 5.5              |
| 6                | 1 50                    | 15.0                           | 2.25                              | 56.00              | 38.66                | 288.09                            | .45953  | 19 59                 | 6                |
| 6.5              | 1 50                    | 15.0                           | 2.44                              | 58.84              | 41.34                | 340.19                            | .53172  | 16 54                 | 6.5              |
| 7                | 1 50                    | 15.0                           | 2.62                              | 61.65              | 43.98                | 397.65                            | .59950  | 14 27                 | 7                |
| 7.5              | 1 50                    | 15.0                           | 2.81                              | 64.36              | 46.50                | 460.00                            | .66276  | 12 29                 | 7.5              |
| 8                | 1 50                    | 15.0                           | 3.00                              | 67.04              | 48.99                | 527.91                            | .72256  | 10 52                 | 8                |
| 8.5              | 1 50                    | 15.0                           | 3.19                              | 69.60              | 51.38                | 600.94                            | .77883  | 9 33                  | 8.5              |
| 9                | 1 50                    | 15.0                           | 3.37                              | 72.20              | 53.80                | 681.16                            | .83325  | 8 25                  | 9                |
| 9.5              | 1 50                    | 15.0                           | 3.56                              | 74.70              | 56.11                | 767.11                            | .88486  | 7 28                  | 9.5              |
| 10               | 1 50                    | 15.0                           | 3.75                              | 77.04              | 58.28                | 858.14                            | .93356  | 6 41                  | 10               |
| 10.5             | 1 50                    | 15.0                           | 3.94                              | 79.51              | 60.57                | 959.00                            | 2.98182 | 5 59                  | 10.5             |
| 11               | 1 50                    | 15.0                           | 4.12                              | 81.82              | 62.69                | 1065.52                           | 3.02756 | 5 23                  | 11               |
| 11.5             | 1 50                    | 15.0                           | 4.31                              | 84.09              | 64.78                | 1180.16                           | 3.07194 | 4 51                  | 11.5             |
| 12               | 1 50                    | 15.0                           | 4.50                              | 86.16              | 66.67                | 1299.93                           | 3.11392 | 4 24                  | 12               |

TRIGONOMETRICAL FUNCTIONS OF THE FROG ANGLES (F).

| Frog Number (n). | Frog Angle (F). | Nat. sin F. | Nat. cos F. | Log sin F. | Log cos F. | Log cot F. | Log vers F. | Frog Number (n). |
|------------------|-----------------|-------------|-------------|------------|------------|------------|-------------|------------------|
| 4                | 14° 15' 00"     | .24615      | .96923      | 9.39120    | 9.98642    | 10.59522   | 8.48811     | 4                |
| 4.5              | 12 40 49        | .21951      | .97561      | .34145     | .98927     | .64782     | .38721      | 4.5              |
| 5                | 11 25 16        | .19802      | .98020      | .29670     | .99131     | .69461     | .29670      | 5                |
| 5.5              | 10 23 20        | .18033      | .98360      | .25606     | .99282     | .73675     | .21467      | 5.5              |
| 6                | 9 31 38         | .16552      | .98621      | .21884     | .99397     | .77513     | .13966      | 6                |
| 6.5              | 8 47 51         | .15294      | .98823      | .18453     | .99486     | .81033     | .07058      | 6.5              |
| 7                | 8 10 16         | .14213      | .98985      | .15268     | .99557     | .84288     | 8.00655     | 7                |
| 7.5              | 7 37 41         | .13274      | .99115      | .12301     | .99614     | .87313     | 7.94691     | 7.5              |
| 8                | 7 09 10         | .12452      | .99222      | .09522     | .99660     | .90138     | .89110      | 8                |
| 8.5              | 6 43 59         | .11724      | .99310      | .06909     | .99699     | .92790     | .83864      | 8.5              |
| 9                | 6 21 35         | .11077      | .99385      | .04442     | .99732     | .95289     | .78915      | 9                |
| 9.5              | 6 01 32         | .10497      | .99448      | 9.02107    | .99759     | .97652     | .74232      | 9.5              |
| 10               | 5 43 29         | .09975      | .99501      | 8.99891    | .99783     | 10.99892   | .69788      | 10               |
| 10.5             | 5 27 09         | .09502      | .99548      | .97781     | .99803     | 11.02021   | .65560      | 10.5             |
| 11               | 5 12 18         | .09072      | .99588      | .95770     | .99820     | .04050     | .61528      | 11               |
| 11.5             | 4 58 45         | .08679      | .99623      | .93848     | .99836     | .05987     | .57676      | 11.5             |
| 12               | 4 46 19         | .08319      | .99653      | 8.92007    | 9.98849    | 11.07842   | 7.53986     | 12               |

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

| Point. | Total<br>Central Angle<br>$\phi$ | Nat. sin $\phi$ | Nat. cos $\phi$ | Log sin $\phi$ | Log cos $\phi$ | Log vers $\phi$ | $x$    | Log $x$ | $y$     |
|--------|----------------------------------|-----------------|-----------------|----------------|----------------|-----------------|--------|---------|---------|
| 1      | 0° 07' 30"                       | .0022           | one             | 7.33878        | 10.00000       | 4.37654         | 0.027  | 8.43568 | 25.000  |
| 2      | 0 22 30                          | .0065           | one             | 7.81590        | 9.99999        | 5.33078         | 0.136  | 9.13465 | 50.000  |
| 3      | 0 45 00                          | .0131           | .9999           | 8.11692        | 9.99996        | 5.93284         | 0.382  | 9.58181 | 74.999  |
| 4      | 1 15 00                          | .0218           | .9997           | 8.33875        | 9.99989        | 6.37653         | 0.818  | 9.91280 | 99.995  |
| 5      | 1 52 30                          | .0327           | .9994           | 8.51480        | 9.99976        | 6.72869         | 1.500  | 0.17602 | 124.985 |
| 6      | 2 37 30                          | .0458           | .9989           | 8.66085        | 9.99954        | 7.02091         | 2.481  | 0.39467 | 149.966 |
| 7      | 3 30 00                          | .0610           | .9981           | 8.78567        | 9.99919        | 7.27072         | 3.817  | 0.58171 | 174.930 |
| 8      | 4 30 00                          | .0784           | .9969           | 8.89464        | 9.99866        | 7.48892         | 5.561  | 0.74514 | 199.870 |
| 9      | 5 37 30                          | .0980           | .9952           | 8.99130        | 9.99790        | 7.68262         | 7.792  | 0.89164 | 224.772 |
| 10     | 6 52 30                          | .1197           | .9928           | 9.07810        | 9.99686        | 7.85675         | 10.489 | 1.02061 | 249.623 |

0° 30'-per-25-foot spiral.

| Deflections from the tangent at the point occupied when the instrument is at--- |           |            |            |            |            |            |            |            |            |            |
|---------------------------------------------------------------------------------|-----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Q                                                                               | 1         | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          | 10         |
| 0° 0' 0"                                                                        | 0° 3' 45" | 0° 13' 07" | 0° 27' 30" | 0° 46' 52" | 1° 11' 15" | 1° 40' 37" | 2° 15' 00" | 2° 54' 22" | 3° 38' 45" | 4° 28' 10" |
| 0 3 45                                                                          | 0 0 00    | 0 07 30    | 0 20 37    | 0 38 45    | 1 01 52    | 1 30 00    | 2 03 07    | 2 41 15    | 3 24 22    | 4 12 30    |
| 0 9 22                                                                          | 0 7 30    | 0 00 00    | 0 11 15    | 0 28 07    | 0 50 00    | 1 16 52    | 1 48 45    | 2 25 37    | 3 07 30    | 3 54 22    |
| 0 17 30                                                                         | 0 16 52   | 0 11 15    | 0 00 00    | 0 15 00    | 0 35 37    | 1 01 15    | 1 31 52    | 2 07 30    | 2 48 07    | 3 33 45    |
| 0 28 07                                                                         | 0 28 45   | 0 24 22    | 0 15 00    | 0 00 00    | 0 18 45    | 0 43 07    | 1 12 30    | 1 46 52    | 2 26 15    | 3 10 37    |
| 0 41 15                                                                         | 0 43 07   | 0 40 00    | 0 31 52    | 0 18 45    | 0 00 00    | 0 22 30    | 0 50 37    | 1 23 45    | 2 01 52    | 2 45 00    |
| 0 56 52                                                                         | 1 00 00   | 0 58 07    | 0 51 15    | 0 39 22    | 0 22 30    | 0 00 00    | 0 26 15    | 0 58 07    | 1 35 00    | 2 16 52    |
| 1 15 00                                                                         | 1 19 22   | 1 18 45    | 1 13 07    | 1 02 30    | 0 46 52    | 0 26 15    | 0 00 00    | 0 30 00    | 1 05 37    | 1 46 15    |
| 1 35 37                                                                         | 1 41 15   | 1 41 52    | 1 37 30    | 1 28 07    | 1 13 45    | 0 54 22    | 0 30 00    | 0 00 00    | 0 33 45    | 1 13 07    |
| 1 58 45                                                                         | 2 05 37   | 2 07 30    | 2 04 22    | 1 56 15    | 1 43 07    | 1 25 00    | 1 01 52    | 0 33 45    | 0 00 00    | 0 37 30    |
| 2 24 20                                                                         | 2 32 30   | 2 35 37    | 2 33 45    | 2 26 52    | 2 15 00    | 1 58 07    | 1 36 15    | 1 09 22    | 0 37 30    | 0 00 00    |

|             |   |   |   |   |   |   |   |   |   |    |
|-------------|---|---|---|---|---|---|---|---|---|----|
| Q           | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| sighting at |   |   |   |   |   |   |   |   |   |    |

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

| Point. | Total<br>Central Angle<br>$\phi$ | Nat. sin $\phi$ | Nat. cos $\phi$ | Log sin $\phi$ | Log cos $\phi$ | Log vers $\phi$ | $x$    | Log $x$ | $y$     |
|--------|----------------------------------|-----------------|-----------------|----------------|----------------|-----------------|--------|---------|---------|
| 1      | 0° 15'                           | .0043           | one             | 7.6398f        | 9.99999        | 4.97860         | .055   | 8.73672 | 25.000  |
| 2      | 0 45                             | .0131           | .9999           | 8.11692        | 9.99996        | 5.93284         | .273   | 9.43616 | 49.999  |
| 3      | 1 30                             | .0262           | .9996           | 8.41792        | 9.99985        | 6.53488         | .763   | 9.88252 | 74.994  |
| 4      | 2 30                             | .0436           | .9990           | 8.63968        | 9.99958        | 6.97853         | 1.636  | 0.21378 | 99.979  |
| 5      | 3 45                             | .0654           | .9978           | 8.81560        | 9.99907        | 7.33063         | 2.999  | 0.47697 | 124.942 |
| 6      | 5 15                             | .0915           | .9958           | 8.96143        | 9.99817        | 7.62274         | 4.960  | 0.69548 | 149.865 |
| 7      | 7 00                             | .1218           | .9923           | 9.08589        | 9.99675        | 7.87238         | 7.628  | 0.88241 | 174.722 |
| 8      | 9 00                             | .1564           | .9877           | 9.19433        | 9.99462        | 8.0903f         | 11.107 | 1.04559 | 199.479 |
| 9      | 11 15                            | .1951           | .9808           | 9.29023        | 9.99157        | 8.28363         | 15.502 | 1.19039 | 224.090 |
| 10     | 13 45                            | .2377           | .9713           | 9.37600        | 9.98737        | 8.45724         | 20.913 | 1.3204f | 248.497 |

1°-per-25-foot spiral.

| — sighting<br>at — |          | Deflections from the tangent at the point occupied when the instrument is at— |            |           |            |            |            |            |            |            |            |
|--------------------|----------|-------------------------------------------------------------------------------|------------|-----------|------------|------------|------------|------------|------------|------------|------------|
| Q                  | Q        | 1                                                                             | 2          | 3         | 4          | 5          | 6          | 7          | 8          | 9          | 10         |
| 0° 0' 0"           | 0° 0' 0" | 0° 07' 30"                                                                    | 0° 26' 15" | 0° 55' 0" | 1° 33' 45" | 2° 22' 30" | 3° 21' 15" | 4° 30' 00" | 5° 48' 47" | 7° 17' 34" | 8° 56' 22" |
| 0 07 30            | 0 18 45  | 0 00 00                                                                       | 0 15 00    | 0 41 15   | 1 17 30    | 2 03 45    | 3 00 00    | 4 06 15    | 5 22 30    | 6 48 47    | 8 25 05    |
| 0 18 45            | 0 35 00  | 0 15 00                                                                       | 0 00 00    | 0 22 30   | 0 56 15    | 1 40 00    | 2 33 45    | 3 37 30    | 4 51 15    | 6 15 02    | 7 48 49    |
| 0 35 00            | 0 56 15  | 0 33 45                                                                       | 0 22 30    | 0 00 00   | 0 30 00    | 1 11 15    | 2 02 30    | 3 03 45    | 4 15 00    | 5 36 15    | 7 07 32    |
| 0 56 15            | 1 22 30  | 0 57 30                                                                       | 0 48 45    | 0 30 00   | 0 00 00    | 0 37 30    | 1 26 15    | 2 25 00    | 3 33 45    | 4 52 30    | 6 21 17    |
| 1 22 30            | 1 53 45  | 1 26 15                                                                       | 1 20 00    | 1 03 45   | 0 37 30    | 0 00 00    | 0 45 00    | 1 41 15    | 2 47 30    | 4 03 45    | 5 30 00    |
| 1 53 45            | 2 30 00  | 2 00 00                                                                       | 1 56 15    | 1 42 30   | 1 18 45    | 0 45 00    | 0 00 00    | 0 52 30    | 1 56 15    | 3 10 00    | 4 33 45    |
| 2 30 00            | 3 11 13  | 2 38 45                                                                       | 2 37 30    | 2 26 15   | 2 05 00    | 1 33 45    | 0 52 30    | 0 00 00    | 1 00 00    | 2 11 15    | 3 32 30    |
| 3 11 13            | 3 57 26  | 3 22 30                                                                       | 3 23 45    | 3 15 00   | 2 56 15    | 2 27 30    | 1 48 45    | 1 00 00    | 0 00 00    | 1 07 30    | 2 26 15    |
| 3 57 26            | 4 48 38  | 4 11 13                                                                       | 4 14 58    | 4 08 45   | 3 52 30    | 3 26 15    | 2 50 00    | 2 03 45    | 1 07 30    | 0 00 00    | 1 15 00    |
| 4 48 38            | 5 04 55  | 5 04 55                                                                       | 5 11 11    | 5 07 28   | 4 53 43    | 4 30 00    | 3 56 15    | 3 12 30    | 2 18 45    | 1 15 00    | 0 00 00    |

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

| Point. | Total<br>Central Angle<br>φ | Nat. sin φ | Nat. cos φ | Log sin φ | Log cos φ | Log vers φ | x      | Log x   | y       |
|--------|-----------------------------|------------|------------|-----------|-----------|------------|--------|---------|---------|
| 1      | 0° 30'                      | .0087      | .9999      | 7.94084   | 9.99998   | 5.58066    | 0.109  | 9.03774 | 25.000  |
| 2      | 1 30                        | .0262      | .9996      | 8.41792   | 9.99985   | 6.53488    | 0.545  | 9.73670 | 49.996  |
| 3      | 3 00                        | .0523      | .9986      | 8.71880   | 9.99946   | 7.13687    | 1.527  | 0.18388 | 74.977  |
| 4      | 5 00                        | .0871      | .9962      | 8.94029   | 9.99834   | 7.58039    | 3.271  | 0.51468 | 99.916  |
| 5      | 7 30                        | .1303      | .9914      | 9.11570   | 9.99627   | 7.93222    | 5.992  | 0.77762 | 124.767 |
| 6      | 10 30                       | .1822      | .9832      | 9.26063   | 9.99266   | 8.22389    | 9.903  | 0.99579 | 149.459 |
| 7      | 14 00                       | .2419      | .9703      | 9.38367   | 9.98690   | 8.47282    | 15.208 | 1.18207 | 173.890 |
| 8      | 18 00                       | .3090      | .9510      | 9.48998   | 9.97820   | 8.68969    | 22.099 | 1.34437 | 197.922 |
| 9      | 22 30                       | .3827      | .9239      | 9.58284   | 9.96561   | 8.88150    | 30.752 | 1.48787 | 221.376 |
| 10     | 27 30                       | .4617      | .8870      | 9.66440   | 9.94793   | 9.05303    | 41.317 | 1.61613 | 244.034 |

2°-per-25-foot spiral.

Deflections from the tangent at the point occupied when the instrument is at—

| sighting<br>at |            | Q          | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8           | 9           | 10          |
|----------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|-------------|-------------|
| Q              | 0° 00' 00" | 0° 00' 00" | 0° 15' 00" | 0° 52' 30" | 1° 50' 00" | 3° 07' 30" | 4° 45' 00" | 6° 42' 30" | 9° 00' 05" | 11° 37' 45" | 14° 35' 30" | 17° 53' 25" |
| 1              | 0 15 00    | 0 00 00    | 0 30 00    | 0 30 00    | 1 22 30    | 2 35 00    | 4 07 30    | 6 00 01    | 8 12 34    | 10 45 10    | 13 37 52    | 16 50 42    |
| 2              | 0 37 30    | 0 30 00    | 0 00 00    | 0 00 00    | 0 45 00    | 1 52 30    | 3 20 00    | 5 07 30    | 7 15 02    | 9 42 36     | 12 30 15    | 15 38 00    |
| 3              | 1 10 00    | 1 07 30    | 0 45 00    | 0 45 00    | 0 00 00    | 1 00 00    | 2 22 30    | 4 05 00    | 6 07 30    | 8 30 04     | 11 12 39    | 14 15 20    |
| 4              | 1 52 30    | 1 55 00    | 1 37 30    | 1 37 30    | 1 00 00    | 0 00 00    | 1 15 00    | 2 52 30    | 4 50 00    | 7 07 30     | 9 45 05     | 12 42 42    |
| 5              | 2 45 00    | 2 52 30    | 2 40 00    | 2 40 00    | 2 07 30    | 1 15 00    | 0 00 00    | 1 30 00    | 3 22 30    | 5 35 00     | 8 07 30     | 11 00 06    |
| 6              | 3 47 28    | 3 59 59    | 3 52 30    | 3 52 30    | 3 25 00    | 2 37 30    | 1 30 00    | 0 00 00    | 1 45 00    | 3 52 30     | 6 20 00     | 9 07 30     |
| 7              | 4 59 53    | 5 17 26    | 5 14 58    | 5 14 58    | 4 52 30    | 4 10 00    | 3 07 30    | 1 45 00    | 0 00 00    | 2 00 00     | 4 22 30     | 7 05 00     |
| 8              | 6 22 15    | 6 44 50    | 6 47 24    | 6 47 24    | 6 29 56    | 5 52 30    | 4 55 00    | 3 37 30    | 2 00 00    | 0 00 00     | 2 15 00     | 4 52 30     |
| 9              | 7 54 30    | 8 22 08    | 8 29 45    | 8 29 45    | 8 17 21    | 7 44 55    | 6 52 30    | 5 40 00    | 4 07 30    | 2 15 00     | 0 00 00     | 2 30 00     |
| 10             | 9 36 35    | 10 09 17   | 10 22 00   | 10 22 00   | 10 14 40   | 9 47 18    | 8 59 54    | 7 52 30    | 6 25 00    | 4 37 30     | 2 30 00     | 0 00 00     |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.       |      |      |      |
|-----|--------|------|------|------|------|------|------|------|------|------|-------------|------|------|------|
| 100 | 00 000 | 043  | 087  | 130  | 173  | 216  | 260  | 303  | 346  | 389  |             |      |      |      |
| 101 | 432    | 475  | 518  | 561  | 604  | 646  | 689  | 732  | 775  | 817  | 43 43 42 41 |      |      |      |
| 102 | 860    | 902  | 945  | 987  | *030 | *072 | *114 | *157 | *199 | *241 | .1 4.3      | 4.3  | 4.2  | 4.1  |
| 103 | 01 283 | 326  | 368  | 410  | 452  | 494  | 536  | 578  | 619  | 661  | .2 8.7      | 8.6  | 8.4  | 8.2  |
| 104 | 703    | 745  | 787  | 828  | 870  | 911  | 953  | 994  | *036 | *077 | .3 13.0     | 12.9 | 12.6 | 12.3 |
| 105 | 02 119 | 160  | 201  | 243  | 284  | 325  | 366  | 407  | 448  | 489  | .4 17.4     | 17.2 | 16.8 | 16.4 |
| 106 | 530    | 571  | 612  | 653  | 694  | 735  | 775  | 816  | 857  | 898  | .5 21.7     | 21.5 | 21.0 | 20.5 |
| 107 | 938    | 979  | *019 | *060 | *100 | *141 | *181 | *221 | *262 | *302 | .6 26.1     | 25.8 | 25.2 | 24.6 |
| 108 | 03 342 | 382  | 422  | 463  | 503  | 543  | 583  | 623  | 663  | 703  | .7 30.4     | 30.1 | 29.4 | 28.7 |
| 109 | 742    | 782  | 822  | 862  | 901  | 941  | 981  | *020 | *060 | *100 | .8 34.8     | 34.4 | 33.6 | 32.8 |
| 110 | 04 139 | 178  | 218  | 257  | 297  | 336  | 375  | 415  | 454  | 493  | .9 39.1     | 38.7 | 37.8 | 36.9 |
| 111 | 532    | 571  | 610  | 649  | 688  | 727  | 766  | 805  | 844  | 883  | 40 40 39 38 |      |      |      |
| 112 | 922    | 960  | 999  | *038 | *076 | *115 | *154 | *192 | *231 | *269 | .1 4.0      | 4.0  | 3.9  | 3.8  |
| 113 | 05 308 | 346  | 384  | 423  | 461  | 499  | 538  | 576  | 614  | 652  | .2 8.1      | 8.0  | 7.8  | 7.6  |
| 114 | 690    | 728  | 766  | 804  | 842  | 880  | 918  | 956  | 994  | *032 | .3 12.1     | 12.0 | 11.7 | 11.4 |
| 115 | 06 070 | 107  | 145  | 183  | 220  | 258  | 296  | 333  | 371  | 408  | .4 16.2     | 16.0 | 15.6 | 15.2 |
| 116 | 446    | 483  | 520  | 558  | 595  | 632  | 670  | 707  | 744  | 781  | .5 20.2     | 20.0 | 19.5 | 19.0 |
| 117 | 818    | 855  | 893  | 930  | 967  | *004 | *040 | *077 | *114 | *151 | .6 24.3     | 24.0 | 23.4 | 22.8 |
| 118 | 07 188 | 225  | 261  | 298  | 335  | 372  | 408  | 445  | 481  | 518  | .7 28.3     | 28.0 | 27.3 | 26.6 |
| 119 | 554    | 591  | 627  | 664  | 700  | 737  | 773  | 809  | 845  | 882  | .8 32.4     | 32.0 | 31.2 | 30.4 |
| 120 | 918    | 954  | 990  | *026 | *062 | *098 | *134 | *170 | *206 | *242 | .9 36.4     | 36.0 | 35.1 | 34.2 |
| 121 | 08 278 | 314  | 350  | 386  | 422  | 457  | 493  | 529  | 564  | 600  | 37 37 36 35 |      |      |      |
| 122 | 636    | 671  | 707  | 742  | 778  | 813  | 849  | 884  | 920  | 955  | .1 3.7      | 3.7  | 3.6  | 3.5  |
| 123 | 990    | *026 | *061 | *096 | *131 | *166 | *202 | *237 | *272 | *307 | .2 7.5      | 7.4  | 7.2  | 7.0  |
| 124 | 09 342 | 377  | 412  | 447  | 482  | 517  | 552  | 586  | 621  | 656  | .3 11.2     | 11.1 | 10.8 | 10.5 |
| 125 | 691    | 725  | 760  | 795  | 830  | 864  | 899  | 933  | 968  | *002 | .4 15.0     | 14.8 | 14.4 | 14.0 |
| 126 | 10 037 | 071  | 106  | 140  | 174  | 209  | 243  | 277  | 312  | 346  | .5 18.7     | 18.5 | 18.0 | 17.5 |
| 127 | 380    | 414  | 448  | 483  | 517  | 551  | 585  | 619  | 653  | 687  | .6 22.5     | 22.2 | 21.6 | 21.0 |
| 128 | 721    | 755  | 789  | 822  | 856  | 890  | 924  | 958  | 991  | *025 | .7 26.2     | 25.9 | 25.2 | 24.5 |
| 129 | 11 059 | 092  | 126  | 160  | 193  | 227  | 260  | 294  | 327  | 361  | .8 30.0     | 29.6 | 28.8 | 28.0 |
| 130 | 394    | 427  | 461  | 494  | 528  | 561  | 594  | 627  | 661  | 694  | .9 33.7     | 33.3 | 32.4 | 31.5 |
| 131 | 727    | 760  | 793  | 826  | 859  | 892  | 925  | 958  | 991  | *024 | 34 34 33 32 |      |      |      |
| 132 | 12 057 | 090  | 123  | 156  | 189  | 221  | 254  | 287  | 320  | 352  | .1 3.4      | 3.4  | 3.3  | 3.2  |
| 133 | 385    | 418  | 450  | 483  | 515  | 548  | 580  | 613  | 645  | 678  | .2 6.9      | 6.8  | 6.6  | 6.4  |
| 134 | 710    | 743  | 775  | 807  | 840  | 872  | 904  | 937  | 969  | *001 | .3 10.3     | 10.2 | 9.9  | 9.6  |
| 135 | 13 033 | 065  | 097  | 130  | 162  | 194  | 226  | 258  | 290  | 322  | .4 13.8     | 13.6 | 13.2 | 12.8 |
| 136 | 354    | 386  | 417  | 449  | 481  | 513  | 545  | 577  | 608  | 640  | .5 17.2     | 17.0 | 16.5 | 16.0 |
| 137 | 672    | 703  | 735  | 767  | 798  | 830  | 862  | 893  | 925  | 956  | .6 20.7     | 20.4 | 19.8 | 19.2 |
| 138 | 988    | *019 | *051 | *082 | *113 | *145 | *176 | *207 | *239 | *270 | .7 24.1     | 23.8 | 23.1 | 22.4 |
| 139 | 14 301 | 332  | 364  | 395  | 426  | 457  | 488  | 519  | 550  | 582  | .8 27.6     | 27.2 | 26.4 | 25.6 |
| 140 | 613    | 644  | 675  | 706  | 736  | 767  | 798  | 829  | 860  | 891  | .9 31.0     | 30.6 | 29.7 | 28.8 |
| 141 | 922    | 952  | 983  | *014 | *045 | *075 | *106 | *137 | *167 | *198 | 31 31 30 29 |      |      |      |
| 142 | 15 229 | 259  | 290  | 320  | 351  | 381  | 412  | 442  | 473  | 503  | .1 3.1      | 3.1  | 3.0  | 2.9  |
| 143 | 533    | 564  | 594  | 624  | 655  | 685  | 715  | 745  | 776  | 806  | .2 6.3      | 6.2  | 6.0  | 5.8  |
| 144 | 836    | 866  | 896  | 926  | 956  | 987  | *017 | *047 | *077 | *107 | .3 9.4      | 9.3  | 9.0  | 8.7  |
| 145 | 16 137 | 166  | 196  | 226  | 256  | 286  | 316  | 346  | 376  | 405  | .4 12.6     | 12.4 | 12.0 | 11.6 |
| 146 | 435    | 465  | 494  | 524  | 554  | 584  | 613  | 643  | 672  | 702  | .5 15.7     | 15.5 | 15.0 | 14.5 |
| 147 | 731    | 761  | 791  | 820  | 849  | 879  | 908  | 938  | 967  | 997  | .6 18.9     | 18.6 | 18.0 | 17.4 |
| 148 | 17 026 | 055  | 085  | 114  | 143  | 172  | 202  | 231  | 260  | 289  | .7 22.0     | 21.7 | 21.0 | 20.3 |
| 149 | 318    | 348  | 377  | 406  | 435  | 464  | 493  | 522  | 551  | 580  | .8 25.2     | 24.8 | 24.0 | 23.2 |
| 150 | 609    | 638  | 667  | 696  | 725  | 753  | 782  | 811  | 840  | 869  | .9 28.3     | 27.9 | 27.0 | 26.1 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.       |      |      |      |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.             |
|-----|--------|-----|------|------|------|------|------|------|------|------|-------------------|
| 150 | 17 609 | 638 | 667  | 696  | 725  | 753  | 782  | 811  | 840  | 869  |                   |
| 151 | 897    | 926 | 955  | 984  | *012 | *041 | *070 | *098 | *127 | *156 |                   |
| 152 | 18 184 | 213 | 241  | 270  | 298  | 327  | 355  | 384  | 412  | 440  | 29 28 27          |
| 153 | 469    | 497 | 526  | 554  | 582  | 611  | 639  | 667  | 695  | 724  | .1 2.0 2.8 2.7    |
| 154 | 752    | 780 | 808  | 836  | 864  | 893  | 921  | 949  | 977  | *005 | .2 5.8 5.6 5.4    |
| 155 | 19 033 | 061 | 089  | 117  | 145  | 173  | 201  | 229  | 256  | 284  | .3 8.7 8.4 8.1    |
| 156 | 312    | 340 | 368  | 396  | 423  | 451  | 479  | 507  | 534  | 562  | .4 11.6 11.2 10.8 |
| 157 | 590    | 617 | 645  | 673  | 700  | 728  | 755  | 783  | 810  | 838  | .5 14.5 14.0 13.5 |
| 158 | 865    | 893 | 920  | 948  | 975  | *003 | *030 | *057 | *085 | *112 | .6 17.4 16.8 16.2 |
| 159 | 20 139 | 167 | 194  | 221  | 249  | 276  | 303  | 330  | 357  | 385  | .7 20.3 19.6 18.9 |
| 160 | 412    | 439 | 466  | 493  | 520  | 547  | 574  | 601  | 628  | 655  | .8 23.2 22.4 21.6 |
| 161 | 682    | 709 | 736  | 763  | 790  | 817  | 844  | 871  | 898  | 924  | .9 26.1 25.2 24.3 |
| 162 | 951    | 978 | *005 | *032 | *058 | *085 | *112 | *139 | *165 | *192 | 26 26             |
| 163 | 21 219 | 245 | 272  | 298  | 325  | 352  | 378  | 405  | 431  | 458  | .1 2.6 2.6        |
| 164 | 484    | 511 | 537  | 564  | 590  | 616  | 643  | 669  | 695  | 722  | .2 5.3 5.2        |
| 165 | 748    | 774 | 801  | 827  | 853  | 880  | 906  | 932  | 958  | 984  | .3 7.9 7.8        |
| 166 | 22 011 | 037 | 063  | 089  | 115  | 141  | 167  | 193  | 219  | 245  | .4 10.6 10.4      |
| 167 | 271    | 297 | 323  | 349  | 375  | 401  | 427  | 453  | 479  | 505  | .5 13.2 13.0      |
| 168 | 531    | 557 | 582  | 608  | 634  | 660  | 686  | 711  | 737  | 763  | .6 15.9 15.6      |
| 169 | 788    | 814 | 840  | 865  | 891  | 917  | 942  | 968  | 994  | *019 | .7 18.5 18.2      |
| 170 | 23 045 | 070 | 096  | 121  | 147  | 172  | 198  | 223  | 249  | 274  | .8 21.2 20.8      |
| 171 | 299    | 325 | 350  | 375  | 401  | 426  | 451  | 477  | 502  | 527  | .9 23.8 23.4      |
| 172 | 553    | 578 | 603  | 628  | 653  | 679  | 704  | 729  | 754  | 779  | 25 25 24          |
| 173 | 804    | 829 | 855  | 880  | 905  | 930  | 955  | 980  | *005 | *030 | .1 2.5 2.5 2.4    |
| 174 | 24 055 | 080 | 105  | 129  | 154  | 179  | 204  | 229  | 254  | 279  | .2 5.1 5.0 4.8    |
| 175 | 304    | 328 | 353  | 378  | 403  | 427  | 452  | 477  | 502  | 526  | .3 7.6 7.5 7.2    |
| 176 | 551    | 576 | 600  | 625  | 650  | 674  | 699  | 723  | 748  | 773  | .4 10.2 10.0 9.6  |
| 177 | 797    | 822 | 846  | 871  | 895  | 920  | 944  | 968  | 993  | *017 | .5 12.7 12.5 12.0 |
| 178 | 25 042 | 066 | 091  | 115  | 139  | 164  | 188  | 212  | 237  | 261  | .6 15.3 15.0 14.4 |
| 179 | 285    | 309 | 334  | 358  | 382  | 406  | 430  | 455  | 479  | 503  | .7 17.8 17.5 16.8 |
| 180 | 527    | 551 | 575  | 599  | 623  | 647  | 672  | 696  | 720  | 744  | .8 20.4 20.0 19.2 |
| 181 | 768    | 792 | 816  | 840  | 863  | 887  | 911  | 935  | 959  | 983  | .9 22.9 22.5 21.6 |
| 182 | 26 007 | 031 | 055  | 078  | 102  | 126  | 150  | 174  | 197  | 221  | 23 23             |
| 183 | 245    | 269 | 292  | 316  | 340  | 363  | 387  | 411  | 434  | 458  | .1 2.3 2.3        |
| 184 | 482    | 505 | 529  | 552  | 576  | 599  | 623  | 646  | 670  | 693  | .2 4.7 4.6        |
| 185 | 717    | 740 | 764  | 787  | 811  | 834  | 858  | 881  | 904  | 928  | .3 7.0 6.9        |
| 186 | 951    | 974 | 998  | *021 | *044 | *068 | *091 | *114 | *137 | *161 | .4 9.4 9.2        |
| 187 | 27 184 | 207 | 230  | 254  | 277  | 300  | 323  | 346  | 369  | 392  | .5 11.7 11.5      |
| 188 | 416    | 439 | 462  | 485  | 508  | 531  | 554  | 577  | 600  | 623  | .6 14.1 13.8      |
| 189 | 646    | 669 | 692  | 715  | 738  | 761  | 784  | 806  | 829  | 852  | .7 16.4 16.1      |
| 190 | 875    | 898 | 921  | 944  | 966  | 989  | *012 | *035 | *058 | *080 | .8 18.8 18.4      |
| 191 | 28 103 | 126 | 149  | 171  | 194  | 217  | 239  | 262  | 285  | 307  | .9 21.1 20.7      |
| 192 | 330    | 352 | 375  | 398  | 420  | 443  | 465  | 488  | 510  | 533  | 22 22 21          |
| 193 | 555    | 578 | 600  | 623  | 645  | 668  | 690  | 713  | 735  | 758  | .1 2.2 2.2 2.1    |
| 194 | 780    | 802 | 825  | 847  | 869  | 892  | 914  | 936  | 959  | 981  | .2 4.5 4.4 4.3    |
| 195 | 29 003 | 025 | 048  | 070  | 092  | 114  | 137  | 159  | 181  | 203  | .3 6.7 6.6 6.4    |
| 196 | 225    | 248 | 270  | 292  | 314  | 336  | 358  | 380  | 402  | 424  | .4 9.0 8.8 8.6    |
| 197 | 446    | 468 | 490  | 512  | 534  | 556  | 578  | 600  | 622  | 644  | .5 11.2 11.0 10.7 |
| 198 | 666    | 688 | 710  | 732  | 754  | 776  | 798  | 820  | 841  | 863  | .6 13.5 13.2 12.9 |
| 199 | 885    | 907 | 929  | 950  | 972  | 994  | *016 | *038 | *059 | *081 | .7 15.7 15.4 15.0 |
| 200 | 30 103 | 124 | 146  | 168  | 190  | 211  | 233  | 254  | 276  | 298  | .8 18.0 17.6 17.2 |
| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.             |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |      |      |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|------|------|
| 200 | 30 103 | 124  | 146  | 168  | 190  | 211  | 233  | 254  | 276  | 298  | 22 21 |      |      |
| 201 | 319    | 341  | 363  | 384  | 406  | 427  | 449  | 470  | 492  | 513  | .1    | 2.2  | 2.1  |
| 202 | 535    | 556  | 578  | 599  | 621  | 642  | 664  | 685  | 707  | 728  | .2    | 4.4  | 4.2  |
| 203 | 749    | 771  | 792  | 813  | 835  | 856  | 878  | 899  | 920  | 941  | .3    | 6.6  | 6.3  |
| 204 | 963    | 984  | *005 | *027 | *048 | *069 | *090 | *112 | *133 | *154 | .4    | 8.8  | 8.4  |
| 205 | 31 175 | 196  | 217  | 239  | 260  | 281  | 302  | 323  | 344  | 365  | .5    | 11.0 | 10.5 |
| 206 | 386    | 408  | 429  | 450  | 471  | 492  | 513  | 534  | 555  | 576  | .6    | 13.2 | 12.6 |
| 207 | 597    | 618  | 639  | 660  | 681  | 702  | 722  | 743  | 764  | 785  | .7    | 15.4 | 14.7 |
| 208 | 806    | 827  | 848  | 869  | 890  | 910  | 931  | 952  | 973  | 994  | .8    | 17.6 | 16.8 |
| 209 | 32 014 | 035  | 056  | 077  | 097  | 118  | 139  | 160  | 180  | 201  | .9    | 19.8 | 18.9 |
| 210 | 222    | 242  | 263  | 284  | 304  | 325  | 346  | 366  | 387  | 407  | 26 20 |      |      |
| 211 | 428    | 449  | 469  | 490  | 510  | 531  | 551  | 572  | 592  | 613  | .1    | 2.6  | 2.0  |
| 212 | 633    | 654  | 674  | 695  | 715  | 736  | 756  | 776  | 797  | 817  | .2    | 4.1  | 4.0  |
| 213 | 838    | 858  | 878  | 899  | 919  | 940  | 960  | 980  | *001 | *021 | .3    | 6.1  | 6.0  |
| 214 | 33 041 | 061  | 082  | 102  | 122  | 142  | 163  | 183  | 203  | 223  | .4    | 8.2  | 8.0  |
| 215 | 244    | 264  | 284  | 304  | 324  | 344  | 365  | 385  | 405  | 425  | .5    | 10.2 | 10.0 |
| 216 | 445    | 465  | 485  | 505  | 525  | 546  | 566  | 586  | 606  | 626  | .6    | 12.3 | 12.0 |
| 217 | 646    | 666  | 686  | 706  | 726  | 746  | 766  | 786  | 806  | 825  | .7    | 14.3 | 14.0 |
| 218 | 845    | 865  | 885  | 905  | 925  | 945  | 965  | 985  | *004 | *024 | .8    | 16.4 | 16.0 |
| 219 | 34 044 | 064  | 084  | 104  | 123  | 143  | 163  | 183  | 203  | 222  | .9    | 18.4 | 18.0 |
| 220 | 242    | 262  | 281  | 301  | 321  | 341  | 360  | 380  | 400  | 419  | 19 19 |      |      |
| 221 | 439    | 459  | 478  | 498  | 518  | 537  | 557  | 576  | 596  | 615  | .1    | 1.9  | 1.9  |
| 222 | 635    | 655  | 674  | 694  | 713  | 733  | 752  | 772  | 791  | 811  | .2    | 3.9  | 3.8  |
| 223 | 830    | 850  | 869  | 889  | 908  | 928  | 947  | 966  | 986  | *005 | .3    | 5.3  | 5.7  |
| 224 | 35 025 | 044  | 063  | 083  | 102  | 121  | 141  | 160  | 179  | 199  | .4    | 7.8  | 7.6  |
| 225 | 218    | 237  | 257  | 276  | 295  | 314  | 334  | 353  | 372  | 391  | .5    | 9.7  | 9.5  |
| 226 | 411    | 430  | 449  | 468  | 487  | 507  | 526  | 545  | 564  | 583  | .6    | 11.7 | 11.4 |
| 227 | 602    | 621  | 641  | 660  | 679  | 698  | 717  | 736  | 755  | 774  | .7    | 13.6 | 13.3 |
| 228 | 793    | 812  | 831  | 850  | 869  | 888  | 907  | 926  | 945  | 964  | .8    | 15.6 | 15.2 |
| 229 | 983    | *002 | *021 | *040 | *059 | *078 | *097 | *116 | *135 | *154 | .9    | 17.5 | 17.1 |
| 230 | 36 173 | 191  | 210  | 229  | 248  | 267  | 286  | 305  | 323  | 342  | 18 18 |      |      |
| 231 | 361    | 380  | 399  | 417  | 436  | 455  | 474  | 492  | 511  | 530  | .1    | 1.8  | 1.8  |
| 232 | 549    | 567  | 586  | 605  | 623  | 642  | 661  | 679  | 698  | 717  | .2    | 3.7  | 3.6  |
| 233 | 735    | 754  | 773  | 791  | 810  | 828  | 847  | 866  | 884  | 903  | .3    | 5.5  | 5.4  |
| 234 | 921    | 940  | 958  | 977  | 996  | *014 | *033 | *051 | *070 | *088 | .4    | 7.4  | 7.2  |
| 235 | 37 107 | 125  | 143  | 162  | 180  | 199  | 217  | 236  | 254  | 273  | .5    | 9.2  | 9.0  |
| 236 | 291    | 309  | 328  | 346  | 364  | 383  | 401  | 420  | 438  | 456  | .6    | 11.1 | 10.8 |
| 237 | 475    | 493  | 511  | 530  | 548  | 566  | 584  | 603  | 621  | 639  | .7    | 12.9 | 12.6 |
| 238 | 657    | 676  | 694  | 712  | 730  | 749  | 767  | 785  | 803  | 821  | .8    | 14.8 | 14.4 |
| 239 | 840    | 858  | 876  | 894  | 912  | 930  | 948  | 967  | 985  | *003 | .9    | 16.6 | 16.2 |
| 240 | 38 021 | 039  | 057  | 075  | 093  | 111  | 129  | 147  | 165  | 183  | 17 17 |      |      |
| 241 | 201    | 219  | 237  | 255  | 273  | 291  | 309  | 327  | 345  | 363  | .1    | 1.7  | 1.7  |
| 242 | 381    | 399  | 417  | 435  | 453  | 471  | 489  | 507  | 525  | 543  | .2    | 3.5  | 3.4  |
| 243 | 560    | 578  | 596  | 614  | 632  | 650  | 667  | 685  | 703  | 721  | .3    | 5.2  | 5.1  |
| 244 | 739    | 757  | 774  | 792  | 810  | 828  | 845  | 863  | 881  | 899  | .4    | 7.0  | 6.8  |
| 245 | 916    | 934  | 952  | 970  | 987  | *005 | *023 | *040 | *058 | *076 | .5    | 8.7  | 8.5  |
| 246 | 39 093 | 111  | 129  | 146  | 164  | 181  | 199  | 217  | 234  | 252  | .6    | 10.5 | 10.2 |
| 247 | 269    | 287  | 305  | 322  | 340  | 357  | 375  | 392  | 410  | 427  | .7    | 12.2 | 11.9 |
| 248 | 445    | 462  | 480  | 497  | 515  | 532  | 550  | 567  | 585  | 602  | .8    | 14.0 | 13.6 |
| 249 | 620    | 637  | 655  | 672  | 689  | 707  | 724  | 742  | 759  | 776  | .9    | 15.7 | 15.3 |
| 250 | 794    | 811  | 828  | 846  | 863  | 881  | 898  | 915  | 933  | 950  | P. P. |      |      |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |      |      |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |      |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|------|
| 250 | 39 794 | 81î  | 828  | 846  | 863  | 881  | 898  | 915  | 933  | 950  |       |      |
| 251 | 967    | 984  | *002 | *019 | *036 | *054 | *071 | *088 | *105 | *123 |       |      |
| 252 | 40 140 | 157  | 174  | 191  | 209  | 226  | 243  | 260  | 277  | 295  |       |      |
| 253 | 312    | 329  | 346  | 363  | 380  | 398  | 415  | 432  | 449  | 466  | 17    | 17   |
| 254 | 483    | 500  | 517  | 534  | 551  | 569  | 586  | 603  | 620  | 637  | .1    | 1.7  |
| 255 | 654    | 671  | 688  | 705  | 722  | 739  | 756  | 773  | 790  | 807  | .2    | 3.5  |
| 256 | 824    | 841  | 858  | 875  | 892  | 908  | 925  | 942  | 959  | 976  | .3    | 5.2  |
| 257 | 993    | *010 | *027 | *044 | *061 | *077 | *094 | *111 | *128 | *145 | .4    | 7.0  |
| 258 | 41 162 | 179  | 195  | 212  | 229  | 246  | 263  | 279  | 296  | 313  | .5    | 8.7  |
| 259 | 330    | 346  | 363  | 380  | 397  | 413  | 430  | 447  | 464  | 480  | .6    | 10.5 |
| 260 | 497    | 514  | 530  | 547  | 564  | 581  | 597  | 614  | 631  | 647  | .7    | 12.2 |
| 261 | 664    | 680  | 697  | 714  | 730  | 747  | 764  | 780  | 797  | 813  | .8    | 14.0 |
| 262 | 830    | 846  | 863  | 880  | 896  | 913  | 929  | 946  | 962  | 979  | .9    | 15.7 |
| 263 | 995    | *012 | *028 | *045 | *061 | *078 | *094 | *111 | *127 | *144 |       |      |
| 264 | 42 166 | 177  | 193  | 209  | 226  | 242  | 259  | 275  | 292  | 308  |       |      |
| 265 | 324    | 341  | 357  | 373  | 390  | 406  | 423  | 439  | 455  | 472  | 16    | 16   |
| 266 | 488    | 504  | 521  | 537  | 553  | 569  | 586  | 602  | 618  | 635  | .1    | 1.6  |
| 267 | 651    | 667  | 683  | 700  | 716  | 732  | 748  | 765  | 781  | 797  | .2    | 3.3  |
| 268 | 813    | 829  | 846  | 862  | 878  | 894  | 910  | 927  | 943  | 959  | .3    | 4.9  |
| 269 | 975    | 991  | *007 | *023 | *040 | *056 | *072 | *088 | *104 | *120 | .4    | 6.6  |
| 270 | 43 136 | 152  | 168  | 184  | 200  | 216  | 233  | 249  | 265  | 281  | .5    | 8.2  |
| 271 | 297    | 313  | 329  | 345  | 361  | 377  | 393  | 409  | 425  | 441  | .6    | 9.9  |
| 272 | 457    | 473  | 489  | 505  | 520  | 536  | 552  | 568  | 584  | 600  | .7    | 11.5 |
| 273 | 616    | 632  | 648  | 664  | 680  | 695  | 711  | 727  | 743  | 759  | .8    | 13.2 |
| 274 | 775    | 791  | 806  | 822  | 838  | 854  | 870  | 886  | 901  | 917  | .9    | 14.8 |
| 275 | 933    | 949  | 965  | 980  | 996  | *012 | *028 | *043 | *059 | *075 |       |      |
| 276 | 44 091 | 106  | 122  | 138  | 154  | 169  | 185  | 201  | 216  | 232  |       |      |
| 277 | 248    | 263  | 279  | 295  | 310  | 326  | 342  | 357  | 373  | 389  | 15    | 15   |
| 278 | 404    | 420  | 435  | 451  | 467  | 482  | 498  | 513  | 529  | 545  | .1    | 1.5  |
| 279 | 560    | 576  | 591  | 607  | 622  | 638  | 653  | 669  | 685  | 700  | .2    | 3.1  |
| 280 | 716    | 731  | 747  | 762  | 778  | 793  | 809  | 824  | 839  | 855  | .3    | 4.6  |
| 281 | 870    | 886  | 901  | 917  | 932  | 948  | 963  | 978  | 994  | *009 | .4    | 6.2  |
| 282 | 45 025 | 040  | 055  | 071  | 086  | 102  | 117  | 132  | 148  | 163  | .5    | 7.7  |
| 283 | 178    | 194  | 209  | 224  | 240  | 255  | 270  | 286  | 301  | 316  | .6    | 9.3  |
| 284 | 332    | 347  | 362  | 377  | 393  | 408  | 423  | 438  | 454  | 469  | .7    | 10.8 |
| 285 | 484    | 499  | 515  | 530  | 545  | 560  | 576  | 591  | 606  | 621  | .8    | 12.4 |
| 286 | 636    | 652  | 667  | 682  | 697  | 712  | 727  | 743  | 758  | 773  | .9    | 13.9 |
| 287 | 788    | 803  | 818  | 833  | 848  | 864  | 879  | 894  | 909  | 924  |       |      |
| 288 | 939    | 954  | 969  | 984  | 999  | *014 | *029 | *044 | *059 | *075 |       |      |
| 289 | 46 090 | 105  | 120  | 135  | 150  | 165  | 180  | 195  | 210  | 225  | 14    | 14   |
| 290 | 240    | 255  | 269  | 284  | 299  | 314  | 329  | 344  | 359  | 374  | .1    | 1.4  |
| 291 | 389    | 404  | 419  | 434  | 449  | 464  | 479  | 493  | 508  | 523  | .2    | 2.9  |
| 292 | 538    | 553  | 568  | 583  | 597  | 612  | 627  | 642  | 657  | 672  | .3    | 4.3  |
| 293 | 687    | 701  | 716  | 731  | 746  | 761  | 775  | 790  | 805  | 820  | .4    | 5.8  |
| 294 | 834    | 849  | 864  | 879  | 894  | 908  | 923  | 938  | 952  | 967  | .5    | 7.2  |
| 295 | 982    | 997  | *011 | *026 | *041 | *055 | *070 | *085 | *100 | *114 | .6    | 8.7  |
| 296 | 47 129 | 144  | 158  | 173  | 188  | 202  | 217  | 232  | 246  | 261  | .7    | 10.1 |
| 297 | 275    | 290  | 305  | 319  | 334  | 348  | 363  | 378  | 392  | 407  | .8    | 11.6 |
| 298 | 421    | 436  | 451  | 465  | 480  | 494  | 509  | 523  | 538  | 552  | .9    | 13.0 |
| 299 | 567    | 581  | 596  | 610  | 625  | 639  | 654  | 668  | 683  | 697  |       |      |
| 300 | 712    | 726  | 741  | 755  | 770  | 784  | 799  | 813  | 828  | 842  |       |      |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |      |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.         | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |  |  |
|------------|--------|------|------|------|------|------|------|------|------|------|-------|--|--|
| <b>300</b> | 47 712 | 726  | 741  | 755  | 770  | 784  | 799  | 813  | 828  | 842  |       |  |  |
| 301        | 856    | 871  | 885  | 900  | 914  | 928  | 943  | 957  | 972  | 986  |       |  |  |
| 302        | 48 000 | 015  | 029  | 044  | 058  | 072  | 087  | 101  | 115  | 130  |       |  |  |
| 303        | 144    | 158  | 173  | 187  | 201  | 216  | 230  | 244  | 259  | 273  |       |  |  |
| 304        | 287    | 301  | 316  | 330  | 344  | 358  | 373  | 387  | 401  | 415  |       |  |  |
| 305        | 430    | 444  | 458  | 472  | 487  | 501  | 515  | 529  | 543  | 558  |       |  |  |
| 306        | 572    | 586  | 600  | 614  | 629  | 643  | 657  | 671  | 685  | 699  |       |  |  |
| 307        | 714    | 728  | 742  | 756  | 770  | 784  | 798  | 812  | 827  | 841  |       |  |  |
| 308        | 855    | 869  | 883  | 897  | 911  | 925  | 939  | 953  | 967  | 982  |       |  |  |
| 309        | 996    | *010 | *024 | *038 | *052 | *066 | *080 | *094 | *108 | *122 |       |  |  |
| <b>310</b> | 49 136 | 150  | 164  | 178  | 192  | 206  | 220  | 234  | 248  | 262  |       |  |  |
| 311        | 276    | 290  | 304  | 318  | 332  | 346  | 359  | 373  | 387  | 401  |       |  |  |
| 312        | 415    | 429  | 443  | 457  | 471  | 485  | 499  | 513  | 526  | 540  |       |  |  |
| 313        | 554    | 568  | 582  | 596  | 610  | 624  | 637  | 651  | 665  | 679  |       |  |  |
| 314        | 693    | 707  | 720  | 734  | 748  | 762  | 776  | 789  | 803  | 817  |       |  |  |
| 315        | 831    | 845  | 858  | 872  | 886  | 900  | 913  | 927  | 941  | 955  |       |  |  |
| 316        | 968    | 982  | 996  | *010 | *023 | *037 | *051 | *065 | *078 | *092 |       |  |  |
| 317        | 50 106 | 119  | 133  | 147  | 160  | 174  | 188  | 201  | 215  | 229  |       |  |  |
| 318        | 242    | 256  | 270  | 283  | 297  | 311  | 324  | 338  | 352  | 365  |       |  |  |
| 319        | 379    | 392  | 406  | 420  | 433  | 447  | 460  | 474  | 488  | 501  |       |  |  |
| <b>320</b> | 515    | 528  | 542  | 555  | 569  | 583  | 596  | 610  | 623  | 637  |       |  |  |
| 321        | 650    | 664  | 677  | 691  | 704  | 718  | 731  | 745  | 758  | 772  |       |  |  |
| 322        | 785    | 799  | 812  | 826  | 839  | 853  | 866  | 880  | 893  | 907  |       |  |  |
| 323        | 920    | 933  | 947  | 960  | 974  | 987  | *001 | *014 | *027 | *041 |       |  |  |
| 324        | 51 054 | 068  | 081  | 094  | 108  | 121  | 135  | 148  | 161  | 175  |       |  |  |
| 325        | 188    | 201  | 215  | 228  | 242  | 255  | 268  | 282  | 295  | 308  |       |  |  |
| 326        | 322    | 335  | 348  | 361  | 375  | 388  | 401  | 415  | 428  | 441  |       |  |  |
| 327        | 455    | 468  | 481  | 494  | 508  | 521  | 534  | 547  | 561  | 574  |       |  |  |
| 328        | 587    | 600  | 614  | 627  | 640  | 653  | 667  | 680  | 693  | 706  |       |  |  |
| 329        | 719    | 733  | 746  | 759  | 772  | 785  | 798  | 812  | 825  | 838  |       |  |  |
| <b>330</b> | 851    | 864  | 877  | 891  | 904  | 917  | 930  | 943  | 956  | 969  |       |  |  |
| 331        | 983    | 996  | *009 | *022 | *035 | *048 | *061 | *074 | *087 | *100 |       |  |  |
| 332        | 52 114 | 127  | 140  | 153  | 166  | 179  | 192  | 205  | 218  | 231  |       |  |  |
| 333        | 244    | 257  | 270  | 283  | 296  | 309  | 322  | 335  | 348  | 361  |       |  |  |
| 334        | 374    | 387  | 400  | 413  | 426  | 439  | 452  | 465  | 478  | 491  |       |  |  |
| 335        | 504    | 517  | 530  | 543  | 556  | 569  | 582  | 595  | 608  | 621  |       |  |  |
| 336        | 634    | 647  | 660  | 672  | 685  | 698  | 711  | 724  | 737  | 750  |       |  |  |
| 337        | 763    | 776  | 789  | 801  | 814  | 827  | 840  | 853  | 866  | 879  |       |  |  |
| 338        | 891    | 904  | 917  | 930  | 943  | 956  | 968  | 981  | 994  | *007 |       |  |  |
| 339        | 53 020 | 033  | 045  | 058  | 071  | 084  | 097  | 109  | 122  | 135  |       |  |  |
| <b>340</b> | 148    | 160  | 173  | 186  | 199  | 211  | 224  | 237  | 250  | 262  |       |  |  |
| 341        | 275    | 288  | 301  | 313  | 326  | 339  | 352  | 364  | 377  | 390  |       |  |  |
| 342        | 402    | 415  | 428  | 440  | 453  | 466  | 478  | 491  | 504  | 516  |       |  |  |
| 343        | 529    | 542  | 554  | 567  | 580  | 592  | 605  | 618  | 630  | 643  |       |  |  |
| 344        | 656    | 668  | 681  | 693  | 706  | 719  | 731  | 744  | 756  | 769  |       |  |  |
| 345        | 782    | 794  | 807  | 819  | 832  | 845  | 857  | 870  | 882  | 895  |       |  |  |
| 346        | 907    | 920  | 932  | 945  | 958  | 970  | 983  | 995  | *008 | *020 |       |  |  |
| 347        | 54 033 | 045  | 058  | 070  | 083  | 095  | 108  | 120  | 133  | 145  |       |  |  |
| 348        | 158    | 170  | 183  | 195  | 208  | 220  | 232  | 245  | 257  | 270  |       |  |  |
| 349        | 282    | 295  | 307  | 320  | 332  | 344  | 357  | 369  | 382  | 394  |       |  |  |
| <b>350</b> | 407    | 419  | 431  | 444  | 456  | 469  | 481  | 493  | 506  | 518  |       |  |  |
| N.         | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |  |  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.         | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |      |
|------------|--------|------|------|------|------|------|------|------|------|------|-------|------|
| <b>350</b> | 54 407 | 419  | 43î  | 444  | 456  | 469  | 481  | 493  | 506  | 518  |       | 12   |
| 351        | 53ô    | 543  | 555  | 568  | 580  | 592  | 605  | 617  | 629  | 642  | .1    | 1.2  |
| 352        | 654    | 666  | 679  | 691  | 703  | 716  | 728  | 740  | 753  | 765  | .2    | 2.5  |
| 353        | 777    | 790  | 802  | 814  | 826  | 839  | 851  | 863  | 876  | 888  | .3    | 3.7  |
| 354        | 900    | 912  | 925  | 937  | 949  | 961  | 974  | 986  | 998  | *010 |       |      |
| 355        | 55 023 | 035  | 047  | 059  | 071  | 084  | 096  | 108  | 120  | 133  | .4    | 5.0  |
| 356        | 145    | 157  | 169  | 181  | 194  | 206  | 218  | 230  | 242  | 254  | .5    | 6.2  |
| 357        | 267    | 279  | 291  | 303  | 315  | 327  | 340  | 352  | 364  | 376  | .6    | 7.5  |
| 358        | 388    | 400  | 412  | 424  | 437  | 449  | 461  | 473  | 485  | 497  | .7    | 8.7  |
| 359        | 509    | 521  | 533  | 545  | 558  | 570  | 582  | 594  | 606  | 618  | .8    | 10.0 |
| <b>360</b> | 630    | 642  | 654  | 666  | 678  | 690  | 702  | 714  | 726  | 738  | .9    | 11.2 |
| 361        | 750    | 762  | 775  | 787  | 799  | 811  | 823  | 835  | 847  | 859  |       | 12   |
| 362        | 871    | 883  | 895  | 907  | 919  | 931  | 943  | 955  | 966  | 978  | .1    | 1.2  |
| 363        | 990    | *002 | *014 | *026 | *038 | *050 | *062 | *074 | *086 | *098 | .2    | 2.4  |
| 364        | 56 110 | 122  | 134  | 146  | 158  | 170  | 181  | 193  | 205  | 217  | .3    | 3.6  |
| 365        | 229    | 241  | 253  | 265  | 277  | 288  | 300  | 312  | 324  | 336  | .4    | 4.8  |
| 366        | 348    | 360  | 372  | 383  | 395  | 407  | 419  | 431  | 443  | 455  | .5    | 6.0  |
| 367        | 466    | 478  | 490  | 502  | 514  | 525  | 537  | 549  | 561  | 573  | .6    | 7.2  |
| 368        | 585    | 596  | 608  | 620  | 632  | 643  | 655  | 667  | 679  | 691  | .7    | 8.4  |
| 369        | 702    | 714  | 726  | 738  | 749  | 761  | 773  | 785  | 796  | 808  | .8    | 9.6  |
| <b>370</b> | 820    | 832  | 843  | 855  | 867  | 879  | 890  | 902  | 914  | 925  | .9    | 10.8 |
| 371        | 937    | 949  | 961  | 972  | 984  | 996  | *007 | *019 | *031 | *042 |       | 11   |
| 372        | 57 054 | 066  | 077  | 089  | 101  | 112  | 124  | 136  | 147  | 159  | .1    | 1.1  |
| 373        | 171    | 182  | 194  | 206  | 217  | 229  | 240  | 252  | 264  | 275  | .2    | 2.3  |
| 374        | 287    | 299  | 310  | 322  | 333  | 345  | 357  | 368  | 380  | 391  | .3    | 3.4  |
| 375        | 403    | 414  | 426  | 438  | 449  | 461  | 472  | 484  | 495  | 507  | .4    | 4.6  |
| 376        | 519    | 530  | 542  | 553  | 565  | 576  | 588  | 599  | 611  | 622  | .5    | 5.7  |
| 377        | 634    | 645  | 657  | 668  | 680  | 691  | 703  | 714  | 726  | 737  | .6    | 6.9  |
| 378        | 749    | 760  | 772  | 783  | 795  | 806  | 818  | 829  | 841  | 852  | .7    | 8.0  |
| 379        | 864    | 875  | 887  | 898  | 909  | 921  | 932  | 944  | 955  | 967  | .8    | 9.2  |
| <b>380</b> | 978    | 990  | *001 | *012 | *024 | *035 | *047 | *058 | *069 | *081 | .9    | 10.3 |
| 381        | 58 092 | 104  | 115  | 126  | 138  | 149  | 161  | 172  | 183  | 195  |       | 11   |
| 382        | 206    | 217  | 229  | 240  | 252  | 263  | 274  | 286  | 297  | 308  | .1    | 1.1  |
| 383        | 320    | 331  | 342  | 354  | 365  | 376  | 388  | 399  | 410  | 422  | .2    | 2.2  |
| 384        | 433    | 444  | 455  | 467  | 478  | 489  | 501  | 512  | 523  | 535  | .3    | 3.3  |
| 385        | 546    | 557  | 568  | 580  | 591  | 602  | 613  | 625  | 636  | 647  | .4    | 4.4  |
| 386        | 658    | 670  | 681  | 692  | 703  | 715  | 726  | 737  | 748  | 760  | .5    | 5.5  |
| 387        | 771    | 782  | 793  | 804  | 816  | 827  | 838  | 849  | 861  | 872  | .6    | 6.6  |
| 388        | 883    | 894  | 905  | 916  | 928  | 939  | 950  | 961  | 972  | 984  | .7    | 7.7  |
| 389        | 995    | *006 | *017 | *028 | *039 | *050 | *062 | *073 | *084 | *095 | .8    | 8.8  |
| <b>390</b> | 59 106 | 117  | 128  | 140  | 151  | 162  | 173  | 184  | 195  | 206  | .9    | 9.9  |
| 391        | 217    | 229  | 240  | 251  | 262  | 273  | 284  | 295  | 306  | 317  |       | 10   |
| 392        | 328    | 339  | 351  | 362  | 373  | 384  | 395  | 406  | 417  | 428  | .1    | 1.0  |
| 393        | 439    | 450  | 461  | 472  | 483  | 494  | 505  | 516  | 527  | 538  | .2    | 2.1  |
| 394        | 549    | 560  | 571  | 582  | 593  | 604  | 615  | 626  | 637  | 648  | .3    | 3.1  |
| 395        | 659    | 670  | 681  | 692  | 703  | 714  | 725  | 736  | 747  | 758  | .4    | 4.2  |
| 396        | 769    | 780  | 791  | 802  | 813  | 824  | 835  | 846  | 857  | 868  | .5    | 5.2  |
| 397        | 879    | 890  | 901  | 912  | 923  | 933  | 944  | 955  | 966  | 977  | .6    | 6.3  |
| 398        | 988    | 999  | *010 | *021 | *032 | *043 | *053 | *064 | *075 | *086 | .7    | 7.3  |
| 399        | 60 097 | 108  | 119  | 130  | 141  | 151  | 162  | 173  | 184  | 195  | .8    | 8.4  |
| <b>400</b> | 206    | 217  | 227  | 238  | 249  | 260  | 271  | 282  | 293  | 303  | .9    | 9.4  |
| N.         | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |      |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |     |
|-----|--------|-----|-----|-----|------|------|------|------|------|------|-------|-----|
| 400 | 60 206 | 217 | 227 | 238 | 249  | 260  | 271  | 282  | 293  | 303  | II    |     |
| 401 | 314    | 325 | 336 | 347 | 357  | 368  | 379  | 390  | 401  | 412  |       |     |
| 402 | 422    | 433 | 444 | 455 | 466  | 476  | 487  | 498  | 509  | 519  |       |     |
| 403 | 530    | 541 | 552 | 563 | 573  | 584  | 595  | 606  | 616  | 627  |       |     |
| 404 | 638    | 649 | 659 | 670 | 681  | 692  | 702  | 713  | 724  | 735  |       |     |
| 405 | 745    | 756 | 767 | 777 | 788  | 799  | 810  | 820  | 831  | 842  |       |     |
| 406 | 852    | 863 | 874 | 884 | 895  | 906  | 916  | 927  | 938  | 949  | .1    | 1.1 |
| 407 | 959    | 970 | 981 | 991 | *002 | *013 | *023 | *034 | *044 | *055 |       | 4.4 |
| 408 | 61 066 | 076 | 087 | 098 | 108  | 119  | 130  | 140  | 151  | 161  |       | 5.5 |
| 409 | 172    | 183 | 193 | 204 | 215  | 225  | 236  | 246  | 257  | 268  |       | 6.6 |
| 410 | 278    | 289 | 299 | 310 | 320  | 331  | 342  | 352  | 363  | 373  |       | 7.7 |
| 411 | 384    | 394 | 405 | 416 | 426  | 437  | 447  | 458  | 468  | 479  |       | 8.8 |
| 412 | 489    | 500 | 511 | 521 | 532  | 542  | 553  | 563  | 574  | 584  | .2    | 9.9 |
| 413 | 595    | 605 | 616 | 626 | 637  | 647  | 658  | 668  | 679  | 689  |       |     |
| 414 | 700    | 710 | 721 | 731 | 742  | 752  | 763  | 773  | 784  | 794  |       |     |
| 415 | 805    | 815 | 825 | 836 | 846  | 857  | 867  | 878  | 888  | 899  |       | 10  |
| 416 | 909    | 920 | 930 | 940 | 951  | 961  | 972  | 982  | 993  | *003 |       | 1.0 |
| 417 | 62 013 | 024 | 034 | 045 | 055  | 065  | 076  | 086  | 097  | 107  |       | 2.1 |
| 418 | 117    | 128 | 138 | 149 | 159  | 169  | 180  | 190  | 200  | 211  | .3    | 3.1 |
| 419 | 221    | 232 | 242 | 252 | 263  | 273  | 283  | 294  | 304  | 314  |       | 4.2 |
| 420 | 325    | 335 | 345 | 356 | 366  | 376  | 387  | 397  | 407  | 418  |       | 5.2 |
| 421 | 428    | 438 | 449 | 459 | 469  | 480  | 490  | 500  | 510  | 521  |       | 6.3 |
| 422 | 531    | 541 | 552 | 562 | 572  | 582  | 593  | 603  | 613  | 624  |       | 7.3 |
| 423 | 634    | 644 | 654 | 665 | 675  | 685  | 695  | 706  | 716  | 726  |       | 8.4 |
| 424 | 736    | 747 | 757 | 767 | 777  | 788  | 798  | 808  | 818  | 828  | .4    | 9.4 |
| 425 | 839    | 849 | 859 | 869 | 879  | 890  | 900  | 910  | 920  | 931  |       |     |
| 426 | 941    | 951 | 961 | 971 | 981  | 992  | *002 | *012 | *022 | *032 |       |     |
| 427 | 63 043 | 053 | 063 | 073 | 083  | 093  | 104  | 114  | 124  | 134  |       | 10  |
| 428 | 144    | 154 | 164 | 175 | 185  | 195  | 205  | 215  | 225  | 235  |       | 1.0 |
| 429 | 245    | 256 | 266 | 276 | 286  | 296  | 306  | 316  | 326  | 336  |       | 2.0 |
| 430 | 347    | 357 | 367 | 377 | 387  | 397  | 407  | 417  | 427  | 437  | .5    | 3.0 |
| 431 | 447    | 458 | 468 | 478 | 488  | 498  | 508  | 518  | 528  | 538  |       | 4.0 |
| 432 | 548    | 558 | 568 | 578 | 588  | 598  | 608  | 618  | 628  | 639  |       | 5.0 |
| 433 | 649    | 659 | 669 | 679 | 689  | 699  | 709  | 719  | 729  | 739  |       | 6.0 |
| 434 | 749    | 759 | 769 | 779 | 789  | 799  | 809  | 819  | 829  | 839  |       | 7.0 |
| 435 | 849    | 859 | 869 | 879 | 889  | 899  | 909  | 919  | 928  | 938  |       | 8.0 |
| 436 | 948    | 958 | 968 | 978 | 988  | 998  | *008 | *018 | *028 | *038 | .6    | 9.0 |
| 437 | 64 048 | 058 | 068 | 078 | 088  | 098  | 107  | 117  | 127  | 137  |       |     |
| 438 | 147    | 157 | 167 | 177 | 187  | 197  | 207  | 217  | 226  | 236  |       |     |
| 439 | 246    | 256 | 266 | 276 | 286  | 296  | 306  | 315  | 325  | 335  |       | 9   |
| 440 | 345    | 355 | 365 | 375 | 384  | 394  | 404  | 414  | 424  | 434  |       | 0.9 |
| 441 | 444    | 453 | 463 | 473 | 483  | 493  | 503  | 512  | 522  | 532  |       | 1.9 |
| 442 | 542    | 552 | 562 | 571 | 581  | 591  | 601  | 611  | 621  | 630  | .7    | 2.8 |
| 443 | 640    | 650 | 660 | 670 | 679  | 689  | 699  | 709  | 718  | 728  |       | 3.8 |
| 444 | 738    | 748 | 758 | 767 | 777  | 787  | 797  | 806  | 816  | 826  |       | 4.7 |
| 445 | 836    | 846 | 855 | 865 | 875  | 885  | 894  | 904  | 914  | 923  |       | 5.7 |
| 446 | 933    | 943 | 953 | 962 | 972  | 982  | 992  | *001 | *011 | *021 |       | 6.6 |
| 447 | 65 031 | 040 | 050 | 060 | 069  | 079  | 089  | 098  | 108  | 118  |       | 7.6 |
| 448 | 128    | 137 | 147 | 157 | 166  | 176  | 186  | 195  | 205  | 215  | .8    | 8.6 |
| 449 | 224    | 234 | 244 | 253 | 263  | 273  | 282  | 292  | 302  | 311  |       | 9.6 |
| 450 | 321    | 331 | 340 | 350 | 360  | 369  | 379  | 389  | 398  | 408  |       |     |
| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |     |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|------|------|------|------|------|------|------|------|------|--------|
| 450 | 65 321 | 331  | 340  | 350  | 360  | 369  | 379  | 389  | 398  | 408  |        |
| 451 | 417    | 427  | 437  | 446  | 456  | 466  | 475  | 485  | 494  | 504  |        |
| 452 | 514    | 523  | 533  | 542  | 552  | 562  | 571  | 581  | 590  | 600  |        |
| 453 | 610    | 619  | 629  | 638  | 648  | 657  | 667  | 677  | 686  | 696  | 10     |
| 454 | 703    | 715  | 724  | 734  | 744  | 753  | 763  | 772  | 782  | 791  | .1 1.0 |
| 455 | 801    | 810  | 820  | 830  | 839  | 849  | 858  | 868  | 877  | 887  | .2 2.0 |
| 456 | 896    | 906  | 915  | 925  | 934  | 944  | 953  | 963  | 972  | 982  | .3 3.0 |
| 457 | 991    | *001 | *010 | *020 | *029 | *039 | *048 | *058 | *067 | *077 | .4 4.0 |
| 458 | 66 086 | 096  | 105  | 115  | 124  | 134  | 143  | 153  | 162  | 172  | .5 5.0 |
| 459 | 181    | 190  | 200  | 209  | 219  | 228  | 238  | 247  | 257  | 266  | .6 6.0 |
| 460 | 276    | 285  | 294  | 304  | 313  | 323  | 332  | 342  | 351  | 360  | .7 7.0 |
| 461 | 370    | 379  | 389  | 398  | 408  | 417  | 426  | 436  | 445  | 455  | .8 8.0 |
| 462 | 464    | 473  | 483  | 492  | 502  | 511  | 520  | 530  | 539  | 548  | .9 9.0 |
| 463 | 558    | 567  | 577  | 586  | 595  | 605  | 614  | 623  | 633  | 642  |        |
| 464 | 652    | 661  | 670  | 680  | 689  | 698  | 708  | 717  | 726  | 736  | 9      |
| 465 | 745    | 754  | 764  | 773  | 782  | 792  | 801  | 810  | 820  | 829  | .1 0.9 |
| 466 | 838    | 848  | 857  | 866  | 876  | 885  | 894  | 904  | 913  | 922  | .2 1.9 |
| 467 | 931    | 941  | 950  | 959  | 969  | 978  | 987  | 996  | *006 | *015 | .3 2.8 |
| 468 | 67 024 | 034  | 043  | 052  | 061  | 071  | 080  | 089  | 099  | 108  |        |
| 469 | 117    | 126  | 136  | 145  | 154  | 163  | 173  | 182  | 191  | 200  | .4 3.8 |
| 470 | 210    | 219  | 228  | 237  | 246  | 256  | 265  | 274  | 283  | 293  | .5 4.7 |
| 471 | 302    | 311  | 320  | 329  | 339  | 348  | 357  | 366  | 376  | 385  | .6 5.7 |
| 472 | 394    | 403  | 412  | 422  | 431  | 440  | 449  | 458  | 467  | 477  | .7 6.6 |
| 473 | 486    | 495  | 504  | 513  | 523  | 532  | 541  | 550  | 559  | 568  | .8 7.6 |
| 474 | 578    | 587  | 596  | 605  | 614  | 623  | 633  | 642  | 651  | 660  | .9 8.5 |
| 475 | 669    | 678  | 687  | 697  | 706  | 715  | 724  | 733  | 742  | 751  |        |
| 476 | 760    | 770  | 779  | 788  | 797  | 806  | 815  | 824  | 833  | 842  |        |
| 477 | 852    | 861  | 870  | 879  | 888  | 897  | 906  | 915  | 924  | 933  | 9      |
| 478 | 943    | 952  | 961  | 970  | 979  | 988  | 997  | *006 | *015 | *024 | .1 0.9 |
| 479 | 68 033 | 042  | 051  | 060  | 070  | 079  | 088  | 097  | 106  | 115  | .2 1.8 |
| 480 | 124    | 133  | 142  | 151  | 160  | 169  | 178  | 187  | 196  | 205  | .3 2.7 |
| 481 | 214    | 223  | 232  | 241  | 250  | 259  | 268  | 277  | 286  | 295  | .4 3.6 |
| 482 | 304    | 313  | 322  | 331  | 340  | 349  | 358  | 367  | 376  | 385  | .5 4.5 |
| 483 | 394    | 403  | 412  | 421  | 430  | 439  | 448  | 457  | 466  | 475  | .6 5.4 |
| 484 | 484    | 493  | 502  | 511  | 520  | 529  | 538  | 547  | 556  | 565  | .7 6.3 |
| 485 | 574    | 583  | 592  | 601  | 610  | 619  | 628  | 637  | 646  | 654  | .8 7.2 |
| 486 | 663    | 672  | 681  | 690  | 699  | 708  | 717  | 726  | 735  | 744  | .9 8.1 |
| 487 | 753    | 762  | 770  | 779  | 788  | 797  | 806  | 815  | 824  | 833  |        |
| 488 | 842    | 851  | 860  | 868  | 877  | 886  | 895  | 904  | 913  | 922  |        |
| 489 | 931    | 940  | 948  | 957  | 966  | 975  | 984  | 993  | *002 | *010 | 8      |
| 490 | 69 019 | 028  | 037  | 046  | 055  | 064  | 073  | 081  | 090  | 099  | .1 0.8 |
| 491 | 108    | 117  | 126  | 134  | 143  | 152  | 161  | 170  | 179  | 187  | .2 1.7 |
| 492 | 196    | 205  | 214  | 223  | 232  | 240  | 249  | 258  | 267  | 276  | .3 2.5 |
| 493 | 284    | 293  | 302  | 311  | 320  | 328  | 337  | 346  | 355  | 364  | .4 3.4 |
| 494 | 372    | 381  | 390  | 399  | 408  | 416  | 425  | 434  | 443  | 451  | .5 4.2 |
| 495 | 460    | 469  | 478  | 487  | 495  | 504  | 513  | 522  | 530  | 539  | .6 5.1 |
| 496 | 548    | 557  | 565  | 574  | 583  | 592  | 600  | 609  | 618  | 627  | .7 5.9 |
| 497 | 635    | 644  | 653  | 662  | 670  | 679  | 688  | 697  | 705  | 714  | .8 6.8 |
| 498 | 723    | 731  | 740  | 749  | 758  | 766  | 775  | 784  | 792  | 801  | .9 7.6 |
| 499 | 810    | 819  | 827  | 836  | 845  | 853  | 862  | 871  | 879  | 888  |        |
| 500 | 897    | 905  | 914  | 923  | 931  | 940  | 949  | 958  | 966  | 975  |        |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|------|------|------|------|------|------|------|------|------|--------|
| 500 | 69 897 | 905  | 914  | 923  | 931  | 940  | 949  | 958  | 966  | 975  |        |
| 501 | 984    | 992  | *001 | *010 | *018 | *027 | *036 | *044 | *053 | *061 |        |
| 502 | 70 070 | 079  | 087  | 096  | 105  | 113  | 122  | 131  | 139  | 148  | 9      |
| 503 | 157    | 165  | 174  | 182  | 191  | 200  | 208  | 217  | 226  | 234  | .1 0.9 |
| 504 | 243    | 251  | 260  | 269  | 277  | 286  | 294  | 303  | 312  | 320  | .2 1.8 |
| 505 | 329    | 337  | 346  | 355  | 363  | 372  | 380  | 389  | 398  | 406  | .3 2.7 |
| 506 | 415    | 423  | 432  | 441  | 449  | 458  | 466  | 475  | 483  | 492  |        |
| 507 | 501    | 509  | 518  | 526  | 535  | 543  | 552  | 560  | 569  | 578  | .4 3.6 |
| 508 | 586    | 595  | 603  | 612  | 620  | 629  | 637  | 646  | 654  | 663  | .5 4.5 |
| 509 | 672    | 680  | 689  | 697  | 706  | 714  | 723  | 731  | 740  | 748  | .6 5.4 |
| 510 | 757    | 765  | 774  | 782  | 791  | 799  | 808  | 816  | 825  | 833  | .7 6.3 |
| 511 | 842    | 850  | 859  | 867  | 876  | 884  | 893  | 901  | 910  | 918  | .8 7.2 |
| 512 | 927    | 935  | 944  | 952  | 961  | 969  | 978  | 986  | 995  | *003 | .9 8.1 |
| 513 | 71 011 | 020  | 028  | 037  | 045  | 054  | 062  | 071  | 079  | 088  |        |
| 514 | 096    | 105  | 113  | 121  | 130  | 138  | 147  | 155  | 164  | 172  | 8      |
| 515 | 180    | 189  | 197  | 206  | 214  | 223  | 231  | 239  | 248  | 256  | .1 0.8 |
| 516 | 265    | 273  | 282  | 290  | 298  | 307  | 315  | 324  | 332  | 340  | .2 1.7 |
| 517 | 349    | 357  | 366  | 374  | 382  | 391  | 399  | 408  | 416  | 424  | .3 2.5 |
| 518 | 433    | 441  | 449  | 458  | 466  | 475  | 483  | 491  | 500  | 508  |        |
| 519 | 516    | 525  | 533  | 542  | 550  | 558  | 567  | 575  | 583  | 592  | .4 3.4 |
| 520 | 600    | 608  | 617  | 625  | 633  | 642  | 650  | 659  | 667  | 675  | .5 4.2 |
| 521 | 684    | 692  | 700  | 709  | 717  | 725  | 734  | 742  | 750  | 758  | .6 5.1 |
| 522 | 767    | 775  | 783  | 792  | 800  | 808  | 817  | 825  | 833  | 842  | .7 5.9 |
| 523 | 850    | 858  | 867  | 875  | 883  | 891  | 900  | 908  | 916  | 925  | .8 6.8 |
| 524 | 933    | 941  | 949  | 958  | 966  | 974  | 983  | 991  | 999  | *007 | .9 7.6 |
| 525 | 72 016 | 024  | 032  | 040  | 049  | 057  | 065  | 074  | 082  | 090  |        |
| 526 | 098    | 107  | 115  | 123  | 131  | 140  | 148  | 156  | 164  | 173  | 8      |
| 527 | 181    | 189  | 197  | 206  | 214  | 222  | 230  | 238  | 247  | 255  | .1 0.8 |
| 528 | 263    | 271  | 280  | 288  | 296  | 304  | 312  | 321  | 329  | 337  | .2 1.6 |
| 529 | 345    | 354  | 362  | 370  | 378  | 386  | 395  | 403  | 411  | 419  | .3 2.4 |
| 530 | 427    | 436  | 444  | 452  | 460  | 468  | 476  | 485  | 493  | 501  |        |
| 531 | 509    | 517  | 526  | 534  | 542  | 550  | 558  | 566  | 575  | 583  | .4 3.2 |
| 532 | 591    | 599  | 607  | 615  | 624  | 632  | 640  | 648  | 656  | 664  | .5 4.0 |
| 533 | 672    | 681  | 689  | 697  | 705  | 713  | 721  | 729  | 738  | 746  | .6 4.8 |
| 534 | 754    | 762  | 770  | 778  | 786  | 795  | 803  | 811  | 819  | 827  | .7 5.6 |
| 535 | 835    | 843  | 851  | 859  | 868  | 876  | 884  | 892  | 900  | 908  | .8 6.4 |
| 536 | 916    | 924  | 932  | 941  | 949  | 957  | 965  | 973  | 981  | 989  | .9 7.2 |
| 537 | 997    | *005 | *013 | *021 | *030 | *038 | *046 | *054 | *062 | *070 |        |
| 538 | 73 078 | 086  | 094  | 102  | 110  | 118  | 126  | 134  | 143  | 151  | 7      |
| 539 | 159    | 167  | 175  | 183  | 191  | 199  | 207  | 215  | 223  | 231  |        |
| 540 | 239    | 247  | 255  | 263  | 271  | 279  | 287  | 295  | 303  | 311  | .1 0.7 |
| 541 | 319    | 328  | 336  | 344  | 352  | 360  | 368  | 376  | 384  | 392  | .2 1.5 |
| 542 | 400    | 408  | 416  | 424  | 432  | 440  | 448  | 456  | 464  | 472  | .3 2.2 |
| 543 | 480    | 488  | 496  | 504  | 512  | 520  | 528  | 536  | 544  | 552  | .4 3.0 |
| 544 | 560    | 568  | 576  | 584  | 592  | 600  | 608  | 615  | 623  | 631  | .5 3.7 |
| 545 | 639    | 647  | 655  | 663  | 671  | 679  | 687  | 695  | 703  | 711  | .6 4.5 |
| 546 | 719    | 727  | 735  | 743  | 751  | 759  | 767  | 775  | 783  | 791  | .7 5.2 |
| 547 | 798    | 806  | 814  | 822  | 830  | 838  | 846  | 854  | 862  | 870  | .8 6.0 |
| 548 | 878    | 886  | 894  | 902  | 909  | 917  | 925  | 933  | 941  | 949  | .9 6.7 |
| 549 | 957    | 965  | 973  | 981  | 989  | 997  | *004 | *012 | *020 | *028 |        |
| 550 | 74 036 | 044  | 052  | 060  | 068  | 075  | 083  | 091  | 099  | 107  |        |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|-----|-----|-----|------|------|------|------|------|------|--------|
| 550 | 74 036 | 044 | 052 | 060 | 068  | 075  | 083  | 091  | 099  | 107  |        |
| 551 | 115    | 123 | 131 | 139 | 146  | 154  | 162  | 170  | 178  | 186  |        |
| 552 | 194    | 202 | 209 | 217 | 225  | 233  | 241  | 249  | 257  | 264  |        |
| 553 | 272    | 280 | 288 | 296 | 304  | 312  | 319  | 327  | 335  | 343  |        |
| 554 | 351    | 359 | 366 | 374 | 382  | 390  | 398  | 406  | 413  | 421  |        |
| 555 | 429    | 437 | 445 | 453 | 460  | 468  | 476  | 484  | 492  | 499  |        |
| 556 | 507    | 515 | 523 | 531 | 538  | 546  | 554  | 562  | 570  | 577  | 8      |
| 557 | 585    | 593 | 601 | 609 | 616  | 624  | 632  | 640  | 648  | 655  | .1 0.8 |
| 558 | 663    | 671 | 679 | 687 | 694  | 702  | 710  | 718  | 725  | 733  | .2 1.6 |
| 559 | 741    | 749 | 756 | 764 | 772  | 780  | 788  | 795  | 803  | 811  | .3 2.4 |
| 560 | 819    | 826 | 834 | 842 | 850  | 857  | 865  | 873  | 881  | 888  | .4 3.2 |
| 561 | 896    | 904 | 912 | 919 | 927  | 935  | 942  | 950  | 958  | 966  | .5 4.0 |
| 562 | 973    | 981 | 989 | 997 | *004 | *012 | *020 | *027 | *035 | *043 | .6 4.8 |
| 563 | 75 051 | 058 | 066 | 074 | 081  | 089  | 097  | 105  | 112  | 120  | .7 5.6 |
| 564 | 128    | 135 | 143 | 151 | 158  | 166  | 174  | 182  | 189  | 197  | .8 6.4 |
| 565 | 205    | 212 | 220 | 228 | 235  | 243  | 251  | 258  | 266  | 274  | .9 7.2 |
| 566 | 281    | 289 | 297 | 304 | 312  | 320  | 327  | 335  | 343  | 350  |        |
| 567 | 358    | 366 | 373 | 381 | 389  | 396  | 404  | 412  | 419  | 427  |        |
| 568 | 435    | 442 | 450 | 458 | 465  | 473  | 480  | 488  | 496  | 503  |        |
| 569 | 511    | 519 | 526 | 534 | 541  | 549  | 557  | 564  | 572  | 580  |        |
| 570 | 587    | 595 | 602 | 610 | 618  | 625  | 633  | 641  | 648  | 656  | 9      |
| 571 | 663    | 671 | 679 | 686 | 694  | 701  | 709  | 717  | 724  | 732  | .1 0.7 |
| 572 | 739    | 747 | 755 | 762 | 770  | 777  | 785  | 792  | 800  | 808  | .2 1.5 |
| 573 | 815    | 823 | 830 | 838 | 846  | 853  | 861  | 868  | 876  | 883  | .3 2.2 |
| 574 | 891    | 899 | 906 | 914 | 921  | 929  | 936  | 944  | 951  | 959  | .4 3.0 |
| 575 | 967    | 974 | 982 | 989 | 997  | *004 | *012 | *019 | *027 | *034 | .5 3.7 |
| 576 | 76 042 | 050 | 057 | 065 | 072  | 080  | 087  | 095  | 102  | 110  | .6 4.5 |
| 577 | 117    | 125 | 132 | 140 | 147  | 155  | 162  | 170  | 178  | 185  |        |
| 578 | 193    | 200 | 208 | 215 | 223  | 230  | 238  | 245  | 253  | 260  | .7 5.2 |
| 579 | 268    | 275 | 283 | 290 | 298  | 305  | 313  | 320  | 328  | 335  | .8 6.0 |
| 580 | 343    | 350 | 358 | 365 | 372  | 380  | 387  | 395  | 402  | 410  | .9 6.7 |
| 581 | 417    | 425 | 432 | 440 | 447  | 455  | 462  | 470  | 477  | 485  |        |
| 582 | 492    | 500 | 507 | 514 | 522  | 529  | 537  | 544  | 552  | 559  |        |
| 583 | 567    | 574 | 582 | 589 | 596  | 604  | 611  | 619  | 626  | 634  |        |
| 584 | 641    | 648 | 656 | 663 | 671  | 678  | 686  | 693  | 700  | 708  |        |
| 585 | 715    | 723 | 730 | 738 | 745  | 752  | 760  | 767  | 775  | 782  |        |
| 586 | 790    | 797 | 804 | 812 | 819  | 827  | 834  | 841  | 849  | 856  | 7      |
| 587 | 864    | 871 | 878 | 886 | 893  | 901  | 908  | 915  | 923  | 930  | .1 0.7 |
| 588 | 937    | 945 | 952 | 960 | 967  | 974  | 982  | 989  | 997  | *004 | .2 1.4 |
| 589 | 77 011 | 019 | 026 | 033 | 041  | 048  | 055  | 063  | 070  | 078  | .3 2.1 |
| 590 | 085    | 092 | 100 | 107 | 114  | 122  | 129  | 136  | 144  | 151  | .4 2.8 |
| 591 | 158    | 166 | 173 | 181 | 188  | 195  | 203  | 210  | 217  | 225  | .5 3.5 |
| 592 | 232    | 239 | 247 | 254 | 261  | 269  | 276  | 283  | 291  | 298  | .6 4.2 |
| 593 | 305    | 313 | 320 | 327 | 335  | 342  | 349  | 356  | 364  | 371  | .7 4.9 |
| 594 | 378    | 386 | 393 | 400 | 408  | 415  | 422  | 430  | 437  | 444  | .8 5.6 |
| 595 | 451    | 459 | 466 | 473 | 481  | 488  | 495  | 503  | 510  | 517  | .9 6.3 |
| 596 | 524    | 532 | 539 | 546 | 554  | 561  | 568  | 575  | 583  | 590  |        |
| 597 | 597    | 604 | 612 | 619 | 626  | 634  | 641  | 648  | 655  | 663  |        |
| 598 | 670    | 677 | 684 | 692 | 699  | 706  | 713  | 721  | 728  | 735  |        |
| 599 | 742    | 750 | 757 | 764 | 771  | 779  | 786  | 793  | 800  | 808  |        |
| 600 | 815    | 822 | 829 | 837 | 844  | 851  | 858  | 866  | 873  | 880  |        |
| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4   | 5   | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|-----|-----|-----|-----|-----|------|------|------|------|--------|
| 600 | 77 815 | 822 | 829 | 837 | 844 | 851 | 858  | 866  | 873  | 880  |        |
| 601 | 887    | 894 | 902 | 909 | 916 | 923 | 931  | 938  | 945  | 952  |        |
| 602 | 959    | 967 | 974 | 981 | 988 | 995 | *003 | *010 | *017 | *024 |        |
| 603 | 78 031 | 039 | 046 | 053 | 060 | 067 | 075  | 082  | 089  | 096  |        |
| 604 | 103    | 111 | 118 | 125 | 132 | 139 | 147  | 154  | 161  | 168  |        |
| 605 | 175    | 182 | 190 | 197 | 204 | 211 | 218  | 226  | 233  | 240  |        |
| 606 | 247    | 254 | 261 | 269 | 276 | 283 | 290  | 297  | 304  | 311  | 7      |
| 607 | 319    | 326 | 333 | 340 | 347 | 354 | 362  | 369  | 376  | 383  | .1 0.7 |
| 608 | 390    | 397 | 404 | 412 | 419 | 426 | 433  | 440  | 447  | 454  | .2 1.5 |
| 609 | 461    | 469 | 476 | 483 | 490 | 497 | 504  | 511  | 518  | 526  | .3 2.2 |
| 610 | 533    | 540 | 547 | 554 | 561 | 568 | 575  | 583  | 590  | 597  | .4 3.0 |
| 611 | 604    | 611 | 618 | 625 | 632 | 639 | 646  | 654  | 661  | 668  | .5 3.7 |
| 612 | 675    | 682 | 689 | 696 | 703 | 710 | 717  | 725  | 732  | 739  | .6 4.5 |
| 613 | 746    | 753 | 760 | 767 | 774 | 781 | 788  | 795  | 802  | 810  | .7 5.2 |
| 614 | 817    | 824 | 831 | 838 | 845 | 852 | 859  | 866  | 873  | 886  | .8 6.0 |
| 615 | 887    | 894 | 901 | 908 | 915 | 923 | 930  | 937  | 944  | 951  | .9 6.7 |
| 616 | 958    | 965 | 972 | 979 | 986 | 993 | *000 | *007 | *014 | *021 |        |
| 617 | 79 028 | 035 | 042 | 049 | 056 | 063 | 070  | 078  | 085  | 092  |        |
| 618 | 099    | 106 | 113 | 120 | 127 | 134 | 141  | 148  | 155  | 162  |        |
| 619 | 169    | 176 | 183 | 190 | 197 | 204 | 211  | 218  | 225  | 232  |        |
| 620 | 239    | 246 | 253 | 260 | 267 | 274 | 281  | 288  | 295  | 302  | 7      |
| 621 | 309    | 316 | 323 | 330 | 337 | 344 | 351  | 358  | 365  | 372  | .1 0.7 |
| 622 | 379    | 386 | 393 | 400 | 407 | 414 | 421  | 428  | 435  | 442  | .2 1.4 |
| 623 | 449    | 456 | 462 | 469 | 476 | 483 | 490  | 497  | 504  | 511  | .3 2.1 |
| 624 | 518    | 525 | 532 | 539 | 546 | 553 | 560  | 567  | 574  | 581  | .4 2.8 |
| 625 | 588    | 595 | 602 | 609 | 616 | 622 | 629  | 636  | 643  | 650  | .5 3.5 |
| 626 | 657    | 664 | 671 | 678 | 685 | 692 | 699  | 706  | 713  | 720  | .6 4.2 |
| 627 | 727    | 733 | 740 | 747 | 754 | 761 | 768  | 775  | 782  | 789  | .7 4.9 |
| 628 | 796    | 803 | 810 | 816 | 823 | 830 | 837  | 844  | 851  | 858  | .8 5.6 |
| 629 | 865    | 872 | 879 | 886 | 892 | 899 | 906  | 913  | 920  | 927  | .9 6.3 |
| 630 | 934    | 941 | 948 | 954 | 961 | 968 | 975  | 982  | 989  | 996  |        |
| 631 | 80 003 | 010 | 016 | 023 | 030 | 037 | 044  | 051  | 058  | 065  |        |
| 632 | 071    | 078 | 085 | 092 | 099 | 106 | 113  | 120  | 126  | 133  |        |
| 633 | 140    | 147 | 154 | 161 | 168 | 174 | 181  | 188  | 195  | 202  |        |
| 634 | 209    | 216 | 222 | 229 | 236 | 243 | 250  | 257  | 263  | 270  |        |
| 635 | 277    | 284 | 291 | 298 | 304 | 311 | 318  | 325  | 332  | 339  |        |
| 636 | 345    | 352 | 359 | 366 | 373 | 380 | 386  | 393  | 400  | 407  | 6      |
| 637 | 414    | 421 | 427 | 434 | 441 | 448 | 455  | 461  | 468  | 475  | .1 0.6 |
| 638 | 482    | 489 | 495 | 502 | 509 | 516 | 523  | 529  | 536  | 543  | .2 1.3 |
| 639 | 550    | 557 | 563 | 570 | 577 | 584 | 591  | 597  | 604  | 611  | .3 1.9 |
| 640 | 618    | 625 | 631 | 638 | 645 | 652 | 658  | 665  | 672  | 679  | .4 2.6 |
| 641 | 686    | 692 | 699 | 706 | 713 | 719 | 726  | 733  | 740  | 746  | .5 3.2 |
| 642 | 753    | 760 | 767 | 774 | 780 | 787 | 794  | 801  | 807  | 814  | .6 3.9 |
| 643 | 821    | 828 | 834 | 841 | 848 | 855 | 862  | 868  | 875  | 882  | .7 4.5 |
| 644 | 888    | 895 | 902 | 909 | 915 | 922 | 929  | 936  | 942  | 949  | .8 5.2 |
| 645 | 956    | 962 | 969 | 976 | 983 | 989 | 996  | *003 | *010 | *016 | .9 5.8 |
| 646 | 81 023 | 030 | 036 | 043 | 050 | 057 | 063  | 070  | 077  | 083  |        |
| 647 | 090    | 097 | 104 | 110 | 117 | 124 | 130  | 137  | 144  | 151  |        |
| 648 | 157    | 164 | 171 | 177 | 184 | 191 | 197  | 204  | 211  | 218  |        |
| 649 | 224    | 231 | 238 | 244 | 251 | 258 | 264  | 271  | 278  | 284  |        |
| 650 | 291    | 298 | 304 | 311 | 318 | 324 | 331  | 338  | 345  | 351  |        |
| N.  | 0      | 1   | 2   | 3   | 4   | 5   | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.         | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|------------|--------|------|------|------|------|------|------|------|------|------|--------|
| <b>650</b> | 81 29î | 298  | 304  | 31î  | 318  | 324  | 33î  | 338  | 345  | 35î  |        |
| 651        | 358    | 365  | 37î  | 378  | 385  | 39î  | 398  | 405  | 41î  | 418  |        |
| 652        | 425    | 43î  | 438  | 444  | 45î  | 458  | 464  | 47î  | 478  | 484  |        |
| 653        | 49î    | 498  | 504  | 51î  | 518  | 524  | 531  | 538  | 544  | 551  |        |
| 654        | 558    | 564  | 571  | 577  | 584  | 591  | 597  | 604  | 611  | 617  |        |
| 655        | 624    | 631  | 637  | 644  | 650  | 657  | 664  | 670  | 677  | 684  |        |
| 656        | 690    | 697  | 703  | 710  | 717  | 723  | 730  | 736  | 743  | 750  | 7      |
| 657        | 756    | 763  | 770  | 776  | 783  | 789  | 796  | 803  | 809  | 816  | .1 0.7 |
| 658        | 822    | 829  | 836  | 842  | 849  | 855  | 862  | 869  | 875  | 882  | .2 1.4 |
| 659        | 888    | 895  | 90î  | 908  | 915  | 92î  | 928  | 934  | 941  | 948  | .3 2.1 |
| <b>660</b> | 954    | 961  | 967  | 974  | 980  | 987  | 994  | *000 | *007 | *013 | .4 2.8 |
| 661        | 82 020 | 026  | 033  | 040  | 046  | 053  | 059  | 066  | 072  | 079  | .5 3.5 |
| 662        | 086    | 092  | 099  | 105  | 112  | 118  | 125  | 131  | 138  | 145  | .6 4.2 |
| 663        | 15î    | 158  | 164  | 171  | 177  | 184  | 190  | 197  | 203  | 210  |        |
| 664        | 217    | 223  | 230  | 236  | 243  | 249  | 256  | 262  | 269  | 275  | .7 4.9 |
| 665        | 282    | 288  | 295  | 302  | 308  | 315  | 321  | 328  | 334  | 341  | .8 5.6 |
| 666        | 347    | 354  | 360  | 367  | 373  | 380  | 386  | 393  | 399  | 406  | .9 6.3 |
| 667        | 412    | 419  | 425  | 432  | 438  | 445  | 451  | 458  | 464  | 471  |        |
| 668        | 477    | 484  | 490  | 497  | 503  | 510  | 516  | 523  | 529  | 536  |        |
| 669        | 542    | 549  | 555  | 562  | 568  | 575  | 581  | 588  | 594  | 601  |        |
| <b>670</b> | 607    | 614  | 620  | 627  | 633  | 640  | 646  | 653  | 659  | 666  | 6      |
| 671        | 672    | 678  | 685  | 691  | 698  | 704  | 711  | 717  | 724  | 730  |        |
| 672        | 737    | 743  | 750  | 756  | 763  | 769  | 775  | 782  | 788  | 795  | .1 0.6 |
| 673        | 80î    | 808  | 814  | 821  | 827  | 834  | 840  | 846  | 853  | 859  | .2 1.3 |
| 674        | 866    | 872  | 879  | 885  | 892  | 898  | 904  | 911  | 917  | 924  | .3 1.9 |
| 675        | 930    | 937  | 943  | 949  | 956  | 962  | 969  | 975  | 982  | 988  |        |
| 676        | 994    | *001 | *007 | *014 | *020 | *027 | *033 | *039 | *046 | *052 | .4 2.6 |
| 677        | 83 059 | 065  | 071  | 078  | 084  | 091  | 097  | 103  | 110  | 116  | .5 3.2 |
| 678        | 123    | 129  | 136  | 142  | 148  | 155  | 161  | 168  | 174  | 180  | .6 3.9 |
| 679        | 187    | 193  | 200  | 206  | 212  | 219  | 225  | 231  | 238  | 244  | .7 4.5 |
| <b>680</b> | 251    | 257  | 263  | 270  | 276  | 283  | 289  | 295  | 302  | 308  | .8 5.2 |
| 681        | 314    | 321  | 327  | 334  | 340  | 346  | 353  | 359  | 365  | 372  | .9 5.8 |
| 682        | 378    | 385  | 391  | 397  | 404  | 410  | 416  | 423  | 429  | 435  |        |
| 683        | 442    | 448  | 455  | 461  | 467  | 474  | 480  | 486  | 493  | 499  |        |
| 684        | 505    | 512  | 518  | 524  | 531  | 537  | 543  | 550  | 556  | 562  |        |
| 685        | 569    | 575  | 581  | 588  | 594  | 600  | 607  | 613  | 619  | 626  | 6      |
| 686        | 632    | 638  | 645  | 651  | 657  | 664  | 670  | 676  | 683  | 689  |        |
| 687        | 695    | 702  | 708  | 714  | 721  | 727  | 733  | 740  | 746  | 752  | .1 0.6 |
| 688        | 759    | 765  | 771  | 778  | 784  | 790  | 796  | 803  | 809  | 815  | .2 1.2 |
| 689        | 822    | 828  | 834  | 841  | 847  | 853  | 859  | 866  | 872  | 878  | .3 1.8 |
| <b>690</b> | 885    | 891  | 897  | 904  | 910  | 916  | 922  | 929  | 935  | 941  |        |
| 691        | 948    | 954  | 960  | 966  | 973  | 979  | 985  | 992  | 998  | *004 | .4 2.4 |
| 692        | 84 010 | 017  | 023  | 029  | 035  | 042  | 048  | 054  | 061  | 067  | .5 3.0 |
| 693        | 073    | 079  | 086  | 092  | 098  | 104  | 111  | 117  | 123  | 129  | .6 3.6 |
| 694        | 136    | 142  | 148  | 154  | 161  | 167  | 173  | 179  | 186  | 192  | .7 4.2 |
| 695        | 198    | 204  | 211  | 217  | 223  | 229  | 236  | 242  | 248  | 254  | .8 4.8 |
| 696        | 261    | 267  | 273  | 279  | 286  | 292  | 298  | 304  | 311  | 317  | .9 5.4 |
| 697        | 323    | 329  | 335  | 342  | 348  | 354  | 360  | 367  | 373  | 379  |        |
| 698        | 385    | 392  | 398  | 404  | 410  | 416  | 423  | 429  | 435  | 441  |        |
| 699        | 447    | 454  | 460  | 466  | 472  | 479  | 485  | 491  | 497  | 503  |        |
| <b>700</b> | 510    | 516  | 522  | 528  | 534  | 541  | 547  | 553  | 559  | 565  |        |
| N.         | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.         | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.    |
|------------|--------|-----|-----|-----|------|------|------|------|------|------|----------|
| <b>700</b> | 84 510 | 516 | 522 | 528 | 534  | 541  | 547  | 553  | 559  | 565  |          |
| 701        | 572    | 578 | 584 | 590 | 596  | 603  | 609  | 615  | 621  | 627  |          |
| 702        | 633    | 640 | 646 | 652 | 658  | 664  | 671  | 677  | 683  | 689  |          |
| 703        | 695    | 701 | 708 | 714 | 720  | 726  | 732  | 739  | 745  | 751  |          |
| 704        | 757    | 763 | 769 | 776 | 782  | 788  | 794  | 800  | 806  | 813  |          |
| 705        | 819    | 825 | 831 | 837 | 843  | 849  | 856  | 862  | 868  | 874  |          |
| 706        | 880    | 886 | 893 | 899 | 905  | 911  | 917  | 923  | 929  | 936  | <b>8</b> |
| 707        | 942    | 948 | 954 | 960 | 966  | 972  | 979  | 985  | 991  | 997  | .1 0.6   |
| 708        | 85 003 | 009 | 015 | 021 | 028  | 034  | 040  | 046  | 052  | 058  | .2 1.3   |
| 709        | 064    | 070 | 077 | 083 | 089  | 095  | 101  | 107  | 113  | 119  | .3 1.9   |
| <b>710</b> | 126    | 132 | 138 | 144 | 150  | 156  | 162  | 168  | 174  | 181  | .4 2.6   |
| 711        | 187    | 193 | 199 | 205 | 211  | 217  | 223  | 229  | 236  | 242  | .5 3.2   |
| 712        | 248    | 254 | 260 | 266 | 272  | 278  | 284  | 290  | 297  | 303  | .6 3.9   |
| 713        | 309    | 315 | 321 | 327 | 333  | 339  | 345  | 351  | 357  | 363  | .7 4.5   |
| 714        | 370    | 376 | 382 | 388 | 394  | 400  | 406  | 412  | 418  | 424  | .8 5.2   |
| 715        | 430    | 436 | 443 | 449 | 455  | 461  | 467  | 473  | 479  | 485  | .9 5.8   |
| 716        | 491    | 497 | 503 | 509 | 515  | 521  | 527  | 533  | 540  | 546  |          |
| 717        | 552    | 558 | 564 | 570 | 576  | 582  | 588  | 594  | 600  | 606  |          |
| 718        | 612    | 618 | 624 | 630 | 636  | 642  | 648  | 655  | 661  | 667  |          |
| 719        | 673    | 679 | 685 | 691 | 697  | 703  | 709  | 715  | 721  | 727  |          |
| <b>720</b> | 733    | 739 | 745 | 751 | 757  | 763  | 769  | 775  | 781  | 787  | <b>6</b> |
| 721        | 793    | 799 | 805 | 811 | 817  | 823  | 829  | 835  | 841  | 847  | .1 0.6   |
| 722        | 853    | 859 | 865 | 872 | 878  | 884  | 890  | 896  | 902  | 908  | .2 1.2   |
| 723        | 914    | 920 | 926 | 932 | 938  | 944  | 950  | 956  | 962  | 968  | .3 1.8   |
| 724        | 974    | 980 | 986 | 992 | 998  | *004 | *010 | *016 | *022 | *028 | .4 2.4   |
| 725        | 86 034 | 040 | 046 | 052 | 058  | 063  | 069  | 075  | 081  | 087  | .5 3.0   |
| 726        | 093    | 099 | 105 | 111 | 117  | 123  | 129  | 135  | 141  | 147  | .6 3.6   |
| 727        | 153    | 159 | 165 | 171 | 177  | 183  | 189  | 195  | 201  | 207  | .7 4.2   |
| 728        | 213    | 219 | 225 | 231 | 237  | 243  | 249  | 255  | 261  | 267  | .8 4.8   |
| 729        | 273    | 278 | 284 | 290 | 296  | 302  | 308  | 314  | 320  | 326  | .9 5.4   |
| <b>730</b> | 332    | 338 | 344 | 350 | 356  | 362  | 368  | 374  | 380  | 386  |          |
| 731        | 391    | 397 | 403 | 409 | 415  | 421  | 427  | 433  | 439  | 445  |          |
| 732        | 451    | 457 | 463 | 469 | 475  | 481  | 486  | 492  | 498  | 504  |          |
| 733        | 510    | 516 | 522 | 528 | 534  | 540  | 546  | 552  | 558  | 563  |          |
| 734        | 569    | 575 | 581 | 587 | 593  | 599  | 605  | 611  | 617  | 623  |          |
| 735        | 628    | 634 | 640 | 646 | 652  | 658  | 664  | 670  | 676  | 682  |          |
| 736        | 688    | 693 | 699 | 705 | 711  | 717  | 723  | 729  | 735  | 741  | <b>5</b> |
| 737        | 746    | 752 | 758 | 764 | 770  | 776  | 782  | 788  | 794  | 800  | .1 0.5   |
| 738        | 805    | 811 | 817 | 823 | 829  | 835  | 841  | 847  | 852  | 858  | .2 1.1   |
| 739        | 864    | 870 | 876 | 882 | 888  | 894  | 899  | 905  | 911  | 917  | .3 1.6   |
| <b>740</b> | 923    | 929 | 935 | 941 | 946  | 952  | 958  | 964  | 970  | 976  | .4 2.2   |
| 741        | 982    | 987 | 993 | 999 | *005 | *011 | *017 | *023 | *028 | *034 | .5 2.7   |
| 742        | 87 040 | 046 | 052 | 058 | 064  | 069  | 075  | 081  | 087  | 093  | .6 3.3   |
| 743        | 099    | 104 | 110 | 116 | 122  | 128  | 134  | 140  | 145  | 151  | .7 3.8   |
| 744        | 157    | 163 | 169 | 175 | 180  | 186  | 192  | 198  | 204  | 210  | .8 4.4   |
| 745        | 215    | 221 | 227 | 233 | 239  | 245  | 250  | 256  | 262  | 268  | .9 4.9   |
| 746        | 274    | 279 | 285 | 291 | 297  | 303  | 309  | 314  | 320  | 326  |          |
| 747        | 332    | 338 | 343 | 349 | 355  | 361  | 367  | 372  | 378  | 384  |          |
| 748        | 390    | 396 | 402 | 407 | 413  | 419  | 425  | 431  | 436  | 442  |          |
| 749        | 448    | 454 | 460 | 465 | 471  | 477  | 483  | 489  | 494  | 500  |          |
| <b>750</b> | 506    | 512 | 517 | 523 | 529  | 535  | 541  | 546  | 552  | 558  |          |
| N.         | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.    |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |     |
|-----|--------|-----|-----|------|------|------|------|------|------|------|-------|-----|
| 750 | 87 506 | 512 | 517 | 523  | 529  | 535  | 541  | 546  | 552  | 558  | 6     |     |
| 751 | 564    | 570 | 575 | 581  | 587  | 593  | 598  | 604  | 610  | 616  |       |     |
| 752 | 622    | 627 | 633 | 639  | 645  | 650  | 656  | 662  | 668  | 673  |       |     |
| 753 | 679    | 685 | 691 | 697  | 702  | 708  | 714  | 720  | 725  | 731  |       |     |
| 754 | 737    | 743 | 748 | 754  | 760  | 766  | 771  | 777  | 783  | 789  |       |     |
| 755 | 794    | 800 | 806 | 812  | 817  | 823  | 829  | 835  | 840  | 846  |       |     |
| 756 | 852    | 858 | 863 | 869  | 875  | 881  | 886  | 892  | 898  | 904  |       |     |
| 757 | 909    | 915 | 921 | 927  | 932  | 938  | 944  | 949  | 955  | 961  |       |     |
| 758 | 967    | 972 | 978 | 984  | 990  | 995  | *001 | *007 | *012 | *018 |       |     |
| 759 | 88 024 | 030 | 035 | 041  | 047  | 053  | 058  | 064  | 070  | 075  | .1    | 0.6 |
| 760 | 081    | 087 | 093 | 098  | 104  | 110  | 115  | 121  | 127  | 133  | .2    | 1.2 |
| 761 | 138    | 144 | 150 | 155  | 161  | 167  | 172  | 178  | 184  | 190  | .3    | 1.8 |
| 762 | 195    | 201 | 207 | 212  | 218  | 224  | 229  | 235  | 241  | 247  | .4    | 2.4 |
| 763 | 252    | 258 | 264 | 269  | 275  | 281  | 286  | 292  | 298  | 303  | .5    | 3.0 |
| 764 | 309    | 315 | 320 | 326  | 332  | 337  | 343  | 349  | 355  | 360  | .6    | 3.6 |
| 765 | 366    | 372 | 377 | 383  | 389  | 394  | 400  | 406  | 411  | 417  | .7    | 4.2 |
| 766 | 423    | 428 | 434 | 440  | 445  | 451  | 457  | 462  | 468  | 474  | .8    | 4.8 |
| 767 | 479    | 485 | 491 | 496  | 502  | 508  | 513  | 519  | 525  | 530  | .9    | 5.4 |
| 768 | 536    | 542 | 547 | 553  | 558  | 564  | 570  | 575  | 581  | 587  |       |     |
| 769 | 592    | 598 | 604 | 609  | 615  | 621  | 626  | 632  | 638  | 643  |       |     |
| 770 | 649    | 654 | 660 | 666  | 671  | 677  | 683  | 688  | 694  | 700  | 5     |     |
| 771 | 705    | 711 | 716 | 722  | 728  | 733  | 739  | 745  | 750  | 756  |       |     |
| 772 | 761    | 767 | 773 | 778  | 784  | 790  | 795  | 801  | 806  | 812  |       |     |
| 773 | 818    | 823 | 829 | 835  | 840  | 846  | 851  | 857  | 863  | 868  |       |     |
| 774 | 874    | 879 | 885 | 891  | 896  | 902  | 907  | 913  | 919  | 924  |       |     |
| 775 | 930    | 936 | 941 | 947  | 952  | 958  | 964  | 969  | 975  | 980  |       |     |
| 776 | 986    | 992 | 997 | *003 | *008 | *014 | *019 | *025 | *031 | *036 |       |     |
| 777 | 89 042 | 047 | 053 | 059  | 064  | 070  | 075  | 081  | 087  | 092  |       |     |
| 778 | 098    | 103 | 109 | 114  | 120  | 126  | 131  | 137  | 142  | 148  | .1    | 0.5 |
| 779 | 153    | 159 | 165 | 170  | 176  | 181  | 187  | 193  | 198  | 204  | .2    | 1.0 |
| 780 | 209    | 215 | 220 | 226  | 231  | 237  | 243  | 248  | 254  | 259  | .3    | 1.5 |
| 781 | 265    | 270 | 276 | 282  | 287  | 293  | 298  | 304  | 309  | 315  | 5     |     |
| 782 | 320    | 326 | 332 | 337  | 343  | 348  | 354  | 359  | 365  | 370  |       |     |
| 783 | 376    | 381 | 387 | 393  | 398  | 404  | 409  | 415  | 420  | 426  |       |     |
| 784 | 431    | 437 | 442 | 448  | 454  | 459  | 465  | 470  | 476  | 481  |       |     |
| 785 | 487    | 492 | 498 | 503  | 509  | 514  | 520  | 525  | 531  | 536  |       |     |
| 786 | 542    | 548 | 553 | 559  | 564  | 570  | 575  | 581  | 586  | 592  |       |     |
| 787 | 597    | 603 | 608 | 614  | 619  | 625  | 630  | 636  | 641  | 647  |       |     |
| 788 | 652    | 658 | 663 | 669  | 674  | 680  | 685  | 691  | 696  | 702  |       |     |
| 789 | 707    | 713 | 718 | 724  | 729  | 735  | 740  | 746  | 751  | 757  |       |     |
| 790 | 762    | 768 | 773 | 779  | 784  | 790  | 795  | 801  | 806  | 812  | .4    | 2.0 |
| 791 | 817    | 823 | 828 | 834  | 839  | 845  | 850  | 856  | 861  | 867  | .5    | 2.5 |
| 792 | 872    | 878 | 883 | 889  | 894  | 900  | 905  | 911  | 916  | 922  | .6    | 3.0 |
| 793 | 927    | 933 | 938 | 943  | 949  | 954  | 960  | 965  | 971  | 976  | .7    | 3.5 |
| 794 | 982    | 987 | 993 | 998  | *004 | *009 | *015 | *020 | *026 | *031 | .8    | 4.0 |
| 795 | 90 036 | 042 | 047 | 053  | 058  | 064  | 069  | 075  | 080  | 086  | .9    | 4.5 |
| 796 | 091    | 097 | 102 | 107  | 113  | 118  | 124  | 129  | 135  | 140  |       |     |
| 797 | 146    | 151 | 156 | 162  | 167  | 173  | 178  | 184  | 189  | 195  |       |     |
| 798 | 200    | 205 | 211 | 216  | 222  | 227  | 233  | 238  | 244  | 249  |       |     |
| 799 | 254    | 260 | 265 | 271  | 276  | 282  | 287  | 292  | 298  | 303  |       |     |
| 800 | 309    | 314 | 320 | 325  | 330  | 336  | 341  | 347  | 352  | 358  |       |     |
| N.  | 0      | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |     |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8    | 9    | P. P.  |
|-----|--------|-----|-----|-----|-----|-----|-----|-----|------|------|--------|
| 800 | 90 309 | 314 | 320 | 325 | 330 | 336 | 341 | 347 | 352  | 358  |        |
| 801 | 363    | 368 | 374 | 379 | 385 | 390 | 396 | 401 | 406  | 412  |        |
| 802 | 417    | 423 | 428 | 433 | 439 | 444 | 450 | 455 | 460  | 466  |        |
| 803 | 471    | 477 | 482 | 488 | 493 | 498 | 504 | 509 | 515  | 520  |        |
| 804 | 525    | 531 | 536 | 542 | 547 | 552 | 558 | 563 | 569  | 574  |        |
| 805 | 579    | 585 | 590 | 596 | 601 | 606 | 612 | 617 | 622  | 628  |        |
| 806 | 633    | 639 | 644 | 649 | 655 | 660 | 666 | 671 | 676  | 682  |        |
| 807 | 687    | 692 | 698 | 703 | 709 | 714 | 719 | 725 | 730  | 736  |        |
| 808 | 741    | 746 | 752 | 757 | 762 | 768 | 773 | 778 | 784  | 789  |        |
| 809 | 795    | 800 | 805 | 811 | 816 | 821 | 827 | 832 | 838  | 843  |        |
| 810 | 848    | 854 | 859 | 864 | 870 | 875 | 880 | 886 | 891  | 896  |        |
| 811 | 902    | 907 | 913 | 918 | 923 | 929 | 934 | 939 | 945  | 950  | 5      |
| 812 | 955    | 961 | 966 | 971 | 977 | 982 | 987 | 993 | 998  | *003 | .1 0.5 |
| 813 | 91 009 | 014 | 019 | 025 | 030 | 036 | 041 | 046 | 052  | 057  | .2 1.1 |
| 814 | 062    | 068 | 073 | 078 | 084 | 089 | 094 | 100 | 105  | 110  | .3 1.6 |
| 815 | 116    | 121 | 126 | 131 | 137 | 142 | 147 | 153 | 158  | 163  | .4 2.2 |
| 816 | 169    | 174 | 179 | 185 | 190 | 195 | 201 | 206 | 211  | 217  | .5 2.7 |
| 817 | 222    | 227 | 233 | 238 | 243 | 249 | 254 | 259 | 264  | 270  | .6 3.3 |
| 818 | 275    | 280 | 286 | 291 | 296 | 302 | 307 | 312 | 318  | 323  | .7 3.8 |
| 819 | 328    | 333 | 339 | 344 | 349 | 355 | 360 | 365 | 371  | 376  | .8 4.4 |
| 820 | 381    | 386 | 392 | 397 | 402 | 408 | 413 | 418 | 423  | 429  | .9 4.9 |
| 821 | 434    | 439 | 445 | 450 | 455 | 461 | 466 | 471 | 476  | 482  |        |
| 822 | 487    | 492 | 497 | 503 | 508 | 513 | 519 | 524 | 529  | 534  |        |
| 823 | 540    | 545 | 550 | 556 | 561 | 566 | 571 | 577 | 582  | 587  |        |
| 824 | 592    | 598 | 603 | 608 | 614 | 619 | 624 | 629 | 635  | 640  |        |
| 825 | 645    | 650 | 656 | 661 | 666 | 671 | 677 | 682 | 687  | 692  |        |
| 826 | 698    | 703 | 708 | 714 | 719 | 724 | 729 | 735 | 740  | 745  |        |
| 827 | 750    | 756 | 761 | 766 | 771 | 777 | 782 | 787 | 792  | 798  |        |
| 828 | 803    | 808 | 813 | 819 | 824 | 829 | 834 | 839 | 845  | 850  |        |
| 829 | 855    | 860 | 866 | 871 | 876 | 881 | 887 | 892 | 897  | 902  |        |
| 830 | 908    | 913 | 918 | 923 | 928 | 934 | 939 | 944 | 949  | 955  | 5      |
| 831 | 960    | 965 | 970 | 976 | 981 | 986 | 991 | 996 | *002 | *007 | .1 0.5 |
| 832 | 92 012 | 017 | 023 | 028 | 033 | 038 | 043 | 049 | 054  | 059  | .2 1.0 |
| 833 | 064    | 069 | 075 | 080 | 085 | 090 | 096 | 101 | 106  | 111  | .3 1.5 |
| 834 | 116    | 122 | 127 | 132 | 137 | 142 | 148 | 153 | 158  | 163  | .4 2.0 |
| 835 | 168    | 174 | 179 | 184 | 189 | 194 | 200 | 205 | 210  | 215  | .5 2.5 |
| 836 | 220    | 226 | 231 | 236 | 241 | 246 | 252 | 257 | 262  | 267  | .6 3.0 |
| 837 | 272    | 277 | 283 | 288 | 293 | 298 | 303 | 309 | 314  | 319  | .7 3.5 |
| 838 | 324    | 329 | 335 | 340 | 345 | 350 | 355 | 360 | 366  | 371  | .8 4.0 |
| 839 | 376    | 381 | 386 | 391 | 397 | 402 | 407 | 412 | 417  | 423  | .9 4.5 |
| 840 | 428    | 433 | 438 | 443 | 448 | 454 | 459 | 464 | 469  | 474  |        |
| 841 | 479    | 485 | 490 | 495 | 500 | 505 | 510 | 515 | 521  | 526  |        |
| 842 | 531    | 536 | 541 | 546 | 552 | 557 | 562 | 567 | 572  | 577  |        |
| 843 | 583    | 588 | 593 | 598 | 603 | 608 | 613 | 619 | 624  | 629  |        |
| 844 | 634    | 639 | 644 | 649 | 655 | 660 | 665 | 670 | 675  | 680  |        |
| 845 | 685    | 691 | 696 | 701 | 706 | 711 | 716 | 721 | 727  | 732  |        |
| 846 | 737    | 742 | 747 | 752 | 757 | 762 | 768 | 773 | 778  | 783  |        |
| 847 | 788    | 793 | 798 | 803 | 809 | 814 | 819 | 824 | 829  | 834  |        |
| 848 | 839    | 844 | 850 | 855 | 860 | 865 | 870 | 875 | 880  | 885  |        |
| 849 | 891    | 896 | 901 | 906 | 911 | 916 | 921 | 926 | 931  | 937  |        |
| 850 | 942    | 947 | 952 | 957 | 962 | 967 | 972 | 977 | 982  | 988  |        |
| N.  | 0      | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|-----|------|------|------|------|------|------|------|------|--------|
| 850 | 92 942 | 947 | 952  | 957  | 962  | 967  | 972  | 977  | 982  | 988  |        |
| 851 | 993    | 998 | *003 | *008 | *013 | *018 | *023 | *028 | *034 | *039 |        |
| 852 | 93 044 | 049 | 054  | 059  | 064  | 069  | 074  | 079  | 084  | 090  |        |
| 853 | 095    | 100 | 105  | 110  | 115  | 120  | 125  | 130  | 135  | 140  |        |
| 854 | 146    | 151 | 156  | 161  | 166  | 171  | 176  | 181  | 186  | 191  |        |
| 855 | 196    | 201 | 207  | 212  | 217  | 222  | 227  | 232  | 237  | 242  |        |
| 856 | 247    | 252 | 257  | 262  | 267  | 272  | 278  | 283  | 288  | 293  | 5      |
| 857 | 298    | 303 | 308  | 313  | 318  | 323  | 328  | 333  | 338  | 343  | .1 0.5 |
| 858 | 348    | 354 | 359  | 364  | 369  | 374  | 379  | 384  | 389  | 394  | .2 1.1 |
| 859 | 399    | 404 | 409  | 414  | 419  | 424  | 429  | 434  | 439  | 445  | .3 1.6 |
| 860 | 450    | 455 | 460  | 465  | 470  | 475  | 480  | 485  | 490  | 495  | .4 2.2 |
| 861 | 500    | 505 | 510  | 515  | 520  | 525  | 530  | 535  | 540  | 545  | .5 2.7 |
| 862 | 550    | 556 | 561  | 566  | 571  | 576  | 581  | 586  | 591  | 596  | .6 3.3 |
| 863 | 601    | 606 | 611  | 616  | 621  | 626  | 631  | 636  | 641  | 646  |        |
| 864 | 651    | 656 | 661  | 666  | 671  | 676  | 681  | 686  | 691  | 696  | .7 3.8 |
| 865 | 701    | 706 | 711  | 716  | 721  | 726  | 731  | 736  | 742  | 747  | .8 4.4 |
| 866 | 752    | 757 | 762  | 767  | 772  | 777  | 782  | 787  | 792  | 797  | .9 4.9 |
| 867 | 802    | 807 | 812  | 817  | 822  | 827  | 832  | 837  | 842  | 847  |        |
| 868 | 852    | 857 | 862  | 867  | 872  | 877  | 882  | 887  | 892  | 897  |        |
| 869 | 902    | 907 | 912  | 917  | 922  | 927  | 932  | 937  | 942  | 947  |        |
| 870 | 952    | 957 | 962  | 967  | 972  | 977  | 982  | 987  | 992  | 997  | 5      |
| 871 | 94 002 | 007 | 012  | 017  | 022  | 026  | 031  | 036  | 041  | 046  | .1 0.5 |
| 872 | 051    | 056 | 061  | 066  | 071  | 076  | 081  | 086  | 091  | 096  | .2 1.0 |
| 873 | 101    | 106 | 111  | 116  | 121  | 126  | 131  | 136  | 141  | 146  | .3 1.5 |
| 874 | 151    | 156 | 161  | 166  | 171  | 176  | 181  | 186  | 191  | 196  |        |
| 875 | 201    | 206 | 210  | 215  | 220  | 225  | 230  | 235  | 240  | 245  | .4 2.0 |
| 876 | 250    | 255 | 260  | 265  | 270  | 275  | 280  | 285  | 290  | 295  | .5 2.5 |
| 877 | 300    | 305 | 310  | 315  | 320  | 324  | 329  | 334  | 339  | 344  | .6 3.0 |
| 878 | 349    | 354 | 359  | 364  | 369  | 374  | 379  | 384  | 389  | 394  | .7 3.5 |
| 879 | 399    | 404 | 409  | 413  | 418  | 423  | 428  | 433  | 438  | 443  | .8 4.0 |
| 880 | 448    | 453 | 458  | 463  | 468  | 473  | 478  | 483  | 487  | 492  | .9 4.5 |
| 881 | 497    | 502 | 507  | 512  | 517  | 522  | 527  | 532  | 537  | 542  |        |
| 882 | 547    | 552 | 556  | 561  | 566  | 571  | 576  | 581  | 586  | 591  |        |
| 883 | 596    | 601 | 606  | 611  | 615  | 620  | 625  | 630  | 635  | 640  |        |
| 884 | 645    | 650 | 655  | 660  | 665  | 670  | 674  | 679  | 684  | 689  |        |
| 885 | 694    | 699 | 704  | 709  | 714  | 719  | 724  | 728  | 733  | 738  | 4      |
| 886 | 743    | 748 | 753  | 758  | 763  | 768  | 773  | 777  | 782  | 787  | .1 0.4 |
| 887 | 792    | 797 | 802  | 807  | 812  | 817  | 821  | 826  | 831  | 836  | .2 0.9 |
| 888 | 841    | 846 | 851  | 856  | 861  | 865  | 870  | 875  | 880  | 885  | .3 1.3 |
| 889 | 890    | 895 | 900  | 905  | 909  | 914  | 919  | 924  | 929  | 934  |        |
| 890 | 939    | 944 | 949  | 953  | 958  | 963  | 968  | 973  | 978  | 983  | .4 1.8 |
| 891 | 988    | 992 | 997  | *002 | *007 | *012 | *017 | *022 | *026 | 031  | .5 2.2 |
| 892 | 95 036 | 041 | 046  | 051  | 056  | 061  | 065  | 070  | 075  | 080  | .6 2.7 |
| 893 | 085    | 090 | 095  | 099  | 104  | 109  | 114  | 119  | 124  | 129  | .7 3.1 |
| 894 | 134    | 138 | 143  | 148  | 153  | 158  | 163  | 167  | 172  | 177  | .8 3.6 |
| 895 | 182    | 187 | 192  | 197  | 201  | 206  | 211  | 216  | 221  | 226  | .9 4.0 |
| 896 | 231    | 235 | 240  | 245  | 250  | 255  | 260  | 264  | 269  | 274  |        |
| 897 | 279    | 284 | 289  | 294  | 298  | 303  | 308  | 313  | 318  | 323  |        |
| 898 | 327    | 332 | 337  | 342  | 347  | 352  | 356  | 361  | 366  | 371  |        |
| 899 | 376    | 381 | 385  | 390  | 395  | 400  | 405  | 410  | 414  | 419  |        |
| 900 | 424    | 429 | 434  | 438  | 443  | 448  | 453  | 458  | 463  | 467  |        |
| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0       | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | P. P.  |
|-----|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 900 | 95 424̂ | 429   | 434   | 438̂  | 443̂  | 448̂  | 453̂  | 458̂  | 463̂  | 467̂  |        |
| 901 | 472̂    | 477̂  | 482̂  | 487̂  | 492̂  | 496̂  | 501̂  | 506̂  | 511̂  | 516̂  |        |
| 902 | 520̂    | 525̂  | 530̂  | 535̂  | 540̂  | 544̂  | 549̂  | 554̂  | 559̂  | 564̂  |        |
| 903 | 569̂    | 573̂  | 578̂  | 583̂  | 588̂  | 593̂  | 597̂  | 602̂  | 607̂  | 612̂  |        |
| 904 | 617̂    | 621̂  | 626̂  | 631̂  | 636̂  | 641̂  | 645̂  | 650̂  | 655̂  | 660̂  |        |
| 905 | 665̂    | 669̂  | 674̂  | 679̂  | 684̂  | 689̂  | 693̂  | 698̂  | 703̂  | 708̂  |        |
| 906 | 713̂    | 717̂  | 722̂  | 727̂  | 732̂  | 737̂  | 741̂  | 746̂  | 751̂  | 756̂  |        |
| 907 | 760̂    | 765̂  | 770̂  | 775̂  | 780̂  | 784̂  | 789̂  | 794̂  | 799̂  | 804̂  |        |
| 908 | 808̂    | 813̂  | 818̂  | 823̂  | 827̂  | 832̂  | 837̂  | 842̂  | 847̂  | 851̂  |        |
| 909 | 856̂    | 861̂  | 866̂  | 870̂  | 875̂  | 880̂  | 885̂  | 890̂  | 894̂  | 899̂  |        |
| 910 | 904̂    | 909̂  | 913̂  | 918̂  | 923̂  | 928̂  | 933̂  | 937̂  | 942̂  | 947̂  |        |
| 911 | 952̂    | 956̂  | 961̂  | 966̂  | 971̂  | 975̂  | 980̂  | 985̂  | 990̂  | 994̂  | 5      |
| 912 | 999̂    | *004̂ | *009̂ | *014̂ | *018̂ | *023̂ | *028̂ | *033̂ | *037̂ | *042̂ | .1 0.5 |
| 913 | 96 047̂ | 052̂  | 056̂  | 061̂  | 066̂  | 071̂  | 075̂  | 080̂  | 085̂  | 090̂  | .2 1.0 |
| 914 | 094̂    | 099̂  | 104̂  | 109̂  | 113̂  | 118̂  | 123̂  | 128̂  | 132̂  | 137̂  | .3 1.5 |
| 915 | 142̂    | 147̂  | 151̂  | 156̂  | 161̂  | 166̂  | 170̂  | 175̂  | 180̂  | 185̂  | .4 2.0 |
| 916 | 189̂    | 194̂  | 199̂  | 204̂  | 208̂  | 213̂  | 218̂  | 222̂  | 227̂  | 232̂  | .5 2.5 |
| 917 | 237̂    | 241̂  | 246̂  | 251̂  | 256̂  | 260̂  | 265̂  | 270̂  | 275̂  | 279̂  | .6 3.0 |
| 918 | 284̂    | 289̂  | 293̂  | 298̂  | 303̂  | 308̂  | 312̂  | 317̂  | 322̂  | 327̂  | .7 3.5 |
| 919 | 331̂    | 336̂  | 341̂  | 345̂  | 350̂  | 355̂  | 360̂  | 364̂  | 369̂  | 374̂  | .8 4.0 |
| 920 | 379̂    | 383̂  | 388̂  | 393̂  | 397̂  | 402̂  | 407̂  | 412̂  | 416̂  | 421̂  | .9 4.5 |
| 921 | 426̂    | 430̂  | 435̂  | 440̂  | 445̂  | 449̂  | 454̂  | 459̂  | 463̂  | 468̂  |        |
| 922 | 473̂    | 478̂  | 482̂  | 487̂  | 492̂  | 496̂  | 501̂  | 506̂  | 511̂  | 515̂  |        |
| 923 | 520̂    | 525̂  | 529̂  | 534̂  | 539̂  | 543̂  | 548̂  | 553̂  | 558̂  | 562̂  |        |
| 924 | 567̂    | 572̂  | 576̂  | 581̂  | 586̂  | 590̂  | 595̂  | 600̂  | 605̂  | 609̂  |        |
| 925 | 614̂    | 619̂  | 623̂  | 628̂  | 633̂  | 637̂  | 642̂  | 647̂  | 651̂  | 656̂  |        |
| 926 | 661̂    | 666̂  | 670̂  | 675̂  | 680̂  | 684̂  | 689̂  | 694̂  | 698̂  | 703̂  |        |
| 927 | 708̂    | 712̂  | 717̂  | 722̂  | 726̂  | 731̂  | 736̂  | 741̂  | 745̂  | 750̂  |        |
| 928 | 755̂    | 759̂  | 764̂  | 769̂  | 773̂  | 778̂  | 783̂  | 787̂  | 792̂  | 797̂  |        |
| 929 | 801̂    | 806̂  | 811̂  | 815̂  | 820̂  | 825̂  | 829̂  | 834̂  | 839̂  | 843̂  |        |
| 930 | 848̂    | 853̂  | 857̂  | 862̂  | 867̂  | 871̂  | 876̂  | 881̂  | 885̂  | 890̂  |        |
| 931 | 895̂    | 899̂  | 904̂  | 909̂  | 913̂  | 918̂  | 923̂  | 927̂  | 932̂  | 937̂  | 4      |
| 932 | 941̂    | 946̂  | 951̂  | 955̂  | 960̂  | 965̂  | 969̂  | 974̂  | 979̂  | 983̂  | .1 0.4 |
| 933 | 988̂    | 993̂  | 997̂  | *002̂ | *007̂ | *011̂ | *016̂ | *020̂ | *025̂ | *030̂ | .2 0.9 |
| 934 | 97 034̂ | 039̂  | 044̂  | 048̂  | 053̂  | 058̂  | 062̂  | 067̂  | 072̂  | 076̂  | .3 1.3 |
| 935 | 081̂    | 086̂  | 090̂  | 095̂  | 099̂  | 104̂  | 109̂  | 113̂  | 118̂  | 123̂  | .4 1.8 |
| 936 | 127̂    | 132̂  | 137̂  | 141̂  | 146̂  | 151̂  | 155̂  | 160̂  | 164̂  | 169̂  | .5 2.2 |
| 937 | 174̂    | 178̂  | 183̂  | 188̂  | 192̂  | 197̂  | 202̂  | 206̂  | 211̂  | 215̂  | .6 2.7 |
| 938 | 220̂    | 225̂  | 229̂  | 234̂  | 239̂  | 243̂  | 248̂  | 252̂  | 257̂  | 262̂  | .7 3.1 |
| 939 | 266̂    | 271̂  | 276̂  | 280̂  | 285̂  | 289̂  | 294̂  | 299̂  | 303̂  | 308̂  | .8 3.6 |
| 940 | 313̂    | 317̂  | 322̂  | 326̂  | 331̂  | 336̂  | 340̂  | 345̂  | 349̂  | 354̂  | .9 4.0 |
| 941 | 359̂    | 363̂  | 368̂  | 373̂  | 377̂  | 382̂  | 386̂  | 391̂  | 396̂  | 400̂  |        |
| 942 | 405̂    | 409̂  | 414̂  | 419̂  | 423̂  | 428̂  | 432̂  | 437̂  | 442̂  | 446̂  |        |
| 943 | 451̂    | 456̂  | 460̂  | 465̂  | 469̂  | 474̂  | 479̂  | 483̂  | 488̂  | 492̂  |        |
| 944 | 497̂    | 502̂  | 506̂  | 511̂  | 515̂  | 520̂  | 525̂  | 529̂  | 534̂  | 538̂  |        |
| 945 | 543̂    | 548̂  | 552̂  | 557̂  | 561̂  | 566̂  | 570̂  | 575̂  | 580̂  | 584̂  |        |
| 946 | 589̂    | 593̂  | 598̂  | 603̂  | 607̂  | 612̂  | 616̂  | 621̂  | 626̂  | 630̂  |        |
| 947 | 635̂    | 639̂  | 644̂  | 649̂  | 653̂  | 658̂  | 662̂  | 667̂  | 671̂  | 676̂  |        |
| 948 | 681̂    | 685̂  | 690̂  | 694̂  | 699̂  | 703̂  | 708̂  | 713̂  | 717̂  | 722̂  |        |
| 949 | 726̂    | 731̂  | 736̂  | 740̂  | 745̂  | 749̂  | 754̂  | 758̂  | 763̂  | 768̂  |        |
| 950 | 772̂    | 777̂  | 781̂  | 786̂  | 790̂  | 795̂  | 800̂  | 804̂  | 809̂  | 813̂  |        |
| N.  | 0       | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.          | 0      | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-------------|--------|-----|-----|------|------|------|------|------|------|------|--------|
| <b>950</b>  | 97 772 | 777 | 781 | 786  | 790  | 795  | 800  | 804  | 809  | 813  |        |
| 951         | 818    | 822 | 827 | 831  | 836  | 841  | 845  | 850  | 854  | 859  |        |
| 952         | 863    | 868 | 873 | 877  | 882  | 886  | 891  | 895  | 900  | 904  |        |
| 953         | 909    | 914 | 918 | 923  | 927  | 932  | 936  | 941  | 945  | 950  |        |
| 954         | 955    | 959 | 964 | 968  | 973  | 977  | 982  | 986  | 991  | 996  |        |
| 955         | 98 000 | 005 | 009 | 014  | 018  | 023  | 027  | 032  | 036  | 041  | 5      |
| 956         | 046    | 050 | 055 | 059  | 064  | 068  | 073  | 077  | 082  | 086  | .1 0.5 |
| 957         | 091    | 095 | 100 | 105  | 109  | 114  | 118  | 123  | 127  | 132  | .2 1.0 |
| 958         | 136    | 141 | 145 | 150  | 154  | 159  | 163  | 168  | 173  | 177  | .3 1.5 |
| 959         | 182    | 186 | 191 | 195  | 200  | 204  | 209  | 213  | 218  | 222  | .4 2.0 |
| <b>960</b>  | 227    | 231 | 236 | 240  | 245  | 249  | 254  | 259  | 263  | 268  | .5 2.5 |
| 961         | 272    | 277 | 281 | 286  | 290  | 295  | 299  | 304  | 308  | 313  | .6 3.0 |
| 962         | 317    | 322 | 326 | 331  | 335  | 340  | 344  | 349  | 353  | 358  | .7 3.5 |
| 963         | 362    | 367 | 371 | 376  | 380  | 385  | 389  | 394  | 398  | 403  | .8 4.0 |
| 964         | 407    | 412 | 416 | 421  | 425  | 430  | 434  | 439  | 443  | 448  | .9 4.5 |
| 965         | 452    | 457 | 461 | 466  | 470  | 475  | 479  | 484  | 488  | 493  |        |
| 966         | 497    | 502 | 506 | 511  | 515  | 520  | 524  | 529  | 533  | 538  |        |
| 967         | 542    | 547 | 551 | 556  | 560  | 565  | 569  | 574  | 578  | 583  |        |
| 968         | 587    | 592 | 596 | 601  | 605  | 610  | 614  | 619  | 623  | 628  |        |
| 969         | 632    | 637 | 641 | 646  | 650  | 655  | 659  | 663  | 668  | 672  |        |
| <b>970</b>  | 677    | 681 | 686 | 690  | 695  | 699  | 704  | 708  | 713  | 717  | 4      |
| 971         | 722    | 726 | 731 | 735  | 740  | 744  | 749  | 753  | 757  | 762  | .1 0.4 |
| 972         | 766    | 771 | 775 | 780  | 784  | 789  | 793  | 798  | 802  | 807  | .2 0.9 |
| 973         | 811    | 815 | 820 | 824  | 829  | 833  | 838  | 842  | 847  | 851  | .3 1.3 |
| 974         | 856    | 860 | 865 | 869  | 873  | 878  | 882  | 887  | 891  | 896  | .4 1.8 |
| 975         | 900    | 905 | 909 | 914  | 918  | 922  | 927  | 931  | 936  | 940  | .5 2.2 |
| 976         | 945    | 949 | 954 | 958  | 963  | 967  | 971  | 976  | 980  | 985  | .6 2.7 |
| 977         | 989    | 994 | 998 | *003 | *007 | *011 | *016 | *020 | *025 | *029 | .7 3.1 |
| 978         | 99 034 | 038 | 043 | 047  | 051  | 056  | 060  | 065  | 069  | 074  | .8 3.6 |
| 979         | 078    | 082 | 087 | 091  | 096  | 100  | 105  | 109  | 113  | 118  | .9 4.0 |
| <b>980</b>  | 122    | 127 | 131 | 136  | 140  | 145  | 149  | 153  | 158  | 162  |        |
| 981         | 167    | 171 | 176 | 180  | 184  | 189  | 193  | 198  | 202  | 206  |        |
| 982         | 211    | 215 | 220 | 224  | 229  | 233  | 237  | 242  | 246  | 251  |        |
| 983         | 255    | 260 | 264 | 268  | 273  | 277  | 282  | 286  | 290  | 295  |        |
| 984         | 299    | 304 | 308 | 312  | 317  | 321  | 326  | 330  | 335  | 339  |        |
| 985         | 343    | 348 | 352 | 357  | 361  | 365  | 370  | 374  | 379  | 383  | 4      |
| 986         | 387    | 392 | 396 | 401  | 405  | 409  | 414  | 418  | 423  | 427  | .1 0.4 |
| 987         | 431    | 436 | 440 | 445  | 449  | 453  | 458  | 462  | 467  | 471  | .2 0.8 |
| 988         | 475    | 480 | 484 | 489  | 493  | 497  | 502  | 506  | 511  | 515  | .3 1.2 |
| 989         | 519    | 524 | 528 | 533  | 537  | 541  | 546  | 550  | 554  | 559  | .4 1.6 |
| <b>990</b>  | 563    | 568 | 572 | 576  | 581  | 585  | 590  | 594  | 598  | 603  | .5 2.0 |
| 991         | 607    | 611 | 616 | 620  | 625  | 629  | 633  | 638  | 642  | 647  | .6 2.4 |
| 992         | 651    | 655 | 660 | 664  | 668  | 673  | 677  | 682  | 686  | 690  | .7 2.8 |
| 993         | 695    | 699 | 703 | 708  | 712  | 717  | 721  | 725  | 730  | 734  | .8 3.2 |
| 994         | 738    | 743 | 747 | 751  | 756  | 760  | 765  | 769  | 773  | 778  | .9 3.6 |
| 995         | 782    | 786 | 791 | 795  | 800  | 804  | 808  | 813  | 817  | 821  |        |
| 996         | 826    | 830 | 834 | 839  | 843  | 847  | 852  | 856  | 861  | 865  |        |
| 997         | 869    | 874 | 878 | 882  | 887  | 891  | 895  | 900  | 904  | 908  |        |
| 998         | 913    | 917 | 922 | 926  | 930  | 935  | 939  | 943  | 948  | 952  |        |
| 999         | 956    | 961 | 965 | 969  | 974  | 978  | 982  | 987  | 991  | 995  |        |
| <b>1000</b> | 00 000 | 004 | 008 | 013  | 017  | 021  | 026  | 030  | 034  | 039  |        |
| N.          | 0      | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.   | 0        | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | P. P. |  |
|------|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|
| 1000 | 000 000  | 043̂  | 087̂  | 130̂  | 173̂  | 217̂  | 260̂  | 304̂  | 347̂  | 390̂  |       |  |
| 01   | 434      | 477̂  | 521̂  | 564̂  | 607̂  | 651̂  | 694̂  | 737̂  | 781̂  | 824̂  |       |  |
| 02   | 867̂     | 911̂  | 954̂  | 997̂  | *041̂ | *084̂ | *127̂ | *171̂ | *214̂ | *257̂ |       |  |
| 03   | 001 301  | 344̂  | 387̂  | 431̂  | 474̂  | 517̂  | 560̂  | 604̂  | 647̂  | 690̂  |       |  |
| 04   | 733̂     | 777̂  | 820̂  | 863̂  | 906̂  | 950̂  | 993̂  | *036̂ | *079̂ | *123̂ |       |  |
| 05   | 002 166  | 209̂  | 252̂  | 295̂  | 339̂  | 382̂  | 425̂  | 468̂  | 511̂  | 555̂  |       |  |
| 06   | 598̂     | 641̂  | 684̂  | 727̂  | 770̂  | 814̂  | 857̂  | 900̂  | 943̂  | 986̂  |       |  |
| 07   | 003 029̂ | 072̂  | 115̂  | 159̂  | 202̂  | 245̂  | 288̂  | 331̂  | 374̂  | 417̂  |       |  |
| 08   | 460̂     | 503̂  | 546̂  | 590̂  | 633̂  | 676̂  | 719̂  | 762̂  | 805̂  | 848̂  |       |  |
| 09   | 891̂     | 934̂  | 977̂  | *020̂ | *063̂ | *106̂ | *149̂ | *192̂ | *235̂ | *278̂ |       |  |
| 1010 | 004 321̂ | 364̂  | 407̂  | 450̂  | 493̂  | 536̂  | 579̂  | 622̂  | 665̂  | 708̂  |       |  |
| 11   | 751̂     | 794̂  | 837̂  | 880̂  | 923̂  | 966̂  | *009̂ | *051̂ | *094̂ | *137̂ |       |  |
| 12   | 005 180̂ | 223̂  | 266̂  | 309̂  | 352̂  | 395̂  | 438̂  | 481̂  | 523̂  | 566̂  |       |  |
| 13   | 609̂     | 652̂  | 695̂  | 738̂  | 781̂  | 824̂  | 866̂  | 909̂  | 952̂  | 995̂  |       |  |
| 14   | 006 038̂ | 081̂  | 123̂  | 166̂  | 209̂  | 252̂  | 295̂  | 337̂  | 380̂  | 423̂  |       |  |
| 15   | 466̂     | 509̂  | 551̂  | 594̂  | 637̂  | 680̂  | 722̂  | 765̂  | 808̂  | 851̂  |       |  |
| 16   | 893̂     | 936̂  | 979̂  | *022̂ | *064̂ | *107̂ | *150̂ | *193̂ | *235̂ | *278̂ |       |  |
| 17   | 007 321̂ | 363̂  | 406̂  | 449̂  | 491̂  | 534̂  | 577̂  | 620̂  | 662̂  | 705̂  |       |  |
| 18   | 748̂     | 790̂  | 833̂  | 875̂  | 918̂  | 961̂  | *003̂ | *046̂ | *089̂ | 131̂  |       |  |
| 19   | 008 174̂ | 217̂  | 259̂  | 302̂  | 344̂  | 387̂  | 430̂  | 472̂  | 515̂  | 557̂  |       |  |
| 1020 | 600̂     | 642̂  | 685̂  | 728̂  | 770̂  | 813̂  | 855̂  | 898̂  | 940̂  | 983̂  |       |  |
| 21   | 009 025̂ | 068̂  | 111̂  | 153̂  | 196̂  | 238̂  | 281̂  | 323̂  | 366̂  | 408̂  |       |  |
| 22   | 451̂     | 493̂  | 536̂  | 578̂  | 621̂  | 663̂  | 706̂  | 748̂  | 790̂  | 833̂  |       |  |
| 23   | 875̂     | 918̂  | 960̂  | *003̂ | *045̂ | *088̂ | *130̂ | *172̂ | *215̂ | *257̂ |       |  |
| 24   | 010 300̂ | 342̂  | 385̂  | 427̂  | 469̂  | 512̂  | 554̂  | 596̂  | 639̂  | 681̂  |       |  |
| 25   | 724̂     | 766̂  | 808̂  | 851̂  | 893̂  | 935̂  | 978̂  | *020̂ | *062̂ | *105̂ |       |  |
| 26   | 011 147̂ | 189̂  | 232̂  | 274̂  | 316̂  | 359̂  | 401̂  | 443̂  | 486̂  | 528̂  |       |  |
| 27   | 570̂     | 612̂  | 655̂  | 697̂  | 739̂  | 782̂  | 824̂  | 866̂  | 908̂  | 951̂  |       |  |
| 28   | 993̂     | *035̂ | *077̂ | *120̂ | *162̂ | *204̂ | *246̂ | *288̂ | *331̂ | *373̂ |       |  |
| 29   | 012 415̂ | 457̂  | 500̂  | 542̂  | 584̂  | 626̂  | 668̂  | 710̂  | 753̂  | 795̂  |       |  |
| 1030 | 837̂     | 879̂  | 921̂  | 963̂  | *006̂ | *048̂ | *090̂ | *132̂ | 174̂  | 216̂  |       |  |
| 31   | 013 258̂ | 301̂  | 343̂  | 385̂  | 427̂  | 469̂  | 511̂  | 553̂  | 595̂  | 637̂  |       |  |
| 32   | 679̂     | 722̂  | 764̂  | 806̂  | 848̂  | 890̂  | 932̂  | 974̂  | *016̂ | *058̂ |       |  |
| 33   | 014 100̂ | 142̂  | 184̂  | 226̂  | 268̂  | 310̂  | 352̂  | 394̂  | 436̂  | 478̂  |       |  |
| 34   | 520̂     | 562̂  | 604̂  | 646̂  | 688̂  | 730̂  | 772̂  | 814̂  | 856̂  | 898̂  |       |  |
| 35   | 940̂     | 982̂  | *024̂ | *066̂ | *108̂ | *150̂ | *192̂ | *234̂ | *276̂ | *318̂ |       |  |
| 36   | 015 360̂ | 401̂  | 443̂  | 485̂  | 527̂  | 569̂  | 611̂  | 653̂  | 695̂  | 737̂  |       |  |
| 37   | 779̂     | 820̂  | 862̂  | 904̂  | 946̂  | 988̂  | *030̂ | *072̂ | *113̂ | 155̂  |       |  |
| 38   | 016 197̂ | 239̂  | 281̂  | 323̂  | 364̂  | 406̂  | 448̂  | 490̂  | 532̂  | 573̂  |       |  |
| 39   | 615̂     | 657̂  | 699̂  | 741̂  | 782̂  | 824̂  | 866̂  | 908̂  | 950̂  | 991̂  |       |  |
| 1040 | 017 033̂ | 075̂  | 117̂  | 158̂  | 200̂  | 242̂  | 284̂  | 325̂  | 367̂  | 409̂  |       |  |
| 41   | 450̂     | 492̂  | 534̂  | 576̂  | 617̂  | 659̂  | 701̂  | 742̂  | 784̂  | 826̂  |       |  |
| 42   | 867̂     | 909̂  | 951̂  | 992̂  | *034̂ | *076̂ | *117̂ | *159̂ | *201̂ | *242̂ |       |  |
| 43   | 018 284̂ | 326̂  | 367̂  | 409̂  | 451̂  | 492̂  | 534̂  | 575̂  | 617̂  | 659̂  |       |  |
| 44   | 700̂     | 742̂  | 783̂  | 825̂  | 867̂  | 908̂  | 950̂  | 991̂  | *033̂ | *074̂ |       |  |
| 45   | 019 116̂ | 158̂  | 199̂  | 241̂  | 282̂  | 324̂  | 365̂  | 407̂  | 448̂  | 490̂  |       |  |
| 46   | 531̂     | 573̂  | 614̂  | 656̂  | 697̂  | 739̂  | 780̂  | 822̂  | 863̂  | 905̂  |       |  |
| 47   | 946̂     | 988̂  | *029̂ | *071̂ | *112̂ | *154̂ | *195̂ | *237̂ | *278̂ | *320̂ |       |  |
| 48   | 020 361̂ | 402̂  | 444̂  | 485̂  | 527̂  | 568̂  | 610̂  | 651̂  | 692̂  | 734̂  |       |  |
| 49   | 775̂     | 817̂  | 858̂  | 899̂  | 941̂  | 982̂  | *024̂ | *065̂ | *106̂ | *148̂ |       |  |
| 1050 | 021 189̂ | 230̂  | 272̂  | 313̂  | 354̂  | 396̂  | 437̂  | 478̂  | 520̂  | 561̂  |       |  |
| N.   | 0        | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | P. P. |  |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.   | 0       | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.   |
|------|---------|------|------|------|------|------|------|------|------|------|---------|
| 1050 | 021 189 | 236  | 272  | 313  | 354  | 396  | 437  | 478  | 520  | 561  |         |
| 51   | 602     | 644  | 685  | 726  | 768  | 809  | 850  | 892  | 933  | 974  |         |
| 52   | 022 015 | 057  | 098  | 139  | 181  | 222  | 263  | 304  | 346  | 387  | 41      |
| 53   | 428     | 469  | 511  | 552  | 593  | 634  | 676  | 717  | 758  | 799  | .1 4.1  |
| 54   | 840     | 882  | 923  | 964  | *005 | *046 | *088 | *129 | *170 | *211 | .2 8.3  |
| 55   | 023 252 | 293  | 335  | 376  | 417  | 458  | 499  | 540  | 581  | 623  | .3 12.4 |
| 56   | 664     | 705  | 746  | 787  | 828  | 869  | 910  | 951  | 993  | *034 | .4 16.6 |
| 57   | 024 075 | 116  | 157  | 198  | 239  | 280  | 321  | 362  | 403  | 444  | .5 20.7 |
| 58   | 485     | 526  | 568  | 609  | 650  | 691  | 732  | 773  | 814  | 855  | .6 24.9 |
| 59   | 896     | 937  | 978  | *019 | *060 | *101 | *142 | *183 | *224 | *265 | .7 29.0 |
| 1060 | 025 306 | 347  | 388  | 429  | 469  | 510  | 551  | 592  | 633  | 674  | .8 33.2 |
| 61   | 715     | 756  | 797  | 838  | 879  | 920  | 961  | *002 | *042 | *083 | .9 37.3 |
| 62   | 026 124 | 165  | 206  | 247  | 288  | 329  | 370  | 410  | 451  | 492  | 41      |
| 63   | 533     | 574  | 615  | 656  | 696  | 737  | 778  | 819  | 860  | 901  | .1 4.1  |
| 64   | 941     | 982  | *023 | *064 | *105 | *145 | *186 | *227 | *268 | *309 | .2 8.2  |
| 65   | 027 349 | 390  | 431  | 472  | 512  | 553  | 594  | 635  | 675  | 716  | .3 12.3 |
| 66   | 757     | 798  | 838  | 879  | 920  | 961  | *001 | *042 | *083 | *123 | .4 16.4 |
| 67   | 028 164 | 205  | 246  | 286  | 327  | 368  | 408  | 449  | 490  | 530  | .5 20.5 |
| 68   | 571     | 612  | 652  | 693  | 734  | 774  | 815  | 856  | 896  | 937  | .6 24.6 |
| 69   | 977     | *018 | *059 | *099 | *140 | *181 | *221 | *262 | *302 | *343 | .7 28.7 |
| 1070 | 029 384 | 424  | 465  | 505  | 546  | 586  | 627  | 668  | 708  | 749  | .8 32.8 |
| 71   | 789     | 830  | 870  | 911  | 951  | 992  | *032 | *073 | *114 | *154 | .9 36.9 |
| 72   | 030 195 | 235  | 276  | 316  | 357  | 397  | 438  | 478  | 519  | 559  | 46      |
| 73   | 599     | 640  | 680  | 721  | 761  | 802  | 842  | 883  | 923  | 964  | .1 4.6  |
| 74   | 031 004 | 044  | 085  | 125  | 166  | 206  | 247  | 287  | 327  | 368  | .2 8.1  |
| 75   | 408     | 449  | 489  | 529  | 570  | 610  | 651  | 691  | 731  | 772  | .3 12.1 |
| 76   | 812     | 852  | 893  | 933  | 973  | *014 | *054 | *094 | *135 | *175 | .4 16.2 |
| 77   | 032 215 | 256  | 296  | 336  | 377  | 417  | 457  | 498  | 538  | 578  | .5 20.2 |
| 78   | 619     | 659  | 699  | 739  | 780  | 820  | 860  | 900  | 941  | 981  | .6 24.3 |
| 79   | 033 021 | 061  | 102  | 142  | 182  | 222  | 263  | 303  | 343  | 383  | .7 28.3 |
| 1080 | 424     | 464  | 504  | 544  | 584  | 625  | 665  | 705  | 745  | 785  | .8 32.4 |
| 81   | 825     | 866  | 906  | 946  | 986  | *026 | *066 | *107 | 147  | 187  | .9 36.4 |
| 82   | 034 227 | 267  | 307  | 347  | 388  | 428  | 468  | 508  | 548  | 588  | 40      |
| 83   | 628     | 668  | 708  | 748  | 789  | 829  | 869  | 909  | 949  | 989  | .1 4.0  |
| 84   | 035 029 | 069  | 109  | 149  | 189  | 229  | 269  | 309  | 349  | 389  | .2 8.0  |
| 85   | 429     | 470  | 510  | 550  | 590  | 630  | 670  | 710  | 750  | 790  | .3 12.0 |
| 86   | 830     | 870  | 910  | 950  | 990  | *029 | *069 | *109 | *149 | *189 | .4 16.0 |
| 87   | 036 229 | 269  | 309  | 349  | 389  | 429  | 469  | 509  | 549  | 589  | .5 20.0 |
| 88   | 629     | 669  | 708  | 748  | 788  | 828  | 868  | 908  | 948  | 988  | .6 24.0 |
| 89   | 037 028 | 068  | 107  | 147  | 187  | 227  | 267  | 307  | 347  | 386  | .7 28.0 |
| 1090 | 426     | 466  | 506  | 546  | 586  | 625  | 665  | 705  | 745  | 785  | .8 32.0 |
| 91   | 825     | 864  | 904  | 944  | 984  | *023 | *063 | *103 | 143  | 183  | .9 36.0 |
| 92   | 038 222 | 262  | 302  | 342  | 381  | 421  | 461  | 501  | 540  | 580  | 39      |
| 93   | 620     | 660  | 699  | 739  | 779  | 819  | 858  | 898  | 938  | 977  | .1 3.9  |
| 94   | 039 017 | 057  | 096  | 136  | 176  | 216  | 255  | 295  | 335  | 374  | .2 7.9  |
| 95   | 414     | 454  | 493  | 533  | 572  | 612  | 652  | 691  | 731  | 771  | .3 11.8 |
| 96   | 810     | 850  | 890  | 929  | 969  | *008 | *048 | *088 | *127 | *167 | .4 15.8 |
| 97   | 040 206 | 246  | 286  | 325  | 365  | 404  | 444  | 483  | 523  | 563  | .5 19.7 |
| 98   | 602     | 642  | 681  | 721  | 760  | 800  | 839  | 879  | 918  | 958  | .6 23.7 |
| 99   | 997     | *037 | *076 | *116 | *155 | *195 | *234 | *274 | *313 | *353 | .7 27.6 |
| 1100 | 041 392 | 432  | 471  | 511  | 550  | 590  | 629  | 669  | 708  | 748  | .8 31.6 |
| N.   | 0       | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.   |



TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES.

| log sin $\phi = \log \phi'' + S.$<br>log tan $\phi = \log \phi'' + T.$ |    | O°       |    | log $\phi' = \log \sin \phi + S'.$<br>log $\phi' = \log \tan \phi + T'.$ |          |    |           |
|------------------------------------------------------------------------|----|----------|----|--------------------------------------------------------------------------|----------|----|-----------|
| "                                                                      | '  | S        | T  | Log. Sin.                                                                | S'       | T' | Log. Tan. |
| 0                                                                      | 0  | 4.685 57 | 57 | — ∞                                                                      | 5.314 42 | 42 | — ∞       |
| 60                                                                     | 1  | 57       | 57 | 6.46 372                                                                 | 42       | 42 | 6.46 372  |
| 120                                                                    | 2  | 57       | 57 | .76 473                                                                  | 42       | 42 | .76 473   |
| 180                                                                    | 3  | 57       | 57 | .94 084                                                                  | 42       | 42 | .94 084   |
| 240                                                                    | 4  | 57       | 57 | 7.06 578                                                                 | 42       | 42 | 7.06 578  |
| 300                                                                    | 5  | 4.685 57 | 57 | 7.16 269                                                                 | 5.314 42 | 42 | 7.16 269  |
| 360                                                                    | 6  | 57       | 57 | .24 187                                                                  | 42       | 42 | .24 188   |
| 420                                                                    | 7  | 57       | 57 | .30 882                                                                  | 42       | 42 | .30 882   |
| 480                                                                    | 8  | 57       | 57 | .36 681                                                                  | 42       | 42 | .36 681   |
| 540                                                                    | 9  | 57       | 57 | .41 797                                                                  | 42       | 42 | .41 797   |
| 600                                                                    | 10 | 4.685 57 | 57 | 7.46 372                                                                 | 5.314 42 | 42 | 7.46 372  |
| 660                                                                    | 11 | 57       | 57 | .50 512                                                                  | 42       | 42 | .50 512   |
| 720                                                                    | 12 | 57       | 57 | .54 290                                                                  | 42       | 42 | .54 291   |
| 780                                                                    | 13 | 57       | 57 | .57 767                                                                  | 42       | 42 | .57 767   |
| 840                                                                    | 14 | 57       | 57 | .60 983                                                                  | 42       | 42 | .60 983   |
| 900                                                                    | 15 | 4.685 57 | 58 | 7.63 981                                                                 | 5.314 42 | 42 | 7.63 982  |
| 960                                                                    | 16 | 57       | 58 | .66 781                                                                  | 42       | 42 | .66 785   |
| 1020                                                                   | 17 | 57       | 58 | .69 417                                                                  | 42       | 42 | .69 418   |
| 1080                                                                   | 18 | 57       | 58 | .71 899                                                                  | 42       | 42 | .71 900   |
| 1140                                                                   | 19 | 57       | 58 | .74 248                                                                  | 42       | 42 | .74 248   |
| 1200                                                                   | 20 | 4.685 57 | 58 | 7.76 473                                                                 | 5.314 43 | 42 | 7.76 476  |
| 1260                                                                   | 21 | 57       | 58 | .78 594                                                                  | 43       | 42 | .78 595   |
| 1320                                                                   | 22 | 57       | 58 | .80 614                                                                  | 43       | 42 | .80 613   |
| 1380                                                                   | 23 | 57       | 58 | .82 545                                                                  | 43       | 42 | .82 546   |
| 1440                                                                   | 24 | 57       | 58 | .84 393                                                                  | 43       | 42 | .84 394   |
| 1500                                                                   | 25 | 4.685 57 | 58 | 7.86 166                                                                 | 5.314 43 | 41 | 7.86 167  |
| 1560                                                                   | 26 | 57       | 58 | .87 869                                                                  | 43       | 41 | .87 871   |
| 1620                                                                   | 27 | 57       | 58 | .89 508                                                                  | 43       | 41 | .89 510   |
| 1680                                                                   | 28 | 57       | 58 | .91 088                                                                  | 43       | 41 | .91 089   |
| 1740                                                                   | 29 | 57       | 58 | .92 612                                                                  | 43       | 41 | .92 613   |
| 1800                                                                   | 30 | 4.685 57 | 58 | 7.94 084                                                                 | 5.314 43 | 41 | 7.94 086  |
| 1860                                                                   | 31 | 57       | 58 | .95 508                                                                  | 43       | 41 | .95 510   |
| 1920                                                                   | 32 | 57       | 58 | .96 887                                                                  | 43       | 41 | .96 889   |
| 1980                                                                   | 33 | 57       | 59 | .98 223                                                                  | 43       | 41 | .98 223   |
| 2040                                                                   | 34 | 57       | 59 | .99 520                                                                  | 43       | 41 | .99 522   |
| 2100                                                                   | 35 | 4.685 56 | 59 | 8.00 778                                                                 | 5.314 43 | 41 | 8.00 781  |
| 2160                                                                   | 36 | 56       | 59 | .02 002                                                                  | 43       | 41 | .02 004   |
| 2220                                                                   | 37 | 56       | 59 | .03 192                                                                  | 43       | 41 | .03 194   |
| 2280                                                                   | 38 | 56       | 59 | .04 350                                                                  | 43       | 40 | .04 352   |
| 2340                                                                   | 39 | 56       | 59 | .05 478                                                                  | 43       | 40 | .05 481   |
| 2400                                                                   | 40 | 4.685 56 | 59 | 8.06 577                                                                 | 5.314 43 | 40 | 8.06 580  |
| 2460                                                                   | 41 | 56       | 59 | .07 650                                                                  | 43       | 40 | .07 653   |
| 2520                                                                   | 42 | 56       | 59 | .08 696                                                                  | 43       | 40 | .08 699   |
| 2580                                                                   | 43 | 56       | 60 | .09 718                                                                  | 43       | 40 | .09 721   |
| 2640                                                                   | 44 | 56       | 60 | .10 716                                                                  | 43       | 40 | .10 720   |
| 2700                                                                   | 45 | 4.685 56 | 60 | 8.11 692                                                                 | 5.314 44 | 40 | 8.11 696  |
| 2760                                                                   | 46 | 56       | 60 | .12 647                                                                  | 44       | 40 | .12 651   |
| 2820                                                                   | 47 | 56       | 60 | .13 581                                                                  | 44       | 40 | .13 585   |
| 2880                                                                   | 48 | 56       | 60 | .14 493                                                                  | 44       | 39 | .14 499   |
| 2940                                                                   | 49 | 56       | 60 | .15 390                                                                  | 44       | 39 | .15 395   |
| 3000                                                                   | 50 | 4.685 56 | 60 | 8.16 268                                                                 | 5.314 44 | 39 | 8.16 272  |
| 3060                                                                   | 51 | 56       | 60 | .17 128                                                                  | 44       | 39 | .17 133   |
| 3120                                                                   | 52 | 56       | 61 | .17 971                                                                  | 44       | 39 | .17 976   |
| 3180                                                                   | 53 | 56       | 61 | .18 798                                                                  | 44       | 39 | .18 803   |
| 3240                                                                   | 54 | 53       | 61 | .19 610                                                                  | 44       | 39 | .19 613   |
| 3300                                                                   | 55 | 4.685 53 | 61 | 8.20 407                                                                 | 5.314 44 | 39 | 8.20 412  |
| 3360                                                                   | 56 | 53       | 61 | .21 189                                                                  | 44       | 38 | .21 193   |
| 3420                                                                   | 57 | 53       | 61 | .21 958                                                                  | 44       | 38 | .21 964   |
| 3480                                                                   | 58 | 53       | 61 | .22 713                                                                  | 44       | 38 | .22 719   |
| 3540                                                                   | 59 | 53       | 62 | .23 453                                                                  | 44       | 38 | .23 462   |

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES.

| log sin $\phi = \log \phi' + S.$<br>log tan $\phi = \log \phi' + T.$ |    | 1°       |    | log $\phi'' = \log \sin \phi + S'.$<br>log $\phi'' = \log \tan \phi + T'.$ |          |    |           |
|----------------------------------------------------------------------|----|----------|----|----------------------------------------------------------------------------|----------|----|-----------|
| "                                                                    | '  | S        | T  | Log. Sin.                                                                  | S'       | T' | Log. Tan. |
| 3600                                                                 | 0  | 4.685 55 | 62 | 8.24 183                                                                   | 5.314 44 | 38 | 8.24 192  |
| 3660                                                                 | 1  | 55       | 62 | .24 903                                                                    | 45       | 38 | .24 910   |
| 3720                                                                 | 2  | 55       | 62 | .25 609                                                                    | 45       | 38 | .25 616   |
| 3780                                                                 | 3  | 55       | 62 | .26 304                                                                    | 45       | 37 | .26 311   |
| 3840                                                                 | 4  | 55       | 62 | .26 988                                                                    | 45       | 37 | .26 993   |
| 3900                                                                 | 5  | 4.685 55 | 62 | 8.27 661                                                                   | 5.314 45 | 37 | 8.27 669  |
| 3960                                                                 | 6  | 55       | 63 | .28 324                                                                    | 45       | 37 | .28 332   |
| 4020                                                                 | 7  | 54       | 63 | .28 977                                                                    | 45       | 37 | .28 983   |
| 4080                                                                 | 8  | 54       | 63 | .29 626                                                                    | 45       | 37 | .29 629   |
| 4140                                                                 | 9  | 54       | 63 | .30 254                                                                    | 45       | 36 | .30 263   |
| 4200                                                                 | 10 | 4.685 54 | 63 | 8.30 876                                                                   | 5.314 45 | 36 | 8.30 886  |
| 4260                                                                 | 11 | 54       | 63 | .31 495                                                                    | 45       | 36 | .31 504   |
| 4320                                                                 | 12 | 54       | 64 | .32 102                                                                    | 45       | 36 | .32 112   |
| 4380                                                                 | 13 | 54       | 64 | .32 701                                                                    | 46       | 36 | .32 711   |
| 4440                                                                 | 14 | 54       | 64 | .33 292                                                                    | 46       | 36 | .33 302   |
| 4500                                                                 | 15 | 4.685 54 | 64 | 8.33 873                                                                   | 5.314 46 | 35 | 8.33 883  |
| 4560                                                                 | 16 | 54       | 64 | .34 456                                                                    | 46       | 35 | .34 461   |
| 4620                                                                 | 17 | 54       | 65 | .35 018                                                                    | 46       | 35 | .35 029   |
| 4680                                                                 | 18 | 54       | 65 | .35 578                                                                    | 46       | 35 | .35 589   |
| 4740                                                                 | 19 | 53       | 65 | .36 131                                                                    | 46       | 35 | .36 143   |
| 4800                                                                 | 20 | 4.685 53 | 65 | 8.36 677                                                                   | 5.314 46 | 34 | 8.36 689  |
| 4860                                                                 | 21 | 53       | 65 | .37 217                                                                    | 46       | 34 | .37 229   |
| 4920                                                                 | 22 | 53       | 65 | .37 750                                                                    | 46       | 34 | .37 762   |
| 4980                                                                 | 23 | 53       | 66 | .38 276                                                                    | 46       | 34 | .38 289   |
| 5040                                                                 | 24 | 53       | 66 | .38 796                                                                    | 47       | 34 | .38 809   |
| 5100                                                                 | 25 | 4.685 53 | 66 | 8.39 310                                                                   | 5.314 47 | 33 | 8.39 323  |
| 5160                                                                 | 26 | 53       | 66 | .39 818                                                                    | 47       | 33 | .39 831   |
| 5220                                                                 | 27 | 53       | 67 | .40 320                                                                    | 47       | 33 | .40 334   |
| 5280                                                                 | 28 | 52       | 67 | .40 816                                                                    | 47       | 33 | .40 830   |
| 5340                                                                 | 29 | 52       | 67 | .41 307                                                                    | 47       | 33 | .41 321   |
| 5400                                                                 | 30 | 4.685 52 | 67 | 8.41 792                                                                   | 5.314 47 | 32 | 8.41 807  |
| 5460                                                                 | 31 | 52       | 67 | .42 271                                                                    | 47       | 32 | .42 287   |
| 5520                                                                 | 32 | 52       | 68 | .42 746                                                                    | 47       | 32 | .42 762   |
| 5580                                                                 | 33 | 52       | 68 | .43 215                                                                    | 48       | 32 | .43 231   |
| 5640                                                                 | 34 | 52       | 68 | .43 680                                                                    | 48       | 31 | .43 696   |
| 5700                                                                 | 35 | 4.685 52 | 68 | 8.44 139                                                                   | 5.314 48 | 31 | 8.44 156  |
| 5760                                                                 | 36 | 52       | 69 | .44 594                                                                    | 48       | 31 | .44 611   |
| 5820                                                                 | 37 | 51       | 69 | .45 044                                                                    | 48       | 31 | .45 061   |
| 5880                                                                 | 38 | 51       | 69 | .45 489                                                                    | 48       | 30 | .45 507   |
| 5940                                                                 | 39 | 51       | 69 | .45 930                                                                    | 48       | 30 | .45 948   |
| 6000                                                                 | 40 | 4.685 51 | 69 | 8.46 366                                                                   | 5.314 48 | 30 | 8.46 385  |
| 6060                                                                 | 41 | 51       | 70 | .46 798                                                                    | 49       | 30 | .46 817   |
| 6120                                                                 | 42 | 51       | 70 | .47 226                                                                    | 49       | 30 | .47 243   |
| 6180                                                                 | 43 | 51       | 70 | .47 650                                                                    | 49       | 29 | .47 669   |
| 6240                                                                 | 44 | 51       | 70 | .48 069                                                                    | 49       | 29 | .48 089   |
| 6300                                                                 | 45 | 4.685 50 | 71 | 8.48 485                                                                   | 5.314 49 | 29 | 8.48 505  |
| 6360                                                                 | 46 | 50       | 71 | .48 896                                                                    | 49       | 28 | .48 917   |
| 6420                                                                 | 47 | 50       | 71 | .49 304                                                                    | 49       | 28 | .49 325   |
| 6480                                                                 | 48 | 50       | 72 | .49 708                                                                    | 49       | 28 | .49 729   |
| 6540                                                                 | 49 | 50       | 72 | .50 108                                                                    | 50       | 28 | .50 130   |
| 6600                                                                 | 50 | 4.685 50 | 72 | 8.50 504                                                                   | 5.314 50 | 27 | 8.50 526  |
| 6660                                                                 | 51 | 50       | 72 | .50 897                                                                    | 50       | 27 | .50 920   |
| 6720                                                                 | 52 | 50       | 73 | .51 286                                                                    | 50       | 27 | .51 310   |
| 6780                                                                 | 53 | 49       | 73 | .51 672                                                                    | 50       | 27 | .51 696   |
| 6840                                                                 | 54 | 49       | 73 | .52 055                                                                    | 50       | 26 | .52 079   |
| 6900                                                                 | 55 | 4.685 49 | 73 | 8.52 434                                                                   | 5.314 50 | 26 | 8.52 458  |
| 6960                                                                 | 56 | 49       | 74 | .52 810                                                                    | 51       | 26 | .52 835   |
| 7020                                                                 | 57 | 49       | 74 | .53 183                                                                    | 51       | 25 | .53 208   |
| 7080                                                                 | 58 | 49       | 74 | .53 552                                                                    | 51       | 25 | .53 578   |
| 7140                                                                 | 59 | 49       | 75 | .53 918                                                                    | 51       | 25 | .53 944   |

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES.

| log sin $\phi = \log \phi'' + S$<br>log tan $\phi = \log \phi'' + T$ |    | $2^\circ$ |    | log $\phi = \log \sin \phi + S'$<br>log $\phi = \log \tan \phi + T'$ |          |    |           |
|----------------------------------------------------------------------|----|-----------|----|----------------------------------------------------------------------|----------|----|-----------|
| "                                                                    | '  | S         | T  | Log. Sin.                                                            | S'       | T' | Log. Tan. |
| 7200                                                                 | 0  | 4.685 48  | 75 | 8.54 282                                                             | 5.314 51 | 25 | 8.54 308  |
| 7260                                                                 | 1  | 48        | 75 | .54 642                                                              | 51       | 24 | .54 669   |
| 7320                                                                 | 2  | 48        | 75 | .54 999                                                              | 51       | 24 | .55 027   |
| 7380                                                                 | 3  | 48        | 76 | .55 354                                                              | 52       | 24 | .55 381   |
| 7440                                                                 | 4  | 48        | 76 | .55 703                                                              | 52       | 23 | .55 733   |
| 7500                                                                 | 5  | 4.685 48  | 76 | 8.56 054                                                             | 5.314 52 | 23 | 8.56 083  |
| 7560                                                                 | 6  | 48        | 77 | .56 400                                                              | 52       | 23 | .56 429   |
| 7620                                                                 | 7  | 47        | 77 | .56 743                                                              | 52       | 22 | .56 772   |
| 7680                                                                 | 8  | 47        | 77 | .57 083                                                              | 52       | 22 | .57 113   |
| 7740                                                                 | 9  | 47        | 78 | .57 421                                                              | 52       | 22 | .57 452   |
| 7800                                                                 | 10 | 4.685 47  | 78 | 8.57 756                                                             | 5.314 53 | 22 | 8.57 787  |
| 7860                                                                 | 11 | 47        | 78 | .58 089                                                              | 53       | 21 | .58 121   |
| 7920                                                                 | 12 | 47        | 79 | .58 419                                                              | 53       | 21 | .58 451   |
| 7980                                                                 | 13 | 46        | 79 | .58 747                                                              | 53       | 21 | .58 779   |
| 8040                                                                 | 14 | 46        | 79 | .59 072                                                              | 53       | 20 | .59 105   |
| 8100                                                                 | 15 | 4.685 46  | 80 | 8.59 395                                                             | 5.314 53 | 20 | 8.59 428  |
| 8160                                                                 | 16 | 46        | 80 | .59 715                                                              | 54       | 20 | .59 749   |
| 8220                                                                 | 17 | 46        | 80 | .60 033                                                              | 54       | 19 | .60 067   |
| 8280                                                                 | 18 | 46        | 81 | .60 349                                                              | 54       | 19 | .60 384   |
| 8340                                                                 | 19 | 45        | 81 | .60 662                                                              | 54       | 19 | .60 698   |
| 8400                                                                 | 20 | 4.685 45  | 81 | 8.60 973                                                             | 5.314 54 | 18 | 8.61 009  |
| 8460                                                                 | 21 | 45        | 82 | .61 282                                                              | 54       | 18 | .61 319   |
| 8520                                                                 | 22 | 45        | 82 | .61 589                                                              | 55       | 18 | .61 626   |
| 8580                                                                 | 23 | 45        | 82 | .61 893                                                              | 55       | 17 | .61 931   |
| 8640                                                                 | 24 | 45        | 83 | .62 196                                                              | 55       | 17 | .62 234   |
| 8700                                                                 | 25 | 4.685 44  | 83 | 8.62 496                                                             | 5.314 55 | 16 | 8.62 535  |
| 8760                                                                 | 26 | 44        | 83 | .62 795                                                              | 55       | 16 | .62 834   |
| 8820                                                                 | 27 | 44        | 84 | .63 091                                                              | 55       | 16 | .63 131   |
| 8880                                                                 | 28 | 44        | 84 | .63 383                                                              | 56       | 15 | .63 423   |
| 8940                                                                 | 29 | 44        | 84 | .63 677                                                              | 56       | 15 | .63 718   |
| 9000                                                                 | 30 | 4.685 43  | 85 | 8.63 968                                                             | 5.314 56 | 15 | 8.64 009  |
| 9060                                                                 | 31 | 43        | 85 | .64 256                                                              | 56       | 14 | .64 298   |
| 9120                                                                 | 32 | 43        | 86 | .64 543                                                              | 56       | 14 | .64 583   |
| 9180                                                                 | 33 | 43        | 86 | .64 827                                                              | 57       | 14 | .64 870   |
| 9240                                                                 | 34 | 43        | 86 | .65 110                                                              | 57       | 13 | .65 153   |
| 9300                                                                 | 35 | 4.685 43  | 87 | 8.65 391                                                             | 5.314 57 | 13 | 8.65 435  |
| 9360                                                                 | 36 | 42        | 87 | .65 670                                                              | 57       | 12 | .65 715   |
| 9420                                                                 | 37 | 42        | 87 | .65 947                                                              | 57       | 12 | .65 993   |
| 9480                                                                 | 38 | 42        | 88 | .66 223                                                              | 58       | 12 | .66 269   |
| 9540                                                                 | 39 | 42        | 88 | .66 497                                                              | 58       | 11 | .66 543   |
| 9600                                                                 | 40 | 4.685 42  | 89 | 8.66 769                                                             | 5.314 58 | 11 | 8.66 816  |
| 9660                                                                 | 41 | 41        | 89 | .67 039                                                              | 58       | 10 | .67 087   |
| 9720                                                                 | 42 | 41        | 89 | .67 308                                                              | 58       | 10 | .67 356   |
| 9780                                                                 | 43 | 41        | 90 | .67 575                                                              | 59       | 10 | .67 624   |
| 9840                                                                 | 44 | 41        | 90 | .67 840                                                              | 59       | 09 | .67 890   |
| 9900                                                                 | 45 | 4.685 41  | 91 | 8.68 104                                                             | 5.314 59 | 09 | 8.68 154  |
| 9960                                                                 | 46 | 40        | 91 | .68 366                                                              | 59       | 08 | .68 417   |
| 10020                                                                | 47 | 40        | 91 | .68 627                                                              | 59       | 08 | .68 678   |
| 10080                                                                | 48 | 40        | 92 | .68 886                                                              | 60       | 08 | .68 938   |
| 10140                                                                | 49 | 40        | 92 | .69 144                                                              | 60       | 07 | .69 196   |
| 10200                                                                | 50 | 4.685 40  | 93 | 8.69 400                                                             | 5.314 60 | 07 | 8.69 453  |
| 10260                                                                | 51 | 39        | 93 | .69 654                                                              | 60       | 06 | .69 708   |
| 10320                                                                | 52 | 39        | 93 | .69 907                                                              | 60       | 06 | .69 961   |
| 10380                                                                | 53 | 39        | 94 | .70 159                                                              | 61       | 06 | .70 214   |
| 10440                                                                | 54 | 39        | 94 | .70 409                                                              | 61       | 05 | .70 464   |
| 10500                                                                | 55 | 4.685 38  | 95 | 8.70 657                                                             | 5.314 61 | 05 | 8.70 714  |
| 10560                                                                | 56 | 38        | 95 | .70 905                                                              | 61       | 04 | .70 962   |
| 10620                                                                | 57 | 38        | 96 | .71 150                                                              | 61       | 04 | .71 208   |
| 10680                                                                | 58 | 38        | 96 | .71 395                                                              | 62       | 03 | .71 453   |
| 10740                                                                | 59 | 38        | 97 | .71 638                                                              | 62       | 03 | .71 697   |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS

0°

| '         | Log. Sin. | D     | Log. Tan. | Com. D. | Log. Cot. | Log. Cos. |           |
|-----------|-----------|-------|-----------|---------|-----------|-----------|-----------|
| <b>0</b>  | — ∞       |       | — ∞       |         | ± ∞       | 0.00 000  | <b>60</b> |
| <b>1</b>  | 6.46 372  | 30103 | 6.46 372  | 30103   | 3.53 627  | 0.00 000  | 59        |
| <b>2</b>  | 6.76 473  | 17609 | 6.76 473  | 17609   | 3.23 524  | 0.00 000  | 58        |
| <b>3</b>  | 6.94 084  | 12494 | 6.94 084  | 12494   | 3.05 913  | 0.00 000  | 57        |
| <b>4</b>  | 7.06 578  | 9691  | 7.06 578  | 9691    | 2.93 421  | 0.00 000  | 56        |
| <b>5</b>  | 7.16 269  | 7918  | 7.16 269  | 7918    | 2.83 730  | 0.00 000  | 55        |
| <b>6</b>  | 7.24 187  | 6695  | 7.24 188  | 6694    | 2.75 812  | 0.00 000  | 54        |
| <b>7</b>  | 7.30 882  | 5799  | 7.30 882  | 5799    | 2.69 117  | 0.00 000  | 53        |
| <b>8</b>  | 7.36 681  | 5113  | 7.36 681  | 5113    | 2.63 318  | 0.00 000  | 52        |
| <b>9</b>  | 7.41 797  | 4573  | 7.41 797  | 4573    | 2.58 203  | 0.00 000  | 51        |
| <b>10</b> | 7.46 372  | 4139  | 7.46 372  | 4139    | 2.53 627  | 0.00 000  | <b>50</b> |
| <b>11</b> | 7.50 512  | 3778  | 7.50 512  | 3779    | 2.49 488  | 0.00 000  | 49        |
| <b>12</b> | 7.54 290  | 3476  | 7.54 291  | 3476    | 2.45 709  | 9.99 999  | 48        |
| <b>13</b> | 7.57 767  | 3218  | 7.57 767  | 3218    | 2.42 233  | 9.99 999  | 47        |
| <b>14</b> | 7.60 983  | 2996  | 7.60 983  | 2996    | 2.39 014  | 9.99 999  | 46        |
| <b>15</b> | 7.63 981  | 2803  | 7.63 982  | 2803    | 2.36 018  | 9.99 999  | 45        |
| <b>16</b> | 7.66 784  | 2633  | 7.66 785  | 2633    | 2.33 215  | 9.99 999  | 44        |
| <b>17</b> | 7.69 417  | 2482  | 7.69 418  | 2482    | 2.30 582  | 9.99 999  | 43        |
| <b>18</b> | 7.71 899  | 2348  | 7.71 900  | 2348    | 2.28 099  | 9.99 999  | 42        |
| <b>19</b> | 7.74 248  | 2227  | 7.74 248  | 2227    | 2.25 751  | 9.99 999  | 41        |
| <b>20</b> | 7.76 473  | 2119  | 7.76 476  | 2119    | 2.23 524  | 9.99 999  | <b>40</b> |
| <b>21</b> | 7.78 594  | 2020  | 7.78 595  | 2020    | 2.21 405  | 9.99 999  | 39        |
| <b>22</b> | 7.80 614  | 1930  | 7.80 613  | 1930    | 2.19 384  | 9.99 999  | 38        |
| <b>23</b> | 7.82 545  | 1848  | 7.82 546  | 1848    | 2.17 454  | 9.99 999  | 37        |
| <b>24</b> | 7.84 393  | 1772  | 7.84 394  | 1773    | 2.15 603  | 9.99 999  | 36        |
| <b>25</b> | 7.86 166  | 1703  | 7.86 167  | 1703    | 2.13 832  | 9.99 999  | 35        |
| <b>26</b> | 7.87 869  | 1639  | 7.87 871  | 1639    | 2.12 129  | 9.99 999  | 34        |
| <b>27</b> | 7.89 508  | 1579  | 7.89 510  | 1579    | 2.10 490  | 9.99 998  | 33        |
| <b>28</b> | 7.91 088  | 1524  | 7.91 089  | 1524    | 2.08 910  | 9.99 998  | 32        |
| <b>29</b> | 7.92 612  | 1472  | 7.92 613  | 1472    | 2.07 386  | 9.99 998  | 31        |
| <b>30</b> | 7.94 084  | 1424  | 7.94 086  | 1424    | 2.05 914  | 9.99 998  | <b>30</b> |
| <b>31</b> | 7.95 508  | 1379  | 7.95 510  | 1379    | 2.04 490  | 9.99 998  | 29        |
| <b>32</b> | 7.96 887  | 1336  | 7.96 889  | 1336    | 2.03 111  | 9.99 998  | 28        |
| <b>33</b> | 7.98 223  | 1296  | 7.98 223  | 1296    | 2.01 774  | 9.99 998  | 27        |
| <b>34</b> | 7.99 520  | 1258  | 7.99 522  | 1259    | 2.00 478  | 9.99 998  | 26        |
| <b>35</b> | 8.00 778  | 1223  | 8.00 781  | 1223    | 1.99 219  | 9.99 997  | 25        |
| <b>36</b> | 8.02 002  | 1190  | 8.02 004  | 1190    | 1.97 993  | 9.99 997  | 24        |
| <b>37</b> | 8.03 192  | 1158  | 8.03 194  | 1158    | 1.96 803  | 9.99 997  | 23        |
| <b>38</b> | 8.04 350  | 1128  | 8.04 352  | 1128    | 1.95 647  | 9.99 997  | 22        |
| <b>39</b> | 8.05 478  | 1099  | 8.05 481  | 1099    | 1.94 519  | 9.99 997  | 21        |
| <b>40</b> | 8.06 577  | 1072  | 8.06 580  | 1072    | 1.93 419  | 9.99 997  | <b>20</b> |
| <b>41</b> | 8.07 650  | 1046  | 8.07 653  | 1046    | 1.92 347  | 9.99 997  | 19        |
| <b>42</b> | 8.08 696  | 1022  | 8.08 699  | 1022    | 1.91 300  | 9.99 997  | 18        |
| <b>43</b> | 8.09 718  | 998   | 8.09 721  | 999     | 1.90 278  | 9.99 996  | 17        |
| <b>44</b> | 8.10 716  | 976   | 8.10 720  | 976     | 1.89 279  | 9.99 996  | 16        |
| <b>45</b> | 8.11 692  | 954   | 8.11 696  | 954     | 1.88 303  | 9.99 996  | 15        |
| <b>46</b> | 8.12 647  | 934   | 8.12 651  | 934     | 1.87 349  | 9.99 996  | 14        |
| <b>47</b> | 8.13 581  | 914   | 8.13 585  | 914     | 1.86 415  | 9.99 996  | 13        |
| <b>48</b> | 8.14 493  | 895   | 8.14 499  | 895     | 1.85 500  | 9.99 996  | 12        |
| <b>49</b> | 8.15 390  | 877   | 8.15 395  | 877     | 1.84 605  | 9.99 995  | 11        |
| <b>50</b> | 8.16 268  | 860   | 8.16 272  | 860     | 1.83 727  | 9.99 995  | <b>10</b> |
| <b>51</b> | 8.17 128  | 843   | 8.17 133  | 843     | 1.82 867  | 9.99 995  | 9         |
| <b>52</b> | 8.17 971  | 827   | 8.17 976  | 827     | 1.82 023  | 9.99 995  | 8         |
| <b>53</b> | 8.18 798  | 811   | 8.18 803  | 812     | 1.81 196  | 9.99 995  | 7         |
| <b>54</b> | 8.19 610  | 797   | 8.19 613  | 797     | 1.80 384  | 9.99 994  | 6         |
| <b>55</b> | 8.20 407  | 782   | 8.20 412  | 783     | 1.79 587  | 9.99 994  | 5         |
| <b>56</b> | 8.21 189  | 768   | 8.21 193  | 768     | 1.78 804  | 9.99 994  | 4         |
| <b>57</b> | 8.21 958  | 753   | 8.21 964  | 753     | 1.78 036  | 9.99 994  | 3         |
| <b>58</b> | 8.22 713  | 742   | 8.22 719  | 742     | 1.77 280  | 9.99 994  | 2         |
| <b>59</b> | 8.23 453  | 730   | 8.23 462  | 730     | 1.76 538  | 9.99 993  | 1         |
| <b>60</b> | 8.24 183  |       | 8.24 192  |         | 1.75 808  | 9.99 993  | <b>0</b>  |
|           | Log. Cos. | D     | Log. Cot. | Com. D. | Log. Tan. | Log. Sin. |           |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

1°

| '         | Log. Sin. | D   | Log. Tan. | Com. D. | Log. Cot. | Log. Cos. |           |
|-----------|-----------|-----|-----------|---------|-----------|-----------|-----------|
| <b>0</b>  | 8.24 183  |     | 8.24 192  |         | 1.75 808  | 9.99 993  | <b>60</b> |
| 1         | 8.24 903  | 718 | 8.24 910  | 718     | 1.75 090  | 9.99 993  | 59        |
| 2         | 8.25 609  | 706 | 8.25 616  | 706     | 1.74 383  | 9.99 993  | 58        |
| 3         | 8.26 304  | 694 | 8.26 311  | 695     | 1.73 688  | 9.99 992  | 57        |
| 4         | 8.26 988  | 684 | 8.26 993  | 684     | 1.73 004  | 9.99 992  | 56        |
| 5         | 8.27 661  | 673 | 8.27 669  | 673     | 1.72 331  | 9.99 992  | 55        |
| 6         | 8.28 324  | 663 | 8.28 332  | 663     | 1.71 667  | 9.99 992  | 54        |
| 7         | 8.28 977  | 653 | 8.28 983  | 653     | 1.71 014  | 9.99 992  | 53        |
| 8         | 8.29 626  | 643 | 8.29 629  | 643     | 1.70 371  | 9.99 991  | 52        |
| 9         | 8.30 254  | 634 | 8.30 263  | 634     | 1.69 736  | 9.99 991  | 51        |
| <b>10</b> | 8.30 879  | 625 | 8.30 888  | 625     | 1.69 111  | 9.99 991  | <b>50</b> |
| 11        | 8.31 493  | 616 | 8.31 504  | 616     | 1.68 493  | 9.99 990  | 49        |
| 12        | 8.32 102  | 607 | 8.32 112  | 607     | 1.67 888  | 9.99 990  | 48        |
| 13        | 8.32 701  | 599 | 8.32 711  | 599     | 1.67 288  | 9.99 990  | 47        |
| 14        | 8.33 292  | 591 | 8.33 302  | 591     | 1.66 697  | 9.99 990  | 46        |
| 15        | 8.33 873  | 583 | 8.33 883  | 583     | 1.66 114  | 9.99 989  | 45        |
| 16        | 8.34 450  | 575 | 8.34 461  | 575     | 1.65 539  | 9.99 989  | 44        |
| 17        | 8.35 018  | 567 | 8.35 029  | 568     | 1.64 971  | 9.99 989  | 43        |
| 18        | 8.35 578  | 560 | 8.35 589  | 560     | 1.64 410  | 9.99 989  | 42        |
| 19        | 8.36 131  | 553 | 8.36 143  | 553     | 1.63 857  | 9.99 988  | 41        |
| <b>20</b> | 8.36 677  | 546 | 8.36 689  | 546     | 1.63 310  | 9.99 988  | <b>40</b> |
| 21        | 8.37 217  | 539 | 8.37 229  | 539     | 1.62 771  | 9.99 988  | 39        |
| 22        | 8.37 750  | 533 | 8.37 762  | 533     | 1.62 238  | 9.99 987  | 38        |
| 23        | 8.38 276  | 526 | 8.38 289  | 527     | 1.61 711  | 9.99 987  | 37        |
| 24        | 8.38 796  | 520 | 8.38 809  | 520     | 1.61 191  | 9.99 987  | 36        |
| 25        | 8.39 310  | 514 | 8.39 323  | 514     | 1.60 676  | 9.99 986  | 35        |
| 26        | 8.39 818  | 508 | 8.39 831  | 508     | 1.60 168  | 9.99 986  | 34        |
| 27        | 8.40 320  | 502 | 8.40 334  | 502     | 1.59 666  | 9.99 986  | 33        |
| 28        | 8.40 816  | 496 | 8.40 830  | 496     | 1.59 169  | 9.99 986  | 32        |
| 29        | 8.41 307  | 491 | 8.41 321  | 491     | 1.58 678  | 9.99 985  | 31        |
| <b>30</b> | 8.41 792  | 485 | 8.41 807  | 485     | 1.58 193  | 9.99 985  | <b>30</b> |
| 31        | 8.42 271  | 479 | 8.42 287  | 480     | 1.57 713  | 9.99 985  | 29        |
| 32        | 8.42 746  | 474 | 8.42 762  | 475     | 1.57 238  | 9.99 984  | 28        |
| 33        | 8.43 213  | 469 | 8.43 231  | 469     | 1.56 768  | 9.99 984  | 27        |
| 34        | 8.43 680  | 464 | 8.43 696  | 464     | 1.56 304  | 9.99 984  | 26        |
| 35        | 8.44 139  | 459 | 8.44 156  | 460     | 1.55 844  | 9.99 983  | 25        |
| 36        | 8.44 594  | 454 | 8.44 611  | 455     | 1.55 389  | 9.99 983  | 24        |
| 37        | 8.45 044  | 450 | 8.45 061  | 450     | 1.54 938  | 9.99 982  | 23        |
| 38        | 8.45 489  | 445 | 8.45 507  | 445     | 1.54 493  | 9.99 982  | 22        |
| 39        | 8.45 930  | 440 | 8.45 948  | 441     | 1.54 052  | 9.99 982  | 21        |
| <b>40</b> | 8.46 366  | 436 | 8.46 385  | 437     | 1.53 615  | 9.99 981  | <b>20</b> |
| 41        | 8.46 798  | 432 | 8.46 817  | 432     | 1.53 183  | 9.99 981  | 19        |
| 42        | 8.47 226  | 428 | 8.47 243  | 428     | 1.52 751  | 9.99 981  | 18        |
| 43        | 8.47 650  | 423 | 8.47 669  | 424     | 1.52 330  | 9.99 980  | 17        |
| 44        | 8.48 069  | 419 | 8.48 089  | 419     | 1.51 911  | 9.99 980  | 16        |
| 45        | 8.48 485  | 415 | 8.48 505  | 416     | 1.51 495  | 9.99 979  | 15        |
| 46        | 8.48 896  | 411 | 8.48 917  | 412     | 1.51 083  | 9.99 979  | 14        |
| 47        | 8.49 304  | 407 | 8.49 325  | 408     | 1.50 675  | 9.99 979  | 13        |
| 48        | 8.49 708  | 404 | 8.49 729  | 404     | 1.50 270  | 9.99 978  | 12        |
| 49        | 8.50 108  | 400 | 8.50 130  | 400     | 1.49 870  | 9.99 978  | 11        |
| <b>50</b> | 8.50 504  | 396 | 8.50 526  | 396     | 1.49 473  | 9.99 978  | <b>10</b> |
| 51        | 8.50 897  | 393 | 8.50 920  | 393     | 1.49 080  | 9.99 977  | 9         |
| 52        | 8.51 286  | 389 | 8.51 310  | 390     | 1.48 690  | 9.99 977  | 8         |
| 53        | 8.51 672  | 386 | 8.51 696  | 386     | 1.48 304  | 9.99 976  | 7         |
| 54        | 8.52 055  | 382 | 8.52 079  | 383     | 1.47 921  | 9.99 976  | 6         |
| 55        | 8.52 434  | 379 | 8.52 458  | 379     | 1.47 541  | 9.99 975  | 5         |
| 56        | 8.52 810  | 375 | 8.52 835  | 376     | 1.47 165  | 9.99 975  | 4         |
| 57        | 8.53 183  | 373 | 8.53 208  | 373     | 1.46 792  | 9.99 975  | 3         |
| 58        | 8.53 552  | 369 | 8.53 578  | 370     | 1.46 422  | 9.99 974  | 2         |
| 59        | 8.53 918  | 366 | 8.53 944  | 366     | 1.46 053  | 9.99 974  | 1         |
| <b>60</b> | 8.54 282  | 363 | 8.54 308  | 364     | 1.45 691  | 9.99 973  | <b>0</b>  |
|           | Log. Cos. | D   | Log. Cot. | Com. D. | Log. Tan. | Log. Sin. | '         |

| '  | Log. Sin. | D   | Log. Tan. | Com. D. | Log. Cot. | Log. Cos. |    |
|----|-----------|-----|-----------|---------|-----------|-----------|----|
| 0  | 8.54 282  | 360 | 8.54 308  | 360     | 1.45 691  | 9.99 973  | 60 |
| 1  | 8.54 642  | 357 | 8.54 669  | 358     | 1.45 331  | 9.99 973  | 59 |
| 2  | 8.54 999  | 354 | 8.55 027  | 354     | 1.44 973  | 9.99 972  | 58 |
| 3  | 8.55 354  | 351 | 8.55 381  | 352     | 1.44 618  | 9.99 972  | 57 |
| 4  | 8.55 703  | 348 | 8.55 733  | 349     | 1.44 266  | 9.99 971  | 56 |
| 5  | 8.56 054  | 346 | 8.56 083  | 346     | 1.43 917  | 9.99 971  | 55 |
| 6  | 8.56 400  | 343 | 8.56 429  | 343     | 1.43 571  | 9.99 971  | 54 |
| 7  | 8.56 743  | 340 | 8.56 772  | 341     | 1.43 227  | 9.99 970  | 53 |
| 8  | 8.57 083  | 338 | 8.57 113  | 338     | 1.42 886  | 9.99 970  | 52 |
| 9  | 8.57 421  | 335 | 8.57 452  | 335     | 1.42 548  | 9.99 969  | 51 |
| 10 | 8.57 756  | 332 | 8.57 787  | 333     | 1.42 212  | 9.99 969  | 50 |
| 11 | 8.58 089  | 330 | 8.58 121  | 330     | 1.41 879  | 9.99 968  | 49 |
| 12 | 8.58 419  | 327 | 8.58 451  | 328     | 1.41 548  | 9.99 968  | 48 |
| 13 | 8.58 747  | 325 | 8.58 779  | 325     | 1.41 220  | 9.99 967  | 47 |
| 14 | 8.59 072  | 323 | 8.59 105  | 323     | 1.40 895  | 9.99 967  | 46 |
| 15 | 8.59 395  | 320 | 8.59 428  | 320     | 1.40 571  | 9.99 966  | 45 |
| 16 | 8.59 715  | 313 | 8.59 749  | 318     | 1.40 251  | 9.99 966  | 44 |
| 17 | 8.60 033  | 316 | 8.60 067  | 316     | 1.39 932  | 9.99 965  | 43 |
| 18 | 8.60 349  | 313 | 8.60 384  | 314     | 1.39 616  | 9.99 965  | 42 |
| 19 | 8.60 662  | 311 | 8.60 698  | 311     | 1.39 302  | 9.99 964  | 41 |
| 20 | 8.60 973  | 309 | 8.61 009  | 309     | 1.38 990  | 9.99 964  | 40 |
| 21 | 8.61 282  | 306 | 8.61 319  | 307     | 1.38 681  | 9.99 963  | 39 |
| 22 | 8.61 589  | 304 | 8.61 626  | 303     | 1.38 374  | 9.99 963  | 38 |
| 23 | 8.61 893  | 302 | 8.61 931  | 300     | 1.38 068  | 9.99 962  | 37 |
| 24 | 8.62 196  | 300 | 8.62 234  | 299     | 1.37 763  | 9.99 962  | 36 |
| 25 | 8.62 496  | 298 | 8.62 535  | 297     | 1.37 465  | 9.99 961  | 35 |
| 26 | 8.62 795  | 296 | 8.62 834  | 294     | 1.37 166  | 9.99 961  | 34 |
| 27 | 8.63 091  | 294 | 8.63 131  | 293     | 1.36 869  | 9.99 960  | 33 |
| 28 | 8.63 383  | 292 | 8.63 423  | 291     | 1.36 574  | 9.99 959  | 32 |
| 29 | 8.63 677  | 290 | 8.63 718  | 288     | 1.36 281  | 9.99 959  | 31 |
| 30 | 8.63 968  | 288 | 8.64 009  | 287     | 1.35 990  | 9.99 958  | 30 |
| 31 | 8.64 256  | 286 | 8.64 298  | 285     | 1.35 702  | 9.99 958  | 29 |
| 32 | 8.64 543  | 284 | 8.64 585  | 283     | 1.35 414  | 9.99 957  | 28 |
| 33 | 8.64 827  | 282 | 8.64 870  | 281     | 1.35 129  | 9.99 957  | 27 |
| 34 | 8.65 110  | 279 | 8.65 153  | 277     | 1.34 846  | 9.99 956  | 26 |
| 35 | 8.65 391  | 277 | 8.65 435  | 276     | 1.34 565  | 9.99 956  | 25 |
| 36 | 8.65 670  | 274 | 8.65 715  | 272     | 1.34 285  | 9.99 955  | 24 |
| 37 | 8.65 947  | 272 | 8.65 993  | 271     | 1.34 007  | 9.99 954  | 23 |
| 38 | 8.66 223  | 269 | 8.66 269  | 269     | 1.33 731  | 9.99 954  | 22 |
| 39 | 8.66 497  | 267 | 8.66 543  | 266     | 1.33 456  | 9.99 953  | 21 |
| 40 | 8.66 769  | 264 | 8.66 816  | 262     | 1.33 184  | 9.99 953  | 20 |
| 41 | 8.67 039  | 262 | 8.67 087  | 261     | 1.32 913  | 9.99 952  | 19 |
| 42 | 8.67 308  | 260 | 8.67 356  | 260     | 1.32 643  | 9.99 952  | 18 |
| 43 | 8.67 575  | 257 | 8.67 624  | 258     | 1.32 376  | 9.99 951  | 17 |
| 44 | 8.67 840  | 255 | 8.67 890  | 254     | 1.32 110  | 9.99 950  | 16 |
| 45 | 8.68 104  | 253 | 8.68 154  | 252     | 1.31 845  | 9.99 950  | 15 |
| 46 | 8.68 366  | 251 | 8.68 417  | 250     | 1.31 583  | 9.99 949  | 14 |
| 47 | 8.68 627  | 248 | 8.68 678  | 249     | 1.31 321  | 9.99 948  | 13 |
| 48 | 8.68 886  | 247 | 8.68 938  | 246     | 1.31 062  | 9.99 948  | 12 |
| 49 | 8.69 144  | 245 | 8.69 196  | 244     | 1.30 803  | 9.99 947  | 11 |
| 50 | 8.69 400  | 243 | 8.69 453  | 242     | 1.30 547  | 9.99 947  | 10 |
| 51 | 8.69 654  | 241 | 8.69 708  | 240     | 1.30 292  | 9.99 946  | 9  |
| 52 | 8.69 907  | 239 | 8.69 961  | 238     | 1.30 038  | 9.99 945  | 8  |
| 53 | 8.70 159  | 237 | 8.70 214  | 236     | 1.29 786  | 9.99 945  | 7  |
| 54 | 8.70 409  | 235 | 8.70 464  | 234     | 1.29 535  | 9.99 944  | 6  |
| 55 | 8.70 657  | 233 | 8.70 714  | 232     | 1.29 286  | 9.99 943  | 5  |
| 56 | 8.70 905  | 231 | 8.70 962  | 230     | 1.29 038  | 9.99 943  | 4  |
| 57 | 8.71 150  | 229 | 8.71 208  | 228     | 1.28 791  | 9.99 942  | 3  |
| 58 | 8.71 395  | 227 | 8.71 453  | 226     | 1.28 546  | 9.99 942  | 2  |
| 59 | 8.71 638  | 225 | 8.71 697  | 224     | 1.28 303  | 9.99 941  | 1  |
| 60 | 8.71 880  | 223 | 8.71 939  | 222     | 1.28 060  | 9.99 940  | 0  |
|    | Log. Cos. | D   | Log. Cot. | Com. D. | Log. Tan. | Log. Sin. | '  |

| °  | Log. Sin. |     | d. | Log. Tan. |     | c. d. | Log. Cot. |  | Log. Cos. |  | P. P. |     |       |       |       |       |     |     |
|----|-----------|-----|----|-----------|-----|-------|-----------|--|-----------|--|-------|-----|-------|-------|-------|-------|-----|-----|
|    |           |     |    |           |     |       |           |  |           |  |       |     |       |       |       |       |     |     |
| 0  | 8.71 880  |     |    | 8.71 939  |     |       | 1.28 060  |  | 9.99 940  |  | 60    |     |       |       |       |       |     |     |
| 1  | 8.72 126  | 246 |    | 8.72 186  | 241 |       | 1.27 819  |  | 9.99 940  |  | 59    |     |       |       |       |       |     |     |
| 2  | 8.72 359  | 239 |    | 8.72 420  | 240 |       | 1.27 579  |  | 9.99 939  |  | 58    |     |       |       |       |       |     |     |
| 3  | 8.72 597  | 237 |    | 8.72 659  | 238 |       | 1.27 341  |  | 9.99 938  |  | 57    |     |       |       |       |       |     |     |
| 4  | 8.72 833  | 236 |    | 8.72 896  | 237 |       | 1.27 104  |  | 9.99 938  |  | 56    |     |       |       |       |       |     |     |
|    |           | 235 |    |           | 235 |       |           |  |           |  |       | 330 | 320   | 310   | 300   |       |     |     |
| 5  | 8.73 069  | 235 |    | 8.73 131  | 235 |       | 1.26 868  |  | 9.99 937  |  | 55    | 6   | 33.0  | 32.0  | 31.0  | 30.0  |     |     |
| 6  | 8.73 302  | 233 |    | 8.73 366  | 235 |       | 1.26 633  |  | 9.99 936  |  | 54    | 7   | 38.5  | 37.5  | 36.5  | 35.0  |     |     |
| 7  | 8.73 535  | 233 |    | 8.73 599  | 233 |       | 1.26 400  |  | 9.99 935  |  | 53    | 8   | 44.0  | 42.0  | 41.0  | 40.0  |     |     |
| 8  | 8.73 766  | 231 |    | 8.73 831  | 232 |       | 1.26 168  |  | 9.99 935  |  | 52    | 9   | 49.5  | 46.0  | 45.0  | 44.0  |     |     |
| 9  | 8.73 997  | 230 |    | 8.74 062  | 231 |       | 1.25 937  |  | 9.99 934  |  | 51    | 10  | 55.0  | 51.0  | 51.0  | 50.0  |     |     |
|    |           | 229 |    |           | 229 |       |           |  |           |  |       | 20  | 110.0 | 106.0 | 103.0 | 100.0 |     |     |
| 10 | 8.74 226  | 227 |    | 8.74 292  | 228 |       | 1.25 708  |  | 9.99 933  |  | 50    | 30  | 175.0 | 160.0 | 155.0 | 150.0 |     |     |
| 11 | 8.74 453  | 226 |    | 8.74 520  | 227 |       | 1.25 479  |  | 9.99 933  |  | 49    | 40  | 220.0 | 213.0 | 206.0 | 200.0 |     |     |
| 12 | 8.74 680  | 225 |    | 8.74 748  | 226 |       | 1.25 252  |  | 9.99 932  |  | 48    | 50  | 275.0 | 266.0 | 258.0 | 250.0 |     |     |
| 13 | 8.74 903  | 224 |    | 8.74 974  | 225 |       | 1.25 026  |  | 9.99 931  |  | 47    |     |       |       |       |       |     |     |
| 14 | 8.75 129  | 223 |    | 8.75 199  | 223 |       | 1.24 801  |  | 9.99 931  |  | 46    |     |       |       |       |       |     |     |
|    |           | 221 |    |           | 221 |       |           |  |           |  |       |     | 290   | 280   | 270   | 260   |     |     |
| 15 | 8.75 353  | 221 |    | 8.75 422  | 223 |       | 1.24 577  |  | 9.99 930  |  | 45    | 6   | 29.0  | 28.0  | 27.0  | 26.0  |     |     |
| 16 | 8.75 574  | 221 |    | 8.75 643  | 223 |       | 1.24 354  |  | 9.99 929  |  | 44    | 7   | 33.5  | 32.5  | 31.5  | 30.0  |     |     |
| 17 | 8.75 795  | 219 |    | 8.75 867  | 221 |       | 1.24 133  |  | 9.99 928  |  | 43    | 8   | 38.0  | 37.0  | 36.0  | 34.0  |     |     |
| 18 | 8.76 015  | 218 |    | 8.76 087  | 220 |       | 1.23 913  |  | 9.99 928  |  | 42    | 9   | 43.5  | 42.0  | 40.5  | 39.0  |     |     |
| 19 | 8.76 233  | 217 |    | 8.76 306  | 219 |       | 1.23 693  |  | 9.99 927  |  | 41    | 10  | 48.0  | 46.0  | 45.0  | 43.0  |     |     |
|    |           | 216 |    |           | 216 |       |           |  |           |  |       | 20  | 96.0  | 93.0  | 90.0  | 86.0  |     |     |
| 20 | 8.76 451  | 215 |    | 8.76 524  | 217 |       | 1.23 473  |  | 9.99 926  |  | 40    | 30  | 145.0 | 140.0 | 135.0 | 130.0 |     |     |
| 21 | 8.76 667  | 214 |    | 8.76 741  | 216 |       | 1.23 258  |  | 9.99 925  |  | 39    | 40  | 193.0 | 186.0 | 180.0 | 173.0 |     |     |
| 22 | 8.76 883  | 213 |    | 8.76 958  | 214 |       | 1.23 042  |  | 9.99 925  |  | 38    | 50  | 241.0 | 233.0 | 225.0 | 216.0 |     |     |
| 23 | 8.77 097  | 212 |    | 8.77 172  | 213 |       | 1.22 827  |  | 9.99 924  |  | 37    |     |       |       |       |       |     |     |
| 24 | 8.77 310  | 211 |    | 8.77 386  | 212 |       | 1.22 613  |  | 9.99 923  |  | 36    |     |       |       |       |       |     |     |
|    |           | 210 |    |           | 210 |       |           |  |           |  |       |     | 250   | 240   | 230   | 220   |     |     |
| 25 | 8.77 522  | 210 |    | 8.77 599  | 212 |       | 1.22 400  |  | 9.99 922  |  | 35    | 6   | 25.0  | 24.0  | 23.0  | 22.0  |     |     |
| 26 | 8.77 733  | 209 |    | 8.77 811  | 211 |       | 1.22 188  |  | 9.99 922  |  | 34    | 7   | 29.5  | 28.0  | 26.5  | 25.0  |     |     |
| 27 | 8.77 943  | 208 |    | 8.78 022  | 210 |       | 1.21 978  |  | 9.99 921  |  | 33    | 8   | 33.5  | 32.0  | 30.5  | 29.0  |     |     |
| 28 | 8.78 152  | 207 |    | 8.78 232  | 209 |       | 1.21 768  |  | 9.99 920  |  | 32    | 9   | 37.5  | 36.0  | 34.5  | 33.0  |     |     |
| 29 | 8.78 360  | 206 |    | 8.78 441  | 208 |       | 1.21 559  |  | 9.99 919  |  | 31    | 10  | 41.0  | 40.0  | 38.0  | 36.0  |     |     |
|    |           | 205 |    |           | 205 |       |           |  |           |  |       | 20  | 83.0  | 80.0  | 76.0  | 73.0  |     |     |
| 30 | 8.78 567  | 204 |    | 8.78 648  | 207 |       | 1.21 351  |  | 9.99 919  |  | 30    | 30  | 125.0 | 120.0 | 115.0 | 110.0 |     |     |
| 31 | 8.78 773  | 203 |    | 8.78 853  | 206 |       | 1.21 144  |  | 9.99 918  |  | 29    | 40  | 166.0 | 160.0 | 153.0 | 146.0 |     |     |
| 32 | 8.78 978  | 202 |    | 8.79 061  | 205 |       | 1.20 938  |  | 9.99 917  |  | 28    | 50  | 208.0 | 200.0 | 191.0 | 183.0 |     |     |
| 33 | 8.79 183  | 201 |    | 8.79 266  | 204 |       | 1.20 734  |  | 9.99 916  |  | 27    |     |       |       |       |       |     |     |
| 34 | 8.79 386  | 200 |    | 8.79 470  | 203 |       | 1.20 530  |  | 9.99 916  |  | 26    |     |       |       |       |       |     |     |
|    |           | 199 |    |           | 199 |       |           |  |           |  |       |     | 210   | 200   | 190   | 180   |     |     |
| 35 | 8.79 588  | 198 |    | 8.79 673  | 198 |       | 1.20 327  |  | 9.99 915  |  | 25    | 6   | 21.0  | 20.0  | 19.0  | 18.0  |     |     |
| 36 | 8.79 789  | 197 |    | 8.79 875  | 197 |       | 1.20 125  |  | 9.99 914  |  | 24    | 7   | 24.5  | 23.0  | 22.0  | 21.0  |     |     |
| 37 | 8.79 989  | 196 |    | 8.80 076  | 196 |       | 1.19 923  |  | 9.99 913  |  | 23    | 8   | 28.0  | 26.0  | 25.0  | 24.0  |     |     |
| 38 | 8.80 189  | 195 |    | 8.80 276  | 195 |       | 1.19 723  |  | 9.99 912  |  | 22    | 9   | 31.5  | 30.0  | 28.5  | 27.0  |     |     |
| 39 | 8.80 387  | 194 |    | 8.80 476  | 194 |       | 1.19 524  |  | 9.99 912  |  | 21    | 10  | 35.0  | 33.0  | 31.0  | 30.0  |     |     |
|    |           | 193 |    |           | 193 |       |           |  |           |  |       | 20  | 70.0  | 66.0  | 63.0  | 60.0  |     |     |
| 40 | 8.80 585  | 192 |    | 8.80 674  | 192 |       | 1.19 326  |  | 9.99 911  |  | 20    | 30  | 105.0 | 100.0 | 95.0  | 90.0  |     |     |
| 41 | 8.80 782  | 191 |    | 8.80 871  | 191 |       | 1.19 128  |  | 9.99 910  |  | 19    | 40  | 140.0 | 133.0 | 126.0 | 120.0 |     |     |
| 42 | 8.80 977  | 190 |    | 8.81 068  | 190 |       | 1.18 931  |  | 9.99 909  |  | 18    | 50  | 175.0 | 166.0 | 158.0 | 150.0 |     |     |
| 43 | 8.81 172  | 189 |    | 8.81 264  | 189 |       | 1.18 736  |  | 9.99 908  |  | 17    |     |       |       |       |       |     |     |
| 44 | 8.81 366  | 188 |    | 8.81 459  | 188 |       | 1.18 541  |  | 9.99 907  |  | 16    |     |       |       |       |       |     |     |
|    |           | 187 |    |           | 187 |       |           |  |           |  |       |     | 9     | 9     | 8     | 7     | 6   | 5   |
| 45 | 8.81 560  | 186 |    | 8.81 653  | 186 |       | 1.18 347  |  | 9.99 907  |  | 15    | 6   | 0.9   | 0.9   | 0.8   | 0.7   | 0.6 | 0.5 |
| 46 | 8.81 752  | 185 |    | 8.81 846  | 185 |       | 1.18 154  |  | 9.99 906  |  | 14    | 7   | 1.1   | 1.0   | 0.9   | 0.8   | 0.7 | 0.6 |
| 47 | 8.81 943  | 184 |    | 8.82 038  | 184 |       | 1.17 961  |  | 9.99 905  |  | 13    | 8   | 1.2   | 1.1   | 1.0   | 0.9   | 0.8 | 0.7 |
| 48 | 8.82 134  | 183 |    | 8.82 230  | 183 |       | 1.17 770  |  | 9.99 904  |  | 12    | 9   | 1.3   | 1.2   | 1.1   | 1.0   | 0.9 | 0.8 |
| 49 | 8.82 324  | 182 |    | 8.82 420  | 182 |       | 1.17 579  |  | 9.99 903  |  | 11    | 10  | 1.4   | 1.3   | 1.2   | 1.1   | 1.0 | 0.9 |
|    |           | 181 |    |           | 181 |       |           |  |           |  |       | 20  | 1.6   | 1.5   | 1.4   | 1.3   | 1.2 | 1.1 |
| 50 | 8.82 513  | 180 |    | 8.82 610  | 180 |       | 1.17 389  |  | 9.99 902  |  | 10    | 30  | 3.1   | 3.0   | 2.9   | 2.8   | 2.7 | 2.6 |
| 51 | 8.82 701  | 179 |    | 8.82 799  | 179 |       | 1.17 201  |  | 9.99 902  |  | 9     | 40  | 4.2   | 4.0   | 3.9   | 3.8   | 3.7 | 3.6 |
| 52 | 8.82 888  | 178 |    | 8.82 987  | 178 |       | 1.17 012  |  | 9.99 901  |  | 8     | 50  | 6.3   | 6.0   | 5.9   | 5.8   | 5.7 | 5.6 |
| 53 | 8.83 075  | 177 |    | 8.83 175  | 177 |       | 1.16 825  |  | 9.99 900  |  | 7     |     |       |       |       |       |     |     |
| 54 | 8.83 260  | 176 |    | 8.83 361  | 176 |       | 1.16 638  |  | 9.99 899  |  | 6     |     |       |       |       |       |     |     |
|    |           | 175 |    |           | 175 |       |           |  |           |  |       |     | 4     | 4     | 3     | 2     | 1   | 0   |
| 55 | 8.83 445  | 174 |    | 8.83 547  | 174 |       | 1.16 453  |  | 9.99 898  |  | 5     | 6   | 0.4   | 0.4   | 0.3   | 0.2   | 0.1 | 0.0 |
| 56 | 8.83 629  | 173 |    | 8.83 732  | 173 |       | 1.16 268  |  | 9.99 897  |  | 4     | 7   | 0.5   | 0.4   | 0.3   | 0.2   | 0.1 | 0.0 |
| 57 | 8.83 813  | 172 |    | 8.83 916  | 172 |       | 1.16 083  |  | 9.99 896  |  | 3     | 8   | 0.6   | 0.5   | 0.4   | 0.3   | 0.2 | 0.1 |
| 58 | 8.83 995  | 171 |    | 8.84 100  | 171 |       | 1.15 900  |  | 9.99 896  |  | 2     | 9   | 0.7   | 0.6   | 0.5   | 0.4   | 0.3 | 0.2 |
| 59 | 8.84 177  | 170 |    | 8.84 282  | 170 |       | 1.15 717  |  | 9.99 895  |  | 1     | 10  | 0.7   | 0.6   | 0.5   | 0.4   | 0.3 | 0.2 |
|    |           | 169 |    |           | 169 |       |           |  |           |  |       | 20  | 1.5   | 1.3   | 1.2   | 1.1   | 1.0 | 0.9 |
| 60 | 8.84 358  | 168 |    | 8.84 464  | 168 |       | 1.15 535  |  | 9.99 894  |  | 0     | 30  | 2.2   | 2.0   | 1.9   | 1.8   | 1.7 | 1.6 |
|    |           | 167 |    |           | 167 |       |           |  |           |  |       | 40  | 3.0   | 2.6   | 2.5   | 2.4   | 2.3 | 2.2 |
|    |           | 166 |    |           | 166 |       |           |  |           |  |       | 50  | 3.7   | 3.3   | 3.2   | 3.1   | 3.0 | 2.9 |
|    |           | 165 |    |           | 165 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 164 |    |           | 164 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 163 |    |           | 163 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 162 |    |           | 162 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 161 |    |           | 161 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 160 |    |           | 160 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 159 |    |           | 159 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 158 |    |           | 158 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 157 |    |           | 157 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 156 |    |           | 156 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 155 |    |           | 155 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 154 |    |           | 154 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 153 |    |           | 153 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 152 |    |           | 152 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 151 |    |           | 151 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 150 |    |           | 150 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 149 |    |           | 149 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 148 |    |           | 148 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 147 |    |           | 147 |       |           |  |           |  |       |     |       |       |       |       |     |     |
|    |           | 146 |    |           | 146 |       |           |  |           |  |       |     |       |       |       |       |     |     |



TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

4°

|    | Log. Sin. | d.  | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P. |  |  |  |
|----|-----------|-----|-----------|-------|-----------|-----------|----|-------|--|--|--|
| 0  | 8.84 358  | 180 | 8.84 464  | 181   | 1.15 535  | 9.99 894  | 60 |       |  |  |  |
| 1  | 8.84 538  | 180 | 8.84 645  | 180   | 1.15 354  | 9.99 893  | 59 |       |  |  |  |
| 2  | 8.84 718  | 178 | 8.84 826  | 179   | 1.15 174  | 9.99 892  | 58 |       |  |  |  |
| 3  | 8.84 897  | 178 | 8.85 005  | 179   | 1.14 994  | 9.99 891  | 57 |       |  |  |  |
| 4  | 8.85 075  | 177 | 8.85 184  | 178   | 1.14 815  | 9.99 890  | 56 |       |  |  |  |
| 5  | 8.85 252  | 176 | 8.85 363  | 177   | 1.14 637  | 9.99 889  | 55 |       |  |  |  |
| 6  | 8.85 429  | 176 | 8.85 540  | 176   | 1.14 459  | 9.99 888  | 54 |       |  |  |  |
| 7  | 8.85 605  | 175 | 8.85 717  | 176   | 1.14 283  | 9.99 888  | 53 |       |  |  |  |
| 8  | 8.85 780  | 174 | 8.85 893  | 175   | 1.14 107  | 9.99 887  | 52 |       |  |  |  |
| 9  | 8.85 954  | 174 | 8.86 068  | 175   | 1.13 931  | 9.99 886  | 51 |       |  |  |  |
| 10 | 8.86 128  | 173 | 8.86 243  | 174   | 1.13 756  | 9.99 885  | 50 |       |  |  |  |
| 11 | 8.86 301  | 172 | 8.86 417  | 173   | 1.13 582  | 9.99 884  | 49 |       |  |  |  |
| 12 | 8.86 474  | 171 | 8.86 590  | 172   | 1.13 409  | 9.99 883  | 48 |       |  |  |  |
| 13 | 8.86 645  | 171 | 8.86 763  | 172   | 1.13 237  | 9.99 882  | 47 |       |  |  |  |
| 14 | 8.86 816  | 170 | 8.86 935  | 171   | 1.13 065  | 9.99 881  | 46 |       |  |  |  |
| 15 | 8.86 987  | 169 | 8.87 106  | 170   | 1.12 893  | 9.99 880  | 45 |       |  |  |  |
| 16 | 8.87 156  | 169 | 8.87 277  | 170   | 1.12 723  | 9.99 879  | 44 |       |  |  |  |
| 17 | 8.87 325  | 168 | 8.87 447  | 169   | 1.12 553  | 9.99 878  | 43 |       |  |  |  |
| 18 | 8.87 494  | 167 | 8.87 616  | 169   | 1.12 384  | 9.99 877  | 42 |       |  |  |  |
| 19 | 8.87 661  | 167 | 8.87 785  | 168   | 1.12 215  | 9.99 876  | 41 |       |  |  |  |
| 20 | 8.87 828  | 166 | 8.87 953  | 167   | 1.12 047  | 9.99 875  | 40 |       |  |  |  |
| 21 | 8.87 995  | 165 | 8.88 120  | 167   | 1.11 880  | 9.99 874  | 39 |       |  |  |  |
| 22 | 8.88 160  | 165 | 8.88 287  | 166   | 1.11 713  | 9.99 874  | 38 |       |  |  |  |
| 23 | 8.88 326  | 164 | 8.88 453  | 165   | 1.11 547  | 9.99 873  | 37 |       |  |  |  |
| 24 | 8.88 490  | 163 | 8.88 618  | 165   | 1.11 381  | 9.99 872  | 36 |       |  |  |  |
| 25 | 8.88 654  | 163 | 8.88 783  | 164   | 1.11 216  | 9.99 871  | 35 |       |  |  |  |
| 26 | 8.88 817  | 162 | 8.88 947  | 163   | 1.11 052  | 9.99 870  | 34 |       |  |  |  |
| 27 | 8.88 980  | 161 | 8.89 111  | 162   | 1.10 889  | 9.99 869  | 33 |       |  |  |  |
| 28 | 8.89 142  | 161 | 8.89 274  | 162   | 1.10 726  | 9.99 868  | 32 |       |  |  |  |
| 29 | 8.89 303  | 160 | 8.89 436  | 161   | 1.10 563  | 9.99 867  | 31 |       |  |  |  |
| 30 | 8.89 464  | 159 | 8.89 598  | 160   | 1.10 401  | 9.99 866  | 30 |       |  |  |  |
| 31 | 8.89 624  | 158 | 8.89 759  | 159   | 1.10 240  | 9.99 865  | 29 |       |  |  |  |
| 32 | 8.89 784  | 158 | 8.89 920  | 159   | 1.10 079  | 9.99 864  | 28 |       |  |  |  |
| 33 | 8.89 943  | 157 | 8.90 080  | 158   | 1.09 919  | 9.99 863  | 27 |       |  |  |  |
| 34 | 8.90 101  | 156 | 8.90 240  | 157   | 1.09 760  | 9.99 862  | 26 |       |  |  |  |
| 35 | 8.90 259  | 156 | 8.90 398  | 156   | 1.09 601  | 9.99 861  | 25 |       |  |  |  |
| 36 | 8.90 417  | 155 | 8.90 557  | 155   | 1.09 443  | 9.99 860  | 24 |       |  |  |  |
| 37 | 8.90 573  | 154 | 8.90 714  | 154   | 1.09 285  | 9.99 859  | 23 |       |  |  |  |
| 38 | 8.90 729  | 153 | 8.90 872  | 153   | 1.09 128  | 9.99 858  | 22 |       |  |  |  |
| 39 | 8.90 885  | 152 | 8.91 028  | 152   | 1.08 971  | 9.99 857  | 21 |       |  |  |  |
| 40 | 8.91 040  | 151 | 8.91 184  | 151   | 1.08 813  | 9.99 856  | 20 |       |  |  |  |
| 41 | 8.91 195  | 150 | 8.91 340  | 150   | 1.08 660  | 9.99 855  | 19 |       |  |  |  |
| 42 | 8.91 349  | 149 | 8.91 495  | 149   | 1.08 505  | 9.99 853  | 18 |       |  |  |  |
| 43 | 8.91 502  | 148 | 8.91 649  | 148   | 1.08 350  | 9.99 852  | 17 |       |  |  |  |
| 44 | 8.91 655  | 147 | 8.91 803  | 147   | 1.08 196  | 9.99 851  | 16 |       |  |  |  |
| 45 | 8.91 807  | 146 | 8.91 957  | 146   | 1.08 043  | 9.99 850  | 15 |       |  |  |  |
| 46 | 8.91 959  | 145 | 8.92 109  | 145   | 1.07 890  | 9.99 849  | 14 |       |  |  |  |
| 47 | 8.92 110  | 144 | 8.92 262  | 144   | 1.07 738  | 9.99 848  | 13 |       |  |  |  |
| 48 | 8.92 261  | 143 | 8.92 413  | 143   | 1.07 586  | 9.99 847  | 12 |       |  |  |  |
| 49 | 8.92 411  | 142 | 8.92 565  | 142   | 1.07 435  | 9.99 846  | 11 |       |  |  |  |
| 50 | 8.92 561  | 141 | 8.92 715  | 141   | 1.07 284  | 9.99 845  | 10 |       |  |  |  |
| 51 | 8.92 710  | 140 | 8.92 866  | 140   | 1.07 134  | 9.99 844  | 9  |       |  |  |  |
| 52 | 8.92 858  | 139 | 8.93 015  | 139   | 1.06 984  | 9.99 843  | 8  |       |  |  |  |
| 53 | 8.93 007  | 138 | 8.93 164  | 138   | 1.06 835  | 9.99 842  | 7  |       |  |  |  |
| 54 | 8.93 154  | 137 | 8.93 313  | 137   | 1.06 686  | 9.99 841  | 6  |       |  |  |  |
| 55 | 8.93 301  | 136 | 8.93 461  | 136   | 1.06 538  | 9.99 840  | 5  |       |  |  |  |
| 56 | 8.93 448  | 135 | 8.93 609  | 135   | 1.06 390  | 9.99 839  | 4  |       |  |  |  |
| 57 | 8.93 594  | 134 | 8.93 756  | 134   | 1.06 243  | 9.99 837  | 3  |       |  |  |  |
| 58 | 8.93 740  | 133 | 8.93 903  | 133   | 1.06 097  | 9.99 836  | 2  |       |  |  |  |
| 59 | 8.93 885  | 132 | 8.94 049  | 132   | 1.05 950  | 9.99 835  | 1  |       |  |  |  |
| 60 | 8.94 029  | 131 | 8.94 195  | 131   | 1.05 805  | 9.99 834  | 0  |       |  |  |  |
|    | Log. Cos. | d.  | Log. Cot. | c. d. | Log. Tan. | Log. Sin. |    | P. P. |  |  |  |

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| °  | P. P.     |     |           |       |           |           |
|----|-----------|-----|-----------|-------|-----------|-----------|
|    | Log. Sin. | d.  | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |
| 0  | 8.94 029  | 144 | 8.94 195  | 145   | 1.05 805  | 9.99 834  |
| 1  | 8.94 174  | 143 | 8.94 340  | 144   | 1.05 659  | 9.99 833  |
| 2  | 8.94 317  | 143 | 8.94 485  | 144   | 1.05 515  | 9.99 832  |
| 3  | 8.94 460  | 143 | 8.94 629  | 144   | 1.05 370  | 9.99 831  |
| 4  | 8.94 603  | 142 | 8.94 773  | 143   | 1.05 226  | 9.99 830  |
| 5  | 8.94 745  | 142 | 8.94 917  | 142   | 1.05 083  | 9.99 829  |
| 6  | 8.94 887  | 141 | 8.95 059  | 142   | 1.04 940  | 9.99 827  |
| 7  | 8.95 028  | 141 | 8.95 202  | 142   | 1.04 798  | 9.99 826  |
| 8  | 8.95 169  | 140 | 8.95 344  | 141   | 1.04 656  | 9.99 825  |
| 9  | 8.95 310  | 140 | 8.95 485  | 141   | 1.04 514  | 9.99 824  |
| 10 | 8.95 450  | 139 | 8.95 626  | 141   | 1.04 373  | 9.99 823  |
| 11 | 8.95 589  | 139 | 8.95 767  | 140   | 1.04 232  | 9.99 822  |
| 12 | 8.95 728  | 138 | 8.95 907  | 140   | 1.04 092  | 9.99 821  |
| 13 | 8.95 867  | 138 | 8.96 047  | 139   | 1.03 952  | 9.99 819  |
| 14 | 8.96 005  | 138 | 8.96 186  | 139   | 1.03 813  | 9.99 818  |
| 15 | 8.96 143  | 137 | 8.96 325  | 138   | 1.03 674  | 9.99 817  |
| 16 | 8.96 280  | 137 | 8.96 464  | 138   | 1.03 536  | 9.99 816  |
| 17 | 8.96 417  | 136 | 8.96 602  | 137   | 1.03 398  | 9.99 815  |
| 18 | 8.96 553  | 136 | 8.96 739  | 137   | 1.03 260  | 9.99 814  |
| 19 | 8.96 689  | 135 | 8.96 876  | 137   | 1.03 123  | 9.99 813  |
| 20 | 8.96 825  | 135 | 8.97 013  | 136   | 1.02 986  | 9.99 811  |
| 21 | 8.96 960  | 134 | 8.97 149  | 136   | 1.02 850  | 9.99 810  |
| 22 | 8.97 094  | 134 | 8.97 285  | 135   | 1.02 714  | 9.99 809  |
| 23 | 8.97 229  | 134 | 8.97 421  | 135   | 1.02 579  | 9.99 808  |
| 24 | 8.97 363  | 133 | 8.97 556  | 134   | 1.02 444  | 9.99 807  |
| 25 | 8.97 496  | 133 | 8.97 690  | 134   | 1.02 309  | 9.99 805  |
| 26 | 8.97 629  | 132 | 8.97 825  | 133   | 1.02 175  | 9.99 804  |
| 27 | 8.97 762  | 132 | 8.97 958  | 133   | 1.02 041  | 9.99 803  |
| 28 | 8.97 894  | 132 | 8.98 092  | 133   | 1.01 908  | 9.99 802  |
| 29 | 8.98 026  | 131 | 8.98 225  | 132   | 1.01 775  | 9.99 801  |
| 30 | 8.98 157  | 131 | 8.98 357  | 132   | 1.01 642  | 9.99 799  |
| 31 | 8.98 288  | 130 | 8.98 490  | 131   | 1.01 510  | 9.99 798  |
| 32 | 8.98 419  | 130 | 8.98 621  | 131   | 1.01 378  | 9.99 797  |
| 33 | 8.98 549  | 130 | 8.98 753  | 131   | 1.01 247  | 9.99 796  |
| 34 | 8.98 679  | 129 | 8.98 884  | 131   | 1.01 116  | 9.99 794  |
| 35 | 8.98 808  | 129 | 8.99 015  | 130   | 1.00 985  | 9.99 793  |
| 36 | 8.98 937  | 128 | 8.99 145  | 130   | 1.00 855  | 9.99 792  |
| 37 | 8.99 066  | 128 | 8.99 275  | 129   | 1.00 725  | 9.99 791  |
| 38 | 8.99 194  | 127 | 8.99 404  | 129   | 1.00 595  | 9.99 789  |
| 39 | 8.99 322  | 127 | 8.99 533  | 129   | 1.00 466  | 9.99 788  |
| 40 | 8.99 449  | 126 | 8.99 662  | 128   | 1.00 337  | 9.99 787  |
| 41 | 8.99 577  | 126 | 8.99 791  | 128   | 1.00 209  | 9.99 786  |
| 42 | 8.99 703  | 126 | 8.99 919  | 127   | 1.00 081  | 9.99 784  |
| 43 | 8.99 830  | 126 | 9.00 046  | 127   | 0.99 953  | 9.99 783  |
| 44 | 8.99 956  | 125 | 9.00 174  | 126   | 0.99 826  | 9.99 782  |
| 45 | 9.00 081  | 125 | 9.00 300  | 126   | 0.99 699  | 9.99 781  |
| 46 | 9.00 207  | 125 | 9.00 427  | 126   | 0.99 573  | 9.99 779  |
| 47 | 9.00 332  | 124 | 9.00 553  | 125   | 0.99 446  | 9.99 778  |
| 48 | 9.00 456  | 124 | 9.00 679  | 125   | 0.99 321  | 9.99 777  |
| 49 | 9.00 580  | 124 | 9.00 804  | 125   | 0.99 195  | 9.99 776  |
| 50 | 9.00 704  | 123 | 9.00 930  | 124   | 0.99 070  | 9.99 774  |
| 51 | 9.00 828  | 123 | 9.01 054  | 124   | 0.98 945  | 9.99 773  |
| 52 | 9.00 951  | 122 | 9.01 179  | 124   | 0.98 821  | 9.99 772  |
| 53 | 9.01 073  | 122 | 9.01 303  | 124   | 0.98 697  | 9.99 770  |
| 54 | 9.01 196  | 122 | 9.01 427  | 123   | 0.98 573  | 9.99 769  |
| 55 | 9.01 318  | 122 | 9.01 550  | 123   | 0.98 450  | 9.99 768  |
| 56 | 9.01 440  | 121 | 9.01 673  | 123   | 0.98 327  | 9.99 766  |
| 57 | 9.01 561  | 121 | 9.01 796  | 122   | 0.98 204  | 9.99 765  |
| 58 | 9.01 682  | 120 | 9.01 918  | 122   | 0.98 081  | 9.99 764  |
| 59 | 9.01 803  | 120 | 9.02 040  | 121   | 0.97 959  | 9.99 763  |
| 60 | 9.01 923  | 120 | 9.02 162  | 121   | 0.97 838  | 9.99 761  |
|    | Log. Cos. | d.  | Log. Cot. | c. d. | Log. Tan. | Log. Sin. |

| 145 | 144   | 143   | 142   | 141   |
|-----|-------|-------|-------|-------|
| 6   | 14.5  | 14.4  | 14.3  | 14.2  |
| 7   | 16.9  | 16.8  | 16.7  | 16.6  |
| 8   | 19.3  | 19.2  | 19.1  | 19.0  |
| 9   | 21.7  | 21.6  | 21.5  | 21.4  |
| 10  | 24.1  | 24.0  | 23.9  | 23.8  |
| 20  | 48.3  | 48.0  | 47.7  | 47.3  |
| 30  | 72.5  | 72.0  | 71.5  | 71.0  |
| 40  | 96.6  | 96.0  | 95.3  | 94.6  |
| 50  | 120.8 | 120.0 | 119.1 | 118.3 |

| 140 | 139   | 138   | 137   | 136   |
|-----|-------|-------|-------|-------|
| 6   | 14.0  | 13.9  | 13.8  | 13.7  |
| 7   | 16.3  | 16.2  | 16.1  | 16.0  |
| 8   | 18.6  | 18.5  | 18.4  | 18.3  |
| 9   | 21.0  | 20.8  | 20.7  | 20.5  |
| 10  | 23.3  | 23.1  | 23.0  | 22.8  |
| 20  | 46.6  | 46.3  | 46.0  | 45.6  |
| 30  | 70.0  | 69.5  | 69.0  | 68.5  |
| 40  | 93.3  | 92.6  | 92.0  | 91.3  |
| 50  | 116.6 | 115.8 | 115.0 | 114.1 |

| 135 | 134   | 133   | 132   |
|-----|-------|-------|-------|
| 6   | 13.5  | 13.4  | 13.3  |
| 7   | 15.7  | 15.6  | 15.5  |
| 8   | 18.0  | 17.8  | 17.7  |
| 9   | 20.2  | 20.1  | 19.9  |
| 10  | 22.5  | 22.3  | 22.1  |
| 20  | 45.0  | 44.6  | 44.3  |
| 30  | 67.5  | 67.0  | 66.5  |
| 40  | 90.0  | 89.3  | 88.6  |
| 50  | 112.5 | 111.6 | 110.8 |

| 131 | 130   | 129   | 128   |
|-----|-------|-------|-------|
| 6   | 13.1  | 13.0  | 12.9  |
| 7   | 15.3  | 15.1  | 15.0  |
| 8   | 17.4  | 17.3  | 17.2  |
| 9   | 19.6  | 19.5  | 19.3  |
| 10  | 21.8  | 21.6  | 21.5  |
| 20  | 43.6  | 43.3  | 43.0  |
| 30  | 65.5  | 65.0  | 64.5  |
| 40  | 87.3  | 86.6  | 86.0  |
| 50  | 109.1 | 108.3 | 107.5 |

| 127 | 126   | 125   | 124   | 123   |
|-----|-------|-------|-------|-------|
| 6   | 12.7  | 12.6  | 12.5  | 12.4  |
| 7   | 14.9  | 14.7  | 14.6  | 14.4  |
| 8   | 16.9  | 16.8  | 16.7  | 16.5  |
| 9   | 19.0  | 18.9  | 18.7  | 18.5  |
| 10  | 21.1  | 21.0  | 20.8  | 20.5  |
| 20  | 42.2  | 42.0  | 41.6  | 41.0  |
| 30  | 63.3  | 63.0  | 62.5  | 61.5  |
| 40  | 84.4  | 84.0  | 83.3  | 82.0  |
| 50  | 105.5 | 105.0 | 104.1 | 103.3 |

| 122 | 121   | 120  | 119  | 118  |
|-----|-------|------|------|------|
| 6   | 12.2  | 12.1 | 12.0 | 11.9 |
| 7   | 14.2  | 14.1 | 14.0 | 13.9 |
| 8   | 16.2  | 16.1 | 16.0 | 15.9 |
| 9   | 18.1  | 18.0 | 17.9 | 17.8 |
| 10  | 20.1  | 20.0 | 19.9 | 19.8 |
| 20  | 40.0  | 39.9 | 39.8 | 39.7 |
| 30  | 60.0  | 59.9 | 59.8 | 59.7 |
| 40  | 80.0  | 79.9 | 79.8 | 79.7 |
| 50  | 100.0 | 99.9 | 99.8 | 99.7 |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

6°

|    | Log. Sin. | d.  | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P. |  |  |  |  |
|----|-----------|-----|-----------|-------|-----------|-----------|----|-------|--|--|--|--|
| 0  | 9.01 923  |     | 9.02 162  | 121   | 0.97 838  | 9.99 761  | 60 |       |  |  |  |  |
| 1  | 9.02 043  | 120 | 9.02 283  | 121   | 0.97 716  | 9.99 760  | 59 |       |  |  |  |  |
| 2  | 9.02 163  | 119 | 9.02 404  | 120   | 0.97 595  | 9.99 759  | 58 |       |  |  |  |  |
| 3  | 9.02 282  | 119 | 9.02 525  | 120   | 0.97 475  | 9.99 757  | 57 |       |  |  |  |  |
| 4  | 9.02 401  | 119 | 9.02 645  | 120   | 0.97 354  | 9.99 756  | 56 |       |  |  |  |  |
| 5  | 9.02 520  | 119 | 9.02 765  | 120   | 0.97 234  | 9.99 754  | 55 |       |  |  |  |  |
| 6  | 9.02 638  | 118 | 9.02 885  | 119   | 0.97 115  | 9.99 753  | 54 |       |  |  |  |  |
| 7  | 9.02 756  | 118 | 9.03 004  | 119   | 0.96 995  | 9.99 752  | 53 |       |  |  |  |  |
| 8  | 9.02 874  | 118 | 9.03 123  | 119   | 0.96 876  | 9.99 750  | 52 |       |  |  |  |  |
| 9  | 9.02 992  | 117 | 9.03 242  | 119   | 0.96 757  | 9.99 749  | 51 |       |  |  |  |  |
| 10 | 9.03 109  | 117 | 9.03 361  | 118   | 0.96 639  | 9.99 748  | 50 |       |  |  |  |  |
| 11 | 9.03 225  | 116 | 9.03 479  | 118   | 0.96 521  | 9.99 746  | 49 |       |  |  |  |  |
| 12 | 9.03 342  | 116 | 9.03 597  | 118   | 0.96 403  | 9.99 745  | 48 |       |  |  |  |  |
| 13 | 9.03 458  | 116 | 9.03 714  | 117   | 0.96 285  | 9.99 744  | 47 |       |  |  |  |  |
| 14 | 9.03 574  | 116 | 9.03 831  | 117   | 0.96 168  | 9.99 742  | 46 |       |  |  |  |  |
| 15 | 9.03 689  | 115 | 9.03 948  | 117   | 0.96 051  | 9.99 741  | 45 |       |  |  |  |  |
| 16 | 9.03 805  | 115 | 9.04 065  | 116   | 0.95 935  | 9.99 739  | 44 |       |  |  |  |  |
| 17 | 9.03 919  | 114 | 9.04 181  | 116   | 0.95 818  | 9.99 738  | 43 |       |  |  |  |  |
| 18 | 9.04 034  | 114 | 9.04 297  | 116   | 0.95 702  | 9.99 737  | 42 |       |  |  |  |  |
| 19 | 9.04 148  | 114 | 9.04 413  | 115   | 0.95 587  | 9.99 735  | 41 |       |  |  |  |  |
| 20 | 9.04 262  | 114 | 9.04 528  | 115   | 0.95 471  | 9.99 734  | 40 |       |  |  |  |  |
| 21 | 9.04 376  | 113 | 9.04 643  | 115   | 0.95 356  | 9.99 732  | 39 |       |  |  |  |  |
| 22 | 9.04 489  | 113 | 9.04 758  | 114   | 0.95 242  | 9.99 731  | 38 |       |  |  |  |  |
| 23 | 9.04 602  | 113 | 9.04 872  | 114   | 0.95 127  | 9.99 730  | 37 |       |  |  |  |  |
| 24 | 9.04 715  | 112 | 9.04 987  | 114   | 0.95 013  | 9.99 728  | 36 |       |  |  |  |  |
| 25 | 9.04 828  | 112 | 9.05 101  | 113   | 0.94 899  | 9.99 727  | 35 |       |  |  |  |  |
| 26 | 9.04 940  | 112 | 9.05 214  | 113   | 0.94 785  | 9.99 725  | 34 |       |  |  |  |  |
| 27 | 9.05 052  | 111 | 9.05 327  | 113   | 0.94 672  | 9.99 724  | 33 |       |  |  |  |  |
| 28 | 9.05 163  | 111 | 9.05 440  | 113   | 0.94 559  | 9.99 723  | 32 |       |  |  |  |  |
| 29 | 9.05 275  | 111 | 9.05 553  | 112   | 0.94 446  | 9.99 721  | 31 |       |  |  |  |  |
| 30 | 9.05 386  | 110 | 9.05 666  | 112   | 0.94 334  | 9.99 720  | 30 |       |  |  |  |  |
| 31 | 9.05 496  | 110 | 9.05 778  | 112   | 0.94 222  | 9.99 718  | 29 |       |  |  |  |  |
| 32 | 9.05 607  | 110 | 9.05 890  | 111   | 0.94 110  | 9.99 717  | 28 |       |  |  |  |  |
| 33 | 9.05 717  | 110 | 9.06 001  | 111   | 0.93 998  | 9.99 715  | 27 |       |  |  |  |  |
| 34 | 9.05 827  | 110 | 9.06 113  | 111   | 0.93 887  | 9.99 714  | 26 |       |  |  |  |  |
| 35 | 9.05 936  | 109 | 9.06 224  | 111   | 0.93 776  | 9.99 712  | 25 |       |  |  |  |  |
| 36 | 9.06 046  | 109 | 9.06 335  | 110   | 0.93 665  | 9.99 711  | 24 |       |  |  |  |  |
| 37 | 9.06 155  | 109 | 9.06 445  | 110   | 0.93 554  | 9.99 710  | 23 |       |  |  |  |  |
| 38 | 9.06 264  | 108 | 9.06 555  | 110   | 0.93 444  | 9.99 708  | 22 |       |  |  |  |  |
| 39 | 9.06 372  | 108 | 9.06 665  | 109   | 0.93 334  | 9.99 707  | 21 |       |  |  |  |  |
| 40 | 9.06 480  | 108 | 9.06 775  | 109   | 0.93 225  | 9.99 705  | 20 |       |  |  |  |  |
| 41 | 9.06 588  | 107 | 9.06 884  | 109   | 0.93 115  | 9.99 704  | 19 |       |  |  |  |  |
| 42 | 9.06 696  | 107 | 9.06 994  | 108   | 0.93 006  | 9.99 702  | 18 |       |  |  |  |  |
| 43 | 9.06 803  | 107 | 9.07 102  | 109   | 0.92 897  | 9.99 701  | 17 |       |  |  |  |  |
| 44 | 9.06 910  | 107 | 9.07 211  | 108   | 0.92 788  | 9.99 699  | 16 |       |  |  |  |  |
| 45 | 9.07 017  | 106 | 9.07 319  | 108   | 0.92 680  | 9.99 698  | 15 |       |  |  |  |  |
| 46 | 9.07 124  | 106 | 9.07 428  | 107   | 0.92 572  | 9.99 696  | 14 |       |  |  |  |  |
| 47 | 9.07 230  | 106 | 9.07 535  | 107   | 0.92 464  | 9.99 695  | 13 |       |  |  |  |  |
| 48 | 9.07 336  | 106 | 9.07 643  | 107   | 0.92 357  | 9.99 693  | 12 |       |  |  |  |  |
| 49 | 9.07 442  | 105 | 9.07 750  | 107   | 0.92 249  | 9.99 692  | 11 |       |  |  |  |  |
| 50 | 9.07 548  | 105 | 9.07 857  | 107   | 0.92 142  | 9.99 690  | 10 |       |  |  |  |  |
| 51 | 9.07 653  | 105 | 9.07 964  | 106   | 0.92 035  | 9.99 689  | 9  |       |  |  |  |  |
| 52 | 9.07 758  | 104 | 9.08 071  | 106   | 0.91 929  | 9.99 687  | 8  |       |  |  |  |  |
| 53 | 9.07 863  | 104 | 9.08 177  | 106   | 0.91 822  | 9.99 686  | 7  |       |  |  |  |  |
| 54 | 9.07 967  | 104 | 9.08 283  | 105   | 0.91 716  | 9.99 684  | 6  |       |  |  |  |  |
| 55 | 9.08 072  | 104 | 9.08 389  | 105   | 0.91 611  | 9.99 683  | 5  |       |  |  |  |  |
| 56 | 9.08 176  | 103 | 9.08 494  | 105   | 0.91 505  | 9.99 681  | 4  |       |  |  |  |  |
| 57 | 9.08 279  | 103 | 9.08 600  | 105   | 0.91 400  | 9.99 679  | 3  |       |  |  |  |  |
| 58 | 9.08 383  | 103 | 9.08 705  | 105   | 0.91 295  | 9.99 678  | 2  |       |  |  |  |  |
| 59 | 9.08 486  | 103 | 9.08 810  | 104   | 0.91 190  | 9.99 676  | 1  |       |  |  |  |  |
| 60 | 9.08 589  | 103 | 9.08 914  |       | 0.91 085  | 9.99 675  | 0  |       |  |  |  |  |
|    | Log. Cos. | d.  | Log. Cot. | c. d. | Log. Tan. | Log. Sin. |    |       |  |  |  |  |

|    |       |       |       |      |
|----|-------|-------|-------|------|
|    | 121   | 120   | 119   | 118  |
| 6  | 12.1  | 12.0  | 11.9  | 11.8 |
| 7  | 14.2  | 14.1  | 14.0  | 13.9 |
| 8  | 16.2  | 16.1  | 16.0  | 15.9 |
| 9  | 18.2  | 18.1  | 18.0  | 17.9 |
| 10 | 20.2  | 20.1  | 20.0  | 19.9 |
| 20 | 40.5  | 40.3  | 40.0  | 39.9 |
| 30 | 60.7  | 60.5  | 60.0  | 59.9 |
| 40 | 81.0  | 80.6  | 80.0  | 79.9 |
| 50 | 101.2 | 100.8 | 100.0 | 99.9 |

|    |      |      |      |
|----|------|------|------|
|    | 117  | 116  | 115  |
| 6  | 11.7 | 11.6 | 11.5 |
| 7  | 13.7 | 13.6 | 13.5 |
| 8  | 15.6 | 15.6 | 15.4 |
| 9  | 17.6 | 17.5 | 17.4 |
| 10 | 19.6 | 19.5 | 19.3 |
| 20 | 39.1 | 39.0 | 38.6 |
| 30 | 58.7 | 58.5 | 58.0 |
| 40 | 78.3 | 78.0 | 77.3 |
| 50 | 97.9 | 97.5 | 96.6 |

|    |      |      |      |      |
|----|------|------|------|------|
|    | 114  | 113  | 112  | 111  |
| 6  | 11.4 | 11.3 | 11.2 | 11.1 |
| 7  | 13.3 | 13.3 | 13.0 | 12.9 |
| 8  | 15.2 | 15.2 | 14.9 | 14.8 |
| 9  | 17.2 | 17.1 | 16.9 | 16.6 |
| 10 | 19.1 | 19.0 | 18.8 | 18.5 |
| 20 | 38.1 | 38.0 | 37.6 | 37.0 |
| 30 | 57.2 | 57.0 | 56.5 | 55.5 |
| 40 | 76.3 | 76.0 | 75.3 | 74.0 |
| 50 | 95.4 | 95.0 | 94.1 | 92.5 |

|    |      |      |      |
|----|------|------|------|
|    | 110  | 109  | 108  |
| 6  | 11.0 | 11.0 | 10.8 |
| 7  | 12.9 | 12.9 | 12.6 |
| 8  | 14.7 | 14.6 | 14.4 |
| 9  | 16.6 | 16.5 | 16.2 |
| 10 | 18.4 | 18.3 | 18.0 |
| 20 | 36.8 | 36.6 | 36.0 |
| 30 | 55.2 | 55.0 | 54.0 |
| 40 | 73.6 | 73.3 | 72.0 |
| 50 | 92.1 | 91.6 | 90.0 |

|    |      |      |      |      |
|----|------|------|------|------|
|    | 107  | 106  | 105  | 104  |
| 6  | 10.7 | 10.6 | 10.5 | 10.4 |
| 7  | 12.5 | 12.5 | 12.2 | 12.1 |
| 8  | 14.3 | 14.2 | 14.1 | 13.8 |
| 9  | 16.1 | 16.0 | 15.9 | 15.6 |
| 10 | 17.9 | 17.8 | 17.6 | 17.3 |
| 20 | 35.8 | 35.6 | 35.3 | 34.6 |
| 30 | 53.7 | 53.5 | 53.0 | 52.0 |
| 40 | 71.6 | 71.3 | 70.6 | 69.3 |
| 50 | 89.6 | 89.1 | 88.3 | 86.6 |

|    |      |      |      |      |
|----|------|------|------|------|
|    | 103  | 102  | 101  | 100  |
| 6  | 10.3 | 10.3 | 10.1 | 10.0 |
| 7  | 12.1 | 12.0 | 11.9 | 11.7 |
| 8  | 13.8 | 13.7 | 13.6 | 13.4 |
| 9  | 15.5 | 15.4 | 15.3 | 15.1 |
| 10 | 17.2 | 17.1 | 17.0 | 16.8 |
| 20 | 34.5 | 34.3 | 34.0 | 33.6 |
| 30 | 51.7 | 51.5 | 51.0 | 50.5 |
| 40 | 69.0 | 68.6 | 68.0 | 67.0 |
| 50 | 86.2 | 85.8 | 85.0 | 83.8 |

|    |       |
|----|-------|
|    | P. P. |
| 6  | 10.3  |
| 7  | 12.1  |
| 8  | 13.8  |
| 9  | 15.5  |
| 10 | 17.2  |
| 20 | 34.5  |
| 30 | 51.7  |
| 40 | 69.0  |
| 50 | 86.2  |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

7°

|    | Log. Sin. | d.  | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P. |  |  |  |
|----|-----------|-----|-----------|-------|-----------|-----------|----|-------|--|--|--|
| 0  | 9.08 589  |     | 9.08 914  |       | 0.91 085  | 9.99 675  | 60 |       |  |  |  |
| 1  | 9.08 692  | 102 | 9.09 018  | 104   | 0.90 981  | 9.99 673  | 59 |       |  |  |  |
| 2  | 9.08 794  | 102 | 9.09 123  | 104   | 0.90 877  | 9.99 672  | 58 |       |  |  |  |
| 3  | 9.08 897  | 102 | 9.09 226  | 103   | 0.90 773  | 9.99 670  | 57 |       |  |  |  |
| 4  | 9.08 999  | 102 | 9.09 330  | 103   | 0.90 670  | 9.99 669  | 56 |       |  |  |  |
| 5  | 9.09 101  | 102 | 9.09 433  | 103   | 0.90 566  | 9.99 667  | 55 |       |  |  |  |
| 6  | 9.09 202  | 101 | 9.09 536  | 103   | 0.90 463  | 9.99 665  | 54 |       |  |  |  |
| 7  | 9.09 303  | 101 | 9.09 639  | 102   | 0.90 360  | 9.99 664  | 53 |       |  |  |  |
| 8  | 9.09 404  | 101 | 9.09 742  | 102   | 0.90 258  | 9.99 662  | 52 |       |  |  |  |
| 9  | 9.09 505  | 100 | 9.09 844  | 102   | 0.90 155  | 9.99 661  | 51 |       |  |  |  |
| 10 | 9.09 605  | 100 | 9.09 947  | 101   | 0.90 053  | 9.99 659  | 50 |       |  |  |  |
| 11 | 9.09 706  | 100 | 9.10 048  | 102   | 0.89 951  | 9.99 658  | 49 |       |  |  |  |
| 12 | 9.09 806  | 100 | 9.10 150  | 101   | 0.89 849  | 9.99 656  | 48 |       |  |  |  |
| 13 | 9.09 906  | 99  | 9.10 252  | 101   | 0.89 748  | 9.99 654  | 47 |       |  |  |  |
| 14 | 9.10 006  | 99  | 9.10 353  | 101   | 0.89 647  | 9.99 653  | 46 |       |  |  |  |
| 15 | 9.10 105  | 99  | 9.10 454  | 101   | 0.89 546  | 9.99 651  | 45 |       |  |  |  |
| 16 | 9.10 205  | 98  | 9.10 555  | 100   | 0.89 445  | 9.99 650  | 44 |       |  |  |  |
| 17 | 9.10 303  | 99  | 9.10 655  | 100   | 0.89 344  | 9.99 648  | 43 |       |  |  |  |
| 18 | 9.10 402  | 98  | 9.10 756  | 100   | 0.89 244  | 9.99 646  | 42 |       |  |  |  |
| 19 | 9.10 501  | 98  | 9.10 856  | 100   | 0.89 144  | 9.99 645  | 41 |       |  |  |  |
| 20 | 9.10 599  | 98  | 9.10 956  | 99    | 0.89 044  | 9.99 643  | 40 |       |  |  |  |
| 21 | 9.10 697  | 97  | 9.11 055  | 99    | 0.88 944  | 9.99 641  | 39 |       |  |  |  |
| 22 | 9.10 795  | 97  | 9.11 155  | 99    | 0.88 845  | 9.99 640  | 38 |       |  |  |  |
| 23 | 9.10 892  | 97  | 9.11 254  | 99    | 0.88 745  | 9.99 638  | 37 |       |  |  |  |
| 24 | 9.10 990  | 97  | 9.11 353  | 98    | 0.88 646  | 9.99 637  | 36 |       |  |  |  |
| 25 | 9.11 087  | 96  | 9.11 452  | 98    | 0.88 548  | 9.99 635  | 35 |       |  |  |  |
| 26 | 9.11 184  | 97  | 9.11 550  | 98    | 0.88 449  | 9.99 633  | 34 |       |  |  |  |
| 27 | 9.11 281  | 96  | 9.11 649  | 98    | 0.88 351  | 9.99 632  | 33 |       |  |  |  |
| 28 | 9.11 377  | 96  | 9.11 747  | 98    | 0.88 253  | 9.99 630  | 32 |       |  |  |  |
| 29 | 9.11 473  | 96  | 9.11 845  | 98    | 0.88 155  | 9.99 628  | 31 |       |  |  |  |
| 30 | 9.11 570  | 95  | 9.11 943  | 97    | 0.88 057  | 9.99 627  | 30 |       |  |  |  |
| 31 | 9.11 665  | 96  | 9.12 040  | 97    | 0.87 959  | 9.99 625  | 29 |       |  |  |  |
| 32 | 9.11 761  | 95  | 9.12 137  | 97    | 0.87 862  | 9.99 623  | 28 |       |  |  |  |
| 33 | 9.11 856  | 95  | 9.12 235  | 96    | 0.87 765  | 9.99 622  | 27 |       |  |  |  |
| 34 | 9.11 952  | 95  | 9.12 331  | 97    | 0.87 668  | 9.99 620  | 26 |       |  |  |  |
| 35 | 9.12 047  | 94  | 9.12 428  | 96    | 0.87 571  | 9.99 618  | 25 |       |  |  |  |
| 36 | 9.12 141  | 94  | 9.12 525  | 96    | 0.87 475  | 9.99 617  | 24 |       |  |  |  |
| 37 | 9.12 236  | 94  | 9.12 621  | 96    | 0.87 379  | 9.99 615  | 23 |       |  |  |  |
| 38 | 9.12 330  | 94  | 9.12 717  | 96    | 0.87 283  | 9.99 613  | 22 |       |  |  |  |
| 39 | 9.12 425  | 93  | 9.12 813  | 95    | 0.87 187  | 9.99 611  | 21 |       |  |  |  |
| 40 | 9.12 518  | 94  | 9.12 908  | 95    | 0.87 091  | 9.99 610  | 20 |       |  |  |  |
| 41 | 9.12 612  | 93  | 9.13 004  | 95    | 0.86 996  | 9.99 608  | 19 |       |  |  |  |
| 42 | 9.12 706  | 93  | 9.13 099  | 95    | 0.86 900  | 9.99 606  | 18 |       |  |  |  |
| 43 | 9.12 799  | 93  | 9.13 194  | 95    | 0.86 805  | 9.99 605  | 17 |       |  |  |  |
| 44 | 9.12 892  | 93  | 9.13 289  | 94    | 0.86 710  | 9.99 603  | 16 |       |  |  |  |
| 45 | 9.12 985  | 92  | 9.13 384  | 94    | 0.86 616  | 9.99 601  | 15 |       |  |  |  |
| 46 | 9.13 078  | 92  | 9.13 478  | 94    | 0.86 521  | 9.99 600  | 14 |       |  |  |  |
| 47 | 9.13 170  | 92  | 9.13 572  | 94    | 0.86 427  | 9.99 598  | 13 |       |  |  |  |
| 48 | 9.13 263  | 92  | 9.13 666  | 94    | 0.86 333  | 9.99 596  | 12 |       |  |  |  |
| 49 | 9.13 355  | 92  | 9.13 760  | 93    | 0.86 239  | 9.99 594  | 11 |       |  |  |  |
| 50 | 9.13 447  | 91  | 9.13 854  | 93    | 0.86 146  | 9.99 593  | 10 |       |  |  |  |
| 51 | 9.13 538  | 92  | 9.13 947  | 93    | 0.86 052  | 9.99 591  | 9  |       |  |  |  |
| 52 | 9.13 630  | 91  | 9.14 041  | 93    | 0.85 959  | 9.99 589  | 8  |       |  |  |  |
| 53 | 9.13 721  | 91  | 9.14 134  | 93    | 0.85 866  | 9.99 587  | 7  |       |  |  |  |
| 54 | 9.13 813  | 90  | 9.14 227  | 92    | 0.85 773  | 9.99 586  | 6  |       |  |  |  |
| 55 | 9.13 903  | 91  | 9.14 319  | 92    | 0.85 680  | 9.99 584  | 5  |       |  |  |  |
| 56 | 9.13 994  | 90  | 9.14 412  | 92    | 0.85 588  | 9.99 582  | 4  |       |  |  |  |
| 57 | 9.14 085  | 90  | 9.14 504  | 92    | 0.85 495  | 9.99 580  | 3  |       |  |  |  |
| 58 | 9.14 175  | 90  | 9.14 596  | 92    | 0.85 403  | 9.99 579  | 2  |       |  |  |  |
| 59 | 9.14 265  | 90  | 9.14 688  | 92    | 0.85 311  | 9.99 577  | 1  |       |  |  |  |
| 60 | 9.14 355  |     | 9.14 780  |       | 0.85 219  | 9.99 575  | 0  |       |  |  |  |
|    | Log. Cos. | d.  | Log. Cot. | c. d. | Log. Tan. | Log. Sin. |    |       |  |  |  |

| P. P.   |      |      |      |
|---------|------|------|------|
| 104     | 103  | 102  | 101  |
| 6 10.4  | 10.3 | 10.2 | 10.1 |
| 7 12.1  | 12.0 | 11.9 | 11.8 |
| 8 13.8  | 13.7 | 13.6 | 13.5 |
| 9 15.6  | 15.4 | 15.3 | 15.1 |
| 10 17.3 | 17.1 | 17.0 | 16.8 |
| 20 34.6 | 34.3 | 34.0 | 33.6 |
| 30 52.0 | 51.5 | 51.0 | 50.5 |
| 40 69.3 | 68.6 | 68.0 | 67.4 |
| 50 86.6 | 85.8 | 85.0 | 84.1 |

| P. P.   |      |      |      |
|---------|------|------|------|
| 100     | 100  | 99   | 98   |
| 6 10.6  | 10.5 | 9.9  | 9.8  |
| 7 11.7  | 11.6 | 11.5 | 11.4 |
| 8 13.4  | 13.3 | 13.2 | 13.0 |
| 9 15.1  | 15.0 | 14.8 | 14.7 |
| 10 16.7 | 16.6 | 16.5 | 16.3 |
| 20 33.5 | 33.3 | 33.0 | 32.6 |
| 30 50.2 | 50.0 | 49.5 | 49.0 |
| 40 67.0 | 66.6 | 66.0 | 65.3 |
| 50 83.7 | 83.3 | 82.5 | 81.6 |

| P. P.   |      |      |      |
|---------|------|------|------|
| 97      | 97   | 96   | 95   |
| 6 9.7   | 9.7  | 9.6  | 9.5  |
| 7 11.4  | 11.3 | 11.2 | 11.1 |
| 8 13.0  | 12.9 | 12.8 | 12.7 |
| 9 14.6  | 14.5 | 14.4 | 14.2 |
| 10 16.2 | 16.1 | 16.0 | 15.8 |
| 20 32.5 | 32.3 | 32.0 | 31.6 |
| 30 48.7 | 48.5 | 48.0 | 47.5 |
| 40 65.0 | 64.6 | 64.0 | 63.3 |
| 50 81.2 | 80.6 | 80.0 | 79.1 |

| P. P.   |      |      |      |
|---------|------|------|------|
| 94      | 94   | 93   | 92   |
| 6 9.4   | 9.4  | 9.3  | 9.2  |
| 7 11.0  | 10.9 | 10.8 | 10.7 |
| 8 12.6  | 12.5 | 12.4 | 12.2 |
| 9 14.2  | 14.1 | 13.9 | 13.8 |
| 10 15.7 | 15.6 | 15.5 | 15.3 |
| 20 31.5 | 31.3 | 31.0 | 30.6 |
| 30 47.2 | 47.0 | 46.8 | 46.4 |
| 40 63.0 | 62.6 | 62.0 | 61.3 |
| 50 78.7 | 78.3 | 77.5 | 76.6 |

| P. P.   |      |      |     |
|---------|------|------|-----|
| 91      | 91   | 90   | 2   |
| 6 9.1   | 9.1  | 9.0  | 0.1 |
| 7 10.7  | 10.6 | 10.5 | 0.2 |
| 8 12.2  | 12.1 | 12.0 | 0.3 |
| 9 13.7  | 13.6 | 13.5 | 0.4 |
| 10 15.2 | 15.1 | 15.0 | 0.5 |
| 20 30.5 | 30.3 | 30.0 | 0.8 |
| 30 45.7 | 45.5 | 45.0 | 1.0 |
| 40 61.0 | 60.6 | 60.0 | 1.2 |
| 50 76.2 | 75.8 | 75.0 | 1.4 |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    |          | Log. Sin. | d.       | Log. Tan. | c. d.    | Log. Cot. | Log. Cos. |  |  | P. P. |  |  |  |
|----|----------|-----------|----------|-----------|----------|-----------|-----------|--|--|-------|--|--|--|
| 0  | 9.19 433 | 80        | 9.19 971 | 81        | 0.80 028 | 9.99 462  | 60        |  |  |       |  |  |  |
| 1  | 9.19 513 | 79        | 9.20 053 | 81        | 0.79 947 | 9.99 460  | 59        |  |  |       |  |  |  |
| 2  | 9.19 592 | 79        | 9.20 134 | 81        | 0.79 865 | 9.99 458  | 58        |  |  |       |  |  |  |
| 3  | 9.19 672 | 79        | 9.20 216 | 81        | 0.79 784 | 9.99 456  | 57        |  |  |       |  |  |  |
| 4  | 9.19 751 | 79        | 9.20 297 | 81        | 0.79 703 | 9.99 454  | 56        |  |  |       |  |  |  |
| 5  | 9.19 830 | 79        | 9.20 378 | 81        | 0.79 622 | 9.99 452  | 55        |  |  |       |  |  |  |
| 6  | 9.19 909 | 79        | 9.20 459 | 81        | 0.79 541 | 9.99 450  | 54        |  |  |       |  |  |  |
| 7  | 9.19 988 | 78        | 9.20 540 | 80        | 0.79 460 | 9.99 448  | 53        |  |  |       |  |  |  |
| 8  | 9.20 066 | 78        | 9.20 620 | 81        | 0.79 379 | 9.99 446  | 52        |  |  |       |  |  |  |
| 9  | 9.20 145 | 78        | 9.20 701 | 80        | 0.79 298 | 9.99 444  | 51        |  |  |       |  |  |  |
| 10 | 9.20 223 | 78        | 9.20 781 | 80        | 0.79 218 | 9.99 442  | 50        |  |  |       |  |  |  |
| 11 | 9.20 301 | 78        | 9.20 862 | 80        | 0.79 138 | 9.99 440  | 49        |  |  |       |  |  |  |
| 12 | 9.20 379 | 78        | 9.20 942 | 80        | 0.79 058 | 9.99 437  | 48        |  |  |       |  |  |  |
| 13 | 9.20 457 | 78        | 9.21 022 | 80        | 0.78 978 | 9.99 435  | 47        |  |  |       |  |  |  |
| 14 | 9.20 535 | 77        | 9.21 102 | 79        | 0.78 898 | 9.99 433  | 46        |  |  |       |  |  |  |
| 15 | 9.20 613 | 77        | 9.21 181 | 79        | 0.78 818 | 9.99 431  | 45        |  |  |       |  |  |  |
| 16 | 9.20 690 | 77        | 9.21 261 | 79        | 0.78 739 | 9.99 429  | 44        |  |  |       |  |  |  |
| 17 | 9.20 768 | 77        | 9.21 340 | 79        | 0.78 659 | 9.99 427  | 43        |  |  |       |  |  |  |
| 18 | 9.20 845 | 77        | 9.21 420 | 79        | 0.78 580 | 9.99 425  | 42        |  |  |       |  |  |  |
| 19 | 9.20 922 | 77        | 9.21 499 | 79        | 0.78 501 | 9.99 423  | 41        |  |  |       |  |  |  |
| 20 | 9.20 999 | 77        | 9.21 578 | 79        | 0.78 422 | 9.99 421  | 40        |  |  |       |  |  |  |
| 21 | 9.21 076 | 76        | 9.21 657 | 78        | 0.78 343 | 9.99 419  | 39        |  |  |       |  |  |  |
| 22 | 9.21 153 | 76        | 9.21 735 | 78        | 0.78 264 | 9.99 417  | 38        |  |  |       |  |  |  |
| 23 | 9.21 230 | 76        | 9.21 814 | 78        | 0.78 186 | 9.99 415  | 37        |  |  |       |  |  |  |
| 24 | 9.21 305 | 76        | 9.21 892 | 78        | 0.78 107 | 9.99 413  | 36        |  |  |       |  |  |  |
| 25 | 9.21 382 | 76        | 9.21 971 | 78        | 0.78 029 | 9.99 411  | 35        |  |  |       |  |  |  |
| 26 | 9.21 458 | 76        | 9.22 049 | 78        | 0.77 951 | 9.99 408  | 34        |  |  |       |  |  |  |
| 27 | 9.21 534 | 75        | 9.22 127 | 78        | 0.77 873 | 9.99 406  | 33        |  |  |       |  |  |  |
| 28 | 9.21 609 | 75        | 9.22 205 | 78        | 0.77 795 | 9.99 404  | 32        |  |  |       |  |  |  |
| 29 | 9.21 683 | 75        | 9.22 283 | 77        | 0.77 717 | 9.99 402  | 31        |  |  |       |  |  |  |
| 30 | 9.21 761 | 75        | 9.22 360 | 77        | 0.77 639 | 9.99 400  | 30        |  |  |       |  |  |  |
| 31 | 9.21 836 | 75        | 9.22 438 | 77        | 0.77 562 | 9.99 398  | 29        |  |  |       |  |  |  |
| 32 | 9.21 911 | 75        | 9.22 515 | 77        | 0.77 484 | 9.99 396  | 28        |  |  |       |  |  |  |
| 33 | 9.21 987 | 75        | 9.22 593 | 77        | 0.77 407 | 9.99 394  | 27        |  |  |       |  |  |  |
| 34 | 9.22 062 | 74        | 9.22 670 | 77        | 0.77 330 | 9.99 392  | 26        |  |  |       |  |  |  |
| 35 | 9.22 136 | 74        | 9.22 747 | 77        | 0.77 253 | 9.99 389  | 25        |  |  |       |  |  |  |
| 36 | 9.22 211 | 74        | 9.22 824 | 76        | 0.77 176 | 9.99 387  | 24        |  |  |       |  |  |  |
| 37 | 9.22 286 | 74        | 9.22 900 | 76        | 0.77 099 | 9.99 385  | 23        |  |  |       |  |  |  |
| 38 | 9.22 360 | 74        | 9.22 977 | 76        | 0.77 022 | 9.99 383  | 22        |  |  |       |  |  |  |
| 39 | 9.22 435 | 74        | 9.23 054 | 76        | 0.76 946 | 9.99 381  | 21        |  |  |       |  |  |  |
| 40 | 9.22 509 | 74        | 9.23 130 | 76        | 0.76 870 | 9.99 379  | 20        |  |  |       |  |  |  |
| 41 | 9.22 583 | 74        | 9.23 206 | 76        | 0.76 793 | 9.99 377  | 19        |  |  |       |  |  |  |
| 42 | 9.22 657 | 74        | 9.23 282 | 76        | 0.76 717 | 9.99 374  | 18        |  |  |       |  |  |  |
| 43 | 9.22 731 | 73        | 9.23 358 | 76        | 0.76 641 | 9.99 372  | 17        |  |  |       |  |  |  |
| 44 | 9.22 805 | 74        | 9.23 434 | 76        | 0.76 565 | 9.99 370  | 16        |  |  |       |  |  |  |
| 45 | 9.22 878 | 73        | 9.23 510 | 73        | 0.76 489 | 9.99 368  | 15        |  |  |       |  |  |  |
| 46 | 9.22 952 | 73        | 9.23 586 | 73        | 0.76 414 | 9.99 366  | 14        |  |  |       |  |  |  |
| 47 | 9.23 025 | 73        | 9.23 661 | 73        | 0.76 338 | 9.99 364  | 13        |  |  |       |  |  |  |
| 48 | 9.23 098 | 73        | 9.23 737 | 73        | 0.76 263 | 9.99 361  | 12        |  |  |       |  |  |  |
| 49 | 9.23 171 | 73        | 9.23 812 | 73        | 0.76 188 | 9.99 359  | 11        |  |  |       |  |  |  |
| 50 | 9.23 244 | 72        | 9.23 887 | 75        | 0.76 113 | 9.99 357  | 10        |  |  |       |  |  |  |
| 51 | 9.23 317 | 72        | 9.23 962 | 75        | 0.76 038 | 9.99 355  | 9         |  |  |       |  |  |  |
| 52 | 9.23 390 | 72        | 9.24 037 | 75        | 0.75 963 | 9.99 353  | 8         |  |  |       |  |  |  |
| 53 | 9.23 462 | 72        | 9.24 112 | 75        | 0.75 888 | 9.99 350  | 7         |  |  |       |  |  |  |
| 54 | 9.23 535 | 72        | 9.24 186 | 74        | 0.75 813 | 9.99 348  | 6         |  |  |       |  |  |  |
| 55 | 9.23 607 | 72        | 9.24 261 | 74        | 0.75 739 | 9.99 346  | 5         |  |  |       |  |  |  |
| 56 | 9.23 679 | 72        | 9.24 335 | 74        | 0.75 664 | 9.99 344  | 4         |  |  |       |  |  |  |
| 57 | 9.23 751 | 72        | 9.24 409 | 74        | 0.75 590 | 9.99 342  | 3         |  |  |       |  |  |  |
| 58 | 9.23 823 | 72        | 9.24 484 | 74        | 0.75 516 | 9.99 339  | 2         |  |  |       |  |  |  |
| 59 | 9.23 895 | 71        | 9.24 558 | 74        | 0.75 442 | 9.99 337  | 1         |  |  |       |  |  |  |
| 60 | 9.23 967 |           | 9.24 632 | 74        | 0.75 368 | 9.99 335  | 0         |  |  |       |  |  |  |
|    |          | Log. Cos. | d.       | Log. Cot. | c. d.    | Log. Tan. | Log. Sin. |  |  | P. P. |  |  |  |

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|    | Log. Sin. | d. | Log. Tan. | e. d. | Log. Cot. | Log. Cos. |    | P. P. |      |      |      |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|------|------|------|
| 0  | 9.23 967  | 71 | 9.24 632  | 73    | 0.75 368  | 9.99 335  | 60 |       |      |      |      |
| 1  | 9.24 038  | 71 | 9.24 703  | 74    | 0.75 294  | 9.99 333  | 59 | 74    | 73   | 73   |      |
| 2  | 9.24 110  | 71 | 9.24 779  | 73    | 0.75 226  | 9.99 330  | 58 | 6     | 7.4  | 7.3  | 7.3  |
| 3  | 9.24 181  | 71 | 9.24 853  | 73    | 0.75 147  | 9.99 328  | 57 | 7     | 8.6  | 8.6  | 8.5  |
| 4  | 9.24 252  | 71 | 9.24 926  | 73    | 0.75 073  | 9.99 326  | 56 | 8     | 9 8  | 9.8  | 9.7  |
| 5  | 9.24 323  | 71 | 9.25 000  | 73    | 0.75 000  | 9.99 324  | 55 | 9     | 11.1 | 11.0 | 10.9 |
| 6  | 9.24 394  | 71 | 9.25 073  | 73    | 0.74 927  | 9.99 321  | 54 | 10    | 12.3 | 12.2 | 12.1 |
| 7  | 9.24 465  | 71 | 9.25 146  | 73    | 0.74 854  | 9.99 319  | 53 | 20    | 24 6 | 24.5 | 24.3 |
| 8  | 9.24 536  | 70 | 9.25 219  | 73    | 0.74 781  | 9.99 317  | 52 | 30    | 37.0 | 36.7 | 36.5 |
| 9  | 9.24 607  | 70 | 9.25 292  | 73    | 0.74 708  | 9.99 315  | 51 | 40    | 49.3 | 49.0 | 48.6 |
| 10 | 9.24 677  | 70 | 9.25 365  | 72    | 0.74 635  | 9.99 312  | 50 | 50    | 61.6 | 61.2 | 60.8 |
| 11 | 9.24 748  | 70 | 9.25 437  | 72    | 0.74 562  | 9.99 310  | 49 | 72    | 72   | 71   | 71   |
| 12 | 9.24 818  | 70 | 9.25 510  | 72    | 0.74 490  | 9.99 308  | 48 | 6     | 7.2  | 7.2  | 7.1  |
| 13 | 9.24 888  | 70 | 9.25 582  | 72    | 0.74 417  | 9.99 306  | 47 | 7     | 8.4  | 8.4  | 8.3  |
| 14 | 9.24 958  | 69 | 9.25 654  | 72    | 0.74 345  | 9.99 303  | 46 | 8     | 9.6  | 9.6  | 9.5  |
| 15 | 9.25 028  | 70 | 9.25 727  | 72    | 0.74 273  | 9.99 301  | 45 | 9     | 10.9 | 10.8 | 10.7 |
| 16 | 9.25 098  | 69 | 9.25 799  | 72    | 0.74 201  | 9.99 299  | 44 | 10    | 12.1 | 12.0 | 11.9 |
| 17 | 9.25 167  | 70 | 9.25 871  | 72    | 0.74 129  | 9.99 296  | 43 | 20    | 24.1 | 24.0 | 23.8 |
| 18 | 9.25 237  | 69 | 9.25 943  | 71    | 0.74 057  | 9.99 294  | 42 | 30    | 36.3 | 36.0 | 35.7 |
| 19 | 9.25 306  | 69 | 9.26 014  | 72    | 0.73 985  | 9.99 292  | 41 | 40    | 48.3 | 48.0 | 47.6 |
| 20 | 9.25 376  | 69 | 9.26 086  | 71    | 0.73 913  | 9.99 290  | 40 | 50    | 60.4 | 60.0 | 59.6 |
| 21 | 9.25 445  | 69 | 9.26 158  | 71    | 0.73 842  | 9.99 287  | 39 | 70    | 70   | 69   | 69   |
| 22 | 9.25 514  | 69 | 9.26 229  | 71    | 0.73 771  | 9.99 285  | 38 | 6     | 7.0  | 7.0  | 6.9  |
| 23 | 9.25 583  | 69 | 9.26 300  | 71    | 0.73 699  | 9.99 283  | 37 | 7     | 8.2  | 8.1  | 8.0  |
| 24 | 9.25 652  | 68 | 9.26 371  | 71    | 0.73 628  | 9.99 280  | 36 | 8     | 9.4  | 9.3  | 9.2  |
| 25 | 9.25 721  | 69 | 9.26 443  | 71    | 0.73 557  | 9.99 278  | 35 | 9     | 10.6 | 10.5 | 10.4 |
| 26 | 9.25 790  | 68 | 9.26 514  | 70    | 0.73 486  | 9.99 276  | 34 | 10    | 11.7 | 11.6 | 11.5 |
| 27 | 9.25 858  | 68 | 9.26 584  | 71    | 0.73 415  | 9.99 273  | 33 | 20    | 23.5 | 23.3 | 23.1 |
| 28 | 9.25 927  | 68 | 9.26 655  | 70    | 0.73 344  | 9.99 271  | 32 | 30    | 35.2 | 35.0 | 34.7 |
| 29 | 9.25 995  | 68 | 9.26 726  | 70    | 0.73 274  | 9.99 269  | 31 | 40    | 47.0 | 46.6 | 46.3 |
| 30 | 9.26 063  | 68 | 9.26 796  | 70    | 0.73 203  | 9.99 266  | 30 | 50    | 58.7 | 58.3 | 57.9 |
| 31 | 9.26 131  | 68 | 9.26 867  | 70    | 0.73 133  | 9.99 264  | 29 | 68    | 68   | 67   | 67   |
| 32 | 9.26 199  | 68 | 9.26 937  | 70    | 0.73 062  | 9.99 262  | 28 | 6     | 6.8  | 6.8  | 6.7  |
| 33 | 9.26 267  | 67 | 9.27 007  | 70    | 0.72 992  | 9.99 259  | 27 | 7     | 8.0  | 7.9  | 7.8  |
| 34 | 9.26 335  | 67 | 9.27 078  | 70    | 0.72 922  | 9.99 257  | 26 | 8     | 9.1  | 9.0  | 8.9  |
| 35 | 9.26 402  | 68 | 9.27 148  | 70    | 0.72 852  | 9.99 255  | 25 | 9     | 10.3 | 10.2 | 10.1 |
| 36 | 9.26 470  | 67 | 9.27 218  | 69    | 0.72 782  | 9.99 252  | 24 | 10    | 11.4 | 11.3 | 11.2 |
| 37 | 9.26 537  | 67 | 9.27 287  | 70    | 0.72 712  | 9.99 250  | 23 | 20    | 22.8 | 22.6 | 22.5 |
| 38 | 9.26 605  | 67 | 9.27 357  | 69    | 0.72 642  | 9.99 248  | 22 | 30    | 34.2 | 34.0 | 33.7 |
| 39 | 9.26 672  | 67 | 9.27 427  | 69    | 0.72 573  | 9.99 245  | 21 | 40    | 45.6 | 45.3 | 45.0 |
| 40 | 9.26 739  | 67 | 9.27 496  | 69    | 0.72 503  | 9.99 243  | 20 | 50    | 57.1 | 56.6 | 56.2 |
| 41 | 9.26 806  | 67 | 9.27 566  | 69    | 0.72 434  | 9.99 240  | 19 | 66    | 66   | 65   | 65   |
| 42 | 9.26 873  | 66 | 9.27 635  | 69    | 0.72 365  | 9.99 238  | 18 | 6     | 6.6  | 6.6  | 6.5  |
| 43 | 9.26 940  | 67 | 9.27 704  | 69    | 0.72 295  | 9.99 236  | 17 | 7     | 7.7  | 7.7  | 7.6  |
| 44 | 9.27 007  | 66 | 9.27 773  | 69    | 0.72 226  | 9.99 233  | 16 | 8     | 8.8  | 8.8  | 8.7  |
| 45 | 9.27 073  | 66 | 9.27 842  | 69    | 0.72 157  | 9.99 231  | 15 | 9     | 10.0 | 9.9  | 9.8  |
| 46 | 9.27 140  | 66 | 9.27 911  | 68    | 0.72 088  | 9.99 228  | 14 | 10    | 11.1 | 11.0 | 10.9 |
| 47 | 9.27 206  | 66 | 9.27 980  | 69    | 0.72 020  | 9.99 226  | 13 | 20    | 22.1 | 22.0 | 21.8 |
| 48 | 9.27 272  | 66 | 9.28 049  | 68    | 0.71 951  | 9.99 224  | 12 | 30    | 33.2 | 33.0 | 32.7 |
| 49 | 9.27 339  | 66 | 9.28 117  | 68    | 0.71 882  | 9.99 221  | 11 | 40    | 44.3 | 44.0 | 43.6 |
| 50 | 9.27 405  | 66 | 9.28 186  | 68    | 0.71 814  | 9.99 219  | 10 | 50    | 55.4 | 55.0 | 54.6 |
| 51 | 9.27 471  | 63 | 9.28 254  | 68    | 0.71 746  | 9.99 216  | 9  | 2     | 2    | 2    | 2    |
| 52 | 9.27 536  | 66 | 9.28 322  | 68    | 0.71 677  | 9.99 214  | 8  | 6     | 0.2  | 0.2  | 0.2  |
| 53 | 9.27 602  | 63 | 9.28 390  | 68    | 0.71 609  | 9.99 212  | 7  | 7     | 0.3  | 0.3  | 0.3  |
| 54 | 9.27 668  | 63 | 9.28 459  | 68    | 0.71 541  | 9.99 209  | 6  | 8     | 0.3  | 0.3  | 0.3  |
| 55 | 9.27 733  | 63 | 9.28 527  | 67    | 0.71 473  | 9.99 207  | 5  | 9     | 0.4  | 0.4  | 0.4  |
| 56 | 9.27 799  | 63 | 9.28 594  | 68    | 0.71 405  | 9.99 204  | 4  | 10    | 0.4  | 0.4  | 0.4  |
| 57 | 9.27 864  | 65 | 9.28 662  | 67    | 0.71 337  | 9.99 202  | 3  | 20    | 0.8  | 0.8  | 0.8  |
| 58 | 9.27 929  | 63 | 9.28 730  | 67    | 0.71 270  | 9.99 199  | 2  | 30    | 1.2  | 1.0  | 1.0  |
| 59 | 9.27 995  | 65 | 9.28 797  | 67    | 0.71 202  | 9.99 197  | 1  | 40    | 1.6  | 1.3  | 1.3  |
| 60 | 9.28 060  |    | 9.28 865  |       | 0.71 135  | 9.99 194  | 0  | 50    | 2.1  | 1.6  | 1.6  |
|    | Log. Cos. | d. | Log. Cot. | e. d. | Log. Tan. | Log. Sin. |    | P. P. |      |      |      |



TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

11°

| °  | Log. Sin. |    | d. | Log. Tan. |       | c. d. | Log. Cot. |  | Log. Cos. | P. P. |                        |
|----|-----------|----|----|-----------|-------|-------|-----------|--|-----------|-------|------------------------|
|    |           |    |    |           |       |       |           |  |           |       |                        |
| 0  | 9.28 060  |    | 65 | 9.28 865  |       | 67    | 0.71 135  |  | 9.99 194  | 60    |                        |
| 1  | 9.28 125  |    | 64 | 9.28 932  |       | 67    | 0.71 067  |  | 9.99 192  | 59    | 67 67                  |
| 2  | 9.28 189  |    | 65 | 9.29 000  |       | 67    | 0.71 000  |  | 9.99 189  | 58    | 6 6.7 6.7              |
| 3  | 9.28 254  |    | 64 | 9.29 067  |       | 67    | 0.70 933  |  | 9.99 187  | 57    | 7 7.9 7.8              |
| 4  | 9.28 319  |    | 64 | 9.29 134  |       | 67    | 0.70 866  |  | 9.99 185  | 56    | 8 9.0 8.9              |
| 5  | 9.28 383  |    | 64 | 9.29 201  |       | 67    | 0.70 798  |  | 9.99 182  | 55    | 9 10.1 10.0            |
| 6  | 9.28 448  |    | 64 | 9.29 268  |       | 66    | 0.70 732  |  | 9.99 180  | 54    | 10 11.2 11.1           |
| 7  | 9.28 512  |    | 64 | 9.29 335  |       | 67    | 0.70 665  |  | 9.99 177  | 53    | 20 22.5 22.3           |
| 8  | 9.28 576  |    | 64 | 9.29 401  |       | 66    | 0.70 598  |  | 9.99 175  | 52    | 30 33.7 33.5           |
| 9  | 9.28 641  |    | 64 | 9.29 468  |       | 67    | 0.70 531  |  | 9.99 172  | 51    | 40 45.0 44.6           |
| 10 | 9.28 705  |    | 64 | 9.29 535  |       | 66    | 0.70 465  |  | 9.99 170  | 50    | 50 56.2 55.8           |
| 11 | 9.28 769  |    | 64 | 9.29 601  |       | 66    | 0.70 398  |  | 9.99 167  | 49    | 66 66 65 65            |
| 12 | 9.28 832  |    | 63 | 9.29 667  |       | 66    | 0.70 332  |  | 9.99 165  | 48    | 6 6.6 6.6 6.5 6.5      |
| 13 | 9.28 896  |    | 63 | 9.29 734  |       | 66    | 0.70 266  |  | 9.99 162  | 47    | 7 7.7 7.7 7.6 7.6      |
| 14 | 9.28 960  |    | 63 | 9.29 800  |       | 66    | 0.70 200  |  | 9.99 160  | 46    | 8 8.8 8.8 8.7 8.6      |
| 15 | 9.29 023  |    | 63 | 9.29 866  |       | 66    | 0.70 134  |  | 9.99 157  | 45    | 9 10.0 9.9 9.8 9.7     |
| 16 | 9.29 087  |    | 63 | 9.29 932  |       | 66    | 0.70 068  |  | 9.99 155  | 44    | 10 11.1 11.0 10.9 10.8 |
| 17 | 9.29 150  |    | 63 | 9.29 998  |       | 66    | 0.70 002  |  | 9.99 152  | 43    | 20 22.1 22.0 21.8 21.6 |
| 18 | 9.29 213  |    | 63 | 9.30 064  |       | 66    | 0.69 936  |  | 9.99 150  | 42    | 30 33.2 33.0 32.7 32.5 |
| 19 | 9.29 277  |    | 63 | 9.30 129  |       | 63    | 0.69 870  |  | 9.99 147  | 41    | 40 44.3 44.0 43.6 43.3 |
| 20 | 9.29 340  |    | 63 | 9.30 195  |       | 63    | 0.69 805  |  | 9.99 145  | 40    | 50 55.4 55.0 54.6 54.1 |
| 21 | 9.29 403  |    | 63 | 9.30 260  |       | 63    | 0.69 739  |  | 9.99 142  | 39    | 64 64 63 63            |
| 22 | 9.29 466  |    | 63 | 9.30 326  |       | 63    | 0.69 674  |  | 9.99 139  | 38    | 6 6.4 6.4 6.3 6.3      |
| 23 | 9.29 528  |    | 63 | 9.30 391  |       | 63    | 0.69 608  |  | 9.99 137  | 37    | 7 7.5 7.4 7.4 7.3      |
| 24 | 9.29 591  |    | 62 | 9.30 456  |       | 63    | 0.69 543  |  | 9.99 134  | 36    | 8 8.6 8.5 8.4 8.4      |
| 25 | 9.29 654  |    | 62 | 9.30 522  |       | 63    | 0.69 478  |  | 9.99 132  | 35    | 9 9.7 9.6 9.5 9.4      |
| 26 | 9.29 716  |    | 62 | 9.30 587  |       | 63    | 0.69 413  |  | 9.99 129  | 34    | 10 10.7 10.6 10.6 10.5 |
| 27 | 9.29 779  |    | 62 | 9.30 652  |       | 63    | 0.69 348  |  | 9.99 127  | 33    | 20 21.5 21.3 21.1 21.0 |
| 28 | 9.29 841  |    | 62 | 9.30 717  |       | 63    | 0.69 283  |  | 9.99 124  | 32    | 30 32.2 32.0 31.7 31.5 |
| 29 | 9.29 903  |    | 62 | 9.30 781  |       | 64    | 0.69 218  |  | 9.99 122  | 31    | 40 43.0 42.6 42.3 42.0 |
| 30 | 9.29 965  |    | 62 | 9.30 846  |       | 65    | 0.69 153  |  | 9.99 119  | 30    | 50 53.7 53.3 52.9 52.5 |
| 31 | 9.30 027  |    | 62 | 9.30 911  |       | 64    | 0.69 089  |  | 9.99 116  | 29    | 62 62 61 61            |
| 32 | 9.30 089  |    | 62 | 9.30 973  |       | 64    | 0.69 024  |  | 9.99 114  | 28    | 6 6.2 6.2 6.1 6.1      |
| 33 | 9.30 151  |    | 61 | 9.31 040  |       | 64    | 0.68 960  |  | 9.99 111  | 27    | 7 7.3 7.2 7.2 7.1      |
| 34 | 9.30 213  |    | 62 | 9.31 104  |       | 64    | 0.68 896  |  | 9.99 109  | 26    | 8 8.3 8.2 8.2 8.1      |
| 35 | 9.30 275  |    | 61 | 9.31 168  |       | 64    | 0.68 831  |  | 9.99 106  | 25    | 9 9.4 9.3 9.2 9.1      |
| 36 | 9.30 336  |    | 61 | 9.31 232  |       | 64    | 0.68 767  |  | 9.99 104  | 24    | 10 10.4 10.3 10.2 10.1 |
| 37 | 9.30 398  |    | 61 | 9.31 297  |       | 64    | 0.68 703  |  | 9.99 101  | 23    | 20 20.8 20.6 20.5 20.3 |
| 38 | 9.30 459  |    | 61 | 9.31 361  |       | 63    | 0.68 639  |  | 9.99 098  | 22    | 30 31.2 31.0 30.7 30.5 |
| 39 | 9.30 520  |    | 61 | 9.31 424  |       | 64    | 0.68 575  |  | 9.99 096  | 21    | 40 41.6 41.3 41.0 40.6 |
| 40 | 9.30 582  |    | 61 | 9.31 488  |       | 64    | 0.68 511  |  | 9.99 093  | 20    | 50 52.1 51.6 51.2 50.8 |
| 41 | 9.30 643  |    | 61 | 9.31 552  |       | 63    | 0.68 447  |  | 9.99 091  | 19    | 60 60 59 59            |
| 42 | 9.30 704  |    | 61 | 9.31 616  |       | 63    | 0.68 384  |  | 9.99 088  | 18    | 6 6.0 6.0 5.9 5.9      |
| 43 | 9.30 765  |    | 61 | 9.31 679  |       | 63    | 0.68 320  |  | 9.99 085  | 17    | 7 7.0 7.0 6.9 6.9      |
| 44 | 9.30 826  |    | 60 | 9.31 743  |       | 63    | 0.68 257  |  | 9.99 083  | 16    | 8 8.0 8.0 7.9 7.9      |
| 45 | 9.30 886  |    | 61 | 9.31 806  |       | 63    | 0.68 193  |  | 9.99 080  | 15    | 9 9.1 9.0 8.9 8.9      |
| 46 | 9.30 947  |    | 60 | 9.31 869  |       | 63    | 0.68 130  |  | 9.99 077  | 14    | 10 10.1 10.0 9.9 9.9   |
| 47 | 9.31 008  |    | 60 | 9.31 933  |       | 63    | 0.68 067  |  | 9.99 075  | 13    | 20 20.1 20.0 19.8 19.8 |
| 48 | 9.31 068  |    | 60 | 9.31 996  |       | 63    | 0.68 004  |  | 9.99 072  | 12    | 30 30.2 30.0 29.7 29.7 |
| 49 | 9.31 129  |    | 60 | 9.32 059  |       | 63    | 0.67 941  |  | 9.99 069  | 11    | 40 40.3 40.0 39.6 39.6 |
| 50 | 9.31 189  |    | 60 | 9.32 122  |       | 63    | 0.67 878  |  | 9.99 067  | 10    | 50 50.4 50.0 49.6 49.6 |
| 51 | 9.31 249  |    | 60 | 9.32 185  |       | 63    | 0.67 815  |  | 9.99 064  | 9     | 3 3 2 2                |
| 52 | 9.31 309  |    | 60 | 9.32 248  |       | 62    | 0.67 752  |  | 9.99 062  | 8     | 6 0.3 0.3 0.2 0.2      |
| 53 | 9.31 370  |    | 59 | 9.32 310  |       | 63    | 0.67 689  |  | 9.99 059  | 7     | 7 0.3 0.3 0.2 0.2      |
| 54 | 9.31 429  |    | 60 | 9.32 373  |       | 62    | 0.67 626  |  | 9.99 056  | 6     | 8 0.4 0.3 0.2 0.2      |
| 55 | 9.31 489  |    | 60 | 9.32 436  |       | 62    | 0.67 564  |  | 9.99 054  | 5     | 9 0.4 0.4 0.3 0.3      |
| 56 | 9.31 549  |    | 59 | 9.32 498  |       | 62    | 0.67 501  |  | 9.99 051  | 4     | 10 0.5 0.4 0.3 0.3     |
| 57 | 9.31 609  |    | 60 | 9.32 560  |       | 62    | 0.67 439  |  | 9.99 048  | 3     | 20 1.0 0.8 0.6 0.6     |
| 58 | 9.31 669  |    | 59 | 9.32 623  |       | 62    | 0.67 377  |  | 9.99 046  | 2     | 30 1.5 1.2 1.0 1.0     |
| 59 | 9.31 728  |    | 59 | 9.32 685  |       | 62    | 0.67 314  |  | 9.99 043  | 1     | 40 2.0 1.6 1.3 1.3     |
| 60 | 9.31 788  |    | 59 | 9.32 747  |       | 62    | 0.67 252  |  | 9.99 040  | 0     | 50 2.5 2.1 1.6 1.6     |
|    | Log. Cos. | d. |    | Log. Cot. | c. d. |       | Log. Tan. |  | Log. Sin. |       | P. P.                  |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P. |  |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|--|--|--|
| 0  | 9.31 788  |    | 9.32 747  | 62    | 0.67 252  | 9.99 046  | 60 |       |  |  |  |
| 1  | 9.31 847  | 59 | 9.32 809  | 62    | 0.67 196  | 9.99 038  | 59 |       |  |  |  |
| 2  | 9.31 906  | 59 | 9.32 871  | 62    | 0.67 128  | 9.99 035  | 58 |       |  |  |  |
| 3  | 9.31 966  | 59 | 9.32 933  | 62    | 0.67 066  | 9.99 032  | 57 |       |  |  |  |
| 4  | 9.32 025  | 59 | 9.32 995  | 61    | 0.67 004  | 9.99 029  | 56 |       |  |  |  |
| 5  | 9.32 084  | 59 | 9.33 057  | 61    | 0.66 943  | 9.99 027  | 55 |       |  |  |  |
| 6  | 9.32 143  | 59 | 9.33 118  | 62    | 0.66 881  | 9.99 024  | 54 |       |  |  |  |
| 7  | 9.32 202  | 58 | 9.33 180  | 61    | 0.66 819  | 9.99 021  | 53 |       |  |  |  |
| 8  | 9.32 260  | 59 | 9.33 242  | 61    | 0.66 758  | 9.99 019  | 52 |       |  |  |  |
| 9  | 9.32 319  | 58 | 9.33 303  | 61    | 0.66 696  | 9.99 016  | 51 |       |  |  |  |
| 10 | 9.32 378  | 58 | 9.33 364  | 61    | 0.66 635  | 9.99 013  | 50 |       |  |  |  |
| 11 | 9.32 436  | 58 | 9.33 426  | 61    | 0.66 574  | 9.99 010  | 49 |       |  |  |  |
| 12 | 9.32 495  | 58 | 9.33 487  | 61    | 0.66 513  | 9.99 008  | 48 |       |  |  |  |
| 13 | 9.32 553  | 58 | 9.33 548  | 61    | 0.66 452  | 9.99 005  | 47 |       |  |  |  |
| 14 | 9.32 611  | 58 | 9.33 609  | 60    | 0.66 390  | 9.99 002  | 46 |       |  |  |  |
| 15 | 9.32 670  | 58 | 9.33 670  | 61    | 0.66 330  | 9.98 999  | 45 |       |  |  |  |
| 16 | 9.32 728  | 58 | 9.33 731  | 61    | 0.66 269  | 9.98 997  | 44 |       |  |  |  |
| 17 | 9.32 786  | 58 | 9.33 792  | 60    | 0.66 208  | 9.98 994  | 43 |       |  |  |  |
| 18 | 9.32 844  | 58 | 9.33 852  | 61    | 0.66 147  | 9.98 991  | 42 |       |  |  |  |
| 19 | 9.32 902  | 58 | 9.33 913  | 60    | 0.66 086  | 9.98 988  | 41 |       |  |  |  |
| 20 | 9.32 960  | 57 | 9.33 974  | 60    | 0.66 026  | 9.98 986  | 40 |       |  |  |  |
| 21 | 9.33 017  | 58 | 9.34 034  | 60    | 0.65 965  | 9.98 983  | 39 |       |  |  |  |
| 22 | 9.33 075  | 57 | 9.34 095  | 60    | 0.65 905  | 9.98 980  | 38 |       |  |  |  |
| 23 | 9.33 133  | 57 | 9.34 155  | 60    | 0.65 845  | 9.98 977  | 37 |       |  |  |  |
| 24 | 9.33 190  | 57 | 9.34 215  | 60    | 0.65 784  | 9.98 975  | 36 |       |  |  |  |
| 25 | 9.33 248  | 57 | 9.34 275  | 60    | 0.65 724  | 9.98 972  | 35 |       |  |  |  |
| 26 | 9.33 305  | 57 | 9.34 336  | 60    | 0.65 664  | 9.98 969  | 34 |       |  |  |  |
| 27 | 9.33 362  | 57 | 9.34 396  | 60    | 0.65 604  | 9.98 966  | 33 |       |  |  |  |
| 28 | 9.33 419  | 57 | 9.34 456  | 59    | 0.65 544  | 9.98 963  | 32 |       |  |  |  |
| 29 | 9.33 476  | 57 | 9.34 513  | 60    | 0.65 484  | 9.98 961  | 31 |       |  |  |  |
| 30 | 9.33 533  | 57 | 9.34 573  | 60    | 0.65 424  | 9.98 958  | 30 |       |  |  |  |
| 31 | 9.33 590  | 57 | 9.34 633  | 59    | 0.65 364  | 9.98 955  | 29 |       |  |  |  |
| 32 | 9.33 647  | 57 | 9.34 695  | 59    | 0.65 305  | 9.98 952  | 28 |       |  |  |  |
| 33 | 9.33 704  | 56 | 9.34 754  | 59    | 0.65 245  | 9.98 949  | 27 |       |  |  |  |
| 34 | 9.33 761  | 56 | 9.34 814  | 59    | 0.65 186  | 9.98 947  | 26 |       |  |  |  |
| 35 | 9.33 817  | 56 | 9.34 873  | 59    | 0.65 126  | 9.98 944  | 25 |       |  |  |  |
| 36 | 9.33 874  | 56 | 9.34 933  | 59    | 0.65 067  | 9.98 941  | 24 |       |  |  |  |
| 37 | 9.33 930  | 56 | 9.34 992  | 59    | 0.65 008  | 9.98 938  | 23 |       |  |  |  |
| 38 | 9.33 987  | 56 | 9.35 051  | 59    | 0.64 948  | 9.98 935  | 22 |       |  |  |  |
| 39 | 9.34 043  | 56 | 9.35 110  | 59    | 0.64 889  | 9.98 933  | 21 |       |  |  |  |
| 40 | 9.34 099  | 56 | 9.35 169  | 59    | 0.64 830  | 9.98 930  | 20 |       |  |  |  |
| 41 | 9.34 156  | 56 | 9.35 228  | 59    | 0.64 771  | 9.98 927  | 19 |       |  |  |  |
| 42 | 9.34 212  | 56 | 9.35 287  | 59    | 0.64 712  | 9.98 924  | 18 |       |  |  |  |
| 43 | 9.34 268  | 56 | 9.35 346  | 59    | 0.64 653  | 9.98 921  | 17 |       |  |  |  |
| 44 | 9.34 324  | 55 | 9.35 405  | 58    | 0.64 594  | 9.98 918  | 16 |       |  |  |  |
| 45 | 9.34 379  | 56 | 9.35 464  | 58    | 0.64 536  | 9.98 915  | 15 |       |  |  |  |
| 46 | 9.34 433  | 55 | 9.35 522  | 58    | 0.64 477  | 9.98 913  | 14 |       |  |  |  |
| 47 | 9.34 491  | 56 | 9.35 581  | 59    | 0.64 418  | 9.98 910  | 13 |       |  |  |  |
| 48 | 9.34 547  | 55 | 9.35 640  | 58    | 0.64 360  | 9.98 907  | 12 |       |  |  |  |
| 49 | 9.34 602  | 55 | 9.35 698  | 58    | 0.64 302  | 9.98 904  | 11 |       |  |  |  |
| 50 | 9.34 658  | 55 | 9.35 756  | 58    | 0.64 243  | 9.98 901  | 10 |       |  |  |  |
| 51 | 9.34 713  | 55 | 9.35 815  | 58    | 0.64 185  | 9.98 898  | 9  |       |  |  |  |
| 52 | 9.34 768  | 55 | 9.35 873  | 58    | 0.64 127  | 9.98 895  | 8  |       |  |  |  |
| 53 | 9.34 824  | 55 | 9.35 931  | 58    | 0.64 068  | 9.98 892  | 7  |       |  |  |  |
| 54 | 9.34 879  | 55 | 9.35 989  | 58    | 0.64 010  | 9.98 890  | 6  |       |  |  |  |
| 55 | 9.34 934  | 55 | 9.36 047  | 58    | 0.63 952  | 9.98 887  | 5  |       |  |  |  |
| 56 | 9.34 989  | 55 | 9.36 105  | 57    | 0.63 894  | 9.98 884  | 4  |       |  |  |  |
| 57 | 9.35 044  | 54 | 9.36 163  | 58    | 0.63 837  | 9.98 881  | 3  |       |  |  |  |
| 58 | 9.35 099  | 55 | 9.36 221  | 57    | 0.63 779  | 9.98 878  | 2  |       |  |  |  |
| 59 | 9.35 154  | 55 | 9.36 278  | 58    | 0.63 721  | 9.98 875  | 1  |       |  |  |  |
| 60 | 9.35 209  | 55 | 9.36 336  | 58    | 0.63 663  | 9.98 872  | 0  |       |  |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | '  | P. P. |  |  |  |



| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P. |      |      |      |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|------|------|------|
| 0  | 9.35 209  |    | 9.36 336  |       | 0.63 663  | 9.98 872  | 60 |       |      |      |      |
| 1  | 9.35 263  | 54 | 9.36 394  | 57    | 0.63 606  | 9.98 866  | 59 |       |      |      |      |
| 2  | 9.35 318  | 54 | 9.36 451  | 57    | 0.63 548  | 9.98 866  | 58 |       |      |      |      |
| 3  | 9.35 372  | 54 | 9.36 509  | 57    | 0.63 491  | 9.98 863  | 57 | 57    | 57   | 56   | 56   |
| 4  | 9.35 427  | 54 | 9.36 566  | 57    | 0.63 433  | 9.98 860  | 56 | 6     | 5.7  | 5.7  | 5.6  |
| 5  | 9.35 481  | 54 | 9.36 623  | 57    | 0.63 376  | 9.98 858  | 55 | 7     | 6.7  | 6.6  | 6.6  |
| 6  | 9.35 536  | 54 | 9.36 681  | 57    | 0.63 319  | 9.98 855  | 54 | 8     | 7.6  | 7.6  | 7.5  |
| 7  | 9.35 590  | 54 | 9.36 738  | 57    | 0.63 262  | 9.98 852  | 53 | 9     | 8.6  | 8.5  | 8.5  |
| 8  | 9.35 644  | 54 | 9.36 795  | 57    | 0.63 204  | 9.98 849  | 52 | 10    | 9.6  | 9.5  | 9.4  |
| 9  | 9.35 698  | 54 | 9.36 852  | 57    | 0.63 147  | 9.98 846  | 51 | 20    | 19.1 | 19.0 | 18.8 |
| 10 | 9.35 752  | 54 | 9.36 909  | 57    | 0.63 090  | 9.98 843  | 50 | 30    | 28.7 | 28.5 | 28.2 |
| 11 | 9.35 806  | 54 | 9.36 966  | 57    | 0.63 033  | 9.98 840  | 49 | 40    | 38.3 | 38.0 | 37.6 |
| 12 | 9.35 860  | 53 | 9.37 023  | 56    | 0.62 977  | 9.98 837  | 48 | 50    | 47.9 | 47.5 | 47.1 |
| 13 | 9.35 914  | 53 | 9.37 080  | 56    | 0.62 920  | 9.98 834  | 47 |       |      |      |      |
| 14 | 9.35 968  | 53 | 9.37 136  | 56    | 0.62 863  | 9.98 831  | 46 | 53    | 55   | 54   | 54   |
| 15 | 9.36 021  | 53 | 9.37 193  | 56    | 0.62 806  | 9.98 828  | 45 | 6     | 5.3  | 5.5  | 5.4  |
| 16 | 9.36 075  | 53 | 9.37 250  | 56    | 0.62 750  | 9.98 825  | 44 | 7     | 6.5  | 6.4  | 6.3  |
| 17 | 9.36 128  | 53 | 9.37 306  | 56    | 0.62 693  | 9.98 822  | 43 | 8     | 7.4  | 7.3  | 7.2  |
| 18 | 9.36 182  | 53 | 9.37 363  | 56    | 0.62 637  | 9.98 819  | 42 | 9     | 8.3  | 8.2  | 8.2  |
| 19 | 9.36 233  | 53 | 9.37 419  | 56    | 0.62 580  | 9.98 816  | 41 | 10    | 9.2  | 9.1  | 9.0  |
| 20 | 9.36 289  | 53 | 9.37 475  | 56    | 0.62 524  | 9.98 813  | 40 | 20    | 18.5 | 18.3 | 18.1 |
| 21 | 9.36 342  | 53 | 9.37 532  | 56    | 0.62 468  | 9.98 810  | 39 | 30    | 27.7 | 27.5 | 27.2 |
| 22 | 9.36 395  | 53 | 9.37 588  | 56    | 0.62 412  | 9.98 807  | 38 | 40    | 37.0 | 36.6 | 36.3 |
| 23 | 9.36 448  | 53 | 9.37 644  | 56    | 0.62 356  | 9.98 804  | 37 | 50    | 46.2 | 45.8 | 45.4 |
| 24 | 9.36 501  | 53 | 9.37 700  | 56    | 0.62 299  | 9.98 801  | 36 |       |      |      |      |
| 25 | 9.36 554  | 53 | 9.37 756  | 56    | 0.62 243  | 9.98 798  | 35 | 53    | 53   | 52   | 52   |
| 26 | 9.36 607  | 53 | 9.37 812  | 56    | 0.62 188  | 9.98 795  | 34 | 6     | 5.3  | 5.3  | 5.2  |
| 27 | 9.36 660  | 53 | 9.37 868  | 56    | 0.62 132  | 9.98 792  | 33 | 7     | 6.2  | 6.2  | 6.1  |
| 28 | 9.36 713  | 53 | 9.37 924  | 56    | 0.62 076  | 9.98 789  | 32 | 8     | 7.1  | 7.0  | 6.9  |
| 29 | 9.36 766  | 53 | 9.37 979  | 56    | 0.62 020  | 9.98 786  | 31 | 9     | 8.0  | 7.9  | 7.8  |
| 30 | 9.36 818  | 53 | 9.38 035  | 56    | 0.61 964  | 9.98 783  | 30 | 10    | 8.9  | 8.8  | 8.7  |
| 31 | 9.36 871  | 52 | 9.38 091  | 55    | 0.61 909  | 9.98 780  | 29 | 20    | 17.8 | 17.6 | 17.5 |
| 32 | 9.36 923  | 52 | 9.38 146  | 55    | 0.61 853  | 9.98 777  | 28 | 30    | 26.7 | 26.5 | 26.2 |
| 33 | 9.36 976  | 52 | 9.38 202  | 55    | 0.61 798  | 9.98 774  | 27 | 40    | 35.6 | 35.3 | 35.0 |
| 34 | 9.37 028  | 52 | 9.38 257  | 55    | 0.61 742  | 9.98 771  | 26 | 50    | 44.6 | 44.1 | 43.7 |
| 35 | 9.37 081  | 52 | 9.38 313  | 55    | 0.61 687  | 9.98 768  | 25 |       |      |      |      |
| 36 | 9.37 133  | 52 | 9.38 368  | 55    | 0.61 632  | 9.98 765  | 24 | 51    | 51   | 51   | 50   |
| 37 | 9.37 185  | 52 | 9.38 423  | 55    | 0.61 576  | 9.98 762  | 23 | 6     | 5.1  | 5.1  | 5.0  |
| 38 | 9.37 237  | 52 | 9.38 478  | 55    | 0.61 521  | 9.98 759  | 22 | 7     | 6.0  | 5.9  | 5.9  |
| 39 | 9.37 289  | 52 | 9.38 533  | 55    | 0.61 466  | 9.98 755  | 21 | 8     | 6.8  | 6.8  | 6.7  |
| 40 | 9.37 341  | 52 | 9.38 589  | 55    | 0.61 411  | 9.98 752  | 20 | 9     | 7.7  | 7.6  | 7.6  |
| 41 | 9.37 393  | 51 | 9.38 644  | 54    | 0.61 356  | 9.98 749  | 19 | 10    | 8.6  | 8.5  | 8.4  |
| 42 | 9.37 445  | 52 | 9.38 698  | 55    | 0.61 301  | 9.98 746  | 18 | 20    | 17.1 | 17.0 | 16.8 |
| 43 | 9.37 497  | 51 | 9.38 753  | 55    | 0.61 246  | 9.98 743  | 17 | 30    | 25.7 | 25.5 | 25.2 |
| 44 | 9.37 548  | 52 | 9.38 808  | 55    | 0.61 191  | 9.98 740  | 16 | 40    | 34.3 | 34.0 | 33.6 |
| 45 | 9.37 600  | 51 | 9.38 863  | 54    | 0.61 137  | 9.98 737  | 15 | 50    | 42.9 | 42.5 | 42.1 |
| 46 | 9.37 652  | 51 | 9.38 918  | 55    | 0.61 082  | 9.98 734  | 14 |       |      |      |      |
| 47 | 9.37 703  | 51 | 9.38 972  | 54    | 0.61 027  | 9.98 731  | 13 | 3     | 3    | 3    | 2    |
| 48 | 9.37 755  | 51 | 9.39 027  | 54    | 0.60 973  | 9.98 728  | 12 | 6     | 0.3  | 0.3  | 0.2  |
| 49 | 9.37 806  | 51 | 9.39 081  | 54    | 0.60 918  | 9.98 725  | 11 | 7     | 0.4  | 0.3  | 0.3  |
| 50 | 9.37 857  | 51 | 9.39 136  | 54    | 0.60 864  | 9.98 721  | 10 | 8     | 0.4  | 0.4  | 0.3  |
| 51 | 9.37 909  | 51 | 9.39 190  | 54    | 0.60 809  | 9.98 718  | 9  | 9     | 0.5  | 0.4  | 0.4  |
| 52 | 9.37 960  | 51 | 9.39 244  | 54    | 0.60 755  | 9.98 715  | 8  | 10    | 0.6  | 0.5  | 0.4  |
| 53 | 9.38 011  | 51 | 9.39 299  | 54    | 0.60 701  | 9.98 712  | 7  | 20    | 1.1  | 1.0  | 0.8  |
| 54 | 9.38 062  | 51 | 9.39 353  | 54    | 0.60 647  | 9.98 709  | 6  | 30    | 1.7  | 1.5  | 1.2  |
| 55 | 9.38 113  | 51 | 9.39 407  | 54    | 0.60 592  | 9.98 706  | 5  | 40    | 2.3  | 2.0  | 1.6  |
| 56 | 9.38 164  | 50 | 9.39 461  | 54    | 0.60 538  | 9.98 703  | 4  | 50    | 2.9  | 2.5  | 2.1  |
| 57 | 9.38 215  | 51 | 9.39 515  | 54    | 0.60 484  | 9.98 700  | 3  |       |      |      |      |
| 58 | 9.38 266  | 51 | 9.39 569  | 54    | 0.60 430  | 9.98 696  | 2  |       |      |      |      |
| 59 | 9.38 317  | 51 | 9.39 623  | 54    | 0.60 376  | 9.98 693  | 1  |       |      |      |      |
| 60 | 9.38 367  | 50 | 9.39 677  | 53    | 0.60 323  | 9.98 690  | 0  |       |      |      |      |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | '  | P. P. |      |      |      |

| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.             |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------------------|
| 0  | 9.38 367  | 50 | 9.39 677  | 54    | 0.60 323  | 9.98 690  | 3  | 60 |                   |
| 1  | 9.38 418  | 50 | 9.39 731  | 53    | 0.60 269  | 9.98 687  | 3  | 59 |                   |
| 2  | 9.38 468  | 50 | 9.39 784  | 54    | 0.60 215  | 9.98 684  | 3  | 58 |                   |
| 3  | 9.38 519  | 50 | 9.39 838  | 53    | 0.60 161  | 9.98 681  | 3  | 57 | 54 53 53          |
| 4  | 9.38 569  | 50 | 9.39 892  | 53    | 0.60 108  | 9.98 678  | 3  | 56 | 6 5.4 5.3 5.3     |
| 5  | 9.38 620  | 50 | 9.39 945  | 53    | 0.60 054  | 9.98 674  | 3  | 55 | 7 6.3 6.2 6.2     |
| 6  | 9.38 670  | 50 | 9.39 999  | 53    | 0.60 001  | 9.98 671  | 3  | 54 | 8 7.2 7.1 7.0     |
| 7  | 9.38 720  | 50 | 9.40 052  | 53    | 0.59 947  | 9.98 668  | 3  | 53 | 9 8.1 8.0 7.9     |
| 8  | 9.38 771  | 50 | 9.40 106  | 53    | 0.59 894  | 9.98 665  | 3  | 52 | 10 9.0 8.9 8.8    |
| 9  | 9.38 821  | 50 | 9.40 159  | 53    | 0.59 841  | 9.98 662  | 3  | 51 | 20 18.0 17.8 17.6 |
| 10 | 9.38 871  | 50 | 9.40 212  | 53    | 0.59 787  | 9.98 658  | 3  | 50 | 30 27.0 26.7 26.5 |
| 11 | 9.38 921  | 50 | 9.40 265  | 53    | 0.59 734  | 9.98 655  | 3  | 49 | 40 36.0 35.6 35.3 |
| 12 | 9.38 971  | 50 | 9.40 318  | 53    | 0.59 681  | 9.98 652  | 3  | 48 | 50 45.0 44.6 44.1 |
| 13 | 9.39 021  | 50 | 9.40 372  | 53    | 0.59 628  | 9.98 649  | 3  | 47 |                   |
| 14 | 9.39 071  | 49 | 9.40 425  | 53    | 0.59 575  | 9.98 646  | 3  | 46 | 52 52 51 51       |
| 15 | 9.39 120  | 50 | 9.40 478  | 53    | 0.59 522  | 9.98 642  | 3  | 45 | 6 5.2 5.2 5.1     |
| 16 | 9.39 170  | 49 | 9.40 531  | 52    | 0.59 469  | 9.98 639  | 3  | 44 | 7 6.1 6.0 5.9     |
| 17 | 9.39 220  | 49 | 9.40 583  | 53    | 0.59 416  | 9.98 636  | 3  | 43 | 8 7.0 6.9 6.8     |
| 18 | 9.39 269  | 49 | 9.40 636  | 52    | 0.59 363  | 9.98 633  | 3  | 42 | 9 7.9 7.8 7.7     |
| 19 | 9.39 319  | 49 | 9.40 689  | 52    | 0.59 311  | 9.98 630  | 3  | 41 | 10 8.7 8.6 8.5    |
| 20 | 9.39 368  | 49 | 9.40 742  | 52    | 0.59 258  | 9.98 626  | 3  | 40 | 20 17.5 17.3 17.1 |
| 21 | 9.39 418  | 49 | 9.40 794  | 52    | 0.59 205  | 9.98 623  | 3  | 39 | 30 26.2 26.0 25.7 |
| 22 | 9.39 467  | 49 | 9.40 847  | 52    | 0.59 153  | 9.98 620  | 3  | 38 | 40 35.0 34.6 34.3 |
| 23 | 9.39 516  | 49 | 9.40 899  | 52    | 0.59 100  | 9.98 617  | 3  | 37 | 50 43.7 43.3 42.9 |
| 24 | 9.39 566  | 49 | 9.40 952  | 52    | 0.59 048  | 9.98 613  | 3  | 36 |                   |
| 25 | 9.39 615  | 49 | 9.41 004  | 52    | 0.58 995  | 9.98 610  | 3  | 35 | 56 50 49 49       |
| 26 | 9.39 664  | 49 | 9.41 057  | 52    | 0.58 943  | 9.98 607  | 3  | 34 | 6 5.0 5.0 4.9     |
| 27 | 9.39 713  | 49 | 9.41 109  | 52    | 0.58 891  | 9.98 604  | 3  | 33 | 7 5.9 5.8 5.7     |
| 28 | 9.39 762  | 49 | 9.41 161  | 52    | 0.58 838  | 9.98 600  | 3  | 32 | 8 6.7 6.6 6.5     |
| 29 | 9.39 811  | 49 | 9.41 213  | 52    | 0.58 786  | 9.98 597  | 3  | 31 | 9 7.6 7.5 7.4     |
| 30 | 9.39 860  | 49 | 9.41 266  | 52    | 0.58 734  | 9.98 594  | 3  | 30 | 10 8.4 8.3 8.2    |
| 31 | 9.39 909  | 48 | 9.41 318  | 52    | 0.58 682  | 9.98 591  | 3  | 29 | 20 16.8 16.6 16.5 |
| 32 | 9.39 957  | 48 | 9.41 370  | 52    | 0.58 630  | 9.98 587  | 3  | 28 | 30 25.2 25.0 24.7 |
| 33 | 9.40 006  | 49 | 9.41 422  | 52    | 9.58 578  | 9.98 584  | 3  | 27 | 40 33.6 33.3 33.0 |
| 34 | 9.40 055  | 48 | 9.41 474  | 51    | 0.58 526  | 9.98 581  | 3  | 26 | 50 42.1 41.6 41.2 |
| 35 | 9.40 103  | 48 | 9.41 525  | 52    | 0.58 474  | 9.98 578  | 3  | 25 |                   |
| 36 | 9.40 152  | 48 | 9.41 577  | 52    | 0.58 422  | 9.98 574  | 3  | 24 | 48 48 47 47       |
| 37 | 9.40 200  | 48 | 9.41 629  | 51    | 0.58 370  | 9.98 571  | 3  | 23 | 6 4.8 4.8 4.7     |
| 38 | 9.40 249  | 48 | 9.41 681  | 51    | 0.58 319  | 9.98 568  | 3  | 22 | 7 5.6 5.6 5.5     |
| 39 | 9.40 297  | 48 | 9.41 732  | 51    | 0.58 267  | 9.98 564  | 3  | 21 | 8 6.4 6.4 6.3     |
| 40 | 9.40 345  | 48 | 9.41 784  | 52    | 0.58 216  | 9.98 561  | 3  | 20 | 9 7.3 7.2 7.1     |
| 41 | 9.40 394  | 48 | 9.41 836  | 51    | 0.58 164  | 9.98 558  | 3  | 19 | 10 8.1 8.0 7.9    |
| 42 | 9.40 442  | 48 | 9.41 887  | 51    | 0.58 112  | 9.98 554  | 3  | 18 | 20 16.1 16.0 15.8 |
| 43 | 9.40 490  | 48 | 9.41 938  | 51    | 0.58 061  | 9.98 551  | 3  | 17 | 30 24.2 24.0 23.7 |
| 44 | 9.40 538  | 48 | 9.41 990  | 51    | 0.58 010  | 9.98 548  | 3  | 16 | 40 32.3 32.0 31.6 |
| 45 | 9.40 586  | 48 | 9.42 041  | 51    | 0.57 958  | 9.98 544  | 3  | 15 | 50 40.4 40.0 39.6 |
| 46 | 9.40 634  | 48 | 9.42 092  | 51    | 0.57 907  | 9.98 541  | 3  | 14 |                   |
| 47 | 9.40 682  | 48 | 9.42 144  | 51    | 0.57 856  | 9.98 538  | 3  | 13 | 3 3               |
| 48 | 9.40 730  | 47 | 9.42 195  | 51    | 0.57 805  | 9.98 534  | 3  | 12 | 6 0.3 0.3 0.3     |
| 49 | 9.40 777  | 48 | 9.42 246  | 51    | 0.57 753  | 9.98 531  | 3  | 11 | 7 0.4 0.4 0.4     |
| 50 | 9.40 825  | 47 | 9.42 297  | 51    | 0.57 702  | 9.98 528  | 3  | 10 | 8 0.4 0.4 0.4     |
| 51 | 9.40 873  | 47 | 9.42 348  | 51    | 0.57 651  | 9.98 524  | 3  | 9  | 9 0.5 0.4 0.4     |
| 52 | 9.40 920  | 47 | 9.42 399  | 51    | 0.57 600  | 9.98 521  | 3  | 8  | 10 0.6 0.5 0.5    |
| 53 | 9.40 968  | 47 | 9.42 450  | 50    | 0.57 549  | 9.98 518  | 3  | 7  | 20 1.1 1.0 1.0    |
| 54 | 9.41 015  | 47 | 9.42 501  | 50    | 0.57 499  | 9.98 514  | 3  | 6  | 30 1.7 1.5 1.5    |
| 55 | 9.41 063  | 47 | 9.42 552  | 50    | 0.57 448  | 9.98 511  | 3  | 5  | 40 2.3 2.0 2.0    |
| 56 | 9.41 110  | 47 | 9.42 602  | 50    | 0.57 397  | 9.98 508  | 3  | 4  | 50 2.9 2.5 2.5    |
| 57 | 9.41 158  | 47 | 9.42 653  | 50    | 0.57 346  | 9.98 504  | 3  | 3  |                   |
| 58 | 9.41 205  | 47 | 9.42 704  | 50    | 0.57 296  | 9.98 501  | 3  | 2  |                   |
| 59 | 9.41 252  | 47 | 9.42 754  | 50    | 0.57 245  | 9.98 498  | 3  | 1  |                   |
| 60 | 9.41 299  | 47 | 9.42 805  | 50    | 0.57 195  | 9.98 494  | 3  | 0  |                   |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | '  | P. P.             |

| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P. |    |      |       |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|----|------|-------|
| 0  | 9.41 299  |    | 9.42 805  |       | 0.57 195  | 9.98 494  |    | 60    |    |      |       |
| 1  | 9.41 346  | 47 | 9.42 856  | 51    | 0.57 144  | 9.98 491  | 33 | 59    |    |      |       |
| 2  | 9.41 394  | 47 | 9.42 906  | 50    | 0.57 094  | 9.98 487  | 33 | 58    |    |      |       |
| 3  | 9.41 441  | 47 | 9.42 956  | 50    | 0.57 043  | 9.98 484  | 33 | 57    |    | 56   | 50    |
| 4  | 9.41 488  | 47 | 9.43 007  | 50    | 0.56 993  | 9.98 481  | 33 | 56    | 6  | 5.6  | 5.0   |
| 5  | 9.41 534  | 46 | 9.43 057  | 50    | 0.56 942  | 9.98 477  | 33 | 55    | 7  | 5.9  | 5.8   |
| 6  | 9.41 581  | 47 | 9.43 107  | 50    | 0.56 892  | 9.98 474  | 33 | 54    | 8  | 6.7  | 6.6   |
| 7  | 9.41 628  | 47 | 9.43 157  | 50    | 0.56 842  | 9.98 470  | 33 | 53    | 9  | 7.6  | 7.5   |
| 8  | 9.41 675  | 46 | 9.43 208  | 50    | 0.56 792  | 9.98 467  | 33 | 52    | 10 | 8.4  | 8.3   |
| 9  | 9.41 721  | 46 | 9.43 258  | 50    | 0.56 742  | 9.98 464  | 33 | 51    | 20 | 16.8 | 16.6  |
| 10 | 9.41 768  | 47 | 9.43 308  | 50    | 0.56 692  | 9.98 460  | 33 | 50    | 30 | 25.2 | 25.0  |
| 11 | 9.41 815  | 46 | 9.43 358  | 50    | 0.56 642  | 9.98 457  | 33 | 49    | 40 | 33.6 | 33.3  |
| 12 | 9.41 861  | 46 | 9.43 408  | 50    | 0.56 592  | 9.98 453  | 33 | 48    | 50 | 42.1 | 41.6  |
| 13 | 9.41 908  | 46 | 9.43 458  | 50    | 0.56 542  | 9.98 450  | 33 | 47    |    |      |       |
| 14 | 9.41 954  | 46 | 9.43 508  | 50    | 0.56 492  | 9.98 446  | 33 | 46    |    |      |       |
| 15 | 9.42 000  | 46 | 9.43 557  | 49    | 0.56 442  | 9.98 443  | 33 | 45    | 49 | 48   |       |
| 16 | 9.42 047  | 46 | 9.43 607  | 50    | 0.56 392  | 9.98 439  | 33 | 44    | 6  | 4.9  | 4.8   |
| 17 | 9.42 093  | 46 | 9.43 657  | 49    | 0.56 343  | 9.98 436  | 33 | 43    | 7  | 5.8  | 5.7   |
| 18 | 9.42 139  | 46 | 9.43 706  | 50    | 0.56 293  | 9.98 433  | 33 | 42    | 8  | 6.6  | 6.5   |
| 19 | 9.42 185  | 46 | 9.43 756  | 50    | 0.56 243  | 9.98 429  | 33 | 41    | 9  | 7.4  | 7.3   |
| 20 | 9.42 232  | 46 | 9.43 806  | 49    | 0.56 194  | 9.98 426  | 33 | 40    | 10 | 8.2  | 8.1   |
| 21 | 9.42 278  | 46 | 9.43 855  | 49    | 0.56 144  | 9.98 422  | 33 | 39    | 20 | 16.5 | 16.3  |
| 22 | 9.42 324  | 43 | 9.43 905  | 49    | 0.56 095  | 9.98 419  | 33 | 38    | 30 | 24.7 | 24.5  |
| 23 | 9.42 369  | 46 | 9.43 954  | 49    | 0.56 045  | 9.98 415  | 33 | 37    | 40 | 33.0 | 32.6  |
| 24 | 9.42 415  | 46 | 9.44 003  | 49    | 0.55 996  | 9.98 412  | 33 | 36    | 50 | 41.2 | 40.8  |
| 25 | 9.42 461  | 46 | 9.44 053  | 49    | 0.55 947  | 9.98 408  | 33 | 35    |    |      |       |
| 26 | 9.42 507  | 43 | 9.44 102  | 49    | 0.55 898  | 9.98 405  | 33 | 34    | 47 | 46   |       |
| 27 | 9.42 553  | 43 | 9.44 151  | 49    | 0.55 848  | 9.98 401  | 33 | 33    | 6  | 4.7  | 4.6   |
| 28 | 9.42 598  | 46 | 9.44 200  | 49    | 0.55 799  | 9.98 398  | 33 | 32    | 7  | 5.5  | 5.5   |
| 29 | 9.42 644  | 43 | 9.44 249  | 49    | 0.55 750  | 9.98 394  | 33 | 31    | 8  | 6.3  | 6.2   |
| 30 | 9.42 690  | 43 | 9.44 299  | 49    | 0.55 701  | 9.98 391  | 33 | 30    | 9  | 7.1  | 7.0   |
| 31 | 9.42 735  | 43 | 9.44 348  | 49    | 0.55 652  | 9.98 387  | 33 | 29    | 10 | 7.9  | 7.8   |
| 32 | 9.42 781  | 43 | 9.44 397  | 49    | 0.55 603  | 9.98 384  | 33 | 28    | 20 | 15.8 | 15.6  |
| 33 | 9.42 826  | 43 | 9.44 446  | 48    | 0.55 554  | 9.98 380  | 33 | 27    | 30 | 23.7 | 23.5  |
| 34 | 9.42 871  | 43 | 9.44 494  | 48    | 0.55 505  | 9.98 377  | 33 | 26    | 40 | 31.6 | 31.3  |
| 35 | 9.42 917  | 43 | 9.44 543  | 49    | 0.55 456  | 9.98 373  | 33 | 25    | 50 | 39.6 | 39.1  |
| 36 | 9.42 962  | 43 | 9.44 592  | 48    | 0.55 407  | 9.98 370  | 33 | 24    |    |      |       |
| 37 | 9.43 007  | 45 | 9.44 641  | 49    | 0.55 359  | 9.98 366  | 33 | 23    | 45 | 45   |       |
| 38 | 9.43 052  | 43 | 9.44 690  | 48    | 0.55 310  | 9.98 363  | 33 | 22    | 6  | 4.5  | 4.4   |
| 39 | 9.43 098  | 45 | 9.44 738  | 48    | 0.55 261  | 9.98 359  | 33 | 21    | 7  | 5.3  | 5.2   |
| 40 | 9.43 143  | 45 | 9.44 787  | 48    | 0.55 213  | 9.98 356  | 33 | 20    | 8  | 6.0  | 6.0   |
| 41 | 9.43 188  | 45 | 9.44 835  | 48    | 0.55 164  | 9.98 352  | 33 | 19    | 9  | 6.8  | 6.7   |
| 42 | 9.43 233  | 45 | 9.44 884  | 48    | 0.55 116  | 9.98 348  | 33 | 18    | 10 | 7.6  | 7.5   |
| 43 | 9.43 278  | 45 | 9.44 932  | 48    | 0.55 067  | 9.98 345  | 33 | 17    | 20 | 15.1 | 15.0  |
| 44 | 9.43 322  | 44 | 9.44 981  | 48    | 0.55 019  | 9.98 341  | 33 | 16    | 30 | 22.7 | 22.5  |
| 45 | 9.43 367  | 45 | 9.45 029  | 48    | 0.54 970  | 9.98 338  | 33 | 15    | 40 | 30.3 | 30.0  |
| 46 | 9.43 412  | 44 | 9.45 077  | 48    | 0.54 922  | 9.98 334  | 33 | 14    | 50 | 37.9 | 37.5  |
| 47 | 9.43 457  | 45 | 9.45 126  | 48    | 0.54 874  | 9.98 331  | 33 | 13    |    |      |       |
| 48 | 9.43 501  | 44 | 9.45 174  | 48    | 0.54 823  | 9.98 327  | 33 | 12    |    |      |       |
| 49 | 9.43 546  | 44 | 9.45 222  | 48    | 0.54 777  | 9.98 324  | 33 | 11    |    |      |       |
| 50 | 9.43 591  | 45 | 9.45 270  | 48    | 0.54 729  | 9.98 320  | 33 | 10    | 4  | 3    |       |
| 51 | 9.43 635  | 44 | 9.45 318  | 48    | 0.54 681  | 9.98 316  | 33 | 9     | 6  | 0.4  | 0.3   |
| 52 | 9.43 680  | 44 | 9.45 367  | 48    | 0.54 633  | 9.98 313  | 33 | 8     | 7  | 0.4  | 0.3   |
| 53 | 9.43 724  | 44 | 9.45 415  | 48    | 0.54 585  | 9.98 309  | 33 | 7     | 8  | 0.5  | 0.4   |
| 54 | 9.43 768  | 44 | 9.45 463  | 47    | 0.54 537  | 9.98 306  | 33 | 6     | 9  | 0.6  | 0.5   |
| 55 | 9.43 813  | 44 | 9.45 510  | 47    | 0.54 489  | 9.98 302  | 33 | 5     | 10 | 0.6  | 0.5   |
| 56 | 9.43 857  | 44 | 9.45 558  | 48    | 0.54 441  | 9.98 298  | 33 | 4     | 20 | 1.3  | 1.1   |
| 57 | 9.43 901  | 44 | 9.45 606  | 47    | 0.54 393  | 9.98 295  | 33 | 3     | 30 | 2.0  | 1.7   |
| 58 | 9.43 945  | 44 | 9.45 654  | 47    | 0.54 346  | 9.98 291  | 33 | 2     | 40 | 2.6  | 2.3   |
| 59 | 9.43 989  | 44 | 9.45 702  | 48    | 0.54 298  | 9.98 288  | 33 | 1     | 50 | 3.3  | 2.9   |
| 60 | 9.44 034  | 44 | 9.45 749  | 47    | 0.54 250  | 9.98 284  | 33 | 0     |    |      |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |       |    |      | P. P. |

| °  | Log. Sin. |  | d. | Log. Tan. |  | c. d. | Log. Cot. |  | Log. Cos. |  | d. | P. P.          |  |  |  |
|----|-----------|--|----|-----------|--|-------|-----------|--|-----------|--|----|----------------|--|--|--|
|    |           |  |    |           |  |       |           |  |           |  |    |                |  |  |  |
| 0  | 9.44 034  |  | 44 | 9.45 749  |  | 48    | 0.54 250  |  | 9.98 284  |  | 3  | 60             |  |  |  |
| 1  | 9.44 078  |  | 44 | 9.45 797  |  | 47    | 0.54 202  |  | 9.98 280  |  | 3  |                |  |  |  |
| 2  | 9.44 122  |  | 44 | 9.45 845  |  | 47    | 0.54 155  |  | 9.98 277  |  | 3  |                |  |  |  |
| 3  | 9.44 166  |  | 43 | 9.45 892  |  | 47    | 0.54 107  |  | 9.98 273  |  | 4  |                |  |  |  |
| 4  | 9.44 209  |  | 44 | 9.45 940  |  | 47    | 0.54 060  |  | 9.98 269  |  | 3  | 48 47 47       |  |  |  |
| 5  | 9.44 253  |  | 44 | 9.45 987  |  | 47    | 0.54 012  |  | 9.98 266  |  | 3  |                |  |  |  |
| 6  | 9.44 297  |  | 43 | 9.46 035  |  | 47    | 0.53 965  |  | 9.98 262  |  | 4  |                |  |  |  |
| 7  | 9.44 341  |  | 43 | 9.46 082  |  | 47    | 0.53 917  |  | 9.98 258  |  | 3  |                |  |  |  |
| 8  | 9.44 384  |  | 44 | 9.46 129  |  | 47    | 0.53 870  |  | 9.98 255  |  | 3  | 50             |  |  |  |
| 9  | 9.44 428  |  | 43 | 9.46 177  |  | 47    | 0.53 823  |  | 9.98 251  |  | 4  |                |  |  |  |
| 10 | 9.44 472  |  | 43 | 9.46 224  |  | 47    | 0.53 776  |  | 9.98 247  |  | 3  |                |  |  |  |
| 11 | 9.44 515  |  | 43 | 9.46 271  |  | 47    | 0.53 728  |  | 9.98 244  |  | 3  |                |  |  |  |
| 12 | 9.44 559  |  | 43 | 9.46 318  |  | 47    | 0.53 681  |  | 9.98 240  |  | 4  | 46 46 45 45    |  |  |  |
| 13 | 9.44 602  |  | 43 | 9.46 366  |  | 47    | 0.53 634  |  | 9.98 236  |  | 3  |                |  |  |  |
| 14 | 9.44 646  |  | 43 | 9.46 413  |  | 47    | 0.53 587  |  | 9.98 233  |  | 4  |                |  |  |  |
| 15 | 9.44 689  |  | 43 | 9.46 460  |  | 47    | 0.53 540  |  | 9.98 229  |  | 3  |                |  |  |  |
| 16 | 9.44 732  |  | 43 | 9.46 507  |  | 47    | 0.53 493  |  | 9.98 225  |  | 4  | 44 43 43       |  |  |  |
| 17 | 9.44 776  |  | 43 | 9.46 554  |  | 47    | 0.53 446  |  | 9.98 222  |  | 3  |                |  |  |  |
| 18 | 9.44 819  |  | 43 | 9.46 601  |  | 46    | 0.53 399  |  | 9.98 218  |  | 4  |                |  |  |  |
| 19 | 9.44 862  |  | 43 | 9.46 647  |  | 47    | 0.53 352  |  | 9.98 214  |  | 3  |                |  |  |  |
| 20 | 9.44 905  |  | 43 | 9.46 694  |  | 47    | 0.53 305  |  | 9.98 211  |  | 4  | 40             |  |  |  |
| 21 | 9.44 948  |  | 43 | 9.46 741  |  | 46    | 0.53 258  |  | 9.98 207  |  | 3  |                |  |  |  |
| 22 | 9.44 991  |  | 43 | 9.46 788  |  | 46    | 0.53 212  |  | 9.98 203  |  | 3  |                |  |  |  |
| 23 | 9.45 034  |  | 43 | 9.46 834  |  | 47    | 0.53 165  |  | 9.98 200  |  | 4  |                |  |  |  |
| 24 | 9.45 077  |  | 43 | 9.46 881  |  | 46    | 0.53 118  |  | 9.98 196  |  | 3  | 35 34 33 31    |  |  |  |
| 25 | 9.45 120  |  | 42 | 9.46 928  |  | 46    | 0.53 072  |  | 9.98 192  |  | 4  |                |  |  |  |
| 26 | 9.45 163  |  | 43 | 9.46 974  |  | 46    | 0.53 025  |  | 9.98 188  |  | 3  |                |  |  |  |
| 27 | 9.45 206  |  | 43 | 9.47 021  |  | 46    | 0.52 979  |  | 9.98 185  |  | 4  | 30             |  |  |  |
| 28 | 9.45 249  |  | 42 | 9.47 067  |  | 46    | 0.52 932  |  | 9.98 181  |  | 3  |                |  |  |  |
| 29 | 9.45 291  |  | 42 | 9.47 114  |  | 46    | 0.52 886  |  | 9.98 177  |  | 4  |                |  |  |  |
| 30 | 9.45 334  |  | 42 | 9.47 160  |  | 46    | 0.52 839  |  | 9.98 173  |  | 3  |                |  |  |  |
| 31 | 9.45 377  |  | 42 | 9.47 207  |  | 46    | 0.52 793  |  | 9.98 170  |  | 4  | 20             |  |  |  |
| 32 | 9.45 419  |  | 42 | 9.47 253  |  | 46    | 0.52 747  |  | 9.98 166  |  | 3  |                |  |  |  |
| 33 | 9.45 462  |  | 42 | 9.47 299  |  | 46    | 0.52 700  |  | 9.98 162  |  | 4  |                |  |  |  |
| 34 | 9.45 504  |  | 42 | 9.47 345  |  | 46    | 0.52 654  |  | 9.98 158  |  | 3  |                |  |  |  |
| 35 | 9.45 547  |  | 42 | 9.47 392  |  | 46    | 0.52 608  |  | 9.98 155  |  | 4  | 25 24 23 22 21 |  |  |  |
| 36 | 9.45 589  |  | 42 | 9.47 438  |  | 46    | 0.52 562  |  | 9.98 151  |  | 3  |                |  |  |  |
| 37 | 9.45 631  |  | 42 | 9.47 484  |  | 46    | 0.52 516  |  | 9.98 147  |  | 4  |                |  |  |  |
| 38 | 9.45 674  |  | 42 | 9.47 530  |  | 46    | 0.52 469  |  | 9.98 143  |  | 3  |                |  |  |  |
| 39 | 9.45 716  |  | 42 | 9.47 576  |  | 46    | 0.52 423  |  | 9.98 140  |  | 4  | 42 42 41 41    |  |  |  |
| 40 | 9.45 758  |  | 42 | 9.47 622  |  | 46    | 0.52 377  |  | 9.98 136  |  | 3  |                |  |  |  |
| 41 | 9.45 800  |  | 42 | 9.47 668  |  | 45    | 0.52 331  |  | 9.98 132  |  | 4  |                |  |  |  |
| 42 | 9.45 842  |  | 42 | 9.47 714  |  | 45    | 0.52 286  |  | 9.98 128  |  | 3  |                |  |  |  |
| 43 | 9.45 885  |  | 42 | 9.47 760  |  | 45    | 0.52 240  |  | 9.98 124  |  | 4  | 20             |  |  |  |
| 44 | 9.45 927  |  | 42 | 9.47 806  |  | 45    | 0.52 194  |  | 9.98 121  |  | 3  |                |  |  |  |
| 45 | 9.45 969  |  | 42 | 9.47 851  |  | 45    | 0.52 148  |  | 9.98 117  |  | 4  |                |  |  |  |
| 46 | 9.46 011  |  | 41 | 9.47 897  |  | 45    | 0.52 102  |  | 9.98 113  |  | 3  |                |  |  |  |
| 47 | 9.46 052  |  | 41 | 9.47 943  |  | 45    | 0.52 057  |  | 9.98 109  |  | 4  | 10             |  |  |  |
| 48 | 9.46 094  |  | 42 | 9.47 989  |  | 45    | 0.52 011  |  | 9.98 105  |  | 3  |                |  |  |  |
| 49 | 9.46 136  |  | 41 | 9.48 034  |  | 45    | 0.51 965  |  | 9.98 102  |  | 4  |                |  |  |  |
| 50 | 9.46 178  |  | 42 | 9.48 080  |  | 45    | 0.51 920  |  | 9.98 098  |  | 3  | 4 3            |  |  |  |
| 51 | 9.46 220  |  | 41 | 9.48 125  |  | 45    | 0.51 874  |  | 9.98 094  |  | 4  |                |  |  |  |
| 52 | 9.46 261  |  | 41 | 9.48 171  |  | 45    | 0.51 829  |  | 9.98 090  |  | 3  |                |  |  |  |
| 53 | 9.46 303  |  | 41 | 9.48 216  |  | 45    | 0.51 783  |  | 9.98 086  |  | 4  |                |  |  |  |
| 54 | 9.46 345  |  | 42 | 9.48 262  |  | 45    | 0.51 738  |  | 9.98 082  |  | 3  | 5              |  |  |  |
| 55 | 9.46 386  |  | 41 | 9.48 307  |  | 45    | 0.51 692  |  | 9.98 079  |  | 4  |                |  |  |  |
| 56 | 9.46 428  |  | 41 | 9.48 353  |  | 45    | 0.51 647  |  | 9.98 075  |  | 3  |                |  |  |  |
| 57 | 9.46 469  |  | 41 | 9.48 398  |  | 45    | 0.51 602  |  | 9.98 071  |  | 4  |                |  |  |  |
| 58 | 9.46 511  |  | 41 | 9.48 443  |  | 45    | 0.51 556  |  | 9.98 067  |  | 3  | 4              |  |  |  |
| 59 | 9.46 552  |  | 41 | 9.48 488  |  | 45    | 0.51 511  |  | 9.98 063  |  | 4  |                |  |  |  |
| 60 | 9.46 593  |  | 41 | 9.48 534  |  | 45    | 0.51 466  |  | 9.98 059  |  | 3  |                |  |  |  |
|    | Log. Cos. |  | d. | Log. Cot. |  | c. d. | Log. Tan. |  | Log. Sin. |  | d. | P. P.          |  |  |  |

|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.                  |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|------------------------|
| 0  | 9.46 593  | 41 | 9.48 534  | 45    | 0.51 466  | 9.98 059  | 3  | 60 |                        |
| 1  | 9.46 635  | 41 | 9.48 579  | 45    | 0.51 421  | 9.98 056  | 3  | 59 |                        |
| 2  | 9.46 676  | 41 | 9.48 624  | 45    | 0.51 376  | 9.98 052  | 4  | 58 |                        |
| 3  | 9.46 717  | 41 | 9.48 669  | 45    | 0.51 330  | 9.98 048  | 4  | 57 | 45° 45 44 44           |
| 4  | 9.46 758  | 41 | 9.48 714  | 45    | 0.51 285  | 9.98 044  | 4  | 56 | 6 4.5 4.5 4.4 4.4      |
| 5  | 9.46 799  | 41 | 9.48 759  | 45    | 0.51 240  | 9.98 040  | 3  | 55 | 7 5.3 5.3 5.2 5.1      |
| 6  | 9.46 840  | 41 | 9.48 804  | 45    | 0.51 195  | 9.98 036  | 4  | 54 | 8 6.0 6.0 5.9 5.8      |
| 7  | 9.46 881  | 41 | 9.48 849  | 44    | 0.51 151  | 9.98 032  | 4  | 53 | 9 6.8 6.7 6.7 6.6      |
| 8  | 9.46 922  | 41 | 9.48 894  | 45    | 0.51 106  | 9.98 028  | 4  | 52 | 10 7.6 7.5 7.4 7.3     |
| 9  | 9.46 963  | 41 | 9.48 939  | 45    | 0.51 061  | 9.98 024  | 4  | 51 | 20 15.1 15.0 14.8 14.6 |
| 10 | 9.47 004  | 41 | 9.48 984  | 45    | 0.51 016  | 9.98 021  | 3  | 50 | 30 22.7 22.5 22.2 22.0 |
| 11 | 9.47 045  | 41 | 9.49 028  | 44    | 0.50 971  | 9.98 017  | 4  | 49 | 40 30.3 30.0 29.6 29.3 |
| 12 | 9.47 086  | 41 | 9.49 073  | 45    | 0.50 926  | 9.98 013  | 4  | 48 | 50 37.9 37.5 37.1 36.6 |
| 13 | 9.47 127  | 40 | 9.49 118  | 44    | 0.50 882  | 9.98 009  | 4  | 47 |                        |
| 14 | 9.47 168  | 41 | 9.49 162  | 44    | 0.50 837  | 9.98 005  | 4  | 46 |                        |
| 15 | 9.47 208  | 40 | 9.49 207  | 45    | 0.50 792  | 9.98 001  | 4  | 45 | 43 43                  |
| 16 | 9.47 249  | 40 | 9.49 252  | 44    | 0.50 748  | 9.97 997  | 3  | 44 | 6 4.3 4.3 4.3          |
| 17 | 9.47 290  | 41 | 9.49 296  | 44    | 0.50 703  | 9.97 993  | 4  | 43 | 7 5.1 5.1 5.0          |
| 18 | 9.47 330  | 40 | 9.49 341  | 44    | 0.50 659  | 9.97 989  | 4  | 42 | 8 5.8 5.7 5.7          |
| 19 | 9.47 371  | 40 | 9.49 385  | 44    | 0.50 614  | 9.97 985  | 4  | 41 | 9 6.5 6.4 6.4          |
| 20 | 9.47 411  | 40 | 9.49 430  | 44    | 0.50 570  | 9.97 981  | 4  | 40 | 10 7.2 7.1 7.1         |
| 21 | 9.47 452  | 40 | 9.49 474  | 44    | 0.50 525  | 9.97 977  | 4  | 39 | 20 14.5 14.3 14.3      |
| 22 | 9.47 492  | 40 | 9.49 518  | 44    | 0.50 481  | 9.97 973  | 4  | 38 | 30 21.7 21.5 21.5      |
| 23 | 9.47 532  | 40 | 9.49 563  | 44    | 0.50 437  | 9.97 969  | 4  | 37 | 40 29.0 28.6 28.6      |
| 24 | 9.47 573  | 40 | 9.49 607  | 44    | 0.50 392  | 9.97 966  | 3  | 36 | 50 36.2 35.8 35.8      |
| 25 | 9.47 613  | 40 | 9.49 651  | 44    | 0.50 348  | 9.97 962  | 4  | 35 |                        |
| 26 | 9.47 653  | 40 | 9.49 695  | 44    | 0.50 304  | 9.97 958  | 4  | 34 | 41 41 40 40            |
| 27 | 9.47 694  | 40 | 9.49 740  | 44    | 0.50 260  | 9.97 954  | 4  | 33 | 6 4.1 4.1 4.0 4.0      |
| 28 | 9.47 734  | 40 | 9.49 784  | 44    | 0.50 216  | 9.97 950  | 4  | 32 | 7 4.8 4.8 4.7 4.6      |
| 29 | 9.47 774  | 40 | 9.49 828  | 44    | 0.50 172  | 9.97 946  | 4  | 31 | 8 5.5 5.4 5.4 5.3      |
| 30 | 9.47 814  | 40 | 9.49 872  | 44    | 0.50 128  | 9.97 942  | 4  | 30 | 9 6.2 6.1 6.1 6.0      |
| 31 | 9.47 854  | 40 | 9.49 916  | 44    | 0.50 083  | 9.97 938  | 4  | 29 | 10 6.9 6.8 6.7 6.6     |
| 32 | 9.47 894  | 40 | 9.49 960  | 44    | 0.50 039  | 9.97 934  | 4  | 28 | 20 13.8 13.6 13.5 13.3 |
| 33 | 9.47 934  | 40 | 9.50 004  | 44    | 0.49 996  | 9.97 930  | 4  | 27 | 30 20.7 20.5 20.2 20.0 |
| 34 | 9.47 974  | 40 | 9.50 048  | 44    | 0.49 952  | 9.97 926  | 4  | 26 | 40 27.6 27.3 27.0 26.6 |
| 35 | 9.48 014  | 40 | 9.50 092  | 44    | 0.49 908  | 9.97 922  | 4  | 25 | 50 34.6 34.1 33.7 33.3 |
| 36 | 9.48 054  | 40 | 9.50 136  | 44    | 0.49 864  | 9.97 918  | 4  | 24 |                        |
| 37 | 9.48 093  | 39 | 9.50 179  | 43    | 0.49 820  | 9.97 914  | 4  | 23 | 36 36 39 38            |
| 38 | 9.48 133  | 40 | 9.50 223  | 44    | 0.49 776  | 9.97 910  | 4  | 22 | 6 3.9 3.9 3.8 3.8      |
| 39 | 9.48 173  | 39 | 9.50 267  | 43    | 0.49 733  | 9.97 906  | 4  | 21 | 7 4.6 4.5 4.5 4.5      |
| 40 | 9.48 213  | 40 | 9.50 311  | 44    | 0.49 689  | 9.97 902  | 4  | 20 | 8 5.2 5.2 5.1 5.1      |
| 41 | 9.48 252  | 39 | 9.50 354  | 43    | 0.49 645  | 9.97 898  | 4  | 19 | 9 5.9 5.8 5.8 5.8      |
| 42 | 9.48 292  | 39 | 9.50 398  | 43    | 0.49 602  | 9.97 894  | 4  | 18 | 10 6.6 6.5 6.4 6.4     |
| 43 | 9.48 331  | 39 | 9.50 442  | 44    | 0.49 558  | 9.97 890  | 4  | 17 | 20 13.1 13.0 12.8 12.8 |
| 44 | 9.48 371  | 39 | 9.50 485  | 43    | 0.49 514  | 9.97 886  | 4  | 16 | 30 19.7 19.5 19.2 19.2 |
| 45 | 9.48 410  | 39 | 9.50 529  | 43    | 0.49 471  | 9.97 881  | 4  | 15 | 40 26.3 26.0 25.6 25.6 |
| 46 | 9.48 450  | 39 | 9.50 572  | 43    | 0.49 427  | 9.97 877  | 4  | 14 | 50 32.9 32.5 32.1 32.1 |
| 47 | 9.48 489  | 39 | 9.50 616  | 43    | 0.49 384  | 9.97 873  | 4  | 13 |                        |
| 48 | 9.48 529  | 39 | 9.50 659  | 43    | 0.49 340  | 9.97 869  | 4  | 12 | 4 4 3                  |
| 49 | 9.48 568  | 39 | 9.50 702  | 43    | 0.49 297  | 9.97 865  | 4  | 11 | 6 0.4 0.4 0.3 0.3      |
| 50 | 9.48 607  | 39 | 9.50 746  | 43    | 0.49 254  | 9.97 861  | 4  | 10 | 7 0.5 0.4 0.4 0.4      |
| 51 | 9.48 646  | 39 | 9.50 789  | 43    | 0.49 210  | 9.97 857  | 4  | 9  | 8 0.6 0.5 0.5 0.5      |
| 52 | 9.48 686  | 39 | 9.50 832  | 43    | 0.49 167  | 9.97 853  | 4  | 8  | 9 0.7 0.6 0.6 0.6      |
| 53 | 9.48 725  | 39 | 9.50 876  | 43    | 0.49 124  | 9.97 849  | 4  | 7  | 10 0.7 0.6 0.6 0.6     |
| 54 | 9.48 764  | 39 | 9.50 919  | 43    | 0.49 081  | 9.97 845  | 4  | 6  | 20 1.5 1.3 1.1 1.1     |
| 55 | 9.48 803  | 39 | 9.50 962  | 43    | 0.49 038  | 9.97 841  | 4  | 5  | 30 2.2 2.0 1.7 1.7     |
| 56 | 9.48 842  | 39 | 9.51 005  | 43    | 0.48 994  | 9.97 837  | 4  | 4  | 40 3.0 2.6 2.3 2.3     |
| 57 | 9.48 881  | 39 | 9.51 048  | 43    | 0.48 951  | 9.97 833  | 4  | 3  | 50 3.7 3.3 2.9 2.9     |
| 58 | 9.48 920  | 39 | 9.51 091  | 43    | 0.48 908  | 9.97 829  | 4  | 2  |                        |
| 59 | 9.48 959  | 39 | 9.51 134  | 43    | 0.48 865  | 9.97 824  | 4  | 1  |                        |
| 60 | 9.48 998  | 38 | 9.51 177  | 43    | 0.48 822  | 9.97 820  | 4  | 0  |                        |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.                  |

|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.48 998  | 39 | 9.51 177  | 43    | 0.48 822  | 9.97 820  | 4  | 60 |       |
| 1  | 9.49 037  | 39 | 9.51 220  | 43    | 0.48 779  | 9.97 816  | 4  | 59 |       |
| 2  | 9.49 076  | 38 | 9.51 263  | 43    | 0.48 736  | 9.97 812  | 4  | 58 |       |
| 3  | 9.49 114  | 39 | 9.51 306  | 43    | 0.48 693  | 9.97 808  | 4  | 57 |       |
| 4  | 9.49 153  | 38 | 9.51 349  | 42    | 0.48 650  | 9.97 804  | 4  | 56 |       |
| 5  | 9.49 192  | 39 | 9.51 392  | 43    | 0.48 608  | 9.97 800  | 4  | 55 |       |
| 6  | 9.49 231  | 38 | 9.51 435  | 42    | 0.48 565  | 9.97 796  | 4  | 54 |       |
| 7  | 9.49 269  | 38 | 9.51 477  | 43    | 0.48 522  | 9.97 792  | 4  | 53 |       |
| 8  | 9.49 308  | 38 | 9.51 520  | 42    | 0.48 479  | 9.97 787  | 4  | 52 |       |
| 9  | 9.49 346  | 38 | 9.51 563  | 42    | 0.48 437  | 9.97 783  | 4  | 51 |       |
| 10 | 9.49 385  | 38 | 9.51 605  | 43    | 0.48 394  | 9.97 779  | 4  | 50 |       |
| 11 | 9.49 423  | 38 | 9.51 648  | 42    | 0.48 351  | 9.97 775  | 4  | 49 |       |
| 12 | 9.49 462  | 38 | 9.51 691  | 42    | 0.48 309  | 9.97 771  | 4  | 48 |       |
| 13 | 9.49 500  | 38 | 9.51 733  | 42    | 0.48 266  | 9.97 767  | 4  | 47 |       |
| 14 | 9.49 539  | 38 | 9.51 776  | 42    | 0.48 224  | 9.97 763  | 4  | 46 |       |
| 15 | 9.49 577  | 38 | 9.51 818  | 42    | 0.48 181  | 9.97 758  | 4  | 45 |       |
| 16 | 9.49 615  | 38 | 9.51 861  | 42    | 0.48 139  | 9.97 754  | 4  | 44 |       |
| 17 | 9.49 653  | 38 | 9.51 903  | 42    | 0.48 096  | 9.97 750  | 4  | 43 |       |
| 18 | 9.49 692  | 38 | 9.51 946  | 42    | 0.48 054  | 9.97 746  | 4  | 42 |       |
| 19 | 9.49 730  | 38 | 9.51 988  | 42    | 0.48 012  | 9.97 742  | 4  | 41 |       |
| 20 | 9.49 768  | 38 | 9.52 030  | 42    | 0.47 969  | 9.97 737  | 4  | 40 |       |
| 21 | 9.49 806  | 38 | 9.52 073  | 42    | 0.47 927  | 9.97 733  | 4  | 39 |       |
| 22 | 9.49 844  | 38 | 9.52 115  | 42    | 0.47 885  | 9.97 729  | 4  | 38 |       |
| 23 | 9.49 882  | 38 | 9.52 157  | 42    | 0.47 842  | 9.97 725  | 4  | 37 |       |
| 24 | 9.49 920  | 38 | 9.52 199  | 42    | 0.47 800  | 9.97 721  | 4  | 36 |       |
| 25 | 9.49 958  | 38 | 9.52 241  | 42    | 0.47 758  | 9.97 716  | 4  | 35 |       |
| 26 | 9.49 996  | 37 | 9.52 284  | 42    | 0.47 716  | 9.97 712  | 4  | 34 |       |
| 27 | 9.50 034  | 38 | 9.52 326  | 42    | 0.47 674  | 9.97 708  | 4  | 33 |       |
| 28 | 9.50 072  | 38 | 9.52 368  | 42    | 0.47 632  | 9.97 704  | 4  | 32 |       |
| 29 | 9.50 110  | 37 | 9.52 410  | 42    | 0.47 590  | 9.97 700  | 4  | 31 |       |
| 30 | 9.50 147  | 38 | 9.52 452  | 42    | 0.47 548  | 9.97 695  | 4  | 30 |       |
| 31 | 9.50 185  | 37 | 9.52 494  | 42    | 0.47 506  | 9.97 691  | 4  | 29 |       |
| 32 | 9.50 223  | 37 | 9.52 536  | 42    | 0.47 464  | 9.97 687  | 4  | 28 |       |
| 33 | 9.50 260  | 38 | 9.52 578  | 41    | 0.47 422  | 9.97 683  | 4  | 27 |       |
| 34 | 9.50 298  | 37 | 9.52 619  | 42    | 0.47 380  | 9.97 678  | 4  | 26 |       |
| 35 | 9.50 336  | 37 | 9.52 661  | 42    | 0.47 338  | 9.97 674  | 4  | 25 |       |
| 36 | 9.50 373  | 37 | 9.52 703  | 41    | 0.47 296  | 9.97 670  | 4  | 24 |       |
| 37 | 9.50 411  | 37 | 9.52 745  | 42    | 0.47 255  | 9.97 666  | 4  | 23 |       |
| 38 | 9.50 448  | 37 | 9.52 787  | 41    | 0.47 213  | 9.97 661  | 4  | 22 |       |
| 39 | 9.50 486  | 37 | 9.52 828  | 41    | 0.47 171  | 9.97 657  | 4  | 21 |       |
| 40 | 9.50 523  | 37 | 9.52 870  | 42    | 0.47 130  | 9.97 653  | 4  | 20 |       |
| 41 | 9.50 561  | 37 | 9.52 912  | 41    | 0.47 088  | 9.97 649  | 4  | 19 |       |
| 42 | 9.50 598  | 37 | 9.52 953  | 41    | 0.47 046  | 9.97 644  | 4  | 18 |       |
| 43 | 9.50 635  | 37 | 9.52 995  | 41    | 0.47 005  | 9.97 640  | 4  | 17 |       |
| 44 | 9.50 672  | 37 | 9.53 036  | 41    | 0.46 963  | 9.97 636  | 4  | 16 |       |
| 45 | 9.50 710  | 37 | 9.53 078  | 41    | 0.46 922  | 9.97 632  | 4  | 15 |       |
| 46 | 9.50 747  | 37 | 9.53 119  | 41    | 0.46 880  | 9.97 627  | 4  | 14 |       |
| 47 | 9.50 784  | 37 | 9.53 161  | 41    | 0.46 839  | 9.97 623  | 4  | 13 |       |
| 48 | 9.50 821  | 37 | 9.53 202  | 41    | 0.46 797  | 9.97 619  | 4  | 12 |       |
| 49 | 9.50 858  | 37 | 9.53 244  | 41    | 0.46 756  | 9.97 614  | 4  | 11 |       |
| 50 | 9.50 895  | 37 | 9.53 285  | 41    | 0.46 714  | 9.97 610  | 4  | 10 |       |
| 51 | 9.50 932  | 37 | 9.53 326  | 41    | 0.46 673  | 9.97 606  | 4  | 9  |       |
| 52 | 9.50 969  | 37 | 9.53 368  | 41    | 0.46 632  | 9.97 601  | 4  | 8  |       |
| 53 | 9.51 006  | 37 | 9.53 409  | 41    | 0.46 591  | 9.97 597  | 4  | 7  |       |
| 54 | 9.51 043  | 37 | 9.53 450  | 41    | 0.46 549  | 9.97 593  | 4  | 6  |       |
| 55 | 9.51 080  | 37 | 9.53 491  | 41    | 0.46 508  | 9.97 588  | 4  | 5  |       |
| 56 | 9.51 117  | 36 | 9.53 533  | 41    | 0.46 467  | 9.97 584  | 4  | 4  |       |
| 57 | 9.51 154  | 37 | 9.53 574  | 41    | 0.46 426  | 9.97 580  | 4  | 3  |       |
| 58 | 9.51 190  | 36 | 9.53 615  | 41    | 0.46 385  | 9.97 575  | 4  | 2  |       |
| 59 | 9.51 227  | 37 | 9.53 656  | 41    | 0.46 344  | 9.97 571  | 4  | 1  |       |
| 60 | 9.51 264  | 36 | 9.53 697  | 41    | 0.46 303  | 9.97 567  | 4  | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

|    |      |      |      |
|----|------|------|------|
|    | 43   | 42   | 42   |
| 6  | 4.3  | 4.2  | 4.2  |
| 7  | 5.0  | 4.9  | 4.9  |
| 8  | 5.7  | 5.6  | 5.6  |
| 9  | 6.4  | 6.4  | 6.3  |
| 10 | 7.1  | 7.1  | 7.0  |
| 20 | 14.3 | 14.1 | 14.0 |
| 30 | 21.5 | 21.2 | 21.0 |
| 40 | 28.6 | 28.3 | 28.0 |
| 50 | 35.8 | 35.4 | 35.0 |

|    |      |      |
|----|------|------|
|    | 41   | 41   |
| 6  | 4.1  | 4.1  |
| 7  | 4.8  | 4.8  |
| 8  | 5.5  | 5.4  |
| 9  | 6.2  | 6.1  |
| 10 | 6.9  | 6.8  |
| 20 | 13.8 | 13.6 |
| 30 | 20.7 | 20.5 |
| 40 | 27.6 | 27.3 |
| 50 | 34.6 | 34.1 |

|    |      |      |      |
|----|------|------|------|
|    | 39   | 38   | 38   |
| 6  | 3.9  | 3.8  | 3.8  |
| 7  | 4.5  | 4.5  | 4.4  |
| 8  | 5.2  | 5.1  | 5.0  |
| 9  | 5.8  | 5.8  | 5.7  |
| 10 | 6.5  | 6.4  | 6.3  |
| 20 | 13.0 | 12.8 | 12.6 |
| 30 | 19.5 | 19.2 | 19.0 |
| 40 | 26.0 | 25.6 | 25.3 |
| 50 | 32.5 | 32.1 | 31.6 |

|    |      |      |      |
|----|------|------|------|
|    | 37   | 37   | 36   |
| 6  | 3.7  | 3.7  | 3.6  |
| 7  | 4.4  | 4.3  | 4.2  |
| 8  | 5.0  | 4.9  | 4.8  |
| 9  | 5.6  | 5.5  | 5.5  |
| 10 | 6.2  | 6.1  | 6.1  |
| 20 | 12.5 | 12.3 | 12.1 |
| 30 | 18.7 | 18.5 | 18.2 |
| 40 | 25.0 | 24.6 | 24.3 |
| 50 | 31.2 | 30.8 | 30.4 |

|    |     |     |
|----|-----|-----|
|    | 4   | 4   |
| 6  | 0.4 | 0.4 |
| 7  | 0.5 | 0.4 |
| 8  | 0.6 | 0.5 |
| 9  | 0.7 | 0.6 |
| 10 | 0.7 | 0.6 |
| 20 | 1.5 | 1.3 |
| 30 | 2.2 | 2.0 |
| 40 | 3.0 | 2.6 |
| 50 | 3.7 | 3.3 |



|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.             |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------------------|
| 0  | 9.51 264  | 37 | 9.53 697  | 41    | 0.46 303  | 9.97 567  | 4  | 60 |                   |
| 1  | 9.51 301  | 36 | 9.53 738  | 41    | 0.46 262  | 9.97 562  | 4  | 59 |                   |
| 2  | 9.51 337  | 36 | 9.53 779  | 41    | 0.46 221  | 9.97 558  | 4  | 58 |                   |
| 3  | 9.51 374  | 36 | 9.53 820  | 41    | 0.46 180  | 9.97 554  | 4  | 57 | 41 46 40          |
| 4  | 9.51 410  | 36 | 9.53 861  | 41    | 0.46 139  | 9.97 549  | 4  | 56 | 6 4.1 4.6 4.0     |
| 5  | 9.51 447  | 36 | 9.53 902  | 41    | 0.46 098  | 9.97 545  | 4  | 55 | 7 4.8 4.7 4.6     |
| 6  | 9.51 483  | 36 | 9.53 943  | 40    | 0.46 057  | 9.97 541  | 4  | 54 | 8 5.4 5.4 5.3     |
| 7  | 9.51 520  | 36 | 9.53 983  | 41    | 0.46 016  | 9.97 536  | 4  | 53 | 9 6.1 6.1 6.0     |
| 8  | 9.51 556  | 36 | 9.54 024  | 41    | 0.45 975  | 9.97 532  | 4  | 52 | 10 6.8 6.7 6.6    |
| 9  | 9.51 593  | 36 | 9.54 065  | 40    | 0.45 934  | 9.97 527  | 4  | 51 | 20 13.6 13.5 13.3 |
| 10 | 9.51 629  | 36 | 9.54 106  | 41    | 0.45 894  | 9.97 523  | 4  | 50 | 30 20.5 20.2 20.0 |
| 11 | 9.51 665  | 36 | 9.54 147  | 40    | 0.45 853  | 9.97 519  | 4  | 49 | 40 27.3 27.0 26.6 |
| 12 | 9.51 702  | 36 | 9.54 187  | 40    | 0.45 812  | 9.97 514  | 4  | 48 | 50 34.1 33.7 33.3 |
| 13 | 9.51 738  | 36 | 9.54 228  | 41    | 0.45 772  | 9.97 510  | 4  | 47 |                   |
| 14 | 9.51 774  | 36 | 9.54 269  | 40    | 0.45 731  | 9.97 505  | 4  | 46 |                   |
| 15 | 9.51 810  | 36 | 9.54 309  | 40    | 0.45 690  | 9.97 501  | 4  | 45 | 39 39             |
| 16 | 9.51 847  | 36 | 9.54 350  | 40    | 0.45 650  | 9.97 497  | 4  | 44 | 6 3.9 3.9         |
| 17 | 9.51 883  | 36 | 9.54 390  | 40    | 0.45 609  | 9.97 492  | 4  | 43 | 7 4.6 4.5         |
| 18 | 9.51 919  | 36 | 9.54 431  | 40    | 0.45 569  | 9.97 488  | 4  | 42 | 8 5.2 5.2         |
| 19 | 9.51 955  | 36 | 9.54 471  | 40    | 0.45 528  | 9.97 483  | 4  | 41 | 9 5.9 5.8         |
| 20 | 9.51 991  | 36 | 9.54 512  | 40    | 0.45 488  | 9.97 479  | 4  | 40 | 10 6.6 6.5        |
| 21 | 9.52 027  | 36 | 9.54 552  | 40    | 0.45 447  | 9.97 475  | 4  | 39 | 20 13.1 13.0      |
| 22 | 9.52 063  | 36 | 9.54 593  | 40    | 0.45 407  | 9.97 470  | 4  | 38 | 30 19.7 19.5      |
| 23 | 9.52 099  | 36 | 9.54 633  | 40    | 0.45 367  | 9.97 466  | 4  | 37 | 40 26.3 26.0      |
| 24 | 9.52 135  | 36 | 9.54 673  | 40    | 0.45 326  | 9.97 461  | 4  | 36 | 50 32.9 32.5      |
| 25 | 9.52 170  | 35 | 9.54 714  | 40    | 0.45 286  | 9.97 457  | 4  | 35 |                   |
| 26 | 9.52 206  | 36 | 9.54 754  | 40    | 0.45 246  | 9.97 452  | 4  | 34 | 37 36 36          |
| 27 | 9.52 242  | 36 | 9.54 794  | 40    | 0.45 205  | 9.97 448  | 4  | 33 | 6 3.7 3.6 3.6     |
| 28 | 9.52 278  | 35 | 9.54 834  | 40    | 0.45 165  | 9.97 443  | 4  | 32 | 7 4.3 4.2 4.2     |
| 29 | 9.52 314  | 36 | 9.54 874  | 40    | 0.45 125  | 9.97 439  | 4  | 31 | 8 4.9 4.8 4.8     |
| 30 | 9.52 349  | 35 | 9.54 915  | 40    | 0.45 085  | 9.97 434  | 4  | 30 | 9 5.5 5.5 5.4     |
| 31 | 9.52 385  | 35 | 9.54 955  | 40    | 0.45 045  | 9.97 430  | 4  | 29 | 10 6.1 6.1 6.0    |
| 32 | 9.52 421  | 35 | 9.54 995  | 40    | 0.45 005  | 9.97 425  | 4  | 28 | 20 12.3 12.1 12.0 |
| 33 | 9.52 456  | 35 | 9.55 035  | 40    | 0.44 965  | 9.97 421  | 4  | 27 | 30 18.5 18.2 18.0 |
| 34 | 9.52 492  | 35 | 9.55 075  | 40    | 0.44 925  | 9.97 416  | 4  | 26 | 40 24.6 24.3 24.0 |
| 35 | 9.52 527  | 35 | 9.55 115  | 39    | 0.44 884  | 9.97 412  | 4  | 25 | 50 30.8 30.4 30.0 |
| 36 | 9.52 563  | 35 | 9.55 155  | 40    | 0.44 845  | 9.97 407  | 4  | 24 |                   |
| 37 | 9.52 598  | 35 | 9.55 195  | 40    | 0.44 805  | 9.97 403  | 4  | 23 | 35 35 34          |
| 38 | 9.52 634  | 35 | 9.55 235  | 40    | 0.44 765  | 9.97 398  | 4  | 22 | 6 3.5 3.5 3.4     |
| 39 | 9.52 669  | 35 | 9.55 275  | 40    | 0.44 725  | 9.97 394  | 4  | 21 | 7 4.1 4.1 4.0     |
| 40 | 9.52 704  | 35 | 9.55 315  | 40    | 0.44 685  | 9.97 389  | 4  | 20 | 8 4.7 4.6 4.6     |
| 41 | 9.52 740  | 35 | 9.55 355  | 39    | 0.44 645  | 9.97 385  | 4  | 19 | 9 5.3 5.2 5.2     |
| 42 | 9.52 775  | 35 | 9.55 394  | 40    | 0.44 605  | 9.97 380  | 4  | 18 | 10 5.9 5.8 5.7    |
| 43 | 9.52 810  | 35 | 9.55 434  | 39    | 0.44 565  | 9.97 376  | 4  | 17 | 20 11.8 11.6 11.5 |
| 44 | 9.52 846  | 35 | 9.55 474  | 40    | 0.44 526  | 9.97 371  | 4  | 16 | 30 17.7 17.5 17.2 |
| 45 | 9.52 881  | 35 | 9.55 514  | 40    | 0.44 486  | 9.97 367  | 4  | 15 | 40 23.6 23.3 23.0 |
| 46 | 9.52 916  | 35 | 9.55 553  | 39    | 0.44 446  | 9.97 362  | 4  | 14 | 50 29.6 29.1 28.7 |
| 47 | 9.52 951  | 35 | 9.55 593  | 39    | 0.44 406  | 9.97 358  | 4  | 13 |                   |
| 48 | 9.52 986  | 35 | 9.55 633  | 39    | 0.44 367  | 9.97 353  | 4  | 12 |                   |
| 49 | 9.53 021  | 35 | 9.55 672  | 39    | 0.44 327  | 9.97 349  | 4  | 11 | 5 4 4             |
| 50 | 9.53 056  | 35 | 9.55 712  | 39    | 0.44 288  | 9.97 344  | 4  | 10 | 6 0.5 0.4 0.4     |
| 51 | 9.53 091  | 35 | 9.55 751  | 40    | 0.44 248  | 9.97 340  | 4  | 9  | 7 0.6 0.5 0.4     |
| 52 | 9.53 126  | 35 | 9.55 791  | 39    | 0.44 208  | 9.97 335  | 4  | 8  | 8 0.6 0.6 0.5     |
| 53 | 9.53 161  | 35 | 9.55 831  | 39    | 0.44 169  | 9.97 330  | 4  | 7  | 9 0.7 0.7 0.6     |
| 54 | 9.53 196  | 35 | 9.55 870  | 39    | 0.44 129  | 9.97 326  | 4  | 6  | 10 0.8 0.7 0.6    |
| 55 | 9.53 231  | 34 | 9.55 909  | 39    | 0.44 090  | 9.97 321  | 4  | 5  | 20 1.6 1.5 1.3    |
| 56 | 9.53 266  | 35 | 9.55 949  | 39    | 0.44 051  | 9.97 317  | 4  | 4  | 30 2.5 2.2 2.0    |
| 57 | 9.53 301  | 35 | 9.55 988  | 39    | 0.44 011  | 9.97 312  | 4  | 3  | 40 3.3 3.0 2.6    |
| 58 | 9.53 335  | 34 | 9.56 028  | 39    | 0.43 972  | 9.97 308  | 4  | 2  | 50 4.1 3.7 3.3    |
| 59 | 9.53 370  | 35 | 9.56 067  | 39    | 0.43 932  | 9.97 303  | 5  | 1  |                   |
| 60 | 9.53 405  | 34 | 9.56 106  | 39    | 0.43 893  | 9.97 298  | 4  | 0  |                   |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.             |

|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |      |      |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|------|------|
| 0  | 9.53 405  | 35 | 9.56 106  | 39    | 0.43 893  | 9.97 298  | 4  | 60 |       |      |      |
| 1  | 9.53 440  | 34 | 9.56 146  | 39    | 0.43 854  | 9.97 294  | 4  | 59 |       |      |      |
| 2  | 9.53 474  | 34 | 9.56 185  | 39    | 0.43 815  | 9.97 289  | 4  | 58 |       |      |      |
| 3  | 9.53 509  | 35 | 9.56 224  | 39    | 0.43 775  | 9.97 285  | 5  | 57 | 39    | 39   |      |
| 4  | 9.53 544  | 34 | 9.56 263  | 39    | 0.43 736  | 9.97 280  | 4  | 56 | 6     | 3.9  | 3.9  |
| 5  | 9.53 578  | 34 | 9.56 303  | 39    | 0.43 697  | 9.97 275  | 4  | 55 | 7     | 3.9  | 4.3  |
| 6  | 9.53 613  | 34 | 9.56 342  | 39    | 0.43 658  | 9.97 271  | 4  | 54 | 8     | 4.6  | 5.2  |
| 7  | 9.53 647  | 34 | 9.56 381  | 39    | 0.43 619  | 9.97 266  | 4  | 53 | 9     | 5.2  | 5.8  |
| 8  | 9.53 682  | 34 | 9.56 420  | 39    | 0.43 580  | 9.97 261  | 5  | 52 | 10    | 5.9  | 6.5  |
| 9  | 9.53 716  | 34 | 9.56 459  | 39    | 0.43 540  | 9.97 257  | 4  | 51 | 20    | 6.6  | 13.0 |
| 10 | 9.53 750  | 34 | 9.56 498  | 39    | 0.43 501  | 9.97 252  | 4  | 50 | 30    | 13.1 | 19.5 |
| 11 | 9.53 785  | 34 | 9.56 537  | 39    | 0.43 462  | 9.97 248  | 4  | 49 | 40    | 19.7 | 26.0 |
| 12 | 9.53 819  | 34 | 9.56 576  | 39    | 0.43 423  | 9.97 243  | 5  | 48 | 50    | 26.3 | 32.5 |
| 13 | 9.53 854  | 34 | 9.56 615  | 38    | 0.43 384  | 9.97 238  | 4  | 47 |       |      |      |
| 14 | 9.53 888  | 34 | 9.56 654  | 39    | 0.43 346  | 9.97 234  | 4  | 46 |       |      |      |
| 15 | 9.53 922  | 34 | 9.56 693  | 39    | 0.43 307  | 9.97 229  | 5  | 45 | 38    | 38   | 37   |
| 16 | 9.53 956  | 34 | 9.56 732  | 39    | 0.43 268  | 9.97 224  | 4  | 44 | 6     | 3.8  | 3.7  |
| 17 | 9.53 990  | 34 | 9.56 771  | 39    | 0.43 229  | 9.97 220  | 4  | 43 | 7     | 4.5  | 4.4  |
| 18 | 9.54 025  | 34 | 9.56 810  | 39    | 0.43 190  | 9.97 215  | 5  | 42 | 8     | 4.1  | 5.0  |
| 19 | 9.54 059  | 34 | 9.56 848  | 38    | 0.43 151  | 9.97 210  | 4  | 41 | 9     | 5.1  | 5.6  |
| 20 | 9.54 093  | 34 | 9.56 887  | 39    | 0.43 112  | 9.97 206  | 4  | 40 | 10    | 5.8  | 6.2  |
| 21 | 9.54 127  | 34 | 9.56 926  | 38    | 0.43 074  | 9.97 201  | 5  | 39 | 20    | 6.4  | 12.5 |
| 22 | 9.54 161  | 34 | 9.56 965  | 39    | 0.43 035  | 9.97 196  | 4  | 38 | 30    | 12.8 | 18.7 |
| 23 | 9.54 195  | 34 | 9.57 003  | 38    | 0.42 996  | 9.97 191  | 5  | 37 | 40    | 19.2 | 25.0 |
| 24 | 9.54 229  | 34 | 9.57 042  | 38    | 0.42 958  | 9.97 187  | 4  | 36 | 50    | 25.6 | 31.2 |
| 25 | 9.54 263  | 33 | 9.57 081  | 39    | 0.42 919  | 9.97 182  | 4  | 35 |       |      |      |
| 26 | 9.54 297  | 34 | 9.57 119  | 38    | 0.42 880  | 9.97 177  | 5  | 34 | 35    | 34   | 34   |
| 27 | 9.54 331  | 34 | 9.57 158  | 38    | 0.42 842  | 9.97 173  | 4  | 33 | 6     | 3.5  | 3.4  |
| 28 | 9.54 365  | 33 | 9.57 196  | 38    | 0.42 803  | 9.97 168  | 5  | 32 | 7     | 4.1  | 3.9  |
| 29 | 9.54 398  | 34 | 9.57 235  | 39    | 0.42 765  | 9.97 163  | 4  | 31 | 8     | 4.6  | 4.5  |
| 30 | 9.54 432  | 34 | 9.57 274  | 38    | 0.42 726  | 9.97 159  | 4  | 30 | 9     | 5.2  | 5.1  |
| 31 | 9.54 466  | 33 | 9.57 312  | 38    | 0.42 687  | 9.97 154  | 5  | 29 | 10    | 5.7  | 5.6  |
| 32 | 9.54 500  | 34 | 9.57 350  | 38    | 0.42 649  | 9.97 149  | 4  | 28 | 20    | 5.8  | 11.3 |
| 33 | 9.54 534  | 33 | 9.57 389  | 38    | 0.42 611  | 9.97 144  | 5  | 27 | 30    | 11.6 | 17.0 |
| 34 | 9.54 567  | 33 | 9.57 427  | 38    | 0.42 572  | 9.97 140  | 4  | 26 | 40    | 17.5 | 22.6 |
| 35 | 9.54 601  | 33 | 9.57 466  | 38    | 0.42 534  | 9.97 135  | 5  | 25 | 50    | 23.3 | 28.3 |
| 36 | 9.54 634  | 33 | 9.57 504  | 38    | 0.42 495  | 9.97 130  | 4  | 24 |       |      |      |
| 37 | 9.54 668  | 34 | 9.57 542  | 38    | 0.42 457  | 9.97 125  | 5  | 23 |       |      |      |
| 38 | 9.54 702  | 33 | 9.57 581  | 38    | 0.42 419  | 9.97 121  | 4  | 22 | 33    | 33   |      |
| 39 | 9.54 735  | 33 | 9.57 619  | 38    | 0.42 380  | 9.97 116  | 5  | 21 | 6     | 3.3  | 3.3  |
| 40 | 9.54 769  | 33 | 9.57 657  | 38    | 0.42 342  | 9.97 111  | 4  | 20 | 7     | 3.9  | 3.8  |
| 41 | 9.54 802  | 33 | 9.57 696  | 38    | 0.42 304  | 9.97 106  | 5  | 19 | 8     | 4.4  | 4.4  |
| 42 | 9.54 836  | 33 | 9.57 734  | 38    | 0.42 266  | 9.97 102  | 4  | 18 | 9     | 5.0  | 4.9  |
| 43 | 9.54 869  | 33 | 9.57 772  | 38    | 0.42 227  | 9.97 097  | 5  | 17 | 10    | 5.6  | 5.5  |
| 44 | 9.54 902  | 33 | 9.57 810  | 38    | 0.42 189  | 9.97 092  | 4  | 16 | 20    | 11.1 | 11.0 |
| 45 | 9.54 936  | 33 | 9.57 848  | 38    | 0.42 151  | 9.97 087  | 5  | 15 | 30    | 16.7 | 16.5 |
| 46 | 9.54 969  | 33 | 9.57 886  | 38    | 0.42 113  | 9.97 082  | 4  | 14 | 40    | 22.3 | 22.0 |
| 47 | 9.55 002  | 33 | 9.57 925  | 38    | 0.42 075  | 9.97 078  | 5  | 13 | 50    | 27.9 | 27.5 |
| 48 | 9.55 036  | 33 | 9.57 963  | 38    | 0.42 037  | 9.97 073  | 4  | 12 |       |      |      |
| 49 | 9.55 069  | 33 | 9.58 001  | 38    | 0.41 999  | 9.97 068  | 5  | 11 |       |      |      |
| 50 | 9.55 102  | 33 | 9.58 039  | 38    | 0.41 961  | 9.97 063  | 4  | 10 | 5     | 4    |      |
| 51 | 9.55 135  | 33 | 9.58 077  | 38    | 0.41 923  | 9.97 058  | 5  | 9  | 6     | 0.5  | 0.4  |
| 52 | 9.55 168  | 33 | 9.58 115  | 38    | 0.41 885  | 9.97 054  | 4  | 8  | 7     | 0.6  | 0.5  |
| 53 | 9.55 202  | 33 | 9.58 153  | 38    | 0.41 847  | 9.97 049  | 5  | 7  | 8     | 0.6  | 0.6  |
| 54 | 9.55 235  | 33 | 9.58 190  | 37    | 0.41 809  | 9.97 044  | 4  | 6  | 9     | 0.7  | 0.7  |
| 55 | 9.55 268  | 33 | 9.58 228  | 38    | 0.41 771  | 9.97 039  | 5  | 5  | 10    | 0.8  | 0.7  |
| 56 | 9.55 301  | 33 | 9.58 266  | 38    | 0.41 733  | 9.97 034  | 4  | 4  | 20    | 1.6  | 1.5  |
| 57 | 9.55 334  | 33 | 9.58 304  | 38    | 0.41 695  | 9.97 029  | 5  | 3  | 30    | 2.5  | 2.2  |
| 58 | 9.55 367  | 33 | 9.58 342  | 37    | 0.41 658  | 9.97 025  | 4  | 2  | 40    | 3.3  | 3.0  |
| 59 | 9.55 400  | 33 | 9.58 380  | 38    | 0.41 620  | 9.97 020  | 5  | 1  | 50    | 4.1  | 3.7  |
| 60 | 9.55 433  | 33 | 9.58 417  | 37    | 0.41 582  | 9.97 015  | 4  | 0  |       |      |      |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |      |      |



| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.             |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------------------|
| 0  | 9.55 433  |    | 9.58 417  | 38    | 0.41 582  | 9.97 015  | 4  | 60 |                   |
| 1  | 9.55 466  | 33 | 9.58 453  | 37    | 0.41 544  | 9.97 010  | 5  | 59 |                   |
| 2  | 9.55 498  | 33 | 9.58 493  | 38    | 0.41 507  | 9.97 005  | 5  | 58 |                   |
| 3  | 9.55 531  | 33 | 9.58 531  | 37    | 0.41 469  | 9.97 000  | 5  | 57 | 38 37 37          |
| 4  | 9.55 564  | 32 | 9.58 568  | 37    | 0.41 431  | 9.96 995  | 4  | 56 | 6 3.8 3.7 3.7     |
| 5  | 9.55 597  | 33 | 9.58 606  | 38    | 0.41 394  | 9.96 991  | 5  | 55 | 7 4.4 4.4 4.3     |
| 6  | 9.55 630  | 32 | 9.58 644  | 37    | 0.41 356  | 9.96 986  | 5  | 54 | 8 5.0 5.0 4.9     |
| 7  | 9.55 662  | 33 | 9.58 681  | 37    | 0.41 318  | 9.96 981  | 5  | 53 | 9 5.7 5.6 5.5     |
| 8  | 9.55 693  | 33 | 9.58 719  | 37    | 0.41 281  | 9.96 976  | 5  | 52 | 10 6.3 6.2 6.1    |
| 9  | 9.55 728  | 32 | 9.58 756  | 37    | 0.41 243  | 9.96 971  | 4  | 51 | 20 12.6 12.5 12.3 |
| 10 | 9.55 760  | 32 | 9.58 794  | 37    | 0.41 206  | 9.96 966  | 5  | 50 | 30 19.0 18.7 18.5 |
| 11 | 9.55 793  | 33 | 9.58 831  | 37    | 0.41 168  | 9.96 961  | 5  | 49 | 40 25.3 25.0 24.6 |
| 12 | 9.55 826  | 32 | 9.58 869  | 37    | 0.41 131  | 9.96 956  | 5  | 48 | 50 31.6 31.2 30.8 |
| 13 | 9.55 858  | 32 | 9.58 906  | 37    | 0.41 093  | 9.96 952  | 4  | 47 |                   |
| 14 | 9.55 891  | 32 | 9.58 944  | 37    | 0.41 056  | 9.96 947  | 5  | 46 |                   |
| 15 | 9.55 923  | 32 | 9.58 981  | 37    | 0.41 018  | 9.96 942  | 5  | 45 | 36 36             |
| 16 | 9.55 956  | 32 | 9.59 019  | 37    | 0.40 981  | 9.96 937  | 5  | 44 | 6 3.6 3.6         |
| 17 | 9.55 988  | 32 | 9.59 056  | 37    | 0.40 944  | 9.96 932  | 5  | 43 | 7 4.2 4.2         |
| 18 | 9.56 020  | 32 | 9.59 093  | 37    | 0.40 906  | 9.96 927  | 5  | 42 | 8 4.8 4.8         |
| 19 | 9.56 053  | 32 | 9.59 131  | 37    | 0.40 869  | 9.96 922  | 4  | 41 | 9 5.5 5.4         |
| 20 | 9.56 085  | 32 | 9.59 168  | 37    | 0.40 832  | 9.96 917  | 5  | 40 | 10 6.1 6.0        |
| 21 | 9.56 118  | 32 | 9.59 205  | 37    | 0.40 794  | 9.96 912  | 5  | 39 | 20 12.1 12.0      |
| 22 | 9.56 150  | 32 | 9.59 242  | 37    | 0.40 757  | 9.96 907  | 5  | 38 | 30 18.2 18.0      |
| 23 | 9.56 182  | 32 | 9.59 280  | 37    | 0.40 720  | 9.96 902  | 5  | 37 | 40 24.3 24.0      |
| 24 | 9.56 214  | 32 | 9.59 317  | 37    | 0.40 683  | 9.96 897  | 5  | 36 | 50 30.4 30.0      |
| 25 | 9.56 247  | 32 | 9.59 354  | 37    | 0.40 646  | 9.96 892  | 5  | 35 |                   |
| 26 | 9.56 279  | 32 | 9.59 391  | 37    | 0.40 608  | 9.96 887  | 5  | 34 | 33 32 32          |
| 27 | 9.56 311  | 32 | 9.59 428  | 37    | 0.40 571  | 9.96 882  | 5  | 33 | 6 3.3 3.2         |
| 28 | 9.56 343  | 32 | 9.59 465  | 37    | 0.40 534  | 9.96 877  | 5  | 32 | 7 3.8 3.7         |
| 29 | 9.56 375  | 32 | 9.59 502  | 37    | 0.40 497  | 9.96 873  | 4  | 31 | 8 4.4 4.2         |
| 30 | 9.56 407  | 32 | 9.59 540  | 37    | 0.40 460  | 9.96 868  | 5  | 30 | 9 4.9 4.8         |
| 31 | 9.56 439  | 32 | 9.59 577  | 37    | 0.40 423  | 9.96 863  | 5  | 29 | 10 5.5 5.3        |
| 32 | 9.56 471  | 32 | 9.59 614  | 37    | 0.40 386  | 9.96 858  | 5  | 28 | 20 11.0 10.8      |
| 33 | 9.56 503  | 32 | 9.59 651  | 37    | 0.40 349  | 9.96 853  | 5  | 27 | 30 16.5 16.2      |
| 34 | 9.56 535  | 32 | 9.59 688  | 37    | 0.40 312  | 9.96 848  | 5  | 26 | 40 22.0 21.6      |
| 35 | 9.56 567  | 32 | 9.59 724  | 36    | 0.40 275  | 9.96 843  | 5  | 25 | 50 27.5 27.1      |
| 36 | 9.56 599  | 32 | 9.59 761  | 37    | 0.40 238  | 9.96 838  | 5  | 24 |                   |
| 37 | 9.56 631  | 31 | 9.59 798  | 37    | 0.40 201  | 9.96 833  | 5  | 23 | 31 31             |
| 38 | 9.56 663  | 31 | 9.59 835  | 37    | 0.40 164  | 9.96 828  | 5  | 22 | 6 3.1 3.1         |
| 39 | 9.56 695  | 32 | 9.59 872  | 36    | 0.40 128  | 9.96 823  | 5  | 21 | 7 3.7 3.6         |
| 40 | 9.56 727  | 32 | 9.59 909  | 37    | 0.40 091  | 9.96 818  | 5  | 20 | 8 4.2 4.1         |
| 41 | 9.56 758  | 31 | 9.59 946  | 37    | 0.40 054  | 9.96 813  | 5  | 19 | 9 4.7 4.6         |
| 42 | 9.56 790  | 32 | 9.59 982  | 36    | 0.40 017  | 9.96 808  | 5  | 18 | 10 5.2 5.1        |
| 43 | 9.56 822  | 31 | 9.60 019  | 37    | 0.39 980  | 9.96 802  | 5  | 17 | 20 10.5 10.3      |
| 44 | 9.56 854  | 32 | 9.60 056  | 36    | 0.39 944  | 9.96 797  | 5  | 16 | 30 15.7 15.5      |
| 45 | 9.56 885  | 31 | 9.60 093  | 37    | 0.39 907  | 9.96 792  | 5  | 15 | 40 21.0 20.6      |
| 46 | 9.56 917  | 31 | 9.60 129  | 36    | 0.39 870  | 9.96 787  | 5  | 14 | 50 26.2 25.8      |
| 47 | 9.56 949  | 32 | 9.60 166  | 37    | 0.39 833  | 9.96 782  | 5  | 13 |                   |
| 48 | 9.56 980  | 31 | 9.60 203  | 36    | 0.39 797  | 9.96 777  | 5  | 12 | 5 5 4             |
| 49 | 9.57 012  | 31 | 9.60 239  | 36    | 0.39 760  | 9.96 772  | 5  | 11 | 6 0.5 0.4         |
| 50 | 9.57 043  | 31 | 9.60 276  | 36    | 0.39 724  | 9.96 767  | 5  | 10 | 7 0.6 0.5         |
| 51 | 9.57 075  | 31 | 9.60 312  | 36    | 0.39 687  | 9.96 762  | 5  | 9  | 8 0.7 0.6         |
| 52 | 9.57 106  | 31 | 9.60 349  | 37    | 0.39 650  | 9.96 757  | 5  | 8  | 9 0.8 0.7         |
| 53 | 9.57 138  | 31 | 9.60 386  | 36    | 0.39 614  | 9.96 752  | 5  | 7  | 10 0.9 0.8        |
| 54 | 9.57 169  | 31 | 9.60 422  | 36    | 0.39 577  | 9.96 747  | 5  | 6  | 20 1.8 1.6        |
| 55 | 9.57 201  | 31 | 9.60 459  | 36    | 0.39 541  | 9.96 742  | 5  | 5  | 30 2.7 2.5        |
| 56 | 9.57 232  | 31 | 9.60 495  | 36    | 0.39 504  | 9.96 737  | 5  | 4  | 40 3.6 3.3        |
| 57 | 9.57 263  | 31 | 9.60 531  | 36    | 0.39 468  | 9.96 732  | 5  | 3  | 50 4.6 4.1        |
| 58 | 9.57 295  | 31 | 9.60 568  | 36    | 0.39 432  | 9.96 727  | 5  | 2  |                   |
| 59 | 9.57 326  | 31 | 9.60 604  | 36    | 0.39 395  | 9.96 721  | 5  | 1  |                   |
| 60 | 9.57 357  | 31 | 9.60 641  | 36    | 0.39 359  | 9.96 716  | 5  | 0  |                   |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.             |

|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.        |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|--------------|
| 0  | 9.57 357  | 31 | 9.60 641  | 36    | 0.39 359  | 9.96 716  | 5  | 60 |              |
| 1  | 9.57 389  | 31 | 9.60 677  | 36    | 0.39 322  | 9.96 711  | 5  | 59 |              |
| 2  | 9.57 420  | 31 | 9.60 713  | 36    | 0.39 286  | 9.96 706  | 5  | 58 |              |
| 3  | 9.57 451  | 31 | 9.60 750  | 36    | 0.39 250  | 9.96 701  | 5  | 57 | 36 36        |
| 4  | 9.57 482  | 31 | 9.60 786  | 36    | 0.39 213  | 9.96 696  | 5  | 56 | 6 3.6 3.6    |
| 5  | 9.57 513  | 31 | 9.60 822  | 36    | 0.39 177  | 9.96 691  | 5  | 55 | 7 4.2 4.2    |
| 6  | 9.57 544  | 31 | 9.60 859  | 36    | 0.39 141  | 9.96 686  | 5  | 54 | 8 4.8 4.8    |
| 7  | 9.57 576  | 31 | 9.60 895  | 36    | 0.39 105  | 9.96 681  | 5  | 53 | 9 5.5 5.4    |
| 8  | 9.57 607  | 31 | 9.60 931  | 36    | 0.39 069  | 9.96 675  | 5  | 52 | 10 6.1 6.0   |
| 9  | 9.57 638  | 31 | 9.60 967  | 36    | 0.39 032  | 9.96 670  | 5  | 51 | 20 12.1 12.0 |
| 10 | 9.57 669  | 31 | 9.61 003  | 36    | 0.38 996  | 9.96 665  | 5  | 50 | 30 18.2 18.0 |
| 11 | 9.57 700  | 31 | 9.61 039  | 36    | 0.38 960  | 9.96 660  | 5  | 49 | 40 24.3 24.0 |
| 12 | 9.57 731  | 31 | 9.61 076  | 36    | 0.38 924  | 9.96 655  | 5  | 48 | 50 30.4 30.0 |
| 13 | 9.57 762  | 30 | 9.61 112  | 36    | 0.38 888  | 9.96 650  | 5  | 47 |              |
| 14 | 9.57 792  | 31 | 9.61 148  | 36    | 0.38 852  | 9.96 644  | 5  | 46 |              |
| 15 | 9.57 823  | 31 | 9.61 184  | 36    | 0.38 816  | 9.96 639  | 5  | 45 | 35 35        |
| 16 | 9.57 854  | 31 | 9.61 220  | 36    | 0.38 780  | 9.96 634  | 5  | 44 | 6 3.5 3.5    |
| 17 | 9.57 885  | 30 | 9.61 256  | 36    | 0.38 744  | 9.96 629  | 5  | 43 | 7 4.1 4.1    |
| 18 | 9.57 916  | 31 | 9.61 292  | 36    | 0.38 708  | 9.96 624  | 5  | 42 | 8 4.7 4.6    |
| 19 | 9.57 947  | 30 | 9.61 328  | 36    | 0.38 672  | 9.96 619  | 5  | 41 | 9 5.3 5.2    |
| 20 | 9.57 977  | 31 | 9.61 364  | 36    | 0.38 636  | 9.96 613  | 5  | 40 | 10 5.9 5.8   |
| 21 | 9.58 008  | 30 | 9.61 400  | 36    | 0.38 600  | 9.96 608  | 5  | 39 | 20 11.8 11.6 |
| 22 | 9.58 039  | 31 | 9.61 436  | 36    | 0.38 564  | 9.96 603  | 5  | 38 | 30 17.7 17.5 |
| 23 | 9.58 070  | 30 | 9.61 472  | 35    | 0.38 528  | 9.96 598  | 5  | 37 | 40 23.6 23.3 |
| 24 | 9.58 100  | 30 | 9.61 507  | 36    | 0.38 492  | 9.96 593  | 5  | 36 | 50 29.6 29.1 |
| 25 | 9.58 131  | 31 | 9.61 543  | 36    | 0.38 456  | 9.96 587  | 5  | 35 |              |
| 26 | 9.58 162  | 30 | 9.61 579  | 35    | 0.38 420  | 9.96 582  | 5  | 34 | 31 31        |
| 27 | 9.58 192  | 30 | 9.61 615  | 36    | 0.38 385  | 9.96 577  | 5  | 33 | 6 3.1 3.1    |
| 28 | 9.58 223  | 30 | 9.61 651  | 36    | 0.38 349  | 9.96 572  | 5  | 32 | 7 3.7 3.6    |
| 29 | 9.58 253  | 30 | 9.61 686  | 35    | 0.38 313  | 9.96 567  | 5  | 31 | 8 4.2 4.1    |
| 30 | 9.58 284  | 30 | 9.61 722  | 36    | 0.38 277  | 9.96 561  | 5  | 30 | 9 4.7 4.6    |
| 31 | 9.58 314  | 30 | 9.61 758  | 35    | 0.38 242  | 9.96 556  | 5  | 29 | 10 5.2 5.1   |
| 32 | 9.58 345  | 30 | 9.61 794  | 36    | 0.38 206  | 9.96 551  | 5  | 28 | 20 10.5 10.3 |
| 33 | 9.58 375  | 30 | 9.61 829  | 35    | 0.38 170  | 9.96 546  | 5  | 27 | 30 15.7 15.5 |
| 34 | 9.58 406  | 30 | 9.61 865  | 36    | 0.38 135  | 9.96 540  | 5  | 26 | 40 21.0 20.6 |
| 35 | 9.58 436  | 30 | 9.61 901  | 36    | 0.38 099  | 9.96 535  | 5  | 25 | 50 26.2 25.8 |
| 36 | 9.58 466  | 30 | 9.61 936  | 35    | 0.38 063  | 9.96 530  | 5  | 24 |              |
| 37 | 9.58 497  | 30 | 9.61 972  | 35    | 0.38 028  | 9.96 525  | 5  | 23 | 36 30 29     |
| 38 | 9.58 527  | 30 | 9.62 007  | 35    | 0.37 992  | 9.96 519  | 5  | 22 | 6 3.0 3.0    |
| 39 | 9.58 557  | 30 | 9.62 043  | 35    | 0.37 957  | 9.96 514  | 5  | 21 | 7 3.5 3.4    |
| 40 | 9.58 587  | 30 | 9.62 078  | 35    | 0.37 921  | 9.96 509  | 5  | 20 | 8 4.0 3.9    |
| 41 | 9.58 618  | 30 | 9.62 114  | 35    | 0.37 886  | 9.96 503  | 5  | 19 | 9 4.6 4.4    |
| 42 | 9.58 648  | 30 | 9.62 149  | 35    | 0.37 850  | 9.96 498  | 5  | 18 | 10 5.1 4.9   |
| 43 | 9.58 678  | 30 | 9.62 185  | 35    | 0.37 815  | 9.96 493  | 5  | 17 | 20 10.1 10.0 |
| 44 | 9.58 708  | 30 | 9.62 220  | 35    | 0.37 779  | 9.96 488  | 5  | 16 | 30 15.2 15.0 |
| 45 | 9.58 738  | 30 | 9.62 256  | 35    | 0.37 744  | 9.96 482  | 5  | 15 | 40 20.3 20.0 |
| 46 | 9.58 769  | 30 | 9.62 291  | 35    | 0.37 708  | 9.96 477  | 5  | 14 | 50 25.4 25.0 |
| 47 | 9.58 799  | 30 | 9.62 327  | 35    | 0.37 673  | 9.96 472  | 5  | 13 |              |
| 48 | 9.58 829  | 30 | 9.62 362  | 35    | 0.37 637  | 9.96 466  | 5  | 12 |              |
| 49 | 9.58 859  | 30 | 9.62 397  | 35    | 0.37 602  | 9.96 461  | 5  | 11 | 5 5          |
| 50 | 9.58 889  | 30 | 9.62 433  | 35    | 0.37 567  | 9.96 456  | 5  | 10 | 6 0.5 0.5    |
| 51 | 9.58 919  | 30 | 9.62 468  | 35    | 0.37 531  | 9.96 450  | 5  | 9  | 7 0.6 0.6    |
| 52 | 9.58 949  | 30 | 9.62 503  | 35    | 0.37 496  | 9.96 445  | 5  | 8  | 8 0.7 0.6    |
| 53 | 9.58 979  | 30 | 9.62 539  | 35    | 0.37 461  | 9.96 440  | 5  | 7  | 9 0.8 0.7    |
| 54 | 9.59 009  | 29 | 9.62 574  | 35    | 0.37 426  | 9.96 434  | 5  | 6  | 10 0.9 0.8   |
| 55 | 9.59 038  | 30 | 9.62 609  | 35    | 0.37 390  | 9.96 429  | 5  | 5  | 20 1.8 1.6   |
| 56 | 9.59 068  | 30 | 9.62 644  | 35    | 0.37 355  | 9.96 424  | 5  | 4  | 30 2.7 2.5   |
| 57 | 9.59 098  | 29 | 9.62 679  | 35    | 0.37 320  | 9.96 418  | 5  | 3  | 40 3.6 3.3   |
| 58 | 9.59 128  | 30 | 9.62 715  | 35    | 0.37 285  | 9.96 413  | 5  | 2  | 50 4.6 4.1   |
| 59 | 9.59 158  | 30 | 9.62 750  | 35    | 0.37 250  | 9.96 408  | 5  | 1  |              |
| 60 | 9.59 188  | 30 | 9.62 785  | 35    | 0.37 215  | 9.96 402  | 5  | 0  |              |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.        |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

23°

| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P. |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|--|--|
| 0  | 9.59 188  |    | 9.62 785  |       | 0.37 215  | 9.96 402  |    |       |  |  |
| 1  | 9.59 217  | 29 | 9.62 820  | 35    | 0.37 179  | 9.96 397  | 5  |       |  |  |
| 2  | 9.59 247  | 30 | 9.62 853  | 35    | 0.37 144  | 9.96 392  | 58 |       |  |  |
| 3  | 9.59 277  | 29 | 9.62 890  | 35    | 0.37 109  | 9.96 386  | 57 |       |  |  |
| 4  | 9.59 306  | 29 | 9.62 923  | 35    | 0.37 074  | 9.96 381  | 56 |       |  |  |
| 5  | 9.59 336  | 30 | 9.62 960  | 35    | 0.37 039  | 9.96 375  | 55 |       |  |  |
| 6  | 9.59 366  | 29 | 9.62 993  | 35    | 0.37 004  | 9.96 370  | 54 |       |  |  |
| 7  | 9.59 393  | 29 | 9.63 030  | 35    | 0.36 969  | 9.96 365  | 53 |       |  |  |
| 8  | 9.59 425  | 29 | 9.63 063  | 35    | 0.36 934  | 9.96 359  | 52 |       |  |  |
| 9  | 9.59 454  | 29 | 9.63 100  | 35    | 0.36 899  | 9.96 354  | 51 |       |  |  |
| 10 | 9.59 484  | 29 | 9.63 135  | 35    | 0.36 864  | 9.96 349  | 50 |       |  |  |
| 11 | 9.59 513  | 29 | 9.63 170  | 35    | 0.36 829  | 9.96 343  | 49 |       |  |  |
| 12 | 9.59 543  | 29 | 9.63 203  | 35    | 0.36 794  | 9.96 338  | 48 |       |  |  |
| 13 | 9.59 572  | 29 | 9.63 240  | 34    | 0.36 760  | 9.96 332  | 47 |       |  |  |
| 14 | 9.59 602  | 29 | 9.63 275  | 35    | 0.36 725  | 9.96 327  | 46 |       |  |  |
| 15 | 9.59 631  | 29 | 9.63 310  | 35    | 0.36 690  | 9.96 321  | 45 |       |  |  |
| 16 | 9.59 661  | 29 | 9.63 344  | 34    | 0.36 655  | 9.96 316  | 44 |       |  |  |
| 17 | 9.59 690  | 29 | 9.63 379  | 35    | 0.36 620  | 9.96 311  | 43 |       |  |  |
| 18 | 9.59 719  | 29 | 9.63 414  | 35    | 0.36 583  | 9.96 305  | 42 |       |  |  |
| 19 | 9.59 749  | 29 | 9.63 449  | 34    | 0.36 551  | 9.96 300  | 41 |       |  |  |
| 20 | 9.59 778  | 29 | 9.63 484  | 35    | 0.36 516  | 9.96 294  | 40 |       |  |  |
| 21 | 9.59 807  | 29 | 9.63 518  | 34    | 0.36 481  | 9.96 289  | 39 |       |  |  |
| 22 | 9.59 837  | 29 | 9.63 553  | 34    | 0.36 447  | 9.96 283  | 38 |       |  |  |
| 23 | 9.59 866  | 29 | 9.63 588  | 35    | 0.36 412  | 9.96 278  | 37 |       |  |  |
| 24 | 9.59 895  | 29 | 9.63 622  | 34    | 0.36 377  | 9.96 272  | 36 |       |  |  |
| 25 | 9.59 924  | 29 | 9.63 657  | 34    | 0.36 343  | 9.96 267  | 35 |       |  |  |
| 26 | 9.59 953  | 29 | 9.63 692  | 35    | 0.36 308  | 9.96 261  | 34 |       |  |  |
| 27 | 9.59 982  | 29 | 9.63 726  | 34    | 0.36 273  | 9.96 256  | 33 |       |  |  |
| 28 | 9.60 012  | 29 | 9.63 761  | 34    | 0.36 239  | 9.96 251  | 32 |       |  |  |
| 29 | 9.60 041  | 29 | 9.63 795  | 34    | 0.36 204  | 9.96 245  | 31 |       |  |  |
| 30 | 9.60 070  | 29 | 9.63 830  | 34    | 0.36 170  | 9.96 240  | 30 |       |  |  |
| 31 | 9.60 099  | 29 | 9.63 864  | 34    | 0.36 135  | 9.96 234  | 29 |       |  |  |
| 32 | 9.60 128  | 29 | 9.63 899  | 34    | 0.36 101  | 9.96 229  | 28 |       |  |  |
| 33 | 9.60 157  | 29 | 9.63 933  | 34    | 0.36 066  | 9.96 223  | 27 |       |  |  |
| 34 | 9.60 186  | 29 | 9.63 968  | 34    | 0.36 032  | 9.96 218  | 26 |       |  |  |
| 35 | 9.60 215  | 29 | 9.64 002  | 34    | 0.35 997  | 9.96 212  | 25 |       |  |  |
| 36 | 9.60 244  | 29 | 9.64 037  | 34    | 0.35 963  | 9.96 206  | 24 |       |  |  |
| 37 | 9.60 273  | 29 | 9.64 071  | 34    | 0.35 928  | 9.96 201  | 23 |       |  |  |
| 38 | 9.60 301  | 29 | 9.64 106  | 34    | 0.35 894  | 9.96 195  | 22 |       |  |  |
| 39 | 9.60 330  | 29 | 9.64 140  | 34    | 0.35 859  | 9.96 190  | 21 |       |  |  |
| 40 | 9.60 359  | 28 | 9.64 174  | 34    | 0.35 825  | 9.96 184  | 20 |       |  |  |
| 41 | 9.60 388  | 29 | 9.64 209  | 34    | 0.35 791  | 9.96 179  | 19 |       |  |  |
| 42 | 9.60 417  | 28 | 9.64 243  | 34    | 0.35 756  | 9.96 173  | 18 |       |  |  |
| 43 | 9.60 445  | 29 | 9.64 277  | 34    | 0.35 722  | 9.96 168  | 17 |       |  |  |
| 44 | 9.60 474  | 29 | 9.64 312  | 34    | 0.35 688  | 9.96 162  | 16 |       |  |  |
| 45 | 9.60 503  | 28 | 9.64 346  | 34    | 0.35 653  | 9.96 157  | 15 |       |  |  |
| 46 | 9.60 532  | 29 | 9.64 380  | 34    | 0.35 619  | 9.96 151  | 14 |       |  |  |
| 47 | 9.60 560  | 28 | 9.64 415  | 34    | 0.35 585  | 9.96 146  | 13 |       |  |  |
| 48 | 9.60 589  | 28 | 9.64 449  | 34    | 0.35 551  | 9.96 140  | 12 |       |  |  |
| 49 | 9.60 618  | 29 | 9.64 483  | 34    | 0.35 517  | 9.96 134  | 11 |       |  |  |
| 50 | 9.60 646  | 28 | 9.64 517  | 34    | 0.35 482  | 9.96 129  | 10 |       |  |  |
| 51 | 9.60 675  | 28 | 9.64 551  | 34    | 0.35 448  | 9.96 123  | 9  |       |  |  |
| 52 | 9.60 703  | 28 | 9.64 585  | 34    | 0.35 414  | 9.96 118  | 8  |       |  |  |
| 53 | 9.60 732  | 28 | 9.64 620  | 34    | 0.35 380  | 9.96 112  | 7  |       |  |  |
| 54 | 9.60 760  | 28 | 9.64 654  | 34    | 0.35 346  | 9.96 106  | 6  |       |  |  |
| 55 | 9.60 789  | 28 | 9.64 688  | 34    | 0.35 312  | 9.96 101  | 5  |       |  |  |
| 56 | 9.60 817  | 28 | 9.64 722  | 34    | 0.35 278  | 9.96 095  | 4  |       |  |  |
| 57 | 9.60 846  | 28 | 9.64 756  | 34    | 0.35 244  | 9.96 090  | 3  |       |  |  |
| 58 | 9.60 874  | 28 | 9.64 790  | 34    | 0.35 209  | 9.96 084  | 2  |       |  |  |
| 59 | 9.60 903  | 28 | 9.64 824  | 34    | 0.35 175  | 9.96 078  | 1  |       |  |  |
| 60 | 9.60 931  | 28 | 9.64 858  | 34    | 0.35 141  | 9.96 073  | 0  |       |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | P. P. |  |  |

|    |      |      |    |  |  |    |  |  |
|----|------|------|----|--|--|----|--|--|
|    |      |      | 35 |  |  | 35 |  |  |
| 6  | 3.3  | 3.5  |    |  |  |    |  |  |
| 7  | 4.1  | 4.1  |    |  |  |    |  |  |
| 8  | 4.7  | 4.6  |    |  |  |    |  |  |
| 9  | 5.3  | 5.2  |    |  |  |    |  |  |
| 10 | 5.9  | 5.8  |    |  |  |    |  |  |
| 20 | 11.8 | 11.6 |    |  |  |    |  |  |
| 30 | 17.7 | 17.5 |    |  |  |    |  |  |
| 40 | 23.6 | 23.3 |    |  |  |    |  |  |
| 50 | 29.6 | 29.1 |    |  |  |    |  |  |
|    |      |      | 34 |  |  | 34 |  |  |
| 6  | 3.4  | 3.4  |    |  |  |    |  |  |
| 7  | 4.0  | 3.9  |    |  |  |    |  |  |
| 8  | 4.6  | 4.5  |    |  |  |    |  |  |
| 9  | 5.2  | 5.1  |    |  |  |    |  |  |
| 10 | 5.7  | 5.6  |    |  |  |    |  |  |
| 20 | 11.5 | 11.3 |    |  |  |    |  |  |
| 30 | 17.2 | 17.0 |    |  |  |    |  |  |
| 40 | 23.0 | 22.6 |    |  |  |    |  |  |
| 50 | 28.7 | 28.3 |    |  |  |    |  |  |
|    |      |      | 30 |  |  |    |  |  |
| 6  | 3.0  |      |    |  |  |    |  |  |
| 7  | 3.5  |      |    |  |  |    |  |  |
| 8  | 4.0  |      |    |  |  |    |  |  |
| 9  | 4.5  |      |    |  |  |    |  |  |
| 10 | 5.0  |      |    |  |  |    |  |  |
| 20 | 10.0 |      |    |  |  |    |  |  |
| 30 | 15.0 |      |    |  |  |    |  |  |
| 40 | 20.0 |      |    |  |  |    |  |  |
| 50 | 25.0 |      |    |  |  |    |  |  |
|    |      |      | 29 |  |  | 28 |  |  |
| 6  | 2.9  | 2.8  |    |  |  |    |  |  |
| 7  | 3.4  | 3.3  |    |  |  |    |  |  |
| 8  | 3.9  | 3.8  |    |  |  |    |  |  |
| 9  | 4.4  | 4.3  |    |  |  |    |  |  |
| 10 | 4.9  | 4.7  |    |  |  |    |  |  |
| 20 | 9.8  | 9.5  |    |  |  |    |  |  |
| 30 | 14.7 | 14.2 |    |  |  |    |  |  |
| 40 | 19.6 | 19.0 |    |  |  |    |  |  |
| 50 | 24.6 | 23.7 |    |  |  |    |  |  |
|    |      |      | 6  |  |  | 5  |  |  |
| 6  | 0.6  | 0.5  |    |  |  |    |  |  |
| 7  | 0.7  | 0.6  |    |  |  |    |  |  |
| 8  | 0.8  | 0.6  |    |  |  |    |  |  |
| 9  | 0.9  | 0.7  |    |  |  |    |  |  |
| 10 | 1.0  | 0.8  |    |  |  |    |  |  |
| 20 | 2.0  | 1.6  |    |  |  |    |  |  |
| 30 | 3.0  | 2.5  |    |  |  |    |  |  |
| 40 | 4.0  | 3.3  |    |  |  |    |  |  |
| 50 | 5.0  | 4.1  |    |  |  |    |  |  |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

24°

|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.   |      |      |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|---------|------|------|
| 0  | 9.60 931  | 28 | 9.64 858  | 34    | 0.35 141  | 9.96 073  | 3  | 60 |         |      |      |
| 1  | 9.60 950  | 28 | 9.64 892  | 34    | 0.35 107  | 9.96 067  | 3  | 59 |         |      |      |
| 2  | 9.60 988  | 28 | 9.64 926  | 33    | 0.35 073  | 9.96 062  | 3  | 58 |         |      |      |
| 3  | 9.61 016  | 28 | 9.64 960  | 34    | 0.35 040  | 9.96 056  | 3  | 57 |         |      |      |
| 4  | 9.61 044  | 28 | 9.64 994  | 34    | 0.35 006  | 9.96 050  | 3  | 56 |         |      |      |
| 5  | 9.61 073  | 28 | 9.65 028  | 34    | 0.34 972  | 9.96 045  | 3  | 55 | 34      | 33   | 33   |
| 6  | 9.61 101  | 28 | 9.65 062  | 34    | 0.34 938  | 9.96 039  | 3  | 54 | 6 3.4   | 3.3  | 3.3  |
| 7  | 9.61 129  | 28 | 9.65 096  | 34    | 0.34 904  | 9.96 033  | 3  | 53 | 7 3.9   | 3.9  | 3.8  |
| 8  | 9.61 157  | 28 | 9.65 129  | 33    | 0.34 870  | 9.96 028  | 3  | 52 | 8 4.5   | 4.4  | 4.4  |
| 9  | 9.61 186  | 28 | 9.65 163  | 34    | 0.34 836  | 9.96 022  | 3  | 51 | 9 5.1   | 5.0  | 4.9  |
| 10 | 9.61 214  | 28 | 9.65 197  | 33    | 0.34 802  | 9.96 016  | 3  | 50 | 10 5.6  | 5.6  | 5.5  |
| 11 | 9.61 242  | 28 | 9.65 231  | 34    | 0.34 769  | 9.96 011  | 3  | 49 | 20 11.3 | 11.1 | 11.0 |
| 12 | 9.61 270  | 28 | 9.65 265  | 34    | 0.34 735  | 9.96 005  | 3  | 48 | 30 17.0 | 16.7 | 16.5 |
| 13 | 9.61 298  | 28 | 9.65 299  | 33    | 0.34 701  | 9.95 999  | 3  | 47 | 40 22.6 | 22.3 | 22.0 |
| 14 | 9.61 326  | 28 | 9.65 332  | 34    | 0.34 667  | 9.95 994  | 3  | 46 | 50 28.3 | 27.9 | 27.5 |
| 15 | 9.61 354  | 28 | 9.65 366  | 33    | 0.34 633  | 9.95 988  | 3  | 45 |         |      |      |
| 16 | 9.61 382  | 28 | 9.65 400  | 33    | 0.34 600  | 9.95 982  | 3  | 44 |         |      |      |
| 17 | 9.61 410  | 28 | 9.65 433  | 33    | 0.34 566  | 9.95 977  | 3  | 43 |         |      |      |
| 18 | 9.61 438  | 28 | 9.65 467  | 34    | 0.34 532  | 9.95 971  | 3  | 42 |         |      |      |
| 19 | 9.61 466  | 28 | 9.65 501  | 33    | 0.34 499  | 9.95 965  | 3  | 41 | 28      | 28   |      |
| 20 | 9.61 494  | 28 | 9.65 535  | 34    | 0.34 465  | 9.95 959  | 3  | 40 | 6 2.8   | 2.8  |      |
| 21 | 9.61 522  | 28 | 9.65 568  | 33    | 0.34 431  | 9.95 954  | 3  | 39 | 7 3.3   | 3.2  |      |
| 22 | 9.61 550  | 28 | 9.65 602  | 33    | 0.34 398  | 9.95 948  | 3  | 38 | 8 3.8   | 3.7  |      |
| 23 | 9.61 578  | 28 | 9.65 635  | 33    | 0.34 364  | 9.95 942  | 3  | 37 | 9 4.3   | 4.2  |      |
| 24 | 9.61 606  | 28 | 9.65 669  | 34    | 0.34 331  | 9.95 937  | 3  | 36 | 10 4.7  | 4.6  |      |
| 25 | 9.61 634  | 27 | 9.65 703  | 33    | 0.34 297  | 9.95 931  | 3  | 35 | 20 9.5  | 9.3  |      |
| 26 | 9.61 661  | 28 | 9.65 736  | 33    | 0.34 263  | 9.95 925  | 3  | 34 | 30 14.2 | 14.0 |      |
| 27 | 9.61 689  | 27 | 9.65 770  | 33    | 0.34 230  | 9.95 919  | 3  | 33 | 40 19.0 | 18.6 |      |
| 28 | 9.61 717  | 28 | 9.65 803  | 33    | 0.34 196  | 9.95 914  | 3  | 32 | 50 23.7 | 23.3 |      |
| 29 | 9.61 745  | 27 | 9.65 837  | 33    | 0.34 163  | 9.95 908  | 3  | 31 |         |      |      |
| 30 | 9.61 772  | 28 | 9.65 870  | 33    | 0.34 129  | 9.95 902  | 3  | 30 |         |      |      |
| 31 | 9.61 800  | 27 | 9.65 904  | 33    | 0.34 096  | 9.95 896  | 3  | 29 |         |      |      |
| 32 | 9.61 828  | 28 | 9.65 937  | 33    | 0.34 062  | 9.95 891  | 3  | 28 |         |      |      |
| 33 | 9.61 856  | 27 | 9.65 971  | 33    | 0.34 029  | 9.95 885  | 3  | 27 | 27      | 27   |      |
| 34 | 9.61 883  | 27 | 9.66 004  | 33    | 0.33 996  | 9.95 879  | 3  | 26 | 6 2.7   | 2.7  |      |
| 35 | 9.61 911  | 27 | 9.66 037  | 33    | 0.33 962  | 9.95 873  | 3  | 25 | 7 3.2   | 3.1  |      |
| 36 | 9.61 938  | 27 | 9.66 071  | 33    | 0.33 929  | 9.95 867  | 3  | 24 | 8 3.6   | 3.6  |      |
| 37 | 9.61 966  | 28 | 9.66 104  | 33    | 0.33 895  | 9.95 862  | 3  | 23 | 9 4.1   | 4.0  |      |
| 38 | 9.61 994  | 27 | 9.66 137  | 33    | 0.33 862  | 9.95 856  | 3  | 22 | 10 4.6  | 4.5  |      |
| 39 | 9.62 021  | 27 | 9.66 171  | 33    | 0.33 829  | 9.95 850  | 3  | 21 | 20 9.1  | 9.0  |      |
| 40 | 9.62 049  | 27 | 9.66 204  | 33    | 0.33 795  | 9.95 844  | 3  | 20 | 30 13.7 | 13.5 |      |
| 41 | 9.62 076  | 27 | 9.66 237  | 33    | 0.33 762  | 9.95 838  | 3  | 19 | 40 18.3 | 18.0 |      |
| 42 | 9.62 104  | 27 | 9.66 271  | 33    | 0.33 729  | 9.95 833  | 3  | 18 | 50 22.9 | 22.5 |      |
| 43 | 9.62 131  | 27 | 9.66 304  | 33    | 0.33 696  | 9.95 827  | 3  | 17 |         |      |      |
| 44 | 9.62 158  | 27 | 9.66 337  | 33    | 0.33 662  | 9.95 821  | 3  | 16 |         |      |      |
| 45 | 9.62 186  | 27 | 9.66 370  | 33    | 0.33 629  | 9.95 815  | 3  | 15 |         |      |      |
| 46 | 9.62 213  | 27 | 9.66 404  | 33    | 0.33 596  | 9.95 809  | 3  | 14 |         |      |      |
| 47 | 9.62 241  | 27 | 9.66 437  | 33    | 0.33 563  | 9.95 804  | 3  | 13 | 6       | 5    |      |
| 48 | 9.62 268  | 27 | 9.66 470  | 33    | 0.33 529  | 9.95 798  | 3  | 12 | 6 0.6   | 0.5  |      |
| 49 | 9.62 295  | 27 | 9.66 503  | 33    | 0.33 496  | 9.95 792  | 3  | 11 | 7 0.7   | 0.6  |      |
| 50 | 9.62 323  | 27 | 9.66 536  | 33    | 0.33 463  | 9.95 786  | 3  | 10 | 8 0.8   | 0.7  |      |
| 51 | 9.62 350  | 27 | 9.66 570  | 33    | 0.33 430  | 9.95 780  | 3  | 9  | 9 0.9   | 0.8  |      |
| 52 | 9.62 377  | 27 | 9.66 603  | 33    | 0.33 397  | 9.95 774  | 3  | 8  | 10 1.0  | 0.9  |      |
| 53 | 9.62 404  | 27 | 9.66 636  | 33    | 0.33 364  | 9.95 768  | 3  | 7  | 20 2.0  | 1.8  |      |
| 54 | 9.62 432  | 27 | 9.66 669  | 33    | 0.33 331  | 9.95 763  | 3  | 6  | 30 3.0  | 2.7  |      |
| 55 | 9.62 459  | 27 | 9.66 702  | 33    | 0.33 298  | 9.95 757  | 3  | 5  | 40 4.0  | 3.6  |      |
| 56 | 9.62 486  | 27 | 9.66 735  | 33    | 0.33 265  | 9.95 751  | 3  | 4  | 50 5.0  | 4.6  |      |
| 57 | 9.62 513  | 27 | 9.66 768  | 33    | 0.33 232  | 9.95 745  | 3  | 3  |         |      |      |
| 58 | 9.62 540  | 27 | 9.66 801  | 33    | 0.33 198  | 9.95 739  | 3  | 2  |         |      |      |
| 59 | 9.62 567  | 27 | 9.66 834  | 33    | 0.33 165  | 9.95 733  | 3  | 1  |         |      |      |
| 60 | 9.62 595  | 27 | 9.66 867  | 33    | 0.33 132  | 9.95 727  | 3  | 0  |         |      |      |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.   |      |      |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|--|--|
| 0  | 9.62 595  |    | 9.66 867  |       | 0.33 132  | 9.95 727  |    | 60 |       |  |  |
| 1  | 9.62 622  | 27 | 9.66 900  | 32    | 0.33 100  | 9.95 721  | 6  | 59 |       |  |  |
| 2  | 9.62 649  | 27 | 9.66 933  | 33    | 0.33 067  | 9.95 716  | 5  | 58 |       |  |  |
| 3  | 9.62 676  | 27 | 9.66 966  | 33    | 0.33 034  | 9.95 710  | 6  | 57 |       |  |  |
| 4  | 9.62 703  | 27 | 9.66 999  | 33    | 0.33 001  | 9.95 704  | 6  | 56 |       |  |  |
| 5  | 9.62 730  | 27 | 9.67 032  | 33    | 0.32 968  | 9.95 698  | 6  | 55 |       |  |  |
| 6  | 9.62 757  | 27 | 9.67 065  | 33    | 0.32 935  | 9.95 692  | 6  | 54 |       |  |  |
| 7  | 9.62 784  | 27 | 9.67 097  | 32    | 0.32 902  | 9.95 686  | 6  | 53 |       |  |  |
| 8  | 9.62 811  | 27 | 9.67 130  | 33    | 0.32 869  | 9.95 680  | 5  | 52 |       |  |  |
| 9  | 9.62 838  | 27 | 9.67 163  | 33    | 0.32 836  | 9.95 674  | 6  | 51 |       |  |  |
| 10 | 9.62 864  | 26 | 9.67 196  | 33    | 0.32 803  | 9.95 668  | 6  | 50 |       |  |  |
| 11 | 9.62 891  | 27 | 9.67 229  | 32    | 0.32 771  | 9.95 662  | 6  | 49 |       |  |  |
| 12 | 9.62 918  | 27 | 9.67 262  | 33    | 0.32 738  | 9.95 656  | 6  | 48 |       |  |  |
| 13 | 9.62 945  | 27 | 9.67 294  | 32    | 0.32 705  | 9.95 650  | 6  | 47 |       |  |  |
| 14 | 9.62 972  | 26 | 9.67 327  | 33    | 0.32 672  | 9.95 644  | 6  | 46 |       |  |  |
| 15 | 9.62 999  | 27 | 9.67 360  | 32    | 0.32 640  | 9.95 638  | 6  | 45 |       |  |  |
| 16 | 9.63 025  | 26 | 9.67 393  | 33    | 0.32 607  | 9.95 632  | 6  | 44 |       |  |  |
| 17 | 9.63 052  | 27 | 9.67 425  | 32    | 0.32 574  | 9.95 627  | 5  | 43 |       |  |  |
| 18 | 9.63 079  | 26 | 9.67 458  | 33    | 0.32 541  | 9.95 621  | 6  | 42 |       |  |  |
| 19 | 9.63 106  | 27 | 9.67 491  | 32    | 0.32 509  | 9.95 615  | 6  | 41 |       |  |  |
| 20 | 9.63 132  | 26 | 9.67 523  | 32    | 0.32 476  | 9.95 609  | 6  | 40 |       |  |  |
| 21 | 9.63 159  | 27 | 9.67 556  | 33    | 0.32 443  | 9.95 603  | 6  | 39 |       |  |  |
| 22 | 9.63 186  | 26 | 9.67 589  | 32    | 0.32 411  | 9.95 597  | 6  | 38 |       |  |  |
| 23 | 9.63 212  | 26 | 9.67 621  | 32    | 0.32 378  | 9.95 591  | 6  | 37 |       |  |  |
| 24 | 9.63 239  | 26 | 9.67 654  | 33    | 0.32 345  | 9.95 585  | 6  | 36 |       |  |  |
| 25 | 9.63 266  | 27 | 9.67 687  | 32    | 0.32 313  | 9.95 579  | 6  | 35 |       |  |  |
| 26 | 9.63 292  | 26 | 9.67 719  | 32    | 0.32 280  | 9.95 573  | 6  | 34 |       |  |  |
| 27 | 9.63 319  | 26 | 9.67 752  | 32    | 0.32 248  | 9.95 567  | 6  | 33 |       |  |  |
| 28 | 9.63 345  | 26 | 9.67 784  | 32    | 0.32 215  | 9.95 561  | 6  | 32 |       |  |  |
| 29 | 9.63 372  | 26 | 9.67 817  | 32    | 0.32 183  | 9.95 555  | 6  | 31 |       |  |  |
| 30 | 9.63 398  | 26 | 9.67 849  | 32    | 0.32 150  | 9.95 549  | 6  | 30 |       |  |  |
| 31 | 9.63 425  | 26 | 9.67 882  | 32    | 0.32 118  | 9.95 543  | 6  | 29 |       |  |  |
| 32 | 9.63 451  | 26 | 9.67 914  | 32    | 0.32 085  | 9.95 537  | 6  | 28 |       |  |  |
| 33 | 9.63 478  | 26 | 9.67 947  | 32    | 0.32 053  | 9.95 530  | 6  | 27 |       |  |  |
| 34 | 9.63 504  | 26 | 9.67 979  | 32    | 0.32 020  | 9.95 524  | 6  | 26 |       |  |  |
| 35 | 9.63 530  | 26 | 9.68 012  | 32    | 0.31 988  | 9.95 518  | 6  | 25 |       |  |  |
| 36 | 9.63 557  | 26 | 9.68 044  | 32    | 0.31 955  | 9.95 512  | 6  | 24 |       |  |  |
| 37 | 9.63 583  | 26 | 9.68 077  | 32    | 0.31 923  | 9.95 506  | 6  | 23 |       |  |  |
| 38 | 9.63 609  | 26 | 9.68 109  | 32    | 0.31 891  | 9.95 500  | 6  | 22 |       |  |  |
| 39 | 9.63 636  | 26 | 9.68 141  | 32    | 0.31 858  | 9.95 494  | 6  | 21 |       |  |  |
| 40 | 9.63 662  | 26 | 9.68 174  | 32    | 0.31 826  | 9.95 488  | 6  | 20 |       |  |  |
| 41 | 9.63 688  | 26 | 9.68 206  | 32    | 0.31 793  | 9.95 482  | 6  | 19 |       |  |  |
| 42 | 9.63 715  | 26 | 9.68 238  | 32    | 0.31 761  | 9.95 476  | 6  | 18 |       |  |  |
| 43 | 9.63 741  | 26 | 9.68 271  | 32    | 0.31 729  | 9.95 470  | 6  | 17 |       |  |  |
| 44 | 9.63 767  | 26 | 9.68 303  | 32    | 0.31 696  | 9.95 464  | 6  | 16 |       |  |  |
| 45 | 9.63 793  | 26 | 9.68 335  | 32    | 0.31 664  | 9.95 458  | 6  | 15 |       |  |  |
| 46 | 9.63 819  | 26 | 9.68 368  | 32    | 0.31 632  | 9.95 452  | 6  | 14 |       |  |  |
| 47 | 9.63 846  | 26 | 9.68 400  | 32    | 0.31 600  | 9.95 445  | 6  | 13 |       |  |  |
| 48 | 9.63 872  | 26 | 9.68 432  | 32    | 0.31 567  | 9.95 439  | 6  | 12 |       |  |  |
| 49 | 9.63 898  | 26 | 9.68 464  | 32    | 0.31 535  | 9.95 433  | 6  | 11 |       |  |  |
| 50 | 9.63 924  | 26 | 9.68 497  | 32    | 0.31 503  | 9.95 427  | 6  | 10 |       |  |  |
| 51 | 9.63 950  | 26 | 9.68 529  | 32    | 0.31 471  | 9.95 421  | 6  | 9  |       |  |  |
| 52 | 9.63 976  | 26 | 9.68 561  | 32    | 0.31 439  | 9.95 415  | 6  | 8  |       |  |  |
| 53 | 9.64 002  | 26 | 9.68 593  | 32    | 0.31 406  | 9.95 409  | 6  | 7  |       |  |  |
| 54 | 9.64 028  | 26 | 9.68 625  | 32    | 0.31 374  | 9.95 403  | 6  | 6  |       |  |  |
| 55 | 9.64 054  | 26 | 9.68 657  | 32    | 0.31 342  | 9.95 397  | 6  | 5  |       |  |  |
| 56 | 9.64 080  | 26 | 9.68 690  | 32    | 0.31 310  | 9.95 390  | 6  | 4  |       |  |  |
| 57 | 9.64 106  | 26 | 9.68 722  | 32    | 0.31 278  | 9.95 384  | 6  | 3  |       |  |  |
| 58 | 9.64 132  | 26 | 9.68 754  | 32    | 0.31 246  | 9.95 378  | 6  | 2  |       |  |  |
| 59 | 9.64 158  | 26 | 9.68 786  | 32    | 0.31 214  | 9.95 372  | 6  | 1  |       |  |  |
| 60 | 9.64 184  | 25 | 9.68 818  | 32    | 0.31 182  | 9.95 366  | 6  | 0  |       |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |  |  |

|    | 33   | 32   | 32   |
|----|------|------|------|
| 6  | 3.3  | 3.2  | 3.2  |
| 7  | 3.8  | 3.8  | 3.7  |
| 8  | 4.4  | 4.3  | 4.2  |
| 9  | 4.9  | 4.9  | 4.8  |
| 10 | 5.5  | 5.4  | 5.3  |
| 20 | 11.0 | 10.8 | 10.6 |
| 30 | 16.5 | 16.2 | 16.0 |
| 40 | 22.0 | 21.6 | 21.3 |
| 50 | 27.5 | 27.1 | 26.6 |

|    | 27   |
|----|------|
| 6  | 2.7  |
| 7  | 3.1  |
| 8  | 3.6  |
| 9  | 4.0  |
| 10 | 4.5  |
| 20 | 9.0  |
| 30 | 13.5 |
| 40 | 18.0 |
| 50 | 22.5 |

|    | 26   | 26   | 25   |
|----|------|------|------|
| 6  | 2.6  | 2.6  | 2.5  |
| 7  | 3.1  | 3.0  | 3.0  |
| 8  | 3.5  | 3.4  | 3.4  |
| 9  | 4.0  | 3.9  | 3.8  |
| 10 | 4.4  | 4.3  | 4.2  |
| 20 | 8.8  | 8.6  | 8.5  |
| 30 | 13.2 | 13.0 | 12.7 |
| 40 | 17.6 | 17.3 | 17.0 |
| 50 | 22.1 | 21.6 | 21.2 |

|    | 6   | 6   | 5   |
|----|-----|-----|-----|
| 6  | 0.6 | 0.6 | 0.5 |
| 7  | 0.7 | 0.7 | 0.6 |
| 8  | 0.8 | 0.8 | 0.7 |
| 9  | 1.0 | 0.9 | 0.8 |
| 10 | 1.1 | 1.0 | 0.9 |
| 20 | 2.1 | 2.0 | 1.8 |
| 30 | 3.2 | 3.0 | 2.7 |
| 40 | 4.3 | 4.0 | 3.6 |
| 50 | 5.4 | 5.0 | 4.6 |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS

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| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |           | P. P.                         |
|----|-----------|----|-----------|-------|-----------|-----------|----|-----------|-------------------------------|
| 0  | 9.64 184  |    | 9.68 818  |       | 0.31 182  | 9.95 366  |    | <b>60</b> |                               |
| 1  | 9.64 210  | 26 | 9.68 850  | 32    | 0.31 150  | 9.95 360  | 6  | 59        |                               |
| 2  | 9.64 236  | 26 | 9.68 882  | 32    | 0.31 117  | 9.95 353  | 6  | 58        |                               |
| 3  | 9.64 262  | 26 | 9.68 914  | 32    | 0.31 085  | 9.95 347  | 6  | 57        |                               |
| 4  | 9.64 287  | 23 | 9.68 946  | 32    | 0.31 053  | 9.95 341  | 6  | 56        |                               |
| 5  | 9.64 313  | 26 | 9.68 978  | 32    | 0.31 021  | 9.95 335  | 6  | 55        | <b>32</b> <b>32</b>           |
| 6  | 9.64 339  | 26 | 9.69 010  | 32    | 0.30 989  | 9.95 329  | 6  | 54        | 6 3.2 3.2                     |
| 7  | 9.64 365  | 23 | 9.69 042  | 32    | 0.30 957  | 9.95 323  | 6  | 53        | 7 3.8 3.7                     |
| 8  | 9.64 391  | 26 | 9.69 074  | 31    | 0.30 926  | 9.95 316  | 6  | 52        | 8 4.3 4.2                     |
| 9  | 9.64 416  | 23 | 9.69 106  | 32    | 0.30 894  | 9.95 310  | 6  | 51        | 9 4.9 4.8                     |
| 10 | 9.64 442  | 26 | 9.69 138  | 32    | 0.30 862  | 9.95 304  | 6  | <b>50</b> | 10 5.4 5.3                    |
| 11 | 9.64 468  | 23 | 9.69 170  | 32    | 0.30 830  | 9.95 298  | 6  | 49        | 20 10.8 10.6                  |
| 12 | 9.64 493  | 23 | 9.69 202  | 32    | 0.30 798  | 9.95 292  | 6  | 48        | 30 16.2 16.0                  |
| 13 | 9.64 519  | 26 | 9.69 234  | 32    | 0.30 766  | 9.95 285  | 6  | 47        | 40 21.6 21.3                  |
| 14 | 9.64 545  | 23 | 9.69 265  | 31    | 0.30 734  | 9.95 279  | 6  | 46        | 50 27.1 26.6                  |
| 15 | 9.64 570  | 23 | 9.69 297  | 32    | 0.30 702  | 9.95 273  | 6  | 45        |                               |
| 16 | 9.64 596  | 23 | 9.69 329  | 32    | 0.30 670  | 9.95 267  | 6  | 44        |                               |
| 17 | 9.64 622  | 26 | 9.69 361  | 31    | 0.30 639  | 9.95 260  | 6  | 43        |                               |
| 18 | 9.64 647  | 23 | 9.69 393  | 32    | 0.30 607  | 9.95 254  | 6  | 42        |                               |
| 19 | 9.64 673  | 23 | 9.69 425  | 32    | 0.30 575  | 9.95 248  | 6  | 41        | <b>31</b> <b>31</b>           |
| 20 | 9.64 698  | 23 | 9.69 456  | 31    | 0.30 543  | 9.95 242  | 6  | <b>40</b> | 6 3.1 3.1                     |
| 21 | 9.64 724  | 23 | 9.69 488  | 32    | 0.30 511  | 9.95 235  | 6  | 39        | 7 3.7 3.6                     |
| 22 | 9.64 749  | 23 | 9.69 520  | 31    | 0.30 480  | 9.95 229  | 6  | 38        | 8 4.2 4.1                     |
| 23 | 9.64 775  | 23 | 9.69 552  | 32    | 0.30 448  | 9.95 223  | 6  | 37        | 9 4.7 4.6                     |
| 24 | 9.64 800  | 23 | 9.69 583  | 31    | 0.30 416  | 9.95 217  | 6  | 36        | 10 5.2 5.1                    |
| 25 | 9.64 826  | 23 | 9.69 615  | 32    | 0.30 384  | 9.95 210  | 6  | 35        | 20 10.5 10.3                  |
| 26 | 9.64 851  | 23 | 9.69 647  | 31    | 0.30 353  | 9.95 204  | 6  | 34        | 30 15.7 15.5                  |
| 27 | 9.64 876  | 23 | 9.69 678  | 31    | 0.30 321  | 9.95 198  | 6  | 33        | 40 21.0 20.6                  |
| 28 | 9.64 902  | 23 | 9.69 710  | 32    | 0.30 289  | 9.95 191  | 6  | 32        | 50 26.2 25.8                  |
| 29 | 9.64 927  | 23 | 9.69 742  | 31    | 0.30 258  | 9.95 185  | 6  | 31        |                               |
| 30 | 9.64 952  | 25 | 9.69 773  | 31    | 0.30 226  | 9.95 179  | 6  | <b>30</b> |                               |
| 31 | 9.64 978  | 23 | 9.69 805  | 32    | 0.30 194  | 9.95 173  | 6  | 29        |                               |
| 32 | 9.65 003  | 23 | 9.69 837  | 31    | 0.30 163  | 9.95 166  | 6  | 28        |                               |
| 33 | 9.65 028  | 25 | 9.69 868  | 31    | 0.30 131  | 9.95 160  | 6  | 27        | <b>26</b> <b>25</b> <b>25</b> |
| 34 | 9.65 054  | 23 | 9.69 900  | 31    | 0.30 100  | 9.95 154  | 6  | 26        | 6 2.6 2.5                     |
| 35 | 9.65 079  | 25 | 9.69 931  | 31    | 0.30 068  | 9.95 147  | 6  | 25        | 7 3.0 2.9                     |
| 36 | 9.65 104  | 23 | 9.69 963  | 31    | 0.30 037  | 9.95 141  | 6  | 24        | 8 3.4 3.3                     |
| 37 | 9.65 129  | 25 | 9.69 994  | 31    | 0.30 005  | 9.95 135  | 6  | 23        | 9 3.9 3.7                     |
| 38 | 9.65 155  | 23 | 9.70 026  | 32    | 0.29 973  | 9.95 128  | 6  | 22        | 10 4.3 4.1                    |
| 39 | 9.65 180  | 25 | 9.70 058  | 31    | 0.29 942  | 9.95 122  | 6  | 21        | 20 8.6 8.3                    |
| 40 | 9.65 205  | 25 | 9.70 089  | 31    | 0.29 910  | 9.95 116  | 6  | <b>20</b> | 30 13.0 12.7                  |
| 41 | 9.65 230  | 23 | 9.70 121  | 31    | 0.29 879  | 9.95 109  | 6  | 19        | 40 17.3 17.0                  |
| 42 | 9.65 255  | 25 | 9.70 152  | 31    | 0.29 847  | 9.95 103  | 6  | 18        | 50 21.6 21.2                  |
| 43 | 9.65 280  | 25 | 9.70 183  | 31    | 0.29 816  | 9.95 097  | 6  | 17        |                               |
| 44 | 9.65 305  | 25 | 9.70 215  | 31    | 0.29 785  | 9.95 090  | 6  | 16        |                               |
| 45 | 9.65 331  | 23 | 9.70 246  | 31    | 0.29 753  | 9.95 084  | 6  | 15        |                               |
| 46 | 9.65 356  | 25 | 9.70 278  | 31    | 0.29 722  | 9.95 078  | 6  | 14        |                               |
| 47 | 9.65 381  | 25 | 9.70 309  | 31    | 0.29 690  | 9.95 071  | 6  | 13        | <b>24</b> <b>6</b> <b>6</b>   |
| 48 | 9.65 406  | 25 | 9.70 341  | 31    | 0.29 659  | 9.95 065  | 6  | 12        | 6 2.4 0.6                     |
| 49 | 9.65 431  | 25 | 9.70 372  | 31    | 0.29 628  | 9.95 058  | 6  | 11        | 7 2.8 0.7                     |
| 50 | 9.65 456  | 25 | 9.70 403  | 31    | 0.29 596  | 9.95 052  | 6  | <b>10</b> | 8 3.2 0.8                     |
| 51 | 9.65 481  | 25 | 9.70 435  | 31    | 0.29 565  | 9.95 046  | 6  | 9         | 9 3.7 1.0                     |
| 52 | 9.65 506  | 25 | 9.70 466  | 31    | 0.29 533  | 9.95 039  | 6  | 8         | 10 4.1 1.1                    |
| 53 | 9.65 530  | 24 | 9.70 497  | 31    | 0.29 502  | 9.95 033  | 6  | 7         | 20 8.1 2.1                    |
| 54 | 9.65 555  | 25 | 9.70 529  | 31    | 0.29 471  | 9.95 026  | 6  | 6         | 30 12.2 3.2                   |
| 55 | 9.65 580  | 25 | 9.70 560  | 31    | 0.29 439  | 9.95 020  | 6  | 5         | 40 16.3 4.3                   |
| 56 | 9.65 605  | 25 | 9.70 591  | 31    | 0.29 408  | 9.95 014  | 6  | 4         | 50 20.4 5.4                   |
| 57 | 9.65 630  | 24 | 9.70 623  | 31    | 0.29 377  | 9.95 007  | 6  | 3         |                               |
| 58 | 9.65 655  | 25 | 9.70 654  | 31    | 0.29 346  | 9.95 001  | 6  | 2         |                               |
| 59 | 9.65 680  | 25 | 9.70 685  | 31    | 0.29 314  | 9.94 994  | 6  | 1         |                               |
| 60 | 9.65 704  | 24 | 9.70 716  | 31    | 0.29 283  | 9.94 988  | 6  | <b>0</b>  |                               |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | '         | P. P.                         |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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| °  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | u. | P. P. |    |      |      |       |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|----|------|------|-------|
| 0  | 9.65704   | 25 | 9.70716   | 31    | 0.29283   | 9.94988   | 6  | 60    |    |      |      |       |
| 1  | 9.65729   | 24 | 9.70748   | 31    | 0.29252   | 9.94981   | 6  | 59    |    |      |      |       |
| 2  | 9.65754   | 25 | 9.70779   | 31    | 0.29221   | 9.94975   | 6  | 58    |    |      |      |       |
| 3  | 9.65779   | 24 | 9.70810   | 31    | 0.29190   | 9.94969   | 6  | 57    |    |      |      |       |
| 4  | 9.65803   | 25 | 9.70841   | 31    | 0.29158   | 9.94962   | 6  | 56    |    |      |      |       |
| 5  | 9.65828   | 24 | 9.70872   | 31    | 0.29127   | 9.94956   | 6  | 55    | 31 | 31   | 30   |       |
| 6  | 9.65853   | 25 | 9.70903   | 31    | 0.29096   | 9.94949   | 6  | 54    | 6  | 3.1  | 3.1  | 3.0   |
| 7  | 9.65878   | 24 | 9.70935   | 31    | 0.29065   | 9.94943   | 6  | 53    | 7  | 3.7  | 3.6  | 3.5   |
| 8  | 9.65902   | 25 | 9.70966   | 31    | 0.29034   | 9.94936   | 6  | 52    | 8  | 4.2  | 4.1  | 4.0   |
| 9  | 9.65927   | 24 | 9.70997   | 31    | 0.29003   | 9.94930   | 6  | 51    | 9  | 4.7  | 4.6  | 4.6   |
| 10 | 9.65951   | 25 | 9.71028   | 31    | 0.28972   | 9.94923   | 6  | 50    | 10 | 5.2  | 5.1  | 5.1   |
| 11 | 9.65976   | 24 | 9.71059   | 31    | 0.28940   | 9.94917   | 6  | 49    | 20 | 10.5 | 10.3 | 10.1  |
| 12 | 9.66001   | 24 | 9.71090   | 31    | 0.28909   | 9.94910   | 6  | 48    | 30 | 15.7 | 15.5 | 15.2  |
| 13 | 9.66023   | 24 | 9.71121   | 31    | 0.28878   | 9.94904   | 6  | 47    | 40 | 21.0 | 20.6 | 20.3  |
| 14 | 9.66050   | 24 | 9.71152   | 31    | 0.28847   | 9.94897   | 6  | 46    | 50 | 26.2 | 25.8 | 25.4  |
| 15 | 9.66074   | 24 | 9.71183   | 31    | 0.28816   | 9.94891   | 6  | 45    |    |      |      |       |
| 16 | 9.66099   | 24 | 9.71214   | 31    | 0.28785   | 9.94884   | 6  | 44    |    |      |      |       |
| 17 | 9.66123   | 24 | 9.71245   | 31    | 0.28754   | 9.94878   | 6  | 43    |    |      |      |       |
| 18 | 9.66148   | 24 | 9.71276   | 31    | 0.28723   | 9.94871   | 6  | 42    |    |      |      |       |
| 19 | 9.66172   | 24 | 9.71307   | 31    | 0.28692   | 9.94865   | 6  | 41    |    |      |      |       |
| 20 | 9.66197   | 24 | 9.71338   | 31    | 0.28661   | 9.94858   | 6  | 40    |    | 25   |      |       |
| 21 | 9.66221   | 24 | 9.71369   | 31    | 0.28630   | 9.94852   | 6  | 39    | 6  | 2.5  |      |       |
| 22 | 9.66246   | 24 | 9.71400   | 31    | 0.28599   | 9.94845   | 6  | 38    | 7  | 2.9  |      |       |
| 23 | 9.66270   | 24 | 9.71431   | 31    | 0.28568   | 9.94839   | 6  | 37    | 8  | 3.3  |      |       |
| 24 | 9.66294   | 24 | 9.71462   | 31    | 0.28537   | 9.94832   | 6  | 36    | 9  | 3.7  |      |       |
| 25 | 9.66319   | 24 | 9.71493   | 31    | 0.28506   | 9.94825   | 7  | 35    | 10 | 4.1  |      |       |
| 26 | 9.66343   | 24 | 9.71524   | 30    | 0.28476   | 9.94819   | 6  | 34    | 20 | 8.3  |      |       |
| 27 | 9.66367   | 24 | 9.71555   | 31    | 0.28445   | 9.94812   | 6  | 33    | 30 | 12.5 |      |       |
| 28 | 9.66392   | 24 | 9.71586   | 31    | 0.28414   | 9.94806   | 6  | 32    | 40 | 16.6 |      |       |
| 29 | 9.66416   | 24 | 9.71617   | 31    | 0.28383   | 9.94799   | 6  | 31    | 50 | 20.8 |      |       |
| 30 | 9.66440   | 24 | 9.71647   | 30    | 0.28352   | 9.94793   | 6  | 30    |    |      |      |       |
| 31 | 9.66465   | 24 | 9.71678   | 31    | 0.28321   | 9.94786   | 6  | 29    |    |      |      |       |
| 32 | 9.66489   | 24 | 9.71709   | 31    | 0.28290   | 9.94779   | 7  | 28    |    |      |      |       |
| 33 | 9.66513   | 24 | 9.71740   | 30    | 0.28260   | 9.94773   | 6  | 27    | 24 | 2.4  | 2.4  | 2.3   |
| 34 | 9.66537   | 24 | 9.71771   | 31    | 0.28229   | 9.94766   | 6  | 26    | 6  | 2.4  | 2.4  | 2.3   |
| 35 | 9.66561   | 24 | 9.71801   | 30    | 0.28198   | 9.94760   | 6  | 25    | 7  | 2.8  | 2.8  | 2.7   |
| 36 | 9.66586   | 24 | 9.71832   | 31    | 0.28167   | 9.94753   | 6  | 24    | 8  | 3.2  | 3.2  | 3.1   |
| 37 | 9.66610   | 24 | 9.71863   | 31    | 0.28136   | 9.94746   | 7  | 23    | 9  | 3.7  | 3.6  | 3.5   |
| 38 | 9.66634   | 24 | 9.71894   | 30    | 0.28106   | 9.94740   | 6  | 22    | 10 | 4.1  | 4.0  | 3.9   |
| 39 | 9.66658   | 24 | 9.71925   | 31    | 0.28075   | 9.94733   | 6  | 21    | 20 | 8.1  | 8.0  | 7.8   |
| 40 | 9.66682   | 24 | 9.71955   | 30    | 0.28044   | 9.94727   | 6  | 20    | 30 | 12.2 | 12.0 | 11.7  |
| 41 | 9.66706   | 24 | 9.71986   | 30    | 0.28014   | 9.94720   | 6  | 19    | 40 | 16.3 | 16.0 | 15.6  |
| 42 | 9.66730   | 24 | 9.72017   | 31    | 0.27983   | 9.94713   | 7  | 18    | 50 | 20.4 | 20.0 | 19.6  |
| 43 | 9.66754   | 24 | 9.72047   | 30    | 0.27952   | 9.94707   | 6  | 17    |    |      |      |       |
| 44 | 9.66778   | 24 | 9.72078   | 31    | 0.27921   | 9.94700   | 6  | 16    |    |      |      |       |
| 45 | 9.66802   | 24 | 9.72109   | 30    | 0.27891   | 9.94693   | 7  | 15    |    |      |      |       |
| 46 | 9.66826   | 24 | 9.72139   | 30    | 0.27860   | 9.94687   | 6  | 14    |    |      |      |       |
| 47 | 9.66850   | 24 | 9.72170   | 30    | 0.27830   | 9.94680   | 6  | 13    |    | 7    | 6    |       |
| 48 | 9.66874   | 24 | 9.72201   | 31    | 0.27799   | 9.94674   | 6  | 12    | 6  | 0.7  | 0.6  |       |
| 49 | 9.66898   | 24 | 9.72231   | 30    | 0.27768   | 9.94667   | 7  | 11    | 7  | 0.8  | 0.7  |       |
| 50 | 9.66922   | 24 | 9.72262   | 30    | 0.27738   | 9.94660   | 6  | 10    | 8  | 0.9  | 0.8  |       |
| 51 | 9.66946   | 24 | 9.72292   | 30    | 0.27707   | 9.94654   | 6  | 9     | 9  | 1.0  | 1.0  |       |
| 52 | 9.66970   | 24 | 9.72323   | 30    | 0.27677   | 9.94647   | 7  | 8     | 10 | 1.1  | 1.1  |       |
| 53 | 9.66994   | 23 | 9.72354   | 31    | 0.27646   | 9.94640   | 6  | 7     | 20 | 2.3  | 2.1  |       |
| 54 | 9.67018   | 24 | 9.72384   | 30    | 0.27615   | 9.94633   | 7  | 6     | 30 | 3.5  | 3.2  |       |
| 55 | 9.67042   | 24 | 9.72415   | 30    | 0.27585   | 9.94627   | 6  | 5     | 40 | 4.6  | 4.3  |       |
| 56 | 9.67066   | 24 | 9.72445   | 30    | 0.27554   | 9.94620   | 6  | 4     | 50 | 5.8  | 5.4  |       |
| 57 | 9.67089   | 23 | 9.72476   | 30    | 0.27524   | 9.94613   | 7  | 3     |    |      |      |       |
| 58 | 9.67113   | 24 | 9.72506   | 30    | 0.27493   | 9.94607   | 6  | 2     |    |      |      |       |
| 59 | 9.67137   | 23 | 9.72537   | 30    | 0.27463   | 9.94600   | 7  | 1     |    |      |      |       |
| 60 | 9.67161   | 24 | 9.72567   | 30    | 0.27432   | 9.94593   | 6  | 0     |    |      |      |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |       |    |      |      | P. P. |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |  | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|--|-------|
| 0  | 9.67 161  | 23 | 9.72 567  | 30    | 0.27 432  | 9.94 593  | 60 |  |       |
| 1  | 9.67 184  | 24 | 9.72 598  | 30    | 0.27 402  | 9.94 587  | 59 |  |       |
| 2  | 9.67 208  | 23 | 9.72 628  | 30    | 0.27 371  | 9.94 580  | 58 |  |       |
| 3  | 9.67 232  | 24 | 9.72 659  | 30    | 0.27 341  | 9.94 573  | 57 |  |       |
| 4  | 9.67 256  | 23 | 9.72 689  | 30    | 0.27 311  | 9.94 566  | 56 |  |       |
| 5  | 9.67 279  | 23 | 9.72 719  | 30    | 0.27 280  | 9.94 560  | 55 |  |       |
| 6  | 9.67 303  | 24 | 9.72 750  | 30    | 0.27 250  | 9.94 553  | 54 |  |       |
| 7  | 9.67 327  | 23 | 9.72 780  | 30    | 0.27 219  | 9.94 546  | 53 |  |       |
| 8  | 9.67 350  | 23 | 9.72 811  | 30    | 0.27 189  | 9.94 539  | 52 |  |       |
| 9  | 9.67 374  | 23 | 9.72 841  | 30    | 0.27 159  | 9.94 533  | 51 |  |       |
| 10 | 9.67 397  | 24 | 9.72 871  | 30    | 0.27 128  | 9.94 526  | 50 |  |       |
| 11 | 9.67 421  | 23 | 9.72 902  | 30    | 0.27 098  | 9.94 519  | 49 |  |       |
| 12 | 9.67 445  | 23 | 9.72 932  | 30    | 0.27 067  | 9.94 512  | 48 |  |       |
| 13 | 9.67 468  | 23 | 9.72 962  | 30    | 0.27 037  | 9.94 506  | 47 |  |       |
| 14 | 9.67 492  | 23 | 9.72 993  | 30    | 0.27 007  | 9.94 499  | 46 |  |       |
| 15 | 9.67 515  | 23 | 9.73 023  | 30    | 0.26 976  | 9.94 492  | 45 |  |       |
| 16 | 9.67 539  | 23 | 9.73 053  | 30    | 0.26 946  | 9.94 485  | 44 |  |       |
| 17 | 9.67 562  | 23 | 9.73 084  | 30    | 0.26 916  | 9.94 478  | 43 |  |       |
| 18 | 9.67 586  | 23 | 9.73 114  | 30    | 0.26 886  | 9.94 472  | 42 |  |       |
| 19 | 9.67 609  | 23 | 9.73 144  | 30    | 0.26 855  | 9.94 465  | 41 |  |       |
| 20 | 9.67 633  | 23 | 9.73 174  | 30    | 0.26 825  | 9.94 458  | 40 |  |       |
| 21 | 9.67 656  | 23 | 9.73 205  | 30    | 0.26 795  | 9.94 451  | 39 |  |       |
| 22 | 9.67 679  | 23 | 9.73 235  | 30    | 0.26 765  | 9.94 444  | 38 |  |       |
| 23 | 9.67 703  | 23 | 9.73 265  | 30    | 0.26 734  | 9.94 437  | 37 |  |       |
| 24 | 9.67 726  | 23 | 9.73 295  | 30    | 0.26 704  | 9.94 431  | 36 |  |       |
| 25 | 9.67 750  | 23 | 9.73 325  | 30    | 0.26 674  | 9.94 424  | 35 |  |       |
| 26 | 9.67 773  | 23 | 9.73 356  | 30    | 0.26 644  | 9.94 417  | 34 |  |       |
| 27 | 9.67 796  | 23 | 9.73 386  | 30    | 0.26 614  | 9.94 410  | 33 |  |       |
| 28 | 9.67 819  | 23 | 9.73 416  | 30    | 0.26 584  | 9.94 403  | 32 |  |       |
| 29 | 9.67 843  | 23 | 9.73 446  | 30    | 0.26 553  | 9.94 396  | 31 |  |       |
| 30 | 9.67 866  | 23 | 9.73 476  | 30    | 0.26 523  | 9.94 390  | 30 |  |       |
| 31 | 9.67 889  | 23 | 9.73 506  | 30    | 0.26 493  | 9.94 383  | 29 |  |       |
| 32 | 9.67 913  | 23 | 9.73 536  | 30    | 0.26 463  | 9.94 376  | 28 |  |       |
| 33 | 9.67 936  | 23 | 9.73 567  | 30    | 0.26 433  | 9.94 369  | 27 |  |       |
| 34 | 9.67 959  | 23 | 9.73 597  | 30    | 0.26 403  | 9.94 362  | 26 |  |       |
| 35 | 9.67 982  | 23 | 9.73 627  | 30    | 0.26 373  | 9.94 355  | 25 |  |       |
| 36 | 9.68 005  | 23 | 9.73 657  | 30    | 0.26 343  | 9.94 348  | 24 |  |       |
| 37 | 9.68 029  | 23 | 9.73 687  | 30    | 0.26 313  | 9.94 341  | 23 |  |       |
| 38 | 9.68 052  | 23 | 9.73 717  | 30    | 0.26 283  | 9.94 335  | 22 |  |       |
| 39 | 9.68 075  | 23 | 9.73 747  | 30    | 0.26 253  | 9.94 328  | 21 |  |       |
| 40 | 9.68 098  | 23 | 9.73 777  | 30    | 0.26 223  | 9.94 321  | 20 |  |       |
| 41 | 9.68 121  | 23 | 9.73 807  | 30    | 0.26 193  | 9.94 314  | 19 |  |       |
| 42 | 9.68 144  | 23 | 9.73 837  | 30    | 0.26 163  | 9.94 307  | 18 |  |       |
| 43 | 9.68 167  | 23 | 9.73 867  | 30    | 0.26 133  | 9.94 300  | 17 |  |       |
| 44 | 9.68 190  | 23 | 9.73 897  | 30    | 0.26 103  | 9.94 293  | 16 |  |       |
| 45 | 9.68 213  | 23 | 9.73 927  | 30    | 0.26 073  | 9.94 286  | 15 |  |       |
| 46 | 9.68 236  | 23 | 9.73 957  | 30    | 0.26 043  | 9.94 279  | 14 |  |       |
| 47 | 9.68 259  | 23 | 9.73 987  | 30    | 0.26 013  | 9.94 272  | 13 |  |       |
| 48 | 9.68 282  | 23 | 9.74 017  | 30    | 0.25 983  | 9.94 265  | 12 |  |       |
| 49 | 9.68 305  | 23 | 9.74 047  | 30    | 0.25 953  | 9.94 258  | 11 |  |       |
| 50 | 9.68 328  | 23 | 9.74 076  | 29    | 0.25 923  | 9.94 251  | 10 |  |       |
| 51 | 9.68 351  | 23 | 9.74 106  | 30    | 0.25 893  | 9.94 245  | 9  |  |       |
| 52 | 9.68 374  | 23 | 9.74 136  | 30    | 0.25 863  | 9.94 238  | 8  |  |       |
| 53 | 9.68 397  | 22 | 9.74 166  | 30    | 0.25 833  | 9.94 231  | 7  |  |       |
| 54 | 9.68 420  | 23 | 9.74 196  | 29    | 0.25 804  | 9.94 224  | 6  |  |       |
| 55 | 9.68 443  | 23 | 9.74 226  | 30    | 0.25 774  | 9.94 217  | 5  |  |       |
| 56 | 9.68 466  | 23 | 9.74 256  | 30    | 0.25 744  | 9.94 210  | 4  |  |       |
| 57 | 9.68 488  | 22 | 9.74 286  | 30    | 0.25 714  | 9.94 203  | 3  |  |       |
| 58 | 9.68 511  | 23 | 9.74 315  | 29    | 0.25 684  | 9.94 196  | 2  |  |       |
| 59 | 9.68 534  | 23 | 9.74 345  | 30    | 0.25 654  | 9.94 189  | 1  |  |       |
| 60 | 9.68 557  | 22 | 9.74 375  | 29    | 0.25 625  | 9.94 182  | 0  |  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |  | P. P. |

|    |      |      |      |
|----|------|------|------|
|    | 36   | 30   | 29   |
| 6  | 3.6  | 3.0  | 2.9  |
| 7  | 3.5  | 3.5  | 3.4  |
| 8  | 4.0  | 4.0  | 3.9  |
| 9  | 4.6  | 4.5  | 4.4  |
| 10 | 5.1  | 5.0  | 4.9  |
| 20 | 10.1 | 10.0 | 9.8  |
| 30 | 15.2 | 15.0 | 14.7 |
| 40 | 20.3 | 20.0 | 19.6 |
| 50 | 25.4 | 25.0 | 24.6 |

|    |      |
|----|------|
|    | 24   |
| 6  | 2.4  |
| 7  | 2.8  |
| 8  | 3.2  |
| 9  | 3.6  |
| 10 | 4.0  |
| 20 | 8.0  |
| 30 | 12.0 |
| 40 | 16.0 |
| 50 | 20.0 |

|    |      |      |      |
|----|------|------|------|
|    | 23   | 23   | 22   |
| 6  | 2.3  | 2.3  | 2.2  |
| 7  | 2.7  | 2.7  | 2.6  |
| 8  | 3.1  | 3.0  | 3.0  |
| 9  | 3.5  | 3.4  | 3.4  |
| 10 | 3.9  | 3.8  | 3.7  |
| 20 | 7.8  | 7.6  | 7.5  |
| 30 | 11.7 | 11.5 | 11.2 |
| 40 | 15.6 | 15.3 | 15.0 |
| 50 | 19.6 | 19.1 | 18.7 |

|    |     |     |
|----|-----|-----|
|    | 7   | 6   |
| 6  | 0.7 | 0.6 |
| 7  | 0.8 | 0.7 |
| 8  | 0.9 | 0.8 |
| 9  | 1.0 | 1.0 |
| 10 | 1.1 | 1.1 |
| 20 | 2.3 | 2.1 |
| 30 | 3.5 | 3.2 |
| 40 | 4.6 | 4.3 |
| 50 | 5.8 | 5.4 |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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| °  | Log. Sin. |    | d. | Log. Tan. |       | c. d. | Log. Cot. |  | Log. Cos. | d. | P. P.                                                                                                                                                                                                                                                                             |       |  |  |
|----|-----------|----|----|-----------|-------|-------|-----------|--|-----------|----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|--|--|
|    |           |    |    |           |       |       |           |  |           |    |                                                                                                                                                                                                                                                                                   |       |  |  |
| 0  | 9.68 557  |    | 23 | 9.74 375  |       | 30    | 0.25 625  |  | 9.94 182  | 7  | <div>60</div> <div>30 29 29</div> <div>6 3.0 2.9 2.9</div> <div>7 3.5 3.4 3.4</div> <div>8 4.0 3.9 3.8</div> <div>9 4.5 4.4 4.3</div> <div>10 5.0 4.9 4.8</div> <div>20 10.0 9.8 9.6</div> <div>30 15.0 14.7 14.5</div> <div>40 20.0 19.6 19.3</div> <div>50 25.0 24.6 24.1</div> |       |  |  |
| 1  | 9.68 580  |    | 22 | 9.74 405  |       | 30    | 0.25 595  |  | 9.94 175  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 2  | 9.68 602  |    | 22 | 9.74 435  |       | 29    | 0.25 565  |  | 9.94 168  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 3  | 9.68 623  |    | 22 | 9.74 464  |       | 30    | 0.25 535  |  | 9.94 161  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 4  | 9.68 648  |    | 23 | 9.74 494  |       | 29    | 0.25 505  |  | 9.94 154  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 5  | 9.68 671  |    | 22 | 9.74 524  |       | 30    | 0.25 476  |  | 9.94 147  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 6  | 9.68 693  |    | 23 | 9.74 554  |       | 29    | 0.25 446  |  | 9.94 140  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 7  | 9.68 716  |    | 22 | 9.74 583  |       | 29    | 0.25 416  |  | 9.94 133  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 8  | 9.68 739  |    | 22 | 9.74 613  |       | 30    | 0.25 387  |  | 9.94 126  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 9  | 9.68 761  |    | 23 | 9.74 643  |       | 29    | 0.25 357  |  | 9.94 118  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 10 | 9.68 784  |    | 22 | 9.74 672  |       | 30    | 0.25 327  |  | 9.94 111  | 7  | <div>50</div> <div>23</div> <div>6 2.3</div> <div>7 2.7</div> <div>8 3.0</div> <div>9 3.4</div> <div>10 3.8</div> <div>20 7.6</div> <div>30 11.5</div> <div>40 15.3</div> <div>50 19.1</div>                                                                                      |       |  |  |
| 11 | 9.68 807  |    | 22 | 9.74 702  |       | 29    | 0.25 297  |  | 9.94 104  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 12 | 9.68 829  |    | 22 | 9.74 732  |       | 29    | 0.25 268  |  | 9.94 097  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 13 | 9.68 852  |    | 22 | 9.74 761  |       | 30    | 0.25 238  |  | 9.94 090  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 14 | 9.68 874  |    | 22 | 9.74 791  |       | 29    | 0.25 208  |  | 9.94 083  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 15 | 9.68 897  |    | 23 | 9.74 821  |       | 29    | 0.25 179  |  | 9.94 076  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 16 | 9.68 920  |    | 22 | 9.74 850  |       | 29    | 0.25 149  |  | 9.94 069  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 17 | 9.68 942  |    | 22 | 9.74 880  |       | 29    | 0.25 120  |  | 9.94 062  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 18 | 9.68 965  |    | 22 | 9.74 909  |       | 30    | 0.25 090  |  | 9.94 055  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 19 | 9.68 987  |    | 23 | 9.74 939  |       | 29    | 0.25 060  |  | 9.94 048  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 20 | 9.69 010  |    | 22 | 9.74 969  |       | 29    | 0.25 031  |  | 9.94 041  | 7  | <div>40</div> <div>22 22 21</div> <div>6 2.2 2.2 2.1</div> <div>7 2.6 2.5 2.5</div> <div>8 3.0 2.9 2.8</div> <div>9 3.4 3.3 3.2</div> <div>10 3.7 3.6 3.6</div> <div>20 7.5 7.3 7.1</div> <div>30 11.2 11.0 10.7</div> <div>40 15.0 14.6 14.3</div> <div>50 18.7 18.3 17.9</div>  |       |  |  |
| 21 | 9.69 032  |    | 22 | 9.74 998  |       | 29    | 0.25 001  |  | 9.94 034  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 22 | 9.69 055  |    | 22 | 9.75 028  |       | 29    | 0.24 972  |  | 9.94 026  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 23 | 9.69 077  |    | 22 | 9.75 057  |       | 29    | 0.24 942  |  | 9.94 019  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 24 | 9.69 099  |    | 22 | 9.75 087  |       | 29    | 0.24 913  |  | 9.94 012  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 25 | 9.69 122  |    | 22 | 9.75 116  |       | 29    | 0.24 883  |  | 9.94 005  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 26 | 9.69 144  |    | 22 | 9.75 146  |       | 29    | 0.24 854  |  | 9.93 998  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 27 | 9.69 167  |    | 22 | 9.75 175  |       | 29    | 0.24 824  |  | 9.93 991  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 28 | 9.69 189  |    | 22 | 9.75 205  |       | 29    | 0.24 795  |  | 9.93 984  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 29 | 9.69 211  |    | 22 | 9.75 234  |       | 29    | 0.24 765  |  | 9.93 977  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 30 | 9.69 234  |    | 22 | 9.75 264  |       | 29    | 0.24 736  |  | 9.93 969  | 7  | <div>30</div> <div>7 7</div> <div>6 0.7 0.7</div> <div>7 0.9 0.8</div> <div>8 1.0 0.9</div> <div>9 1.1 1.0</div> <div>10 1.2 1.1</div> <div>20 2.5 2.3</div> <div>30 3.7 3.5</div> <div>40 5.0 4.6</div> <div>50 6.2 5.8</div>                                                    |       |  |  |
| 31 | 9.69 256  |    | 22 | 9.75 293  |       | 29    | 0.24 706  |  | 9.93 962  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 32 | 9.69 278  |    | 22 | 9.75 323  |       | 29    | 0.24 677  |  | 9.93 955  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 33 | 9.69 301  |    | 22 | 9.75 352  |       | 29    | 0.24 647  |  | 9.93 948  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 34 | 9.69 323  |    | 22 | 9.75 382  |       | 29    | 0.24 618  |  | 9.93 941  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 35 | 9.69 345  |    | 22 | 9.75 411  |       | 29    | 0.24 588  |  | 9.93 934  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 36 | 9.69 367  |    | 22 | 9.75 441  |       | 29    | 0.24 559  |  | 9.93 926  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 37 | 9.69 390  |    | 22 | 9.75 470  |       | 29    | 0.24 529  |  | 9.93 919  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 38 | 9.69 412  |    | 22 | 9.75 499  |       | 29    | 0.24 500  |  | 9.93 912  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 39 | 9.69 434  |    | 22 | 9.75 529  |       | 29    | 0.24 471  |  | 9.93 905  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 40 | 9.69 456  |    | 22 | 9.75 558  |       | 29    | 0.24 441  |  | 9.93 898  | 7  | <div>20</div> <div>19 18 17 16</div> <div>15 14 13 12 11</div> <div>10 9 8 7 6</div> <div>5 4 3 2 1</div> <div>0</div>                                                                                                                                                            |       |  |  |
| 41 | 9.69 478  |    | 22 | 9.75 588  |       | 29    | 0.24 412  |  | 9.93 891  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 42 | 9.69 500  |    | 22 | 9.75 617  |       | 29    | 0.24 383  |  | 9.93 883  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 43 | 9.69 523  |    | 22 | 9.75 646  |       | 29    | 0.24 353  |  | 9.93 876  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 44 | 9.69 545  |    | 22 | 9.75 676  |       | 29    | 0.24 324  |  | 9.93 869  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 45 | 9.69 567  |    | 22 | 9.75 705  |       | 29    | 0.24 295  |  | 9.93 862  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 46 | 9.69 589  |    | 22 | 9.75 734  |       | 29    | 0.24 265  |  | 9.93 854  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 47 | 9.69 611  |    | 22 | 9.75 764  |       | 29    | 0.24 236  |  | 9.93 847  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 48 | 9.69 633  |    | 22 | 9.75 793  |       | 29    | 0.24 207  |  | 9.93 840  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 49 | 9.69 655  |    | 22 | 9.75 822  |       | 29    | 0.24 177  |  | 9.93 833  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 50 | 9.69 677  |    | 22 | 9.75 851  |       | 29    | 0.24 148  |  | 9.93 826  | 7  | <div>10</div> <div>9 8 7 6</div> <div>5 4 3 2 1</div> <div>0</div>                                                                                                                                                                                                                |       |  |  |
| 51 | 9.69 699  |    | 22 | 9.75 881  |       | 29    | 0.24 119  |  | 9.93 818  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 52 | 9.69 721  |    | 22 | 9.75 910  |       | 29    | 0.24 090  |  | 9.93 811  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 53 | 9.69 743  |    | 22 | 9.75 939  |       | 29    | 0.24 060  |  | 9.93 804  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 54 | 9.69 765  |    | 22 | 9.75 968  |       | 29    | 0.24 031  |  | 9.93 796  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 55 | 9.69 787  |    | 22 | 9.75 998  |       | 29    | 0.24 002  |  | 9.93 789  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 56 | 9.69 809  |    | 22 | 9.76 027  |       | 29    | 0.23 973  |  | 9.93 782  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 57 | 9.69 831  |    | 21 | 9.76 056  |       | 29    | 0.23 943  |  | 9.93 775  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 58 | 9.69 853  |    | 22 | 9.76 085  |       | 29    | 0.23 914  |  | 9.93 767  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 59 | 9.69 875  |    | 22 | 9.76 115  |       | 29    | 0.23 885  |  | 9.93 760  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
| 60 | 9.69 897  |    | 22 | 9.76 144  |       | 29    | 0.23 856  |  | 9.93 753  | 7  |                                                                                                                                                                                                                                                                                   |       |  |  |
|    | Log. Cos. | d. |    | Log. Cot. | c. d. |       | Log. Tan. |  | Log. Sin. | d. |                                                                                                                                                                                                                                                                                   | P. P. |  |  |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |      |      |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|------|------|--|
| 0  | 9.69 897  |    | 9.76 144  |       | 0.23 856  | 9.93 753  |    | 60 |       |      |      |  |
| 1  | 9.69 919  | 22 | 9.76 173  | 29    | 0.23 827  | 9.93 746  | 7  | 59 |       |      |      |  |
| 2  | 9.69 940  | 21 | 9.76 202  | 29    | 0.23 797  | 9.93 738  | 7  | 58 |       |      |      |  |
| 3  | 9.69 962  | 22 | 9.76 231  | 29    | 0.23 768  | 9.93 731  | 7  | 57 |       |      |      |  |
| 4  | 9.69 984  | 22 | 9.76 260  | 29    | 0.23 739  | 9.93 724  | 7  | 56 |       |      |      |  |
| 5  | 9.70 006  | 21 | 9.76 289  | 29    | 0.23 710  | 9.93 716  | 7  | 55 |       |      |      |  |
| 6  | 9.70 028  | 22 | 9.76 319  | 29    | 0.23 681  | 9.93 709  | 7  | 54 |       |      |      |  |
| 7  | 9.70 050  | 21 | 9.76 348  | 29    | 0.23 652  | 9.93 702  | 7  | 53 |       |      |      |  |
| 8  | 9.70 071  | 22 | 9.76 377  | 29    | 0.23 623  | 9.93 694  | 7  | 52 |       |      |      |  |
| 9  | 9.70 093  | 21 | 9.76 406  | 29    | 0.23 594  | 9.93 687  | 7  | 51 |       |      |      |  |
| 10 | 9.70 115  | 22 | 9.76 435  | 29    | 0.23 565  | 9.93 680  | 7  | 50 | 26    | 2.9  | 2.8  |  |
| 11 | 9.70 137  | 21 | 9.76 464  | 29    | 0.23 535  | 9.93 672  | 7  | 49 | 7     | 3.4  | 3.3  |  |
| 12 | 9.70 158  | 21 | 9.76 493  | 29    | 0.23 506  | 9.93 665  | 7  | 48 | 8     | 3.9  | 3.8  |  |
| 13 | 9.70 180  | 22 | 9.76 522  | 29    | 0.23 477  | 9.93 658  | 7  | 47 | 9     | 4.4  | 4.3  |  |
| 14 | 9.70 202  | 21 | 9.76 551  | 29    | 0.23 448  | 9.93 650  | 7  | 46 | 10    | 4.9  | 4.8  |  |
| 15 | 9.70 223  | 21 | 9.76 580  | 29    | 0.23 419  | 9.93 643  | 7  | 45 | 20    | 9.8  | 9.6  |  |
| 16 | 9.70 245  | 21 | 9.76 609  | 29    | 0.23 390  | 9.93 635  | 7  | 44 | 30    | 14.7 | 14.5 |  |
| 17 | 9.70 267  | 22 | 9.76 638  | 29    | 0.23 361  | 9.93 628  | 7  | 43 | 40    | 19.6 | 19.3 |  |
| 18 | 9.70 288  | 21 | 9.76 667  | 29    | 0.23 332  | 9.93 621  | 7  | 42 | 50    | 24.6 | 24.1 |  |
| 19 | 9.70 310  | 21 | 9.76 696  | 29    | 0.23 303  | 9.93 613  | 7  | 41 |       |      |      |  |
| 20 | 9.70 331  | 22 | 9.76 725  | 29    | 0.23 274  | 9.93 606  | 7  | 40 |       |      |      |  |
| 21 | 9.70 353  | 21 | 9.76 754  | 29    | 0.23 245  | 9.93 599  | 7  | 39 |       |      |      |  |
| 22 | 9.70 375  | 21 | 9.76 783  | 29    | 0.23 216  | 9.93 591  | 7  | 38 |       |      |      |  |
| 23 | 9.70 396  | 21 | 9.76 812  | 29    | 0.23 187  | 9.93 584  | 7  | 37 |       |      |      |  |
| 24 | 9.70 418  | 21 | 9.76 841  | 29    | 0.23 158  | 9.93 576  | 7  | 36 |       |      |      |  |
| 25 | 9.70 439  | 21 | 9.76 870  | 29    | 0.23 129  | 9.93 569  | 7  | 35 |       |      |      |  |
| 26 | 9.70 461  | 21 | 9.76 899  | 28    | 0.23 101  | 9.93 562  | 7  | 34 | 22    | 2.2  | 2.1  |  |
| 27 | 9.70 482  | 21 | 9.76 928  | 29    | 0.23 072  | 9.93 554  | 7  | 33 | 7     | 2.5  | 2.4  |  |
| 28 | 9.70 504  | 21 | 9.76 957  | 29    | 0.23 043  | 9.93 547  | 7  | 32 | 8     | 2.9  | 2.8  |  |
| 29 | 9.70 525  | 21 | 9.76 986  | 29    | 0.23 014  | 9.93 539  | 7  | 31 | 9     | 3.3  | 3.1  |  |
| 30 | 9.70 547  | 21 | 9.77 015  | 29    | 0.22 985  | 9.93 532  | 7  | 30 | 10    | 3.6  | 3.5  |  |
| 31 | 9.70 568  | 21 | 9.77 043  | 28    | 0.22 956  | 9.93 524  | 7  | 29 | 20    | 7.3  | 7.0  |  |
| 32 | 9.70 590  | 21 | 9.77 072  | 29    | 0.22 927  | 9.93 517  | 7  | 28 | 30    | 11.0 | 10.5 |  |
| 33 | 9.70 611  | 21 | 9.77 101  | 29    | 0.22 898  | 9.93 509  | 7  | 27 | 40    | 14.6 | 14.0 |  |
| 34 | 9.70 632  | 21 | 9.77 130  | 29    | 0.22 869  | 9.93 502  | 7  | 26 | 50    | 18.3 | 17.5 |  |
| 35 | 9.70 654  | 21 | 9.77 159  | 28    | 0.22 841  | 9.93 495  | 7  | 25 |       |      |      |  |
| 36 | 9.70 675  | 21 | 9.77 188  | 29    | 0.22 812  | 9.93 487  | 7  | 24 |       |      |      |  |
| 37 | 9.70 696  | 21 | 9.77 217  | 29    | 0.22 783  | 9.93 480  | 7  | 23 |       |      |      |  |
| 38 | 9.70 718  | 21 | 9.77 245  | 28    | 0.22 754  | 9.93 472  | 7  | 22 |       |      |      |  |
| 39 | 9.70 739  | 21 | 9.77 274  | 29    | 0.22 725  | 9.93 465  | 7  | 21 |       |      |      |  |
| 40 | 9.70 760  | 21 | 9.77 303  | 29    | 0.22 696  | 9.93 457  | 7  | 20 |       |      |      |  |
| 41 | 9.70 782  | 21 | 9.77 332  | 28    | 0.22 668  | 9.93 450  | 7  | 19 |       |      |      |  |
| 42 | 9.70 803  | 21 | 9.77 361  | 29    | 0.22 639  | 9.93 442  | 7  | 18 |       |      |      |  |
| 43 | 9.70 824  | 21 | 9.77 389  | 28    | 0.22 610  | 9.93 435  | 7  | 17 |       |      |      |  |
| 44 | 9.70 846  | 21 | 9.77 418  | 29    | 0.22 581  | 9.93 427  | 7  | 16 | 8     | 0.8  | 0.7  |  |
| 45 | 9.70 867  | 21 | 9.77 447  | 28    | 0.22 553  | 9.93 420  | 7  | 15 | 7     | 0.9  | 0.8  |  |
| 46 | 9.70 888  | 21 | 9.77 476  | 29    | 0.22 524  | 9.93 412  | 7  | 14 | 8     | 1.0  | 0.9  |  |
| 47 | 9.70 909  | 21 | 9.77 504  | 28    | 0.22 495  | 9.93 405  | 7  | 13 | 9     | 1.2  | 1.1  |  |
| 48 | 9.70 930  | 21 | 9.77 533  | 29    | 0.22 466  | 9.93 397  | 7  | 12 | 10    | 1.3  | 1.1  |  |
| 49 | 9.70 952  | 21 | 9.77 562  | 28    | 0.22 438  | 9.93 390  | 7  | 11 | 20    | 2.6  | 2.3  |  |
| 50 | 9.70 973  | 21 | 9.77 591  | 29    | 0.22 409  | 9.93 382  | 8  | 10 | 30    | 4.0  | 3.5  |  |
| 51 | 9.70 994  | 21 | 9.77 619  | 28    | 0.22 380  | 9.93 374  | 7  | 9  | 40    | 5.3  | 4.6  |  |
| 52 | 9.71 015  | 21 | 9.77 648  | 29    | 0.22 352  | 9.93 367  | 7  | 8  | 50    | 6.6  | 5.8  |  |
| 53 | 9.71 036  | 21 | 9.77 677  | 28    | 0.22 323  | 9.93 359  | 7  | 7  |       |      |      |  |
| 54 | 9.71 057  | 21 | 9.77 705  | 28    | 0.22 294  | 9.93 352  | 7  | 6  |       |      |      |  |
| 55 | 9.71 078  | 21 | 9.77 734  | 28    | 0.22 266  | 9.93 344  | 7  | 5  |       |      |      |  |
| 56 | 9.71 099  | 21 | 9.77 763  | 29    | 0.22 237  | 9.93 337  | 7  | 4  |       |      |      |  |
| 57 | 9.71 121  | 21 | 9.77 791  | 28    | 0.22 208  | 9.93 329  | 7  | 3  |       |      |      |  |
| 58 | 9.71 142  | 21 | 9.77 820  | 28    | 0.22 180  | 9.93 321  | 8  | 2  |       |      |      |  |
| 59 | 9.71 163  | 21 | 9.77 849  | 29    | 0.22 151  | 9.93 314  | 7  | 1  |       |      |      |  |
| 60 | 9.71 184  | 21 | 9.77 877  | 28    | 0.22 122  | 9.93 306  | 7  | 0  |       |      |      |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |      |      |  |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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| °  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P. |  |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|--|--|--|
| 0  | 9.71 184  |    | 9.77 877  | 28    | 0.22 122  | 9.93 306  | 7  | 60    |  |  |  |
| 1  | 9.71 205  | 21 | 9.77 906  | 28    | 0.22 094  | 9.93 299  | 7  | 59    |  |  |  |
| 2  | 9.71 226  | 21 | 9.77 934  | 28    | 0.22 065  | 9.93 291  | 7  | 58    |  |  |  |
| 3  | 9.71 247  | 21 | 9.77 963  | 28    | 0.22 037  | 9.93 284  | 7  | 57    |  |  |  |
| 4  | 9.71 268  | 21 | 9.77 992  | 29    | 0.22 008  | 9.93 276  | 8  | 56    |  |  |  |
| 5  | 9.71 289  | 21 | 9.78 020  | 28    | 0.21 979  | 9.93 268  | 7  | 55    |  |  |  |
| 6  | 9.71 310  | 21 | 9.78 049  | 28    | 0.21 951  | 9.93 261  | 7  | 54    |  |  |  |
| 7  | 9.71 331  | 20 | 9.78 077  | 28    | 0.21 922  | 9.93 253  | 8  | 53    |  |  |  |
| 8  | 9.71 351  | 21 | 9.78 106  | 28    | 0.21 894  | 9.93 245  | 7  | 52    |  |  |  |
| 9  | 9.71 372  | 21 | 9.78 134  | 28    | 0.21 865  | 9.93 238  | 7  | 51    |  |  |  |
| 10 | 9.71 393  | 21 | 9.78 163  | 28    | 0.21 837  | 9.93 230  | 7  | 50    |  |  |  |
| 11 | 9.71 414  | 20 | 9.78 191  | 28    | 0.21 808  | 9.93 223  | 8  | 49    |  |  |  |
| 12 | 9.71 435  | 21 | 9.78 220  | 28    | 0.21 780  | 9.93 215  | 7  | 48    |  |  |  |
| 13 | 9.71 456  | 21 | 9.78 248  | 28    | 0.21 751  | 9.93 207  | 7  | 47    |  |  |  |
| 14 | 9.71 477  | 21 | 9.78 277  | 28    | 0.21 723  | 9.93 200  | 8  | 46    |  |  |  |
| 15 | 9.71 498  | 20 | 9.78 305  | 28    | 0.21 694  | 9.93 192  | 7  | 45    |  |  |  |
| 16 | 9.71 518  | 21 | 9.78 334  | 28    | 0.21 666  | 9.93 184  | 7  | 44    |  |  |  |
| 17 | 9.71 539  | 20 | 9.78 362  | 28    | 0.21 637  | 9.93 177  | 8  | 43    |  |  |  |
| 18 | 9.71 560  | 21 | 9.78 391  | 28    | 0.21 609  | 9.93 169  | 7  | 42    |  |  |  |
| 19 | 9.71 581  | 20 | 9.78 419  | 28    | 0.21 580  | 9.93 161  | 8  | 41    |  |  |  |
| 20 | 9.71 601  | 21 | 9.78 448  | 28    | 0.21 552  | 9.93 153  | 7  | 40    |  |  |  |
| 21 | 9.71 622  | 20 | 9.78 476  | 28    | 0.21 523  | 9.93 146  | 7  | 39    |  |  |  |
| 22 | 9.71 643  | 21 | 9.78 505  | 28    | 0.21 495  | 9.93 138  | 8  | 38    |  |  |  |
| 23 | 9.71 664  | 20 | 9.78 533  | 28    | 0.21 467  | 9.93 130  | 7  | 37    |  |  |  |
| 24 | 9.71 684  | 21 | 9.78 561  | 28    | 0.21 438  | 9.93 123  | 8  | 36    |  |  |  |
| 25 | 9.71 705  | 20 | 9.78 590  | 28    | 0.21 410  | 9.93 115  | 7  | 35    |  |  |  |
| 26 | 9.71 726  | 20 | 9.78 618  | 28    | 0.21 381  | 9.93 107  | 7  | 34    |  |  |  |
| 27 | 9.71 746  | 21 | 9.78 647  | 28    | 0.21 353  | 9.93 100  | 8  | 33    |  |  |  |
| 28 | 9.71 767  | 20 | 9.78 675  | 28    | 0.21 325  | 9.93 092  | 7  | 32    |  |  |  |
| 29 | 9.71 788  | 20 | 9.78 703  | 28    | 0.21 296  | 9.93 084  | 8  | 31    |  |  |  |
| 30 | 9.71 808  | 20 | 9.78 732  | 28    | 0.21 268  | 9.93 076  | 7  | 30    |  |  |  |
| 31 | 9.71 829  | 20 | 9.78 760  | 28    | 0.21 239  | 9.93 069  | 7  | 29    |  |  |  |
| 32 | 9.71 849  | 20 | 9.78 788  | 28    | 0.21 211  | 9.93 061  | 8  | 28    |  |  |  |
| 33 | 9.71 870  | 21 | 9.78 817  | 28    | 0.21 183  | 9.93 053  | 7  | 27    |  |  |  |
| 34 | 9.71 891  | 20 | 9.78 845  | 28    | 0.21 154  | 9.93 045  | 8  | 26    |  |  |  |
| 35 | 9.71 911  | 20 | 9.78 873  | 28    | 0.21 126  | 9.93 038  | 7  | 25    |  |  |  |
| 36 | 9.71 932  | 20 | 9.78 902  | 28    | 0.21 098  | 9.93 030  | 8  | 24    |  |  |  |
| 37 | 9.71 952  | 20 | 9.78 930  | 28    | 0.21 070  | 9.93 022  | 7  | 23    |  |  |  |
| 38 | 9.71 973  | 20 | 9.78 958  | 28    | 0.21 041  | 9.93 014  | 8  | 22    |  |  |  |
| 39 | 9.71 993  | 20 | 9.78 987  | 28    | 0.21 013  | 9.93 006  | 7  | 21    |  |  |  |
| 40 | 9.72 014  | 20 | 9.79 015  | 28    | 0.20 985  | 9.92 999  | 8  | 20    |  |  |  |
| 41 | 9.72 034  | 20 | 9.79 043  | 28    | 0.20 956  | 9.92 991  | 7  | 19    |  |  |  |
| 42 | 9.72 055  | 20 | 9.79 071  | 28    | 0.20 928  | 9.92 983  | 8  | 18    |  |  |  |
| 43 | 9.72 075  | 20 | 9.79 100  | 28    | 0.20 900  | 9.92 975  | 8  | 17    |  |  |  |
| 44 | 9.72 096  | 20 | 9.79 128  | 28    | 0.20 872  | 9.92 967  | 8  | 16    |  |  |  |
| 45 | 9.72 116  | 20 | 9.79 156  | 28    | 0.20 843  | 9.92 960  | 7  | 15    |  |  |  |
| 46 | 9.72 136  | 20 | 9.79 184  | 28    | 0.20 815  | 9.92 952  | 8  | 14    |  |  |  |
| 47 | 9.72 157  | 20 | 9.79 213  | 28    | 0.20 787  | 9.92 944  | 8  | 13    |  |  |  |
| 48 | 9.72 177  | 20 | 9.79 241  | 28    | 0.20 759  | 9.92 936  | 7  | 12    |  |  |  |
| 49 | 9.72 198  | 20 | 9.79 269  | 28    | 0.20 731  | 9.92 928  | 8  | 11    |  |  |  |
| 50 | 9.72 218  | 20 | 9.79 297  | 28    | 0.20 702  | 9.92 920  | 8  | 10    |  |  |  |
| 51 | 9.72 238  | 20 | 9.79 325  | 28    | 0.20 674  | 9.92 913  | 7  | 9     |  |  |  |
| 52 | 9.72 259  | 20 | 9.79 354  | 28    | 0.20 646  | 9.92 905  | 8  | 8     |  |  |  |
| 53 | 9.72 279  | 20 | 9.79 382  | 28    | 0.20 618  | 9.92 897  | 8  | 7     |  |  |  |
| 54 | 9.72 299  | 20 | 9.79 410  | 28    | 0.20 590  | 9.92 889  | 7  | 6     |  |  |  |
| 55 | 9.72 319  | 20 | 9.79 438  | 28    | 0.20 561  | 9.92 881  | 8  | 5     |  |  |  |
| 56 | 9.72 340  | 20 | 9.79 466  | 28    | 0.20 533  | 9.92 873  | 8  | 4     |  |  |  |
| 57 | 9.72 360  | 20 | 9.79 494  | 28    | 0.20 505  | 9.92 865  | 7  | 3     |  |  |  |
| 58 | 9.72 380  | 20 | 9.79 522  | 28    | 0.20 477  | 9.92 858  | 8  | 2     |  |  |  |
| 59 | 9.72 400  | 20 | 9.79 551  | 28    | 0.20 449  | 9.92 850  | 8  | 1     |  |  |  |
| 60 | 9.72 421  | 20 | 9.79 579  | 28    | 0.20 421  | 9.92 842  | 8  | 0     |  |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | P. P. |  |  |  |

|    | 29   | 28   | 28   |
|----|------|------|------|
| 6  | 2.9  | 2.8  | 2.8  |
| 7  | 3.4  | 3.3  | 3.2  |
| 8  | 3.8  | 3.8  | 3.7  |
| 9  | 4.3  | 4.3  | 4.2  |
| 10 | 4.8  | 4.7  | 4.6  |
| 20 | 9.6  | 9.5  | 9.3  |
| 30 | 14.5 | 14.2 | 14.0 |
| 40 | 19.3 | 19.0 | 18.6 |
| 50 | 24.1 | 23.7 | 23.3 |

|    | 21   | 20   | 20   |
|----|------|------|------|
| 6  | 2.1  | 2.6  | 2.0  |
| 7  | 2.4  | 2.4  | 2.3  |
| 8  | 2.8  | 2.7  | 2.6  |
| 9  | 3.1  | 3.1  | 3.0  |
| 10 | 3.5  | 3.4  | 3.3  |
| 20 | 7.0  | 6.8  | 6.6  |
| 30 | 10.5 | 10.2 | 10.0 |
| 40 | 14.0 | 13.6 | 13.3 |
| 50 | 17.5 | 17.1 | 16.6 |

|    | 8   | 7   |
|----|-----|-----|
| 6  | 0.8 | 0.7 |
| 7  | 0.9 | 0.9 |
| 8  | 1.0 | 1.0 |
| 9  | 1.2 | 1.1 |
| 10 | 1.3 | 1.2 |
| 20 | 2.6 | 2.5 |
| 30 | 4.0 | 3.7 |
| 40 | 5.3 | 5.0 |
| 50 | 6.6 | 6.2 |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS  
32°

| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |      |      |      |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|------|------|------|
| 0  | 9.72 421  |    | 9.79 579  |       | 0.20 421  | 9.92 842  |    | 60 |       |      |      |      |
| 1  | 9.72 441  | 20 | 9.79 607  | 28    | 0.20 393  | 9.92 834  | 8  | 59 |       |      |      |      |
| 2  | 9.72 461  | 20 | 9.79 635  | 28    | 0.20 365  | 9.92 826  | 8  | 58 |       |      |      |      |
| 3  | 9.72 481  | 20 | 9.79 663  | 28    | 0.20 337  | 9.92 818  | 8  | 57 |       |      |      |      |
| 4  | 9.72 501  | 20 | 9.79 691  | 28    | 0.20 308  | 9.92 810  | 8  | 56 |       |      |      |      |
| 5  | 9.72 522  | 20 | 9.79 719  | 28    | 0.20 280  | 9.92 802  | 8  | 55 |       |      |      |      |
| 6  | 9.72 542  | 20 | 9.79 747  | 28    | 0.20 252  | 9.92 794  | 8  | 54 |       |      |      |      |
| 7  | 9.72 562  | 20 | 9.79 775  | 28    | 0.20 224  | 9.92 786  | 8  | 53 |       |      |      |      |
| 8  | 9.72 582  | 20 | 9.79 803  | 28    | 0.20 196  | 9.92 778  | 8  | 52 |       |      |      |      |
| 9  | 9.72 602  | 20 | 9.79 831  | 28    | 0.20 168  | 9.92 771  | 8  | 51 |       |      |      |      |
| 10 | 9.72 622  | 20 | 9.79 859  | 28    | 0.20 140  | 9.92 763  | 8  | 50 | 28    | 28   | 27   |      |
| 11 | 9.72 642  | 20 | 9.79 887  | 28    | 0.20 112  | 9.92 755  | 8  | 49 | 6     | 2.8  | 2.8  | 2.7  |
| 12 | 9.72 662  | 20 | 9.79 915  | 28    | 0.20 084  | 9.92 747  | 8  | 48 | 7     | 3.3  | 3.2  | 3.2  |
| 13 | 9.72 682  | 20 | 9.79 943  | 28    | 0.20 056  | 9.92 739  | 8  | 47 | 8     | 3.8  | 3.7  | 3.6  |
| 14 | 9.72 702  | 20 | 9.79 971  | 28    | 0.20 028  | 9.92 731  | 8  | 46 | 9     | 4.3  | 4.2  | 4.1  |
| 15 | 9.72 723  | 20 | 9.79 999  | 28    | 0.20 000  | 9.92 723  | 8  | 45 | 10    | 4.7  | 4.6  | 4.6  |
| 16 | 9.72 743  | 20 | 9.80 027  | 28    | 0.19 972  | 9.92 715  | 8  | 44 | 20    | 9.5  | 9.3  | 9.1  |
| 17 | 9.72 763  | 20 | 9.80 055  | 28    | 0.19 944  | 9.92 707  | 8  | 43 | 30    | 14.2 | 14.0 | 13.7 |
| 18 | 9.72 783  | 19 | 9.80 083  | 28    | 0.19 916  | 9.92 699  | 8  | 42 | 40    | 19.0 | 18.6 | 18.3 |
| 19 | 9.72 802  | 20 | 9.80 111  | 28    | 0.19 888  | 9.92 691  | 8  | 41 | 50    | 23.7 | 23.3 | 22.9 |
| 20 | 9.72 822  | 20 | 9.80 139  | 28    | 0.19 860  | 9.92 683  | 8  | 40 |       |      |      |      |
| 21 | 9.72 842  | 20 | 9.80 167  | 28    | 0.19 832  | 9.92 675  | 8  | 39 |       |      |      |      |
| 22 | 9.72 862  | 20 | 9.80 195  | 28    | 0.19 804  | 9.92 667  | 8  | 38 |       |      |      |      |
| 23 | 9.72 882  | 20 | 9.80 223  | 28    | 0.19 776  | 9.92 659  | 8  | 37 |       |      |      |      |
| 24 | 9.72 902  | 20 | 9.80 251  | 28    | 0.19 748  | 9.92 651  | 8  | 36 |       |      |      |      |
| 25 | 9.72 922  | 19 | 9.80 279  | 27    | 0.19 721  | 9.92 643  | 8  | 35 |       |      |      |      |
| 26 | 9.72 942  | 20 | 9.80 307  | 28    | 0.19 693  | 9.92 635  | 8  | 34 | 20    | 2.0  | 2.0  | 1.9  |
| 27 | 9.72 962  | 20 | 9.80 335  | 28    | 0.19 665  | 9.92 627  | 8  | 33 | 6     | 2.6  | 2.6  | 2.6  |
| 28 | 9.72 982  | 20 | 9.80 363  | 28    | 0.19 637  | 9.92 619  | 8  | 32 | 7     | 2.4  | 2.3  | 2.3  |
| 29 | 9.73 002  | 19 | 9.80 391  | 27    | 0.19 609  | 9.92 611  | 8  | 31 | 8     | 2.7  | 2.6  | 2.6  |
| 30 | 9.73 021  | 20 | 9.80 418  | 28    | 0.19 581  | 9.92 603  | 8  | 30 | 9     | 3.1  | 3.0  | 2.9  |
| 31 | 9.73 041  | 20 | 9.80 446  | 28    | 0.19 553  | 9.92 595  | 8  | 29 | 10    | 3.4  | 3.3  | 3.2  |
| 32 | 9.73 061  | 19 | 9.80 474  | 28    | 0.19 525  | 9.92 587  | 8  | 28 | 20    | 6.8  | 6.6  | 6.5  |
| 33 | 9.73 081  | 20 | 9.80 502  | 27    | 0.19 497  | 9.92 579  | 8  | 27 | 30    | 10.2 | 10.0 | 9.7  |
| 34 | 9.73 101  | 19 | 9.80 530  | 28    | 0.19 470  | 9.92 570  | 8  | 26 | 40    | 13.6 | 13.3 | 13.0 |
| 35 | 9.73 120  | 20 | 9.80 558  | 28    | 0.19 442  | 9.92 562  | 8  | 25 | 50    | 17.1 | 16.6 | 16.2 |
| 36 | 9.73 140  | 19 | 9.80 586  | 27    | 0.19 414  | 9.92 554  | 8  | 24 |       |      |      |      |
| 37 | 9.73 160  | 20 | 9.80 613  | 28    | 0.19 386  | 9.92 546  | 8  | 23 |       |      |      |      |
| 38 | 9.73 180  | 19 | 9.80 641  | 28    | 0.19 358  | 9.92 538  | 8  | 22 |       |      |      |      |
| 39 | 9.73 199  | 20 | 9.80 669  | 27    | 0.19 330  | 9.92 530  | 8  | 21 |       |      |      |      |
| 40 | 9.73 219  | 19 | 9.80 697  | 28    | 0.19 303  | 9.92 522  | 8  | 20 |       |      |      |      |
| 41 | 9.73 239  | 20 | 9.80 725  | 27    | 0.19 275  | 9.92 514  | 8  | 19 |       |      |      |      |
| 42 | 9.73 258  | 19 | 9.80 752  | 28    | 0.19 247  | 9.92 506  | 8  | 18 |       |      |      |      |
| 43 | 9.73 278  | 20 | 9.80 780  | 28    | 0.19 219  | 9.92 498  | 8  | 17 |       |      |      |      |
| 44 | 9.73 298  | 19 | 9.80 808  | 27    | 0.19 191  | 9.92 489  | 8  | 16 | 8     | 8    | 7    |      |
| 45 | 9.73 317  | 20 | 9.80 836  | 28    | 0.19 164  | 9.92 481  | 8  | 15 | 6     | 0.8  | 0.8  | 0.7  |
| 46 | 9.73 337  | 19 | 9.80 864  | 27    | 0.19 136  | 9.92 473  | 8  | 14 | 7     | 1.0  | 0.9  | 0.9  |
| 47 | 9.73 357  | 20 | 9.80 891  | 28    | 0.19 108  | 9.92 465  | 8  | 13 | 8     | 1.1  | 1.0  | 1.0  |
| 48 | 9.73 376  | 19 | 9.80 919  | 27    | 0.19 080  | 9.92 457  | 8  | 12 | 9     | 1.3  | 1.2  | 1.1  |
| 49 | 9.73 396  | 20 | 9.80 947  | 28    | 0.19 053  | 9.92 449  | 8  | 11 | 10    | 1.4  | 1.3  | 1.2  |
| 50 | 9.73 415  | 19 | 9.80 975  | 27    | 0.19 025  | 9.92 441  | 8  | 10 | 20    | 2.8  | 2.6  | 2.5  |
| 51 | 9.73 435  | 20 | 9.81 002  | 28    | 0.18 997  | 9.92 433  | 8  | 9  | 30    | 4.2  | 4.0  | 3.7  |
| 52 | 9.73 455  | 19 | 9.81 030  | 27    | 0.18 970  | 9.92 424  | 8  | 8  | 40    | 5.6  | 5.3  | 5.0  |
| 53 | 9.73 474  | 20 | 9.81 058  | 28    | 0.18 942  | 9.92 416  | 8  | 7  | 50    | 7.1  | 6.6  | 6.2  |
| 54 | 9.73 494  | 19 | 9.81 085  | 27    | 0.18 914  | 9.92 408  | 8  | 6  |       |      |      |      |
| 55 | 9.73 513  | 20 | 9.81 113  | 28    | 0.18 886  | 9.92 400  | 8  | 5  |       |      |      |      |
| 56 | 9.73 533  | 19 | 9.81 141  | 27    | 0.18 859  | 9.92 392  | 8  | 4  |       |      |      |      |
| 57 | 9.73 552  | 20 | 9.81 168  | 28    | 0.18 831  | 9.92 383  | 8  | 3  |       |      |      |      |
| 58 | 9.73 572  | 19 | 9.81 196  | 27    | 0.18 803  | 9.92 375  | 8  | 2  |       |      |      |      |
| 59 | 9.73 591  | 20 | 9.81 224  | 28    | 0.18 776  | 9.92 367  | 8  | 1  |       |      |      |      |
| 60 | 9.73 611  | 19 | 9.81 251  | 27    | 0.18 748  | 9.92 359  | 8  | 0  |       |      |      |      |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |      |      |      |

|    |           |    |           |       |           |           |    | P. P. |  |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|--|--|--|
| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |       |  |  |  |
| 0  | 9.73 611  | 10 | 9.81 251  | 28    | 0.18 748  | 9.92 359  | 8  | 60    |  |  |  |
| 1  | 9.73 630  | 10 | 9.81 270  | 27    | 0.18 720  | 9.92 351  | 8  | 59    |  |  |  |
| 2  | 9.73 650  | 10 | 9.81 307  | 27    | 0.18 693  | 9.92 342  | 8  | 58    |  |  |  |
| 3  | 9.73 669  | 10 | 9.81 334  | 28    | 0.18 665  | 9.92 334  | 8  | 57    |  |  |  |
| 4  | 9.73 688  | 10 | 9.81 362  | 27    | 0.18 637  | 9.92 326  | 8  | 56    |  |  |  |
| 5  | 9.73 708  | 10 | 9.81 390  | 27    | 0.18 610  | 9.92 318  | 8  | 55    |  |  |  |
| 6  | 9.73 727  | 10 | 9.81 417  | 27    | 0.18 582  | 9.92 310  | 8  | 54    |  |  |  |
| 7  | 9.73 746  | 10 | 9.81 445  | 28    | 0.18 555  | 9.92 301  | 8  | 53    |  |  |  |
| 8  | 9.73 766  | 10 | 9.81 473  | 27    | 0.18 527  | 9.92 293  | 8  | 52    |  |  |  |
| 9  | 9.73 785  | 10 | 9.81 500  | 27    | 0.18 499  | 9.92 285  | 8  | 51    |  |  |  |
| 10 | 9.73 805  | 10 | 9.81 528  | 27    | 0.18 472  | 9.92 277  | 8  | 50    |  |  |  |
| 11 | 9.73 824  | 10 | 9.81 555  | 27    | 0.18 444  | 9.92 268  | 8  | 49    |  |  |  |
| 12 | 9.73 843  | 10 | 9.81 583  | 27    | 0.18 417  | 9.92 260  | 8  | 48    |  |  |  |
| 13 | 9.73 862  | 10 | 9.81 610  | 27    | 0.18 389  | 9.92 252  | 8  | 47    |  |  |  |
| 14 | 9.73 882  | 10 | 9.81 638  | 28    | 0.18 362  | 9.92 244  | 8  | 46    |  |  |  |
| 15 | 9.73 901  | 10 | 9.81 666  | 27    | 0.18 334  | 9.92 235  | 8  | 45    |  |  |  |
| 16 | 9.73 920  | 10 | 9.81 693  | 27    | 0.18 306  | 9.92 227  | 8  | 44    |  |  |  |
| 17 | 9.73 940  | 10 | 9.81 721  | 27    | 0.18 279  | 9.92 219  | 8  | 43    |  |  |  |
| 18 | 9.73 959  | 10 | 9.81 748  | 27    | 0.18 251  | 9.92 210  | 8  | 42    |  |  |  |
| 19 | 9.73 978  | 10 | 9.81 776  | 27    | 0.18 224  | 9.92 202  | 8  | 41    |  |  |  |
| 20 | 9.73 997  | 10 | 9.81 803  | 27    | 0.18 196  | 9.92 194  | 8  | 40    |  |  |  |
| 21 | 9.74 016  | 10 | 9.81 831  | 27    | 0.18 169  | 9.92 185  | 8  | 39    |  |  |  |
| 22 | 9.74 036  | 10 | 9.81 858  | 27    | 0.18 141  | 9.92 177  | 8  | 38    |  |  |  |
| 23 | 9.74 055  | 10 | 9.81 886  | 27    | 0.18 114  | 9.92 169  | 8  | 37    |  |  |  |
| 24 | 9.74 074  | 10 | 9.81 913  | 27    | 0.18 086  | 9.92 160  | 8  | 36    |  |  |  |
| 25 | 9.74 093  | 10 | 9.81 941  | 27    | 0.18 059  | 9.92 152  | 8  | 35    |  |  |  |
| 26 | 9.74 112  | 10 | 9.81 968  | 27    | 0.18 031  | 9.92 144  | 8  | 34    |  |  |  |
| 27 | 9.74 131  | 10 | 9.81 996  | 27    | 0.18 004  | 9.92 135  | 8  | 33    |  |  |  |
| 28 | 9.74 151  | 10 | 9.82 023  | 27    | 0.17 976  | 9.92 127  | 8  | 32    |  |  |  |
| 29 | 9.74 170  | 10 | 9.82 051  | 27    | 0.17 949  | 9.92 119  | 8  | 31    |  |  |  |
| 30 | 9.74 189  | 10 | 9.82 078  | 27    | 0.17 921  | 9.92 110  | 8  | 30    |  |  |  |
| 31 | 9.74 208  | 10 | 9.82 105  | 27    | 0.17 894  | 9.92 102  | 8  | 29    |  |  |  |
| 32 | 9.74 227  | 10 | 9.82 133  | 27    | 0.17 867  | 9.92 094  | 8  | 28    |  |  |  |
| 33 | 9.74 246  | 10 | 9.82 160  | 27    | 0.17 839  | 9.92 085  | 8  | 27    |  |  |  |
| 34 | 9.74 265  | 10 | 9.82 188  | 27    | 0.17 812  | 9.92 077  | 8  | 26    |  |  |  |
| 35 | 9.74 284  | 10 | 9.82 215  | 27    | 0.17 784  | 9.92 069  | 8  | 25    |  |  |  |
| 36 | 9.74 303  | 10 | 9.82 243  | 27    | 0.17 757  | 9.92 060  | 8  | 24    |  |  |  |
| 37 | 9.74 322  | 10 | 9.82 270  | 27    | 0.17 729  | 9.92 052  | 8  | 23    |  |  |  |
| 38 | 9.74 341  | 10 | 9.82 297  | 27    | 0.17 702  | 9.92 043  | 8  | 22    |  |  |  |
| 39 | 9.74 360  | 10 | 9.82 325  | 27    | 0.17 675  | 9.92 035  | 8  | 21    |  |  |  |
| 40 | 9.74 379  | 10 | 9.82 352  | 27    | 0.17 647  | 9.92 027  | 8  | 20    |  |  |  |
| 41 | 9.74 398  | 10 | 9.82 380  | 27    | 0.17 620  | 9.92 018  | 8  | 19    |  |  |  |
| 42 | 9.74 417  | 10 | 9.82 407  | 27    | 0.17 593  | 9.92 010  | 8  | 18    |  |  |  |
| 43 | 9.74 436  | 10 | 9.82 434  | 27    | 0.17 565  | 9.92 001  | 8  | 17    |  |  |  |
| 44 | 9.74 455  | 10 | 9.82 462  | 27    | 0.17 538  | 9.91 993  | 8  | 16    |  |  |  |
| 45 | 9.74 474  | 10 | 9.82 489  | 27    | 0.17 510  | 9.91 984  | 8  | 15    |  |  |  |
| 46 | 9.74 493  | 10 | 9.82 516  | 27    | 0.17 483  | 9.91 976  | 8  | 14    |  |  |  |
| 47 | 9.74 511  | 10 | 9.82 544  | 27    | 0.17 456  | 9.91 967  | 8  | 13    |  |  |  |
| 48 | 9.74 530  | 10 | 9.82 571  | 27    | 0.17 428  | 9.91 959  | 8  | 12    |  |  |  |
| 49 | 9.74 549  | 10 | 9.82 598  | 27    | 0.17 401  | 9.91 951  | 8  | 11    |  |  |  |
| 50 | 9.74 568  | 10 | 9.82 626  | 27    | 0.17 374  | 9.91 942  | 8  | 10    |  |  |  |
| 51 | 9.74 587  | 10 | 9.82 653  | 27    | 0.17 347  | 9.91 934  | 8  | 9     |  |  |  |
| 52 | 9.74 606  | 10 | 9.82 680  | 27    | 0.17 319  | 9.91 925  | 8  | 8     |  |  |  |
| 53 | 9.74 625  | 10 | 9.82 708  | 27    | 0.17 292  | 9.91 917  | 8  | 7     |  |  |  |
| 54 | 9.74 643  | 10 | 9.82 735  | 27    | 0.17 265  | 9.91 908  | 8  | 6     |  |  |  |
| 55 | 9.74 662  | 10 | 9.82 762  | 27    | 0.17 237  | 9.91 900  | 8  | 5     |  |  |  |
| 56 | 9.74 681  | 10 | 9.82 789  | 27    | 0.17 210  | 9.91 891  | 8  | 4     |  |  |  |
| 57 | 9.74 700  | 10 | 9.82 817  | 27    | 0.17 183  | 9.91 883  | 8  | 3     |  |  |  |
| 58 | 9.74 718  | 10 | 9.82 844  | 27    | 0.17 156  | 9.91 874  | 8  | 2     |  |  |  |
| 59 | 9.74 737  | 10 | 9.82 871  | 27    | 0.17 128  | 9.91 866  | 8  | 1     |  |  |  |
| 60 | 9.74 756  | 10 | 9.82 898  | 27    | 0.17 101  | 9.91 857  | 8  | 0     |  |  |  |
|    |           |    |           |       |           |           |    | P. P. |  |  |  |
|    |           |    |           |       |           |           |    |       |  |  |  |
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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |  |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|--|--|--|
| 0  | 9.74 756  | 19 | 9.82 898  | 27    | 0.17 101  | 9.91 857  | 8  | 60 |       |  |  |  |
| 1  | 9.74 775  | 18 | 9.82 926  | 27    | 0.17 074  | 9.91 849  | 8  | 59 |       |  |  |  |
| 2  | 9.74 793  | 19 | 9.82 953  | 27    | 0.17 047  | 9.91 840  | 8  | 58 |       |  |  |  |
| 3  | 9.74 812  | 18 | 9.82 980  | 27    | 0.17 019  | 9.91 832  | 8  | 57 |       |  |  |  |
| 4  | 9.74 831  | 18 | 9.83 007  | 27    | 0.16 992  | 9.91 823  | 9  | 56 |       |  |  |  |
| 5  | 9.74 849  | 19 | 9.83 035  | 27    | 0.16 965  | 9.91 814  | 8  | 55 |       |  |  |  |
| 6  | 9.74 868  | 18 | 9.83 062  | 27    | 0.16 938  | 9.91 806  | 8  | 54 |       |  |  |  |
| 7  | 9.74 887  | 18 | 9.83 089  | 27    | 0.16 910  | 9.91 797  | 8  | 53 |       |  |  |  |
| 8  | 9.74 903  | 19 | 9.83 116  | 27    | 0.16 883  | 9.91 789  | 8  | 52 |       |  |  |  |
| 9  | 9.74 924  | 18 | 9.83 143  | 27    | 0.16 856  | 9.91 780  | 8  | 51 |       |  |  |  |
| 10 | 9.74 943  | 18 | 9.83 171  | 27    | 0.16 829  | 9.91 772  | 8  | 50 |       |  |  |  |
| 11 | 9.74 961  | 18 | 9.83 198  | 27    | 0.16 802  | 9.91 763  | 8  | 49 |       |  |  |  |
| 12 | 9.74 980  | 18 | 9.83 223  | 27    | 0.16 774  | 9.91 755  | 9  | 48 |       |  |  |  |
| 13 | 9.74 998  | 18 | 9.83 252  | 27    | 0.16 747  | 9.91 746  | 8  | 47 |       |  |  |  |
| 14 | 9.75 017  | 19 | 9.83 279  | 27    | 0.16 720  | 9.91 737  | 8  | 46 |       |  |  |  |
| 15 | 9.75 036  | 18 | 9.83 307  | 27    | 0.16 693  | 9.91 729  | 8  | 45 |       |  |  |  |
| 16 | 9.75 054  | 18 | 9.83 334  | 27    | 0.16 666  | 9.91 720  | 8  | 44 |       |  |  |  |
| 17 | 9.75 073  | 18 | 9.83 361  | 27    | 0.16 639  | 9.91 712  | 8  | 43 |       |  |  |  |
| 18 | 9.75 091  | 18 | 9.83 388  | 27    | 0.16 612  | 9.91 703  | 8  | 42 |       |  |  |  |
| 19 | 9.75 110  | 18 | 9.83 413  | 27    | 0.16 584  | 9.91 694  | 8  | 41 |       |  |  |  |
| 20 | 9.75 128  | 18 | 9.83 442  | 27    | 0.16 557  | 9.91 686  | 8  | 40 |       |  |  |  |
| 21 | 9.75 147  | 18 | 9.83 469  | 27    | 0.16 530  | 9.91 677  | 8  | 39 |       |  |  |  |
| 22 | 9.75 163  | 18 | 9.83 496  | 27    | 0.16 503  | 9.91 668  | 8  | 38 |       |  |  |  |
| 23 | 9.75 184  | 18 | 9.83 524  | 27    | 0.16 476  | 9.91 660  | 8  | 37 |       |  |  |  |
| 24 | 9.75 202  | 18 | 9.83 551  | 27    | 0.16 449  | 9.91 651  | 9  | 36 |       |  |  |  |
| 25 | 9.75 221  | 18 | 9.83 578  | 27    | 0.16 422  | 9.91 642  | 8  | 35 |       |  |  |  |
| 26 | 9.75 239  | 18 | 9.83 605  | 27    | 0.16 395  | 9.91 634  | 8  | 34 |       |  |  |  |
| 27 | 9.75 257  | 18 | 9.83 632  | 27    | 0.16 368  | 9.91 625  | 8  | 33 |       |  |  |  |
| 28 | 9.75 276  | 18 | 9.83 659  | 27    | 0.16 340  | 9.91 616  | 8  | 32 |       |  |  |  |
| 29 | 9.75 294  | 18 | 9.83 686  | 27    | 0.16 313  | 9.91 608  | 8  | 31 |       |  |  |  |
| 30 | 9.75 313  | 18 | 9.83 713  | 27    | 0.16 286  | 9.91 599  | 8  | 30 |       |  |  |  |
| 31 | 9.75 331  | 18 | 9.83 740  | 27    | 0.16 259  | 9.91 590  | 8  | 29 |       |  |  |  |
| 32 | 9.75 349  | 18 | 9.83 767  | 27    | 0.16 232  | 9.91 582  | 8  | 28 |       |  |  |  |
| 33 | 9.75 368  | 18 | 9.83 794  | 27    | 0.16 205  | 9.91 573  | 8  | 27 |       |  |  |  |
| 34 | 9.75 386  | 18 | 9.83 821  | 27    | 0.16 178  | 9.91 564  | 8  | 26 |       |  |  |  |
| 35 | 9.75 404  | 18 | 9.83 848  | 27    | 0.16 151  | 9.91 556  | 8  | 25 |       |  |  |  |
| 36 | 9.75 423  | 18 | 9.83 873  | 27    | 0.16 124  | 9.91 547  | 8  | 24 |       |  |  |  |
| 37 | 9.75 441  | 18 | 9.83 902  | 27    | 0.16 097  | 9.91 538  | 8  | 23 |       |  |  |  |
| 38 | 9.75 459  | 18 | 9.83 929  | 27    | 0.16 070  | 9.91 529  | 8  | 22 |       |  |  |  |
| 39 | 9.75 478  | 18 | 9.83 957  | 27    | 0.16 043  | 9.91 521  | 8  | 21 |       |  |  |  |
| 40 | 9.75 496  | 18 | 9.83 984  | 27    | 0.16 016  | 9.91 512  | 8  | 20 |       |  |  |  |
| 41 | 9.75 514  | 18 | 9.84 011  | 27    | 0.15 989  | 9.91 503  | 8  | 19 |       |  |  |  |
| 42 | 9.75 532  | 18 | 9.84 038  | 27    | 0.15 962  | 9.91 495  | 8  | 18 |       |  |  |  |
| 43 | 9.75 551  | 18 | 9.84 065  | 27    | 0.15 935  | 9.91 486  | 8  | 17 |       |  |  |  |
| 44 | 9.75 569  | 18 | 9.84 091  | 26    | 0.15 908  | 9.91 477  | 8  | 16 |       |  |  |  |
| 45 | 9.75 587  | 18 | 9.84 118  | 27    | 0.15 881  | 9.91 468  | 8  | 15 |       |  |  |  |
| 46 | 9.75 603  | 18 | 9.84 145  | 27    | 0.15 854  | 9.91 460  | 8  | 14 |       |  |  |  |
| 47 | 9.75 623  | 18 | 9.84 172  | 27    | 0.15 827  | 9.91 451  | 8  | 13 |       |  |  |  |
| 48 | 9.75 642  | 18 | 9.84 199  | 27    | 0.15 800  | 9.91 442  | 8  | 12 |       |  |  |  |
| 49 | 9.75 660  | 18 | 9.84 226  | 27    | 0.15 773  | 9.91 433  | 8  | 11 |       |  |  |  |
| 50 | 9.75 678  | 18 | 9.84 253  | 27    | 0.15 746  | 9.91 424  | 8  | 10 |       |  |  |  |
| 51 | 9.75 696  | 18 | 9.84 280  | 27    | 0.15 719  | 9.91 416  | 8  | 9  |       |  |  |  |
| 52 | 9.75 714  | 18 | 9.84 307  | 27    | 0.15 692  | 9.91 407  | 8  | 8  |       |  |  |  |
| 53 | 9.75 732  | 18 | 9.84 334  | 27    | 0.15 665  | 9.91 398  | 8  | 7  |       |  |  |  |
| 54 | 9.75 750  | 18 | 9.84 361  | 26    | 0.15 639  | 9.91 389  | 8  | 6  |       |  |  |  |
| 55 | 9.75 769  | 18 | 9.84 388  | 27    | 0.15 612  | 9.91 380  | 8  | 5  |       |  |  |  |
| 56 | 9.75 787  | 18 | 9.84 415  | 27    | 0.15 585  | 9.91 372  | 8  | 4  |       |  |  |  |
| 57 | 9.75 805  | 18 | 9.84 442  | 27    | 0.15 558  | 9.91 363  | 8  | 3  |       |  |  |  |
| 58 | 9.75 823  | 18 | 9.84 469  | 27    | 0.15 531  | 9.91 354  | 8  | 2  |       |  |  |  |
| 59 | 9.75 841  | 18 | 9.84 496  | 27    | 0.15 504  | 9.91 345  | 8  | 1  |       |  |  |  |
| 60 | 9.75 859  | 18 | 9.84 522  | 26    | 0.15 477  | 9.91 336  | 9  | 0  |       |  |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |  |  |  |

|    | 27   | 27   | 26   |
|----|------|------|------|
| 6  | 2.7  | 2.7  | 2.6  |
| 7  | 3.2  | 3.1  | 3.1  |
| 8  | 3.6  | 3.6  | 3.5  |
| 9  | 4.1  | 4.0  | 4.0  |
| 10 | 4.6  | 4.5  | 4.4  |
| 20 | 9.1  | 9.0  | 8.8  |
| 30 | 13.7 | 13.5 | 13.2 |
| 40 | 18.3 | 18.0 | 17.6 |
| 50 | 22.9 | 22.5 | 22.1 |

|    | 19   | 18   | 18   |
|----|------|------|------|
| 6  | 1.9  | 1.8  | 1.8  |
| 7  | 2.2  | 2.1  | 2.1  |
| 8  | 2.5  | 2.4  | 2.4  |
| 9  | 2.8  | 2.8  | 2.7  |
| 10 | 3.1  | 3.1  | 3.0  |
| 20 | 6.3  | 6.1  | 6.0  |
| 30 | 9.5  | 9.2  | 9.0  |
| 40 | 12.6 | 12.3 | 12.0 |
| 50 | 15.8 | 15.4 | 15.0 |

|    | 9   | 8   |
|----|-----|-----|
| 6  | 0.9 | 0.8 |
| 7  | 1.6 | 1.0 |
| 8  | 1.2 | 1.1 |
| 9  | 1.3 | 1.3 |
| 10 | 1.5 | 1.4 |
| 20 | 3.0 | 2.8 |
| 30 | 4.5 | 4.2 |
| 40 | 6.0 | 5.6 |
| 50 | 7.5 | 7.1 |



TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

35°

|           |           |    |           |       |           |           |    |    |  | P. P. |           |       |           |           |    |   |       |
|-----------|-----------|----|-----------|-------|-----------|-----------|----|----|--|-------|-----------|-------|-----------|-----------|----|---|-------|
| /         | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    |  |       |           |       |           |           |    |   |       |
| 0         | 9.75 859  | 18 | 9.84 522  | 27    | 0.15 477  | 9.91 336  | 9  | 60 |  |       |           |       |           |           |    |   |       |
| 1         | 9.75 877  | 18 | 9.84 549  | 27    | 0.15 450  | 9.91 327  | 9  | 59 |  |       |           |       |           |           |    |   |       |
| 2         | 9.75 895  | 18 | 9.84 576  | 27    | 0.15 423  | 9.91 318  | 9  | 58 |  |       |           |       |           |           |    |   |       |
| 3         | 9.75 913  | 18 | 9.84 603  | 26    | 0.15 396  | 9.91 310  | 8  | 57 |  |       |           |       |           |           |    |   |       |
| 4         | 9.75 931  | 18 | 9.84 630  | 27    | 0.15 370  | 9.91 301  | 9  | 56 |  |       |           |       |           |           |    |   |       |
| 5         | 9.75 949  | 18 | 9.84 657  | 27    | 0.15 343  | 9.91 292  | 9  |    |  |       |           |       |           |           |    |   |       |
| 6         | 9.75 967  | 18 | 9.84 684  | 27    | 0.15 316  | 9.91 283  | 8  | 55 |  |       |           |       |           |           |    |   |       |
| 7         | 9.75 985  | 18 | 9.84 711  | 27    | 0.15 289  | 9.91 274  | 9  | 54 |  |       |           |       |           |           |    |   |       |
| 8         | 9.76 003  | 18 | 9.84 737  | 26    | 0.15 262  | 9.91 265  | 9  | 53 |  |       |           |       |           |           |    |   |       |
| 9         | 9.76 021  | 18 | 9.84 764  | 27    | 0.15 235  | 9.91 256  | 9  | 52 |  |       |           |       |           |           |    |   |       |
| 10        | 9.76 039  | 18 | 9.84 791  | 27    | 0.15 208  | 9.91 247  | 9  | 51 |  |       |           |       |           |           |    |   |       |
| 11        | 9.76 057  | 18 | 9.84 818  | 26    | 0.15 182  | 9.91 239  | 8  | 50 |  |       |           |       |           |           |    |   |       |
| 12        | 9.76 075  | 18 | 9.84 845  | 27    | 0.15 155  | 9.91 230  | 9  | 49 |  |       |           |       |           |           |    |   |       |
| 13        | 9.76 092  | 17 | 9.84 871  | 26    | 0.15 128  | 9.91 221  | 9  | 48 |  |       |           |       |           |           |    |   |       |
| 14        | 9.76 110  | 18 | 9.84 898  | 27    | 0.15 101  | 9.91 212  | 9  | 47 |  |       |           |       |           |           |    |   |       |
| 15        | 9.76 128  | 18 | 9.84 925  | 27    | 0.15 074  | 9.91 203  | 9  | 46 |  |       |           |       |           |           |    |   |       |
| 16        | 9.76 146  | 17 | 9.84 952  | 26    | 0.15 048  | 9.91 194  | 9  | 45 |  |       |           |       |           |           |    |   |       |
| 17        | 9.76 164  | 18 | 9.84 979  | 27    | 0.15 021  | 9.91 185  | 8  | 44 |  |       |           |       |           |           |    |   |       |
| 18        | 9.76 182  | 18 | 9.85 005  | 26    | 0.14 994  | 9.91 176  | 9  | 43 |  |       |           |       |           |           |    |   |       |
| 19        | 9.76 200  | 18 | 9.85 032  | 27    | 0.14 967  | 9.91 167  | 9  | 42 |  |       |           |       |           |           |    |   |       |
| 20        | 9.76 217  | 17 | 9.85 059  | 27    | 0.14 940  | 9.91 158  | 9  | 41 |  |       |           |       |           |           |    |   |       |
| 21        | 9.76 235  | 18 | 9.85 086  | 26    | 0.14 914  | 9.91 149  | 9  | 40 |  |       |           |       |           |           |    |   |       |
| 22        | 9.76 253  | 18 | 9.85 113  | 27    | 0.14 887  | 9.91 140  | 9  | 39 |  |       |           |       |           |           |    |   |       |
| 23        | 9.76 271  | 17 | 9.85 139  | 26    | 0.14 860  | 9.91 131  | 9  | 38 |  |       |           |       |           |           |    |   |       |
| 24        | 9.76 289  | 18 | 9.85 166  | 27    | 0.14 833  | 9.91 122  | 9  | 37 |  |       |           |       |           |           |    |   |       |
| 25        | 9.76 306  | 17 | 9.85 193  | 26    | 0.14 807  | 9.91 113  | 9  | 36 |  |       |           |       |           |           |    |   |       |
| 26        | 9.76 324  | 18 | 9.85 220  | 27    | 0.14 780  | 9.91 104  | 9  | 35 |  |       |           |       |           |           |    |   |       |
| 27        | 9.76 342  | 17 | 9.85 246  | 26    | 0.14 753  | 9.91 095  | 9  | 34 |  |       |           |       |           |           |    |   |       |
| 28        | 9.76 360  | 18 | 9.85 273  | 27    | 0.14 726  | 9.91 086  | 9  | 33 |  |       |           |       |           |           |    |   |       |
| 29        | 9.76 377  | 17 | 9.85 300  | 26    | 0.14 700  | 9.91 077  | 9  | 32 |  |       |           |       |           |           |    |   |       |
| 30        | 9.76 395  | 18 | 9.85 327  | 27    | 0.14 673  | 9.91 068  | 9  | 31 |  |       |           |       |           |           |    |   |       |
| 31        | 9.76 413  | 17 | 9.85 353  | 26    | 0.14 646  | 9.91 059  | 9  | 30 |  |       |           |       |           |           |    |   |       |
| 32        | 9.76 431  | 18 | 9.85 380  | 27    | 0.14 620  | 9.91 050  | 9  | 29 |  |       |           |       |           |           |    |   |       |
| 33        | 9.76 448  | 17 | 9.85 407  | 26    | 0.14 593  | 9.91 041  | 9  | 28 |  |       |           |       |           |           |    |   |       |
| 34        | 9.76 466  | 17 | 9.85 433  | 26    | 0.14 566  | 9.91 032  | 9  | 27 |  |       |           |       |           |           |    |   |       |
| 35        | 9.76 484  | 18 | 9.85 460  | 27    | 0.14 539  | 9.91 023  | 9  | 26 |  |       |           |       |           |           |    |   |       |
| 36        | 9.76 501  | 17 | 9.85 487  | 26    | 0.14 513  | 9.91 014  | 9  | 25 |  |       |           |       |           |           |    |   |       |
| 37        | 9.76 519  | 17 | 9.85 513  | 26    | 0.14 486  | 9.91 005  | 9  | 24 |  |       |           |       |           |           |    |   |       |
| 38        | 9.76 536  | 17 | 9.85 540  | 27    | 0.14 459  | 9.90 996  | 9  | 23 |  |       |           |       |           |           |    |   |       |
| 39        | 9.76 554  | 18 | 9.85 567  | 26    | 0.14 433  | 9.90 987  | 9  | 22 |  |       |           |       |           |           |    |   |       |
| 40        | 9.76 572  | 17 | 9.85 594  | 27    | 0.14 406  | 9.90 978  | 9  | 21 |  |       |           |       |           |           |    |   |       |
| 41        | 9.76 589  | 17 | 9.85 620  | 26    | 0.14 379  | 9.90 969  | 9  | 20 |  |       |           |       |           |           |    |   |       |
| 42        | 9.76 607  | 17 | 9.85 647  | 26    | 0.14 353  | 9.90 960  | 9  | 19 |  |       |           |       |           |           |    |   |       |
| 43        | 9.76 624  | 17 | 9.85 673  | 26    | 0.14 326  | 9.90 951  | 9  | 18 |  |       |           |       |           |           |    |   |       |
| 44        | 9.76 642  | 18 | 9.85 700  | 27    | 0.14 299  | 9.90 942  | 9  | 17 |  |       |           |       |           |           |    |   |       |
| 45        | 9.76 660  | 17 | 9.85 727  | 26    | 0.14 273  | 9.90 933  | 9  | 16 |  |       |           |       |           |           |    |   |       |
| 46        | 9.76 677  | 17 | 9.85 753  | 26    | 0.14 246  | 9.90 923  | 9  | 15 |  |       |           |       |           |           |    |   |       |
| 47        | 9.76 695  | 17 | 9.85 780  | 27    | 0.14 219  | 9.90 914  | 9  | 14 |  |       |           |       |           |           |    |   |       |
| 48        | 9.76 712  | 17 | 9.85 807  | 26    | 0.14 193  | 9.90 905  | 9  | 13 |  |       |           |       |           |           |    |   |       |
| 49        | 9.76 730  | 17 | 9.85 833  | 26    | 0.14 166  | 9.90 896  | 9  | 12 |  |       |           |       |           |           |    |   |       |
| 50        | 9.76 747  | 17 | 9.85 860  | 26    | 0.14 140  | 9.90 887  | 9  | 11 |  |       |           |       |           |           |    |   |       |
| 51        | 9.76 765  | 17 | 9.85 887  | 27    | 0.14 113  | 9.90 878  | 9  | 10 |  |       |           |       |           |           |    |   |       |
| 52        | 9.76 782  | 17 | 9.85 913  | 26    | 0.14 086  | 9.90 869  | 9  | 9  |  |       |           |       |           |           |    |   |       |
| 53        | 9.76 800  | 17 | 9.85 940  | 26    | 0.14 060  | 9.90 860  | 9  | 8  |  |       |           |       |           |           |    |   |       |
| 54        | 9.76 817  | 17 | 9.85 966  | 26    | 0.14 033  | 9.90 850  | 9  | 7  |  |       |           |       |           |           |    |   |       |
| 55        | 9.76 835  | 17 | 9.85 993  | 26    | 0.14 007  | 9.90 841  | 9  | 6  |  |       |           |       |           |           |    |   |       |
| 56        | 9.76 852  | 17 | 9.86 020  | 27    | 0.13 980  | 9.90 832  | 9  | 5  |  |       |           |       |           |           |    |   |       |
| 57        | 9.76 869  | 17 | 9.86 046  | 26    | 0.13 953  | 9.90 823  | 9  | 4  |  |       |           |       |           |           |    |   |       |
| 58        | 9.76 887  | 17 | 9.86 073  | 26    | 0.13 927  | 9.90 814  | 9  | 3  |  |       |           |       |           |           |    |   |       |
| 59        | 9.76 904  | 17 | 9.86 099  | 26    | 0.13 900  | 9.90 805  | 9  | 2  |  |       |           |       |           |           |    |   |       |
| 60        | 9.76 922  | 17 | 9.86 126  | 26    | 0.13 874  | 9.90 796  | 9  | 1  |  |       |           |       |           |           |    |   |       |
| Log. Cos. |           |    |           |       |           |           |    |    |  | d.    | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | / |       |
|           |           |    |           |       |           |           |    |    |  |       |           |       |           |           |    |   | P. P. |

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|    |           |    |           |       |           |           |    | P. P. |  |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|--|--|--|
| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |       |  |  |  |
| 0  | 9.76 922  | 17 | 9.86 126  | 26    | 0.13 874  | 9.90 796  | 9  | 60    |  |  |  |
| 1  | 9.76 939  | 17 | 9.86 152  | 26    | 0.13 847  | 9.90 786  | 9  | 59    |  |  |  |
| 2  | 9.76 956  | 17 | 9.86 179  | 27    | 0.13 821  | 9.90 777  | 9  | 58    |  |  |  |
| 3  | 9.76 974  | 17 | 9.86 206  | 26    | 0.13 794  | 9.90 768  | 9  | 57    |  |  |  |
| 4  | 9.76 991  | 17 | 9.86 232  | 26    | 0.13 767  | 9.90 759  | 9  | 56    |  |  |  |
| 5  | 9.77 008  | 17 | 9.86 259  | 26    | 0.13 741  | 9.90 750  | 9  | 55    |  |  |  |
| 6  | 9.77 026  | 17 | 9.86 285  | 26    | 0.13 714  | 9.90 740  | 9  | 54    |  |  |  |
| 7  | 9.77 043  | 17 | 9.86 312  | 26    | 0.13 688  | 9.90 731  | 9  | 53    |  |  |  |
| 8  | 9.77 060  | 17 | 9.86 338  | 26    | 0.13 661  | 9.90 722  | 9  | 52    |  |  |  |
| 9  | 9.77 078  | 17 | 9.86 365  | 26    | 0.13 635  | 9.90 713  | 9  | 51    |  |  |  |
| 10 | 9.77 095  | 17 | 9.86 391  | 26    | 0.13 608  | 9.90 703  | 9  | 50    |  |  |  |
| 11 | 9.77 112  | 17 | 9.86 418  | 26    | 0.13 582  | 9.90 694  | 9  | 49    |  |  |  |
| 12 | 9.77 130  | 17 | 9.86 444  | 26    | 0.13 555  | 9.90 685  | 9  | 48    |  |  |  |
| 13 | 9.77 147  | 17 | 9.86 471  | 26    | 0.13 529  | 9.90 676  | 9  | 47    |  |  |  |
| 14 | 9.77 164  | 17 | 9.86 497  | 26    | 0.13 502  | 9.90 666  | 9  | 46    |  |  |  |
| 15 | 9.77 181  | 17 | 9.86 524  | 26    | 0.13 476  | 9.90 657  | 9  | 45    |  |  |  |
| 16 | 9.77 198  | 17 | 9.86 550  | 26    | 0.13 449  | 9.90 648  | 9  | 44    |  |  |  |
| 17 | 9.77 216  | 17 | 9.86 577  | 26    | 0.13 423  | 9.90 639  | 9  | 43    |  |  |  |
| 18 | 9.77 233  | 17 | 9.86 603  | 26    | 0.13 396  | 9.90 629  | 9  | 42    |  |  |  |
| 19 | 9.77 250  | 17 | 9.86 630  | 26    | 0.13 370  | 9.90 620  | 9  | 41    |  |  |  |
| 20 | 9.77 267  | 17 | 9.86 656  | 26    | 0.13 343  | 9.90 611  | 9  | 40    |  |  |  |
| 21 | 9.77 284  | 17 | 9.86 683  | 26    | 0.13 317  | 9.90 602  | 9  | 39    |  |  |  |
| 22 | 9.77 302  | 17 | 9.86 709  | 26    | 0.13 290  | 9.90 592  | 9  | 38    |  |  |  |
| 23 | 9.77 319  | 17 | 9.86 736  | 26    | 0.13 264  | 9.90 583  | 9  | 37    |  |  |  |
| 24 | 9.77 336  | 17 | 9.86 762  | 26    | 0.13 237  | 9.90 574  | 9  | 36    |  |  |  |
| 25 | 9.77 353  | 17 | 9.86 788  | 26    | 0.13 211  | 9.90 564  | 9  | 35    |  |  |  |
| 26 | 9.77 370  | 17 | 9.86 815  | 26    | 0.13 185  | 9.90 555  | 9  | 34    |  |  |  |
| 27 | 9.77 387  | 17 | 9.86 841  | 26    | 0.13 158  | 9.90 546  | 9  | 33    |  |  |  |
| 28 | 9.77 404  | 17 | 9.86 868  | 26    | 0.13 132  | 9.90 536  | 9  | 32    |  |  |  |
| 29 | 9.77 421  | 17 | 9.86 894  | 26    | 0.13 105  | 9.90 527  | 9  | 31    |  |  |  |
| 30 | 9.77 439  | 17 | 9.86 921  | 26    | 0.13 079  | 9.90 518  | 9  | 30    |  |  |  |
| 31 | 9.77 456  | 17 | 9.86 947  | 26    | 0.13 052  | 9.90 508  | 9  | 29    |  |  |  |
| 32 | 9.77 473  | 17 | 9.86 973  | 26    | 0.13 026  | 9.90 499  | 9  | 28    |  |  |  |
| 33 | 9.77 490  | 17 | 9.87 000  | 26    | 0.13 000  | 9.90 490  | 9  | 27    |  |  |  |
| 34 | 9.77 507  | 17 | 9.87 026  | 26    | 0.12 973  | 9.90 480  | 9  | 26    |  |  |  |
| 35 | 9.77 524  | 17 | 9.87 053  | 26    | 0.12 947  | 9.90 471  | 9  | 25    |  |  |  |
| 36 | 9.77 541  | 17 | 9.87 079  | 26    | 0.12 920  | 9.90 461  | 9  | 24    |  |  |  |
| 37 | 9.77 558  | 17 | 9.87 105  | 26    | 0.12 894  | 9.90 452  | 9  | 23    |  |  |  |
| 38 | 9.77 575  | 17 | 9.87 132  | 26    | 0.12 868  | 9.90 443  | 9  | 22    |  |  |  |
| 39 | 9.77 592  | 17 | 9.87 158  | 26    | 0.12 841  | 9.90 433  | 9  | 21    |  |  |  |
| 40 | 9.77 609  | 17 | 9.87 185  | 26    | 0.12 815  | 9.90 424  | 9  | 20    |  |  |  |
| 41 | 9.77 626  | 17 | 9.87 211  | 26    | 0.12 789  | 9.90 414  | 9  | 19    |  |  |  |
| 42 | 9.77 643  | 17 | 9.87 237  | 26    | 0.12 762  | 9.90 405  | 9  | 18    |  |  |  |
| 43 | 9.77 660  | 17 | 9.87 264  | 26    | 0.12 736  | 9.90 396  | 9  | 17    |  |  |  |
| 44 | 9.77 677  | 16 | 9.87 290  | 26    | 0.12 709  | 9.90 386  | 9  | 16    |  |  |  |
| 45 | 9.77 693  | 17 | 9.87 316  | 26    | 0.12 683  | 9.90 377  | 9  | 15    |  |  |  |
| 46 | 9.77 710  | 17 | 9.87 343  | 26    | 0.12 657  | 9.90 367  | 9  | 14    |  |  |  |
| 47 | 9.77 727  | 17 | 9.87 369  | 26    | 0.12 630  | 9.90 358  | 9  | 13    |  |  |  |
| 48 | 9.77 744  | 17 | 9.87 395  | 26    | 0.12 604  | 9.90 348  | 9  | 12    |  |  |  |
| 49 | 9.77 761  | 16 | 9.87 422  | 26    | 0.12 578  | 9.90 339  | 9  | 11    |  |  |  |
| 50 | 9.77 778  | 17 | 9.87 448  | 26    | 0.12 551  | 9.90 330  | 9  | 10    |  |  |  |
| 51 | 9.77 795  | 17 | 9.87 474  | 26    | 0.12 525  | 9.90 320  | 9  | 9     |  |  |  |
| 52 | 9.77 812  | 16 | 9.87 501  | 26    | 0.12 499  | 9.90 311  | 9  | 8     |  |  |  |
| 53 | 9.77 828  | 17 | 9.87 527  | 26    | 0.12 472  | 9.90 301  | 9  | 7     |  |  |  |
| 54 | 9.77 845  | 17 | 9.87 553  | 26    | 0.12 446  | 9.90 292  | 9  | 6     |  |  |  |
| 55 | 9.77 862  | 16 | 9.87 580  | 26    | 0.12 420  | 9.90 282  | 9  | 5     |  |  |  |
| 56 | 9.77 879  | 17 | 9.87 606  | 26    | 0.12 393  | 9.90 273  | 9  | 4     |  |  |  |
| 57 | 9.77 896  | 17 | 9.87 632  | 26    | 0.12 367  | 9.90 263  | 9  | 3     |  |  |  |
| 58 | 9.77 913  | 16 | 9.87 659  | 26    | 0.12 341  | 9.90 254  | 9  | 2     |  |  |  |
| 59 | 9.77 929  | 17 | 9.87 685  | 26    | 0.12 315  | 9.90 244  | 9  | 1     |  |  |  |
| 60 | 9.77 946  | 17 | 9.87 711  | 26    | 0.12 288  | 9.90 235  | 9  | 0     |  |  |  |
|    |           |    |           |       |           |           |    | P. P. |  |  |  |



TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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| °  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.77 946  | 16 | 9.87 711  | 26    | 0.12 288  | 9.90 235  | 9  | 60 |       |
| 1  | 9.77 963  | 17 | 9.87 737  | 26    | 0.12 262  | 9.90 225  | 9  | 59 |       |
| 2  | 9.77 980  | 16 | 9.87 764  | 26    | 0.12 236  | 9.90 216  | 9  | 58 |       |
| 3  | 9.77 996  | 17 | 9.87 790  | 26    | 0.12 209  | 9.90 206  | 10 | 57 |       |
| 4  | 9.78 013  | 16 | 9.87 816  | 26    | 0.12 183  | 9.90 196  | 9  | 56 |       |
| 5  | 9.78 030  | 16 | 9.87 843  | 26    | 0.12 157  | 9.90 187  | 9  | 55 |       |
| 6  | 9.78 046  | 17 | 9.87 869  | 26    | 0.12 131  | 9.90 177  | 9  | 54 |       |
| 7  | 9.78 063  | 16 | 9.87 895  | 26    | 0.12 104  | 9.90 168  | 9  | 53 |       |
| 8  | 9.78 080  | 17 | 9.87 921  | 26    | 0.12 078  | 9.90 158  | 9  | 52 |       |
| 9  | 9.78 097  | 16 | 9.87 948  | 26    | 0.12 052  | 9.90 149  | 9  | 51 |       |
| 10 | 9.78 113  | 16 | 9.87 974  | 26    | 0.12 026  | 9.90 139  | 9  | 50 |       |
| 11 | 9.78 130  | 17 | 9.88 000  | 26    | 0.11 999  | 9.90 130  | 10 | 49 |       |
| 12 | 9.78 147  | 16 | 9.88 026  | 26    | 0.11 973  | 9.90 120  | 9  | 48 |       |
| 13 | 9.78 163  | 16 | 9.88 053  | 26    | 0.11 947  | 9.90 110  | 9  | 47 |       |
| 14 | 9.78 180  | 16 | 9.88 079  | 26    | 0.11 921  | 9.90 101  | 9  | 46 |       |
| 15 | 9.78 196  | 16 | 9.88 105  | 26    | 0.11 895  | 9.90 091  | 9  | 45 |       |
| 16 | 9.78 213  | 17 | 9.88 131  | 26    | 0.11 868  | 9.90 082  | 10 | 44 |       |
| 17 | 9.78 230  | 16 | 9.88 157  | 26    | 0.11 842  | 9.90 072  | 9  | 43 |       |
| 18 | 9.78 246  | 16 | 9.88 184  | 26    | 0.11 816  | 9.90 062  | 9  | 42 |       |
| 19 | 9.78 263  | 16 | 9.88 210  | 26    | 0.11 790  | 9.90 053  | 9  | 41 |       |
| 20 | 9.78 279  | 16 | 9.88 236  | 26    | 0.11 763  | 9.90 043  | 10 | 40 |       |
| 21 | 9.78 296  | 16 | 9.88 262  | 26    | 0.11 737  | 9.90 033  | 9  | 39 |       |
| 22 | 9.78 312  | 16 | 9.88 288  | 26    | 0.11 711  | 9.90 024  | 9  | 38 |       |
| 23 | 9.78 329  | 17 | 9.88 315  | 26    | 0.11 685  | 9.90 014  | 10 | 37 |       |
| 24 | 9.78 346  | 16 | 9.88 341  | 26    | 0.11 659  | 9.90 004  | 9  | 36 |       |
| 25 | 9.78 362  | 16 | 9.88 367  | 26    | 0.11 633  | 9.89 995  | 9  | 35 |       |
| 26 | 9.78 379  | 16 | 9.88 393  | 26    | 0.11 606  | 9.89 985  | 10 | 34 |       |
| 27 | 9.78 395  | 16 | 9.88 419  | 26    | 0.11 580  | 9.89 975  | 9  | 33 |       |
| 28 | 9.78 412  | 16 | 9.88 445  | 26    | 0.11 554  | 9.89 966  | 9  | 32 |       |
| 29 | 9.78 428  | 16 | 9.88 472  | 26    | 0.11 528  | 9.89 956  | 10 | 31 |       |
| 30 | 9.78 444  | 16 | 9.88 498  | 26    | 0.11 502  | 9.89 946  | 9  | 30 |       |
| 31 | 9.78 461  | 16 | 9.88 524  | 26    | 0.11 476  | 9.89 937  | 9  | 29 |       |
| 32 | 9.78 477  | 16 | 9.88 550  | 26    | 0.11 449  | 9.89 927  | 10 | 28 |       |
| 33 | 9.78 494  | 16 | 9.88 576  | 26    | 0.11 423  | 9.89 917  | 9  | 27 |       |
| 34 | 9.78 510  | 16 | 9.88 602  | 26    | 0.11 397  | 9.89 908  | 10 | 26 |       |
| 35 | 9.78 527  | 16 | 9.88 629  | 26    | 0.11 371  | 9.89 898  | 9  | 25 |       |
| 36 | 9.78 543  | 16 | 9.88 655  | 26    | 0.11 345  | 9.89 888  | 10 | 24 |       |
| 37 | 9.78 559  | 16 | 9.88 681  | 26    | 0.11 319  | 9.89 878  | 9  | 23 |       |
| 38 | 9.78 576  | 16 | 9.88 707  | 26    | 0.11 293  | 9.89 869  | 10 | 22 |       |
| 39 | 9.78 592  | 16 | 9.88 733  | 26    | 0.11 266  | 9.89 859  | 9  | 21 |       |
| 40 | 9.78 609  | 16 | 9.88 759  | 26    | 0.11 240  | 9.89 849  | 10 | 20 |       |
| 41 | 9.78 625  | 16 | 9.88 785  | 26    | 0.11 214  | 9.89 839  | 9  | 19 |       |
| 42 | 9.78 641  | 16 | 9.88 811  | 26    | 0.11 188  | 9.89 830  | 10 | 18 |       |
| 43 | 9.78 658  | 16 | 9.88 838  | 26    | 0.11 162  | 9.89 820  | 9  | 17 |       |
| 44 | 9.78 674  | 16 | 9.88 864  | 26    | 0.11 136  | 9.89 810  | 10 | 16 |       |
| 45 | 9.78 690  | 16 | 9.88 890  | 26    | 0.11 110  | 9.89 800  | 9  | 15 |       |
| 46 | 9.78 707  | 16 | 9.88 916  | 26    | 0.11 084  | 9.89 791  | 10 | 14 |       |
| 47 | 9.78 723  | 16 | 9.88 942  | 26    | 0.11 058  | 9.89 781  | 9  | 13 |       |
| 48 | 9.78 739  | 16 | 9.88 968  | 26    | 0.11 032  | 9.89 771  | 10 | 12 |       |
| 49 | 9.78 755  | 16 | 9.88 994  | 26    | 0.11 005  | 9.89 761  | 9  | 11 |       |
| 50 | 9.78 772  | 16 | 9.89 020  | 26    | 0.10 979  | 9.89 751  | 10 | 10 |       |
| 51 | 9.78 788  | 16 | 9.89 046  | 26    | 0.10 953  | 9.89 742  | 9  | 9  |       |
| 52 | 9.78 804  | 16 | 9.89 072  | 26    | 0.10 927  | 9.89 732  | 10 | 8  |       |
| 53 | 9.78 821  | 16 | 9.89 098  | 26    | 0.10 901  | 9.89 722  | 9  | 7  |       |
| 54 | 9.78 837  | 16 | 9.89 124  | 26    | 0.10 875  | 9.89 712  | 10 | 6  |       |
| 55 | 9.78 853  | 16 | 9.89 150  | 26    | 0.10 849  | 9.89 702  | 9  | 5  |       |
| 56 | 9.78 869  | 16 | 9.89 177  | 26    | 0.10 823  | 9.89 692  | 10 | 4  |       |
| 57 | 9.78 885  | 16 | 9.89 203  | 26    | 0.10 797  | 9.89 683  | 9  | 3  |       |
| 58 | 9.78 902  | 16 | 9.89 229  | 26    | 0.10 771  | 9.89 673  | 10 | 2  |       |
| 59 | 9.78 918  | 16 | 9.89 255  | 26    | 0.10 745  | 9.89 663  | 9  | 1  |       |
| 60 | 9.78 934  | 16 | 9.89 281  | 26    | 0.10 719  | 9.89 653  | 10 | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

26

26

|    |      |      |
|----|------|------|
| 6  | 2.6  | 2.6  |
| 7  | 3.1  | 3.0  |
| 8  | 3.5  | 3.4  |
| 9  | 4.0  | 3.9  |
| 10 | 4.4  | 4.3  |
| 20 | 8.8  | 8.6  |
| 30 | 13.2 | 13.0 |
| 40 | 17.6 | 17.3 |
| 50 | 22.1 | 21.6 |

17

16

16

|    |      |      |      |
|----|------|------|------|
| 6  | 1.7  | 1.6  | 1.6  |
| 7  | 2.0  | 1.9  | 1.8  |
| 8  | 2.2  | 2.2  | 2.1  |
| 9  | 2.5  | 2.5  | 2.4  |
| 10 | 2.8  | 2.7  | 2.6  |
| 20 | 5.6  | 5.5  | 5.3  |
| 30 | 8.5  | 8.2  | 8.0  |
| 40 | 11.3 | 11.0 | 10.6 |
| 50 | 14.1 | 13.7 | 13.3 |

10

9

|    |     |     |
|----|-----|-----|
| 6  | 1.0 | 0.9 |
| 7  | 1.1 | 1.1 |
| 8  | 1.3 | 1.2 |
| 9  | 1.5 | 1.4 |
| 10 | 1.6 | 1.6 |
| 20 | 3.3 | 3.1 |
| 30 | 5.0 | 4.7 |
| 40 | 6.6 | 6.3 |
| 50 | 8.3 | 7.9 |

52°

385

| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |  | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|--|-------|
| 0  | 9.78 934  | 16 | 9.89 281  | 26    | 0.10 719  | 9.89 653  | 60 |  |       |
| 1  | 9.78 950  | 16 | 9.89 307  | 26    | 0.10 693  | 9.89 643  | 59 |  |       |
| 2  | 9.78 966  | 16 | 9.89 333  | 26    | 0.10 667  | 9.89 633  | 58 |  |       |
| 3  | 9.78 982  | 16 | 9.89 359  | 26    | 0.10 641  | 9.89 623  | 57 |  |       |
| 4  | 9.78 999  | 16 | 9.89 385  | 26    | 0.10 615  | 9.89 613  | 56 |  |       |
| 5  | 9.79 015  | 16 | 9.89 411  | 26    | 0.10 589  | 9.89 604  | 55 |  |       |
| 6  | 9.79 031  | 16 | 9.89 437  | 26    | 0.10 563  | 9.89 594  | 54 |  |       |
| 7  | 9.79 047  | 16 | 9.89 463  | 26    | 0.10 537  | 9.89 584  | 53 |  |       |
| 8  | 9.79 063  | 16 | 9.89 489  | 26    | 0.10 511  | 9.89 574  | 52 |  |       |
| 9  | 9.79 079  | 16 | 9.89 515  | 26    | 0.10 485  | 9.89 564  | 51 |  |       |
| 10 | 9.79 095  | 16 | 9.89 541  | 26    | 0.10 459  | 9.89 554  | 50 |  |       |
| 11 | 9.79 111  | 16 | 9.89 567  | 26    | 0.10 433  | 9.89 544  | 49 |  |       |
| 12 | 9.79 127  | 16 | 9.89 593  | 26    | 0.10 407  | 9.89 534  | 48 |  |       |
| 13 | 9.79 143  | 16 | 9.89 619  | 26    | 0.10 381  | 9.89 524  | 47 |  |       |
| 14 | 9.79 159  | 16 | 9.89 645  | 26    | 0.10 355  | 9.89 514  | 46 |  |       |
| 15 | 9.79 175  | 16 | 9.89 671  | 26    | 0.10 329  | 9.89 504  | 45 |  |       |
| 16 | 9.79 191  | 16 | 9.89 697  | 26    | 0.10 303  | 9.89 494  | 44 |  |       |
| 17 | 9.79 207  | 16 | 9.89 723  | 26    | 0.10 277  | 9.89 484  | 43 |  |       |
| 18 | 9.79 223  | 16 | 9.89 749  | 26    | 0.10 251  | 9.89 474  | 42 |  |       |
| 19 | 9.79 239  | 16 | 9.89 775  | 26    | 0.10 225  | 9.89 464  | 41 |  |       |
| 20 | 9.79 255  | 16 | 9.89 801  | 26    | 0.10 199  | 9.89 454  | 40 |  |       |
| 21 | 9.79 271  | 16 | 9.89 827  | 26    | 0.10 173  | 9.89 444  | 39 |  |       |
| 22 | 9.79 287  | 16 | 9.89 853  | 26    | 0.10 147  | 9.89 434  | 38 |  |       |
| 23 | 9.79 303  | 16 | 9.89 879  | 26    | 0.10 121  | 9.89 424  | 37 |  |       |
| 24 | 9.79 319  | 16 | 9.89 905  | 26    | 0.10 095  | 9.89 414  | 36 |  |       |
| 25 | 9.79 335  | 16 | 9.89 931  | 26    | 0.10 069  | 9.89 404  | 35 |  |       |
| 26 | 9.79 351  | 16 | 9.89 957  | 23    | 0.10 043  | 9.89 394  | 34 |  |       |
| 27 | 9.79 367  | 13 | 9.89 983  | 26    | 0.10 017  | 9.89 384  | 33 |  |       |
| 28 | 9.79 383  | 16 | 9.90 008  | 26    | 0.09 991  | 9.89 374  | 32 |  |       |
| 29 | 9.79 399  | 16 | 9.90 034  | 26    | 0.09 965  | 9.89 364  | 31 |  |       |
| 30 | 9.79 415  | 16 | 9.90 060  | 26    | 0.09 939  | 9.89 354  | 30 |  |       |
| 31 | 9.79 431  | 13 | 9.90 086  | 26    | 0.09 913  | 9.89 344  | 29 |  |       |
| 32 | 9.79 446  | 16 | 9.90 112  | 26    | 0.09 887  | 9.89 334  | 28 |  |       |
| 33 | 9.79 462  | 16 | 9.90 138  | 23    | 0.09 861  | 9.89 324  | 27 |  |       |
| 34 | 9.79 478  | 13 | 9.90 164  | 26    | 0.09 836  | 9.89 314  | 26 |  |       |
| 35 | 9.79 494  | 16 | 9.90 190  | 26    | 0.09 810  | 9.89 304  | 25 |  |       |
| 36 | 9.79 510  | 16 | 9.90 216  | 26    | 0.09 784  | 9.89 294  | 24 |  |       |
| 37 | 9.79 526  | 13 | 9.90 242  | 26    | 0.09 758  | 9.89 284  | 23 |  |       |
| 38 | 9.79 541  | 16 | 9.90 268  | 26    | 0.09 732  | 9.89 274  | 22 |  |       |
| 39 | 9.79 557  | 16 | 9.90 294  | 23    | 0.09 706  | 9.89 264  | 21 |  |       |
| 40 | 9.79 573  | 13 | 9.90 319  | 26    | 0.09 680  | 9.89 253  | 20 |  |       |
| 41 | 9.79 589  | 16 | 9.90 345  | 26    | 0.09 654  | 9.89 243  | 19 |  |       |
| 42 | 9.79 605  | 13 | 9.90 371  | 26    | 0.09 628  | 9.89 233  | 18 |  |       |
| 43 | 9.79 620  | 16 | 9.90 397  | 23    | 0.09 602  | 9.89 223  | 17 |  |       |
| 44 | 9.79 636  | 13 | 9.90 423  | 26    | 0.09 577  | 9.89 213  | 16 |  |       |
| 45 | 9.79 652  | 16 | 9.90 449  | 26    | 0.09 551  | 9.89 203  | 15 |  |       |
| 46 | 9.79 668  | 13 | 9.90 475  | 26    | 0.09 525  | 9.89 193  | 14 |  |       |
| 47 | 9.79 683  | 16 | 9.90 501  | 23    | 0.09 499  | 9.89 182  | 13 |  |       |
| 48 | 9.79 699  | 13 | 9.90 526  | 26    | 0.09 473  | 9.89 172  | 12 |  |       |
| 49 | 9.79 715  | 13 | 9.90 552  | 26    | 0.09 447  | 9.89 162  | 11 |  |       |
| 50 | 9.79 730  | 16 | 9.90 578  | 26    | 0.09 421  | 9.89 152  | 10 |  |       |
| 51 | 9.79 746  | 13 | 9.90 604  | 23    | 0.09 395  | 9.89 142  | 9  |  |       |
| 52 | 9.79 762  | 16 | 9.90 630  | 26    | 0.09 370  | 9.89 132  | 8  |  |       |
| 53 | 9.79 777  | 13 | 9.90 656  | 26    | 0.09 344  | 9.89 121  | 7  |  |       |
| 54 | 9.79 793  | 16 | 9.90 682  | 23    | 0.09 318  | 9.89 111  | 6  |  |       |
| 55 | 9.79 809  | 13 | 9.90 707  | 26    | 0.09 292  | 9.89 101  | 5  |  |       |
| 56 | 9.79 824  | 16 | 9.90 733  | 26    | 0.09 266  | 9.89 091  | 4  |  |       |
| 57 | 9.79 840  | 13 | 9.90 759  | 26    | 0.09 240  | 9.89 081  | 3  |  |       |
| 58 | 9.79 856  | 13 | 9.90 785  | 23    | 0.09 214  | 9.89 070  | 2  |  |       |
| 59 | 9.79 871  | 13 | 9.90 811  | 26    | 0.09 189  | 9.89 060  | 1  |  |       |
| 60 | 9.79 887  | 13 | 9.90 837  | 26    | 0.09 163  | 9.89 050  | 0  |  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |  | P. P. |

|    |      |      |
|----|------|------|
|    | 26   | 23   |
| 6  | 2.6  | 2.5  |
| 7  | 3.0  | 3.0  |
| 8  | 3.4  | 3.4  |
| 9  | 3.9  | 3.8  |
| 10 | 4.3  | 4.2  |
| 20 | 8.6  | 8.5  |
| 30 | 13.0 | 12.7 |
| 40 | 17.3 | 17.0 |
| 50 | 21.6 | 21.2 |

|    |      |      |
|----|------|------|
|    | 16   | 15   |
| 6  | 1.6  | 1.5  |
| 7  | 1.9  | 1.8  |
| 8  | 2.2  | 2.0  |
| 9  | 2.5  | 2.3  |
| 10 | 2.7  | 2.6  |
| 20 | 5.5  | 5.1  |
| 30 | 8.2  | 7.7  |
| 40 | 11.0 | 10.3 |
| 50 | 13.7 | 12.9 |

|    |     |     |
|----|-----|-----|
|    | 10  | 9   |
| 6  | 1.0 | 0.9 |
| 7  | 1.2 | 1.1 |
| 8  | 1.4 | 1.2 |
| 9  | 1.6 | 1.4 |
| 10 | 1.7 | 1.6 |
| 20 | 3.5 | 3.1 |
| 30 | 5.2 | 4.7 |
| 40 | 7.0 | 6.3 |
| 50 | 8.7 | 7.9 |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

39°

|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|--|--|
| 0  | 9.79 887  | 16 | 9.90 837  | 26    | 0.09 163  | 9.89 050  | 10 | 60 |       |  |  |
| 1  | 9.79 903  | 15 | 9.90 863  | 25    | 0.09 137  | 9.89 040  | 10 | 59 |       |  |  |
| 2  | 9.79 918  | 15 | 9.90 888  | 26    | 0.09 111  | 9.89 030  | 10 | 58 |       |  |  |
| 3  | 9.79 934  | 15 | 9.90 914  | 25    | 0.09 085  | 9.89 019  | 10 | 57 |       |  |  |
| 4  | 9.79 949  | 15 | 9.90 940  | 26    | 0.09 060  | 9.89 009  | 10 | 56 |       |  |  |
| 5  | 9.79 965  | 15 | 9.90 966  | 26    | 0.09 034  | 9.88 999  | 10 | 55 |       |  |  |
| 6  | 9.79 980  | 15 | 9.90 992  | 25    | 0.09 008  | 9.88 989  | 10 | 54 |       |  |  |
| 7  | 9.79 996  | 15 | 9.91 017  | 26    | 0.08 982  | 9.88 978  | 10 | 53 |       |  |  |
| 8  | 9.80 011  | 15 | 9.91 043  | 26    | 0.08 956  | 9.88 968  | 10 | 52 |       |  |  |
| 9  | 9.80 027  | 15 | 9.91 069  | 25    | 0.08 930  | 9.88 958  | 10 | 51 |       |  |  |
| 10 | 9.80 042  | 15 | 9.91 095  | 26    | 0.08 905  | 9.88 947  | 10 | 50 |       |  |  |
| 11 | 9.80 058  | 15 | 9.91 121  | 25    | 0.08 879  | 9.88 937  | 10 | 49 |       |  |  |
| 12 | 9.80 073  | 15 | 9.91 146  | 26    | 0.08 853  | 9.88 927  | 10 | 48 |       |  |  |
| 13 | 9.80 089  | 15 | 9.91 172  | 25    | 0.08 827  | 9.88 917  | 10 | 47 |       |  |  |
| 14 | 9.80 104  | 15 | 9.91 198  | 26    | 0.08 802  | 9.88 906  | 10 | 46 |       |  |  |
| 15 | 9.80 120  | 15 | 9.91 224  | 26    | 0.08 776  | 9.88 896  | 10 | 45 |       |  |  |
| 16 | 9.80 135  | 15 | 9.91 250  | 25    | 0.08 750  | 9.88 886  | 10 | 44 |       |  |  |
| 17 | 9.80 151  | 15 | 9.91 275  | 26    | 0.08 724  | 9.88 875  | 10 | 43 |       |  |  |
| 18 | 9.80 166  | 15 | 9.91 301  | 25    | 0.08 698  | 9.88 865  | 10 | 42 |       |  |  |
| 19 | 9.80 182  | 15 | 9.91 327  | 26    | 0.08 673  | 9.88 855  | 10 | 41 |       |  |  |
| 20 | 9.80 197  | 15 | 9.91 353  | 25    | 0.08 647  | 9.88 844  | 10 | 40 |       |  |  |
| 21 | 9.80 213  | 15 | 9.91 378  | 26    | 0.08 621  | 9.88 834  | 10 | 39 |       |  |  |
| 22 | 9.80 228  | 15 | 9.91 404  | 25    | 0.08 595  | 9.88 823  | 10 | 38 |       |  |  |
| 23 | 9.80 243  | 15 | 9.91 430  | 26    | 0.08 570  | 9.88 813  | 10 | 37 |       |  |  |
| 24 | 9.80 259  | 15 | 9.91 456  | 25    | 0.08 544  | 9.88 803  | 10 | 36 |       |  |  |
| 25 | 9.80 274  | 15 | 9.91 481  | 26    | 0.08 518  | 9.88 792  | 10 | 35 |       |  |  |
| 26 | 9.80 289  | 15 | 9.91 507  | 25    | 0.08 492  | 9.88 782  | 10 | 34 |       |  |  |
| 27 | 9.80 305  | 15 | 9.91 533  | 26    | 0.08 467  | 9.88 772  | 10 | 33 |       |  |  |
| 28 | 9.80 320  | 15 | 9.91 559  | 25    | 0.08 441  | 9.88 761  | 10 | 32 |       |  |  |
| 29 | 9.80 335  | 15 | 9.91 584  | 26    | 0.08 415  | 9.88 751  | 10 | 31 |       |  |  |
| 30 | 9.80 351  | 15 | 9.91 610  | 25    | 0.08 389  | 9.88 740  | 10 | 30 |       |  |  |
| 31 | 9.80 366  | 15 | 9.91 636  | 26    | 0.08 364  | 9.88 730  | 10 | 29 |       |  |  |
| 32 | 9.80 381  | 15 | 9.91 662  | 25    | 0.08 338  | 9.88 720  | 10 | 28 |       |  |  |
| 33 | 9.80 397  | 15 | 9.91 687  | 26    | 0.08 312  | 9.88 709  | 10 | 27 |       |  |  |
| 34 | 9.80 412  | 15 | 9.91 713  | 25    | 0.08 286  | 9.88 699  | 10 | 26 |       |  |  |
| 35 | 9.80 427  | 15 | 9.91 739  | 26    | 0.08 261  | 9.88 688  | 10 | 25 |       |  |  |
| 36 | 9.80 443  | 15 | 9.91 765  | 25    | 0.08 235  | 9.88 678  | 10 | 24 |       |  |  |
| 37 | 9.80 458  | 15 | 9.91 790  | 26    | 0.08 209  | 9.88 667  | 10 | 23 |       |  |  |
| 38 | 9.80 473  | 15 | 9.91 816  | 25    | 0.08 183  | 9.88 657  | 10 | 22 |       |  |  |
| 39 | 9.80 488  | 15 | 9.91 842  | 26    | 0.08 158  | 9.88 646  | 10 | 21 |       |  |  |
| 40 | 9.80 504  | 15 | 9.91 867  | 25    | 0.08 132  | 9.88 636  | 10 | 20 |       |  |  |
| 41 | 9.80 519  | 15 | 9.91 893  | 26    | 0.08 106  | 9.88 625  | 10 | 19 |       |  |  |
| 42 | 9.80 534  | 15 | 9.91 919  | 25    | 0.08 081  | 9.88 615  | 10 | 18 |       |  |  |
| 43 | 9.80 549  | 15 | 9.91 945  | 26    | 0.08 055  | 9.88 604  | 10 | 17 |       |  |  |
| 44 | 9.80 564  | 15 | 9.91 970  | 25    | 0.08 029  | 9.88 594  | 10 | 16 |       |  |  |
| 45 | 9.80 580  | 15 | 9.91 996  | 26    | 0.08 004  | 9.88 583  | 10 | 15 |       |  |  |
| 46 | 9.80 595  | 15 | 9.92 022  | 25    | 0.07 978  | 9.88 573  | 10 | 14 |       |  |  |
| 47 | 9.80 610  | 15 | 9.92 047  | 26    | 0.07 952  | 9.88 562  | 10 | 13 |       |  |  |
| 48 | 9.80 625  | 15 | 9.92 073  | 25    | 0.07 926  | 9.88 552  | 10 | 12 |       |  |  |
| 49 | 9.80 640  | 15 | 9.92 099  | 26    | 0.07 901  | 9.88 541  | 10 | 11 |       |  |  |
| 50 | 9.80 655  | 15 | 9.92 124  | 25    | 0.07 875  | 9.88 531  | 10 | 10 |       |  |  |
| 51 | 9.80 671  | 15 | 9.92 150  | 26    | 0.07 849  | 9.88 520  | 10 | 9  |       |  |  |
| 52 | 9.80 686  | 15 | 9.92 176  | 25    | 0.07 824  | 9.88 510  | 10 | 8  |       |  |  |
| 53 | 9.80 701  | 15 | 9.92 201  | 26    | 0.07 798  | 9.88 499  | 10 | 7  |       |  |  |
| 54 | 9.80 716  | 15 | 9.92 227  | 25    | 0.07 772  | 9.88 489  | 10 | 6  |       |  |  |
| 55 | 9.80 731  | 15 | 9.92 253  | 26    | 0.07 747  | 9.88 478  | 10 | 5  |       |  |  |
| 56 | 9.80 746  | 15 | 9.92 278  | 25    | 0.07 721  | 9.88 467  | 10 | 4  |       |  |  |
| 57 | 9.80 761  | 15 | 9.92 304  | 26    | 0.07 695  | 9.88 457  | 10 | 3  |       |  |  |
| 58 | 9.80 776  | 15 | 9.92 330  | 25    | 0.07 670  | 9.88 446  | 10 | 2  |       |  |  |
| 59 | 9.80 791  | 15 | 9.92 355  | 26    | 0.07 644  | 9.88 436  | 10 | 1  |       |  |  |
| 60 | 9.80 806  | 15 | 9.92 381  | 26    | 0.07 618  | 9.88 425  | 10 | 0  |       |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |  |  |

|    |      |      |
|----|------|------|
|    | 26   | 25   |
| 6  | 2.6  | 2.3  |
| 7  | 3.0  | 3.0  |
| 8  | 3.4  | 3.4  |
| 9  | 3.9  | 3.8  |
| 10 | 4.3  | 4.2  |
| 20 | 8.6  | 8.5  |
| 30 | 13.0 | 12.7 |
| 40 | 17.3 | 17.0 |
| 50 | 21.6 | 21.2 |

|    |      |      |      |
|----|------|------|------|
|    | 16   | 15   | 15   |
| 6  | 1.6  | 1.5  | 1.5  |
| 7  | 1.8  | 1.8  | 1.7  |
| 8  | 2.1  | 2.0  | 2.0  |
| 9  | 2.4  | 2.3  | 2.2  |
| 10 | 2.6  | 2.6  | 2.5  |
| 20 | 5.3  | 5.1  | 5.0  |
| 30 | 8.0  | 7.7  | 7.5  |
| 40 | 10.6 | 10.3 | 10.0 |
| 50 | 13.3 | 12.9 | 12.5 |

|    |     |     |     |
|----|-----|-----|-----|
|    | 11  | 10  | 10  |
| 6  | 1.1 | 1.0 | 1.0 |
| 7  | 1.3 | 1.2 | 1.1 |
| 8  | 1.4 | 1.4 | 1.3 |
| 9  | 1.6 | 1.6 | 1.5 |
| 10 | 1.8 | 1.7 | 1.6 |
| 20 | 3.6 | 3.5 | 3.3 |
| 30 | 5.5 | 5.2 | 5.0 |
| 40 | 7.3 | 7.0 | 6.6 |
| 50 | 9.1 | 8.7 | 8.3 |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

40°

| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P. |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|--|--|
| 0  | 9.80 806  |    | 9.92 381  |       | 0.07 618  | 9.88 423  |    |       |  |  |
| 1  | 9.80 822  | 15 | 9.92 407  | 25    | 0.07 593  | 9.88 415  | 10 | 60    |  |  |
| 2  | 9.80 837  | 15 | 9.92 432  | 25    | 0.07 567  | 9.88 404  | 11 | 59    |  |  |
| 3  | 9.80 852  | 15 | 9.92 458  | 26    | 0.07 541  | 9.88 393  | 10 | 58    |  |  |
| 4  | 9.80 867  | 15 | 9.92 484  | 25    | 0.07 516  | 9.88 383  | 10 | 57    |  |  |
|    |           |    |           |       |           |           | 10 | 56    |  |  |
| 5  | 9.80 882  | 15 | 9.92 509  | 25    | 0.07 490  | 9.88 372  | 10 |       |  |  |
| 6  | 9.80 897  | 15 | 9.92 535  | 25    | 0.07 465  | 9.88 361  | 11 | 55    |  |  |
| 7  | 9.80 912  | 15 | 9.92 561  | 26    | 0.07 439  | 9.88 351  | 10 | 54    |  |  |
| 8  | 9.80 927  | 15 | 9.92 586  | 25    | 0.07 413  | 9.88 340  | 10 | 53    |  |  |
| 9  | 9.80 942  | 15 | 9.92 612  | 25    | 0.07 388  | 9.88 329  | 11 | 52    |  |  |
|    |           |    |           |       |           |           | 10 | 51    |  |  |
| 10 | 9.80 957  | 15 | 9.92 638  | 26    | 0.07 362  | 9.88 319  | 10 |       |  |  |
| 11 | 9.80 972  | 15 | 9.92 663  | 25    | 0.07 336  | 9.88 308  | 10 | 50    |  |  |
| 12 | 9.80 987  | 15 | 9.92 689  | 25    | 0.07 311  | 9.88 297  | 11 | 49    |  |  |
| 13 | 9.81 001  | 14 | 9.92 714  | 25    | 0.07 285  | 9.88 287  | 10 | 48    |  |  |
| 14 | 9.81 016  | 15 | 9.92 740  | 26    | 0.07 259  | 9.88 276  | 10 | 47    |  |  |
|    |           |    |           |       |           |           | 11 | 46    |  |  |
| 15 | 9.81 031  | 15 | 9.92 766  | 25    | 0.07 234  | 9.88 265  | 10 |       |  |  |
| 16 | 9.81 046  | 15 | 9.92 791  | 25    | 0.07 208  | 9.88 255  | 10 | 45    |  |  |
| 17 | 9.81 061  | 15 | 9.92 817  | 25    | 0.07 183  | 9.88 244  | 10 | 44    |  |  |
| 18 | 9.81 076  | 15 | 9.92 842  | 25    | 0.07 157  | 9.88 233  | 11 | 43    |  |  |
| 19 | 9.81 091  | 14 | 9.92 868  | 26    | 0.07 131  | 9.88 223  | 10 | 42    |  |  |
|    |           |    |           |       |           |           | 11 | 41    |  |  |
| 20 | 9.81 106  | 15 | 9.92 894  | 25    | 0.07 106  | 9.88 212  | 10 |       |  |  |
| 21 | 9.81 121  | 15 | 9.92 919  | 25    | 0.07 080  | 9.88 201  | 10 | 40    |  |  |
| 22 | 9.81 136  | 14 | 9.92 945  | 26    | 0.07 055  | 9.88 190  | 11 | 39    |  |  |
| 23 | 9.81 150  | 15 | 9.92 971  | 25    | 0.07 029  | 9.88 180  | 10 | 38    |  |  |
| 24 | 9.81 165  | 15 | 9.92 996  | 25    | 0.07 003  | 9.88 169  | 11 | 37    |  |  |
|    |           |    |           |       |           |           | 10 | 36    |  |  |
| 25 | 9.81 180  | 14 | 9.93 022  | 25    | 0.06 978  | 9.88 158  | 11 |       |  |  |
| 26 | 9.81 195  | 15 | 9.93 047  | 25    | 0.06 952  | 9.88 147  | 10 | 35    |  |  |
| 27 | 9.81 210  | 15 | 9.93 073  | 25    | 0.06 927  | 9.88 137  | 11 | 34    |  |  |
| 28 | 9.81 225  | 14 | 9.93 098  | 26    | 0.06 901  | 9.88 126  | 10 | 33    |  |  |
| 29 | 9.81 239  | 15 | 9.93 124  | 25    | 0.06 875  | 9.88 115  | 10 | 32    |  |  |
|    |           |    |           |       |           |           | 11 | 31    |  |  |
| 30 | 9.81 254  | 14 | 9.93 150  | 25    | 0.06 850  | 9.88 104  | 10 |       |  |  |
| 31 | 9.81 269  | 15 | 9.93 175  | 25    | 0.06 824  | 9.88 094  | 11 | 30    |  |  |
| 32 | 9.81 284  | 15 | 9.93 201  | 25    | 0.06 799  | 9.88 083  | 10 | 29    |  |  |
| 33 | 9.81 299  | 14 | 9.93 226  | 25    | 0.06 773  | 9.88 072  | 11 | 28    |  |  |
| 34 | 9.81 313  | 15 | 9.93 252  | 26    | 0.06 748  | 9.88 061  | 10 | 27    |  |  |
|    |           |    |           |       |           |           | 11 | 26    |  |  |
| 35 | 9.81 328  | 14 | 9.93 278  | 25    | 0.06 722  | 9.88 050  | 10 |       |  |  |
| 36 | 9.81 343  | 15 | 9.93 303  | 25    | 0.06 696  | 9.88 039  | 11 | 25    |  |  |
| 37 | 9.81 358  | 14 | 9.93 329  | 25    | 0.06 671  | 9.88 029  | 10 | 24    |  |  |
| 38 | 9.81 372  | 14 | 9.93 354  | 25    | 0.06 645  | 9.88 018  | 11 | 23    |  |  |
| 39 | 9.81 387  | 15 | 9.93 380  | 25    | 0.06 620  | 9.88 007  | 10 | 22    |  |  |
|    |           |    |           |       |           |           | 11 | 21    |  |  |
| 40 | 9.81 402  | 14 | 9.93 405  | 25    | 0.06 594  | 9.87 996  | 10 |       |  |  |
| 41 | 9.81 416  | 15 | 9.93 431  | 25    | 0.06 569  | 9.87 985  | 11 | 20    |  |  |
| 42 | 9.81 431  | 14 | 9.93 456  | 25    | 0.06 543  | 9.87 974  | 10 | 19    |  |  |
| 43 | 9.81 446  | 14 | 9.93 482  | 26    | 0.06 518  | 9.87 963  | 11 | 18    |  |  |
| 44 | 9.81 460  | 15 | 9.93 508  | 25    | 0.06 492  | 9.87 953  | 10 | 17    |  |  |
|    |           |    |           |       |           |           | 11 | 16    |  |  |
| 45 | 9.81 475  | 14 | 9.93 533  | 25    | 0.06 466  | 9.87 942  | 10 |       |  |  |
| 46 | 9.81 490  | 14 | 9.93 559  | 25    | 0.06 441  | 9.87 931  | 11 | 15    |  |  |
| 47 | 9.81 504  | 15 | 9.93 584  | 25    | 0.06 415  | 9.87 920  | 10 | 14    |  |  |
| 48 | 9.81 519  | 14 | 9.93 610  | 25    | 0.06 390  | 9.87 909  | 11 | 13    |  |  |
| 49 | 9.81 534  | 14 | 9.93 635  | 25    | 0.06 364  | 9.87 898  | 10 | 12    |  |  |
|    |           |    |           |       |           |           | 11 | 11    |  |  |
| 50 | 9.81 548  | 14 | 9.93 661  | 25    | 0.06 339  | 9.87 887  | 10 |       |  |  |
| 51 | 9.81 563  | 15 | 9.93 686  | 25    | 0.06 313  | 9.87 876  | 11 | 10    |  |  |
| 52 | 9.81 578  | 14 | 9.93 712  | 25    | 0.06 288  | 9.87 865  | 10 | 9     |  |  |
| 53 | 9.81 592  | 14 | 9.93 737  | 25    | 0.06 262  | 9.87 854  | 11 | 8     |  |  |
| 54 | 9.81 607  | 14 | 9.93 763  | 25    | 0.06 237  | 9.87 844  | 10 | 7     |  |  |
|    |           |    |           |       |           |           | 11 | 6     |  |  |
| 55 | 9.81 621  | 14 | 9.93 788  | 25    | 0.06 211  | 9.87 833  | 10 |       |  |  |
| 56 | 9.81 636  | 14 | 9.93 814  | 26    | 0.06 186  | 9.87 822  | 11 | 5     |  |  |
| 57 | 9.81 650  | 14 | 9.93 840  | 25    | 0.06 160  | 9.87 811  | 10 | 4     |  |  |
| 58 | 9.81 665  | 15 | 9.93 865  | 25    | 0.06 134  | 9.87 800  | 11 | 3     |  |  |
| 59 | 9.81 680  | 14 | 9.93 891  | 25    | 0.06 109  | 9.87 789  | 10 | 2     |  |  |
|    |           |    |           |       |           |           | 11 | 1     |  |  |
| 60 | 9.81 694  | 14 | 9.93 916  | 25    | 0.06 083  | 9.87 778  | 10 |       |  |  |
|    |           |    |           |       |           |           | 11 | 0     |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | P. P. |  |  |

|    | 26   | 25   |
|----|------|------|
| 6  | 2.6  | 2.5  |
| 7  | 3.0  | 3.0  |
| 8  | 3.4  | 3.4  |
| 9  | 3.8  | 3.8  |
| 10 | 4.2  | 4.2  |
| 20 | 8.5  | 8.5  |
| 30 | 12.7 | 12.7 |
| 40 | 17.0 | 17.0 |
| 50 | 21.2 | 21.2 |

|    | 15   | 15   | 14   |
|----|------|------|------|
| 6  | 1.5  | 1.5  | 1.4  |
| 7  | 1.7  | 1.7  | 1.7  |
| 8  | 2.0  | 2.0  | 1.9  |
| 9  | 2.2  | 2.2  | 2.2  |
| 10 | 2.4  | 2.4  | 2.4  |
| 20 | 4.8  | 4.8  | 4.8  |
| 30 | 7.2  | 7.2  | 7.2  |
| 40 | 9.6  | 9.6  | 9.6  |
| 50 | 12.1 | 12.1 | 12.1 |

|    | 11  | 10  |
|----|-----|-----|
| 6  | 1.1 | 1.0 |
| 7  | 1.2 | 1.2 |
| 8  | 1.4 | 1.4 |
| 9  | 1.6 | 1.6 |
| 10 | 1.7 | 1.7 |
| 20 | 3.5 | 3.5 |
| 30 | 5.2 | 5.2 |
| 40 | 7.0 | 7.0 |
| 50 | 8.7 | 8.7 |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS. 41°

| °  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|--|--|
| 0  | 9.81 694  | 14 | 9.93 916  | 23    | 0.06 083  | 9.87 778  | 11 | 60 |       |  |  |
| 1  | 9.81 709  | 14 | 9.93 942  | 23    | 0.06 058  | 9.87 767  | 11 | 59 |       |  |  |
| 2  | 9.81 723  | 14 | 9.93 967  | 23    | 0.06 032  | 9.87 756  | 11 | 58 |       |  |  |
| 3  | 9.81 738  | 14 | 9.93 993  | 23    | 0.06 007  | 9.87 745  | 11 | 57 |       |  |  |
| 4  | 9.81 752  | 14 | 9.94 018  | 23    | 0.05 981  | 9.87 734  | 11 | 56 |       |  |  |
| 5  | 9.81 767  | 14 | 9.94 044  | 23    | 0.05 956  | 9.87 723  | 11 | 55 |       |  |  |
| 6  | 9.81 781  | 14 | 9.94 069  | 23    | 0.05 930  | 9.87 712  | 11 | 54 |       |  |  |
| 7  | 9.81 796  | 14 | 9.94 095  | 23    | 0.05 905  | 9.87 701  | 11 | 53 |       |  |  |
| 8  | 9.81 810  | 14 | 9.94 120  | 23    | 0.05 879  | 9.87 690  | 11 | 52 |       |  |  |
| 9  | 9.81 824  | 14 | 9.94 146  | 23    | 0.05 854  | 9.87 679  | 11 | 51 |       |  |  |
| 10 | 9.81 839  | 14 | 9.94 171  | 23    | 0.05 828  | 9.87 668  | 11 | 50 |       |  |  |
| 11 | 9.81 853  | 14 | 9.94 197  | 23    | 0.05 803  | 9.87 657  | 11 | 49 |       |  |  |
| 12 | 9.81 868  | 14 | 9.94 222  | 23    | 0.05 777  | 9.87 645  | 11 | 48 |       |  |  |
| 13 | 9.81 882  | 14 | 9.94 248  | 23    | 0.05 752  | 9.87 634  | 11 | 47 |       |  |  |
| 14 | 9.81 897  | 14 | 9.94 273  | 23    | 0.05 726  | 9.87 623  | 11 | 46 |       |  |  |
| 15 | 9.81 911  | 14 | 9.94 299  | 23    | 0.05 701  | 9.87 612  | 11 | 45 |       |  |  |
| 16 | 9.81 925  | 14 | 9.94 324  | 23    | 0.05 675  | 9.87 601  | 11 | 44 |       |  |  |
| 17 | 9.81 940  | 14 | 9.94 350  | 23    | 0.05 650  | 9.87 590  | 11 | 43 |       |  |  |
| 18 | 9.81 954  | 14 | 9.94 375  | 23    | 0.05 625  | 9.87 579  | 11 | 42 |       |  |  |
| 19 | 9.81 969  | 14 | 9.94 400  | 23    | 0.05 599  | 9.87 568  | 11 | 41 |       |  |  |
| 20 | 9.81 983  | 14 | 9.94 426  | 23    | 0.05 574  | 9.87 557  | 11 | 40 |       |  |  |
| 21 | 9.81 997  | 14 | 9.94 451  | 23    | 0.05 548  | 9.87 546  | 11 | 39 |       |  |  |
| 22 | 9.82 012  | 14 | 9.94 477  | 23    | 0.05 523  | 9.87 535  | 11 | 38 |       |  |  |
| 23 | 9.82 026  | 14 | 9.94 502  | 23    | 0.05 497  | 9.87 523  | 11 | 37 |       |  |  |
| 24 | 9.82 040  | 14 | 9.94 528  | 23    | 0.05 472  | 9.87 512  | 11 | 36 |       |  |  |
| 25 | 9.82 055  | 14 | 9.94 553  | 23    | 0.05 446  | 9.87 501  | 11 | 35 |       |  |  |
| 26 | 9.82 069  | 14 | 9.94 579  | 23    | 0.05 421  | 9.87 490  | 11 | 34 |       |  |  |
| 27 | 9.82 083  | 14 | 9.94 604  | 23    | 0.05 395  | 9.87 479  | 11 | 33 |       |  |  |
| 28 | 9.82 098  | 14 | 9.94 630  | 23    | 0.05 370  | 9.87 468  | 11 | 32 |       |  |  |
| 29 | 9.82 112  | 14 | 9.94 655  | 23    | 0.05 344  | 9.87 457  | 11 | 31 |       |  |  |
| 30 | 9.82 126  | 14 | 9.94 681  | 23    | 0.05 319  | 9.87 445  | 11 | 30 |       |  |  |
| 31 | 9.82 140  | 14 | 9.94 706  | 23    | 0.05 293  | 9.87 434  | 11 | 29 |       |  |  |
| 32 | 9.82 155  | 14 | 9.94 732  | 23    | 0.05 268  | 9.87 423  | 11 | 28 |       |  |  |
| 33 | 9.82 169  | 14 | 9.94 757  | 23    | 0.05 243  | 9.87 412  | 11 | 27 |       |  |  |
| 34 | 9.82 183  | 14 | 9.94 782  | 23    | 0.05 217  | 9.87 401  | 11 | 26 |       |  |  |
| 35 | 9.82 197  | 14 | 9.94 808  | 23    | 0.05 192  | 9.87 389  | 11 | 25 |       |  |  |
| 36 | 9.82 212  | 14 | 9.94 833  | 23    | 0.05 166  | 9.87 378  | 11 | 24 |       |  |  |
| 37 | 9.82 226  | 14 | 9.94 859  | 23    | 0.05 141  | 9.87 367  | 11 | 23 |       |  |  |
| 38 | 9.82 240  | 14 | 9.94 884  | 23    | 0.05 115  | 9.87 356  | 11 | 22 |       |  |  |
| 39 | 9.82 254  | 14 | 9.94 910  | 23    | 0.05 090  | 9.87 345  | 11 | 21 |       |  |  |
| 40 | 9.82 269  | 14 | 9.94 935  | 23    | 0.05 064  | 9.87 333  | 11 | 20 |       |  |  |
| 41 | 9.82 283  | 14 | 9.94 961  | 23    | 0.05 039  | 9.87 322  | 11 | 19 |       |  |  |
| 42 | 9.82 297  | 14 | 9.94 986  | 23    | 0.05 014  | 9.87 311  | 11 | 18 |       |  |  |
| 43 | 9.82 311  | 14 | 9.95 011  | 23    | 0.04 988  | 9.87 300  | 11 | 17 |       |  |  |
| 44 | 9.82 325  | 14 | 9.95 037  | 23    | 0.04 963  | 9.87 288  | 11 | 16 |       |  |  |
| 45 | 9.82 339  | 14 | 9.95 062  | 23    | 0.04 937  | 9.87 277  | 11 | 15 |       |  |  |
| 46 | 9.82 354  | 14 | 9.95 083  | 23    | 0.04 912  | 9.87 266  | 11 | 14 |       |  |  |
| 47 | 9.82 368  | 14 | 9.95 113  | 23    | 0.04 886  | 9.87 254  | 11 | 13 |       |  |  |
| 48 | 9.82 382  | 14 | 9.95 139  | 23    | 0.04 861  | 9.87 243  | 11 | 12 |       |  |  |
| 49 | 9.82 396  | 14 | 9.95 164  | 23    | 0.04 836  | 9.87 232  | 11 | 11 |       |  |  |
| 50 | 9.82 410  | 14 | 9.95 189  | 23    | 0.04 810  | 9.87 221  | 11 | 10 |       |  |  |
| 51 | 9.82 424  | 14 | 9.95 215  | 23    | 0.04 785  | 9.87 209  | 11 | 9  |       |  |  |
| 52 | 9.82 438  | 14 | 9.95 240  | 23    | 0.04 759  | 9.87 198  | 11 | 8  |       |  |  |
| 53 | 9.82 452  | 14 | 9.95 266  | 23    | 0.04 734  | 9.87 187  | 11 | 7  |       |  |  |
| 54 | 9.82 467  | 14 | 9.95 291  | 23    | 0.04 708  | 9.87 175  | 11 | 6  |       |  |  |
| 55 | 9.82 481  | 14 | 9.95 316  | 23    | 0.04 683  | 9.87 164  | 11 | 5  |       |  |  |
| 56 | 9.82 495  | 14 | 9.95 342  | 23    | 0.04 658  | 9.87 153  | 11 | 4  |       |  |  |
| 57 | 9.82 509  | 14 | 9.95 367  | 23    | 0.04 632  | 9.87 141  | 11 | 3  |       |  |  |
| 58 | 9.82 523  | 14 | 9.95 393  | 23    | 0.04 607  | 9.87 130  | 11 | 2  |       |  |  |
| 59 | 9.82 537  | 14 | 9.95 418  | 23    | 0.04 581  | 9.87 118  | 11 | 1  |       |  |  |
| 60 | 9.82 551  | 14 | 9.95 443  | 23    | 0.04 556  | 9.87 107  | 11 | 0  |       |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |  |  |

2525

62.52.5  
73.02.9  
83.43.3  
93.83.7  
104.24.1  
208.58.3  
3012.712.5  
4017.016.6  
5021.220.8

1414

61.41.4  
71.71.6  
81.91.8  
92.22.1  
102.42.3  
204.84.6  
307.27.0  
409.69.3  
5012.111.6

1111

61.11.1  
71.31.3  
81.51.4  
91.71.6  
101.91.8  
203.83.6  
305.75.5  
407.67.3  
509.69.1

|    |           |    |           |       |           |           |    | P. P. |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|--|
| '  | Log. Sin. | d  | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |       |  |
| 0  | 9.82 551  | 14 | 9.95 443  | 23    | 0.04 556  | 9.87 107  | 11 | 60    |  |
| 1  | 9.82 565  | 14 | 9.95 469  | 23    | 0.04 531  | 9.87 096  | 11 | 59    |  |
| 2  | 9.82 579  | 14 | 9.95 494  | 23    | 0.04 505  | 9.87 084  | 11 | 58    |  |
| 3  | 9.82 593  | 14 | 9.95 520  | 23    | 0.04 480  | 9.87 073  | 11 | 57    |  |
| 4  | 9.82 607  | 14 | 9.95 545  | 23    | 0.04 454  | 9.87 062  | 11 | 56    |  |
| 5  | 9.82 621  | 14 | 9.95 571  | 23    | 0.04 429  | 9.87 050  | 11 | 55    |  |
| 6  | 9.82 635  | 14 | 9.95 596  | 23    | 0.04 404  | 9.87 039  | 11 | 54    |  |
| 7  | 9.82 649  | 14 | 9.95 621  | 23    | 0.04 378  | 9.87 027  | 11 | 53    |  |
| 8  | 9.82 663  | 14 | 9.95 647  | 23    | 0.04 353  | 9.87 016  | 11 | 52    |  |
| 9  | 9.82 677  | 14 | 9.95 672  | 23    | 0.04 327  | 9.87 004  | 11 | 51    |  |
| 10 | 9.82 691  | 14 | 9.95 697  | 23    | 0.04 302  | 9.86 993  | 11 | 50    |  |
| 11 | 9.82 705  | 14 | 9.95 723  | 23    | 0.04 277  | 9.86 982  | 11 | 49    |  |
| 12 | 9.82 719  | 14 | 9.95 748  | 23    | 0.04 251  | 9.86 970  | 11 | 48    |  |
| 13 | 9.82 733  | 13 | 9.95 774  | 23    | 0.04 226  | 9.86 959  | 11 | 47    |  |
| 14 | 9.82 746  | 14 | 9.95 799  | 23    | 0.04 200  | 9.86 947  | 11 | 46    |  |
| 15 | 9.82 760  | 14 | 9.95 824  | 23    | 0.04 175  | 9.86 936  | 11 | 45    |  |
| 16 | 9.82 774  | 14 | 9.95 850  | 23    | 0.04 150  | 9.86 924  | 11 | 44    |  |
| 17 | 9.82 788  | 14 | 9.95 875  | 23    | 0.04 124  | 9.86 913  | 11 | 43    |  |
| 18 | 9.82 802  | 13 | 9.95 901  | 23    | 0.04 099  | 9.86 901  | 11 | 42    |  |
| 19 | 9.82 816  | 14 | 9.95 926  | 23    | 0.04 074  | 9.86 890  | 11 | 41    |  |
| 20 | 9.82 830  | 14 | 9.95 951  | 23    | 0.04 048  | 9.86 878  | 11 | 40    |  |
| 21 | 9.82 844  | 14 | 9.95 977  | 23    | 0.04 023  | 9.86 867  | 11 | 39    |  |
| 22 | 9.82 858  | 13 | 9.96 002  | 23    | 0.03 997  | 9.86 855  | 11 | 38    |  |
| 23 | 9.82 871  | 14 | 9.96 027  | 23    | 0.03 972  | 9.86 844  | 11 | 37    |  |
| 24 | 9.82 885  | 14 | 9.96 053  | 23    | 0.03 947  | 9.86 832  | 11 | 36    |  |
| 25 | 9.82 899  | 13 | 9.96 078  | 23    | 0.03 921  | 9.86 821  | 11 | 35    |  |
| 26 | 9.82 913  | 14 | 9.96 104  | 23    | 0.03 896  | 9.86 809  | 11 | 34    |  |
| 27 | 9.82 927  | 13 | 9.96 129  | 23    | 0.03 871  | 9.86 798  | 12 | 33    |  |
| 28 | 9.82 940  | 14 | 9.96 154  | 23    | 0.03 845  | 9.86 786  | 11 | 32    |  |
| 29 | 9.82 954  | 14 | 9.96 180  | 23    | 0.03 820  | 9.86 774  | 11 | 31    |  |
| 30 | 9.82 968  | 13 | 9.96 205  | 23    | 0.03 795  | 9.86 763  | 11 | 30    |  |
| 31 | 9.82 982  | 14 | 9.96 230  | 23    | 0.03 769  | 9.86 751  | 11 | 29    |  |
| 32 | 9.82 996  | 13 | 9.96 256  | 23    | 0.03 744  | 9.86 740  | 11 | 28    |  |
| 33 | 9.83 009  | 14 | 9.96 281  | 23    | 0.03 718  | 9.86 728  | 12 | 27    |  |
| 34 | 9.83 023  | 13 | 9.96 306  | 23    | 0.03 693  | 9.86 716  | 11 | 26    |  |
| 35 | 9.83 037  | 14 | 9.96 332  | 23    | 0.03 668  | 9.86 705  | 11 | 25    |  |
| 36 | 9.83 051  | 13 | 9.96 357  | 23    | 0.03 642  | 9.86 693  | 11 | 24    |  |
| 37 | 9.83 064  | 14 | 9.96 383  | 23    | 0.03 617  | 9.86 682  | 11 | 23    |  |
| 38 | 9.83 078  | 13 | 9.96 408  | 23    | 0.03 592  | 9.86 670  | 11 | 22    |  |
| 39 | 9.83 092  | 14 | 9.96 433  | 23    | 0.03 566  | 9.86 658  | 12 | 21    |  |
| 40 | 9.83 106  | 13 | 9.96 459  | 23    | 0.03 541  | 9.86 647  | 11 | 20    |  |
| 41 | 9.83 119  | 13 | 9.96 484  | 23    | 0.03 516  | 9.86 635  | 12 | 19    |  |
| 42 | 9.83 133  | 14 | 9.96 509  | 23    | 0.03 490  | 9.86 623  | 11 | 18    |  |
| 43 | 9.83 147  | 13 | 9.96 535  | 23    | 0.03 465  | 9.86 612  | 11 | 17    |  |
| 44 | 9.83 160  | 13 | 9.96 560  | 23    | 0.03 440  | 9.86 600  | 12 | 16    |  |
| 45 | 9.83 174  | 14 | 9.96 585  | 23    | 0.03 414  | 9.86 588  | 11 | 15    |  |
| 46 | 9.83 188  | 13 | 9.96 611  | 23    | 0.03 389  | 9.86 577  | 11 | 14    |  |
| 47 | 9.83 201  | 13 | 9.96 636  | 23    | 0.03 364  | 9.86 565  | 12 | 13    |  |
| 48 | 9.83 215  | 14 | 9.96 661  | 23    | 0.03 338  | 9.86 553  | 11 | 12    |  |
| 49 | 9.83 229  | 13 | 9.96 687  | 23    | 0.03 313  | 9.86 542  | 12 | 11    |  |
| 50 | 9.83 242  | 13 | 9.96 712  | 23    | 0.03 287  | 9.86 530  | 11 | 10    |  |
| 51 | 9.83 256  | 13 | 9.96 737  | 23    | 0.03 262  | 9.86 518  | 11 | 9     |  |
| 52 | 9.83 269  | 13 | 9.96 763  | 23    | 0.03 237  | 9.86 507  | 11 | 8     |  |
| 53 | 9.83 283  | 14 | 9.96 788  | 23    | 0.03 211  | 9.86 495  | 12 | 7     |  |
| 54 | 9.83 297  | 13 | 9.96 813  | 23    | 0.03 186  | 9.86 483  | 11 | 6     |  |
| 55 | 9.83 310  | 13 | 9.96 839  | 23    | 0.03 161  | 9.86 471  | 12 | .5    |  |
| 56 | 9.83 324  | 13 | 9.96 864  | 23    | 0.03 135  | 9.86 460  | 11 | 4     |  |
| 57 | 9.83 337  | 13 | 9.96 889  | 23    | 0.03 110  | 9.86 448  | 12 | 3     |  |
| 58 | 9.83 351  | 13 | 9.96 915  | 23    | 0.03 085  | 9.86 436  | 11 | 2     |  |
| 59 | 9.83 365  | 14 | 9.96 940  | 23    | 0.03 059  | 9.86 424  | 12 | 1     |  |
| 60 | 9.83 378  | 13 | 9.96 965  | 25    | 0.03 034  | 9.86 412  | 12 | 0     |  |
|    |           |    |           |       |           |           |    | P. P. |  |

|    | 23   | 25   |
|----|------|------|
| 6  | 2.3  | 2.5  |
| 7  | 3.0  | 2.9  |
| 8  | 3.4  | 3.3  |
| 9  | 3.8  | 3.7  |
| 10 | 4.2  | 4.1  |
| 20 | 8.5  | 8.3  |
| 30 | 12.7 | 12.5 |
| 40 | 17.0 | 16.6 |
| 50 | 21.2 | 20.8 |

|    | 14   | 13   |
|----|------|------|
| 6  | 1.4  | 1.3  |
| 7  | 1.6  | 1.6  |
| 8  | 1.8  | 1.8  |
| 9  | 2.1  | 2.0  |
| 10 | 2.3  | 2.2  |
| 20 | 4.6  | 4.5  |
| 30 | 7.0  | 6.7  |
| 40 | 9.3  | 9.0  |
| 50 | 11.6 | 11.2 |

|    | 12   | 11  | 11  |
|----|------|-----|-----|
| 6  | 1.2  | 1.1 | 1.1 |
| 7  | 1.4  | 1.3 | 1.3 |
| 8  | 1.6  | 1.5 | 1.4 |
| 9  | 1.8  | 1.7 | 1.6 |
| 10 | 2.0  | 1.9 | 1.8 |
| 20 | 4.0  | 3.8 | 3.6 |
| 30 | 6.0  | 5.7 | 5.5 |
| 40 | 8.0  | 7.6 | 7.3 |
| 50 | 10.0 | 9.6 | 9.1 |



TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

43°

|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P. |       |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|-------|--|--|
| 0  | 9.83 378  | 13 | 9.96 905  | 25    | 0.03 034  | 9.86 412  | 11 | 60    |       |  |  |
| 1  | 9.83 392  | 13 | 9.96 991  | 25    | 0.03 009  | 9.86 401  | 12 | 59    |       |  |  |
| 2  | 9.83 405  | 13 | 9.97 016  | 25    | 0.02 984  | 9.86 389  | 11 | 58    |       |  |  |
| 3  | 9.83 419  | 13 | 9.97 041  | 25    | 0.02 958  | 9.86 377  | 12 | 57    |       |  |  |
| 4  | 9.83 432  | 13 | 9.97 067  | 25    | 0.02 933  | 9.86 365  | 11 | 56    |       |  |  |
| 5  | 9.83 446  | 13 | 9.97 092  | 25    | 0.02 908  | 9.86 354  | 12 | 55    |       |  |  |
| 6  | 9.83 459  | 13 | 9.97 117  | 25    | 0.02 882  | 9.86 342  | 12 | 54    |       |  |  |
| 7  | 9.83 473  | 13 | 9.97 143  | 25    | 0.02 857  | 9.86 330  | 11 | 53    |       |  |  |
| 8  | 9.83 486  | 13 | 9.97 168  | 25    | 0.02 832  | 9.86 318  | 12 | 52    |       |  |  |
| 9  | 9.83 500  | 13 | 9.97 193  | 25    | 0.02 806  | 9.86 306  | 12 | 51    |       |  |  |
| 10 | 9.83 513  | 13 | 9.97 219  | 25    | 0.02 781  | 9.86 294  | 12 | 50    |       |  |  |
| 11 | 9.83 527  | 13 | 9.97 244  | 25    | 0.02 756  | 9.86 282  | 11 | 49    |       |  |  |
| 12 | 9.83 540  | 13 | 9.97 269  | 25    | 0.02 730  | 9.86 271  | 12 | 48    |       |  |  |
| 13 | 9.83 554  | 13 | 9.97 295  | 25    | 0.02 705  | 9.86 259  | 12 | 47    |       |  |  |
| 14 | 9.83 567  | 13 | 9.97 320  | 25    | 0.02 680  | 9.86 247  | 11 | 46    |       |  |  |
| 15 | 9.83 580  | 13 | 9.97 345  | 25    | 0.02 654  | 9.86 235  | 12 | 45    |       |  |  |
| 16 | 9.83 594  | 13 | 9.97 370  | 25    | 0.02 629  | 9.86 223  | 12 | 44    |       |  |  |
| 17 | 9.83 607  | 13 | 9.97 396  | 25    | 0.02 604  | 9.86 211  | 12 | 43    |       |  |  |
| 18 | 9.83 621  | 13 | 9.97 421  | 25    | 0.02 578  | 9.86 199  | 12 | 42    |       |  |  |
| 19 | 9.83 634  | 13 | 9.97 446  | 25    | 0.02 553  | 9.86 187  | 11 | 41    |       |  |  |
| 20 | 9.83 647  | 13 | 9.97 472  | 25    | 0.02 528  | 9.86 176  | 12 | 40    |       |  |  |
| 21 | 9.83 661  | 13 | 9.97 497  | 25    | 0.02 502  | 9.86 164  | 12 | 39    |       |  |  |
| 22 | 9.83 674  | 13 | 9.97 522  | 25    | 0.02 477  | 9.86 152  | 12 | 38    |       |  |  |
| 23 | 9.83 688  | 13 | 9.97 548  | 25    | 0.02 452  | 9.86 140  | 12 | 37    |       |  |  |
| 24 | 9.83 701  | 13 | 9.97 573  | 25    | 0.02 427  | 9.86 128  | 12 | 36    |       |  |  |
| 25 | 9.83 714  | 13 | 9.97 598  | 25    | 0.02 401  | 9.86 116  | 12 | 35    |       |  |  |
| 26 | 9.83 728  | 13 | 9.97 624  | 25    | 0.02 376  | 9.86 104  | 12 | 34    |       |  |  |
| 27 | 9.83 741  | 13 | 9.97 649  | 25    | 0.02 351  | 9.86 092  | 12 | 33    |       |  |  |
| 28 | 9.83 754  | 13 | 9.97 674  | 25    | 0.02 325  | 9.86 080  | 12 | 32    |       |  |  |
| 29 | 9.83 768  | 13 | 9.97 699  | 25    | 0.02 300  | 9.86 068  | 12 | 31    |       |  |  |
| 30 | 9.83 781  | 13 | 9.97 725  | 25    | 0.02 275  | 9.86 056  | 12 | 30    |       |  |  |
| 31 | 9.83 794  | 13 | 9.97 750  | 25    | 0.02 249  | 9.86 044  | 12 | 29    |       |  |  |
| 32 | 9.83 808  | 13 | 9.97 775  | 25    | 0.02 224  | 9.86 032  | 12 | 28    |       |  |  |
| 33 | 9.83 821  | 13 | 9.97 801  | 25    | 0.02 199  | 9.86 020  | 12 | 27    |       |  |  |
| 34 | 9.83 834  | 13 | 9.97 826  | 25    | 0.02 174  | 9.86 008  | 12 | 26    |       |  |  |
| 35 | 9.83 847  | 13 | 9.97 851  | 25    | 0.02 148  | 9.85 996  | 12 | 25    |       |  |  |
| 36 | 9.83 861  | 13 | 9.97 877  | 25    | 0.02 123  | 9.85 984  | 12 | 24    |       |  |  |
| 37 | 9.83 874  | 13 | 9.97 902  | 25    | 0.02 098  | 9.85 972  | 12 | 23    |       |  |  |
| 38 | 9.83 887  | 13 | 9.97 927  | 25    | 0.02 072  | 9.85 960  | 12 | 22    |       |  |  |
| 39 | 9.83 900  | 13 | 9.97 952  | 25    | 0.02 047  | 9.85 948  | 12 | 21    |       |  |  |
| 40 | 9.83 914  | 13 | 9.97 978  | 25    | 0.02 022  | 9.85 936  | 12 | 20    |       |  |  |
| 41 | 9.83 927  | 13 | 9.98 003  | 25    | 0.01 996  | 9.85 924  | 12 | 19    |       |  |  |
| 42 | 9.83 940  | 13 | 9.98 028  | 25    | 0.01 971  | 9.85 912  | 12 | 18    |       |  |  |
| 43 | 9.83 953  | 13 | 9.98 054  | 25    | 0.01 946  | 9.85 900  | 12 | 17    |       |  |  |
| 44 | 9.83 967  | 13 | 9.98 079  | 25    | 0.01 921  | 9.85 887  | 12 | 16    |       |  |  |
| 45 | 9.83 980  | 13 | 9.98 104  | 25    | 0.01 895  | 9.85 875  | 12 | 15    |       |  |  |
| 46 | 9.83 993  | 13 | 9.98 129  | 25    | 0.01 870  | 9.85 863  | 12 | 14    |       |  |  |
| 47 | 9.84 006  | 13 | 9.98 155  | 25    | 0.01 845  | 9.85 851  | 12 | 13    |       |  |  |
| 48 | 9.84 019  | 13 | 9.98 180  | 25    | 0.01 819  | 9.85 839  | 12 | 12    |       |  |  |
| 49 | 9.84 033  | 13 | 9.98 205  | 25    | 0.01 794  | 9.85 827  | 12 | 11    |       |  |  |
| 50 | 9.84 046  | 13 | 9.98 231  | 25    | 0.01 769  | 9.85 815  | 12 | 10    |       |  |  |
| 51 | 9.84 059  | 13 | 9.98 256  | 25    | 0.01 744  | 9.85 803  | 12 | 9     |       |  |  |
| 52 | 9.84 072  | 13 | 9.98 281  | 25    | 0.01 718  | 9.85 791  | 12 | 8     |       |  |  |
| 53 | 9.84 085  | 13 | 9.98 306  | 25    | 0.01 693  | 9.85 778  | 12 | 7     |       |  |  |
| 54 | 9.84 098  | 13 | 9.98 332  | 25    | 0.01 668  | 9.85 766  | 12 | 6     |       |  |  |
| 55 | 9.84 111  | 13 | 9.98 357  | 25    | 0.01 642  | 9.85 754  | 12 | 5     |       |  |  |
| 56 | 9.84 124  | 13 | 9.98 382  | 25    | 0.01 617  | 9.85 742  | 12 | 4     |       |  |  |
| 57 | 9.84 138  | 13 | 9.98 408  | 25    | 0.01 592  | 9.85 730  | 12 | 3     |       |  |  |
| 58 | 9.84 151  | 13 | 9.98 433  | 25    | 0.01 567  | 9.85 718  | 12 | 2     |       |  |  |
| 59 | 9.84 164  | 13 | 9.98 458  | 25    | 0.01 541  | 9.85 705  | 12 | 1     |       |  |  |
| 60 | 9.84 177  | 13 | 9.98 483  | 25    | 0.01 516  | 9.85 693  | 12 | 0     |       |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |       | P. P. |  |  |

|    |      |      |
|----|------|------|
|    | 25   | 25   |
| 6  | 2.5  | 2.5  |
| 7  | 3.0  | 2.9  |
| 8  | 3.4  | 3.3  |
| 9  | 3.8  | 3.7  |
| 10 | 4.2  | 4.1  |
| 20 | 8.5  | 8.3  |
| 30 | 12.7 | 12.5 |
| 40 | 17.0 | 16.6 |
| 50 | 21.2 | 20.8 |

|    |      |      |
|----|------|------|
|    | 13   | 13   |
| 6  | 1.3  | 1.3  |
| 7  | 1.6  | 1.5  |
| 8  | 1.8  | 1.7  |
| 9  | 2.0  | 1.9  |
| 10 | 2.2  | 2.1  |
| 20 | 4.5  | 4.3  |
| 30 | 6.7  | 6.5  |
| 40 | 9.0  | 8.6  |
| 50 | 11.2 | 10.8 |

|    |      |      |     |
|----|------|------|-----|
|    | 12   | 12   | 11  |
| 6  | 1.2  | 1.2  | 1.1 |
| 7  | 1.4  | 1.4  | 1.3 |
| 8  | 1.6  | 1.6  | 1.5 |
| 9  | 1.9  | 1.8  | 1.7 |
| 10 | 2.1  | 2.0  | 1.9 |
| 20 | 4.1  | 4.0  | 3.8 |
| 30 | 6.2  | 6.0  | 5.7 |
| 40 | 8.3  | 8.0  | 7.6 |
| 50 | 10.4 | 10.0 | 9.6 |

| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.84 177  |    | 9.98 483  |       | 0.01 516  | 9.85 693  |    | 60 |       |
| 1  | 9.84 190  | 13 | 9.98 509  | 25    | 0.01 491  | 9.85 681  | 12 | 59 |       |
| 2  | 9.84 203  | 13 | 9.98 534  | 25    | 0.01 465  | 9.85 669  | 12 | 58 |       |
| 3  | 9.84 216  | 13 | 9.98 559  | 25    | 0.01 440  | 9.85 657  | 12 | 57 |       |
| 4  | 9.84 229  | 13 | 9.98 585  | 25    | 0.01 415  | 9.85 644  | 12 | 56 |       |
| 5  | 9.84 242  | 13 | 9.98 610  | 25    | 0.01 390  | 9.85 632  | 12 | 55 |       |
| 6  | 9.84 255  | 13 | 9.98 635  | 25    | 0.01 364  | 9.85 620  | 12 | 54 |       |
| 7  | 9.84 268  | 13 | 9.98 660  | 25    | 0.01 339  | 9.85 608  | 12 | 53 |       |
| 8  | 9.84 281  | 13 | 9.98 686  | 25    | 0.01 314  | 9.85 595  | 12 | 52 |       |
| 9  | 9.84 294  | 13 | 9.98 711  | 25    | 0.01 289  | 9.85 583  | 12 | 51 |       |
| 10 | 9.84 307  | 13 | 9.98 736  | 25    | 0.01 263  | 9.85 571  | 12 | 50 |       |
| 11 | 9.84 320  | 13 | 9.98 762  | 25    | 0.01 238  | 9.85 559  | 12 | 49 |       |
| 12 | 9.84 333  | 13 | 9.98 787  | 25    | 0.01 213  | 9.85 546  | 12 | 48 |       |
| 13 | 9.84 346  | 13 | 9.98 812  | 25    | 0.01 187  | 9.85 534  | 12 | 47 |       |
| 14 | 9.84 359  | 13 | 9.98 837  | 25    | 0.01 162  | 9.85 522  | 12 | 46 |       |
| 15 | 9.84 372  | 13 | 9.98 863  | 25    | 0.01 137  | 9.85 509  | 12 | 45 |       |
| 16 | 9.84 385  | 13 | 9.98 888  | 25    | 0.01 112  | 9.85 497  | 12 | 44 |       |
| 17 | 9.84 398  | 13 | 9.98 913  | 25    | 0.01 086  | 9.85 485  | 12 | 43 |       |
| 18 | 9.84 411  | 13 | 9.98 938  | 25    | 0.01 061  | 9.85 472  | 12 | 42 |       |
| 19 | 9.84 424  | 12 | 9.98 964  | 25    | 0.01 036  | 9.85 460  | 12 | 41 |       |
| 20 | 9.84 437  | 13 | 9.98 989  | 25    | 0.01 010  | 9.85 448  | 12 | 40 |       |
| 21 | 9.84 450  | 13 | 9.99 014  | 25    | 0.00 985  | 9.85 435  | 12 | 39 |       |
| 22 | 9.84 463  | 13 | 9.99 040  | 25    | 0.00 960  | 9.85 423  | 12 | 38 |       |
| 23 | 9.84 476  | 13 | 9.99 065  | 25    | 0.00 935  | 9.85 411  | 12 | 37 |       |
| 24 | 9.84 489  | 13 | 9.99 090  | 25    | 0.00 909  | 9.85 398  | 12 | 36 |       |
| 25 | 9.84 502  | 12 | 9.99 115  | 25    | 0.00 884  | 9.85 386  | 12 | 35 |       |
| 26 | 9.84 514  | 13 | 9.99 141  | 25    | 0.00 859  | 9.85 374  | 12 | 34 |       |
| 27 | 9.84 527  | 13 | 9.99 166  | 25    | 0.00 834  | 9.85 361  | 12 | 33 |       |
| 28 | 9.84 540  | 13 | 9.99 191  | 25    | 0.00 808  | 9.85 349  | 12 | 32 |       |
| 29 | 9.84 553  | 12 | 9.99 216  | 25    | 0.00 783  | 9.85 336  | 12 | 31 |       |
| 30 | 9.84 566  | 13 | 9.99 242  | 25    | 0.00 758  | 9.85 324  | 12 | 30 |       |
| 31 | 9.84 579  | 13 | 9.99 267  | 25    | 0.00 733  | 9.85 312  | 12 | 29 |       |
| 32 | 9.84 592  | 13 | 9.99 292  | 25    | 0.00 707  | 9.85 299  | 12 | 28 |       |
| 33 | 9.84 604  | 13 | 9.99 318  | 25    | 0.00 682  | 9.85 287  | 12 | 27 |       |
| 34 | 9.84 617  | 13 | 9.99 343  | 25    | 0.00 657  | 9.85 274  | 12 | 26 |       |
| 35 | 9.84 630  | 12 | 9.99 368  | 25    | 0.00 631  | 9.85 262  | 12 | 25 |       |
| 36 | 9.84 643  | 13 | 9.99 393  | 25    | 0.00 606  | 9.85 249  | 12 | 24 |       |
| 37 | 9.84 656  | 13 | 9.99 419  | 25    | 0.00 581  | 9.85 237  | 12 | 23 |       |
| 38 | 9.84 669  | 12 | 9.99 444  | 25    | 0.00 556  | 9.85 224  | 12 | 22 |       |
| 39 | 9.84 681  | 13 | 9.99 469  | 25    | 0.00 530  | 9.85 212  | 12 | 21 |       |
| 40 | 9.84 694  | 12 | 9.99 494  | 25    | 0.00 505  | 9.85 199  | 12 | 20 |       |
| 41 | 9.84 707  | 13 | 9.99 520  | 25    | 0.00 480  | 9.85 187  | 12 | 19 |       |
| 42 | 9.84 720  | 12 | 9.99 545  | 25    | 0.00 455  | 9.85 174  | 12 | 18 |       |
| 43 | 9.84 732  | 13 | 9.99 570  | 25    | 0.00 430  | 9.85 162  | 12 | 17 |       |
| 44 | 9.84 745  | 12 | 9.99 595  | 25    | 0.00 404  | 9.85 149  | 12 | 16 |       |
| 45 | 9.84 758  | 13 | 9.99 621  | 25    | 0.00 379  | 9.85 137  | 12 | 15 |       |
| 46 | 9.84 771  | 12 | 9.99 646  | 25    | 0.00 353  | 9.85 124  | 12 | 14 |       |
| 47 | 9.84 783  | 13 | 9.99 671  | 25    | 0.00 328  | 9.85 112  | 12 | 13 |       |
| 48 | 9.84 796  | 12 | 9.99 697  | 25    | 0.00 303  | 9.85 099  | 12 | 12 |       |
| 49 | 9.84 809  | 13 | 9.99 722  | 25    | 0.00 278  | 9.85 087  | 12 | 11 |       |
| 50 | 9.84 822  | 12 | 9.99 747  | 25    | 0.00 252  | 9.85 074  | 12 | 10 |       |
| 51 | 9.84 834  | 12 | 9.99 772  | 25    | 0.00 227  | 9.85 062  | 12 | 9  |       |
| 52 | 9.84 847  | 13 | 9.99 798  | 25    | 0.00 202  | 9.85 049  | 12 | 8  |       |
| 53 | 9.84 860  | 12 | 9.99 823  | 25    | 0.00 177  | 9.85 037  | 13 | 7  |       |
| 54 | 9.84 872  | 12 | 9.99 848  | 25    | 0.00 151  | 9.85 024  | 12 | 6  |       |
| 55 | 9.84 885  | 13 | 9.99 873  | 25    | 0.00 126  | 9.85 011  | 12 | 5  |       |
| 56 | 9.84 898  | 12 | 9.99 899  | 25    | 0.00 101  | 9.84 999  | 12 | 4  |       |
| 57 | 9.84 910  | 12 | 9.99 924  | 25    | 0.00 076  | 9.84 986  | 12 | 3  |       |
| 58 | 9.84 923  | 13 | 9.99 949  | 25    | 0.00 050  | 9.84 974  | 13 | 2  |       |
| 59 | 9.84 936  | 12 | 9.99 974  | 25    | 0.00 025  | 9.84 961  | 12 | 1  |       |
| 60 | 9.84 948  |    | 0.00 000  |       | 0.00 000  | 9.84 948  |    | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

|    |      |      |
|----|------|------|
|    | 25   | 25   |
| 6  | 2.5  | 2.5  |
| 7  | 3.0  | 2.9  |
| 8  | 3.4  | 3.3  |
| 9  | 3.8  | 3.7  |
| 10 | 4.2  | 4.1  |
| 20 | 8.5  | 8.3  |
| 30 | 12.7 | 12.5 |
| 40 | 17.0 | 16.6 |
| 50 | 21.2 | 20.8 |

|    |      |      |
|----|------|------|
|    | 13   | 13   |
| 6  | 1.3  | 1.3  |
| 7  | 1.6  | 1.5  |
| 8  | 1.8  | 1.7  |
| 9  | 2.0  | 1.9  |
| 10 | 2.2  | 2.1  |
| 20 | 4.5  | 4.3  |
| 30 | 6.7  | 6.5  |
| 40 | 9.0  | 8.6  |
| 50 | 11.2 | 10.8 |

|    |      |      |
|----|------|------|
|    | 12   | 12   |
| 6  | 1.2  | 1.2  |
| 7  | 1.4  | 1.4  |
| 8  | 1.6  | 1.6  |
| 9  | 1.9  | 1.8  |
| 10 | 2.1  | 2.0  |
| 20 | 4.1  | 4.0  |
| 30 | 6.2  | 6.0  |
| 40 | 8.3  | 8.0  |
| 50 | 10.4 | 10.0 |



TABLE VIII.

LOGARITHMIC VERSED SINES AND EXTERNAL  
SECANTS.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

0°

1°

| '  | Log. Vers. | D     | Log. Exsec. | D     | Log. Vers. | D    | Log. Exsec. | D    | '  |
|----|------------|-------|-------------|-------|------------|------|-------------|------|----|
| 0  | —∞         |       | —∞          |       | 6.18271    | 1435 | 6.18278     | 1436 | 0  |
| 1  | 2.62642    | 60206 | 2.62642     | 60206 | .19707     | 1412 | .19714      | 1412 | 1  |
| 2  | 3.22848    | 35218 | 3.22848     | 35218 | .21119     | 1389 | .21126      | 1390 | 2  |
| 3  | 3.58066    | 24987 | 3.58066     | 24987 | .22509     | 1368 | .22516      | 1368 | 3  |
| 4  | 3.83054    |       | 3.83054     |       | .23877     | 1346 | .23884      | 1347 | 4  |
| 5  | 4.02436    | 19382 | 4.02436     | 19382 | 6.25223    | 1326 | 6.25231     | 1326 | 5  |
| 6  | .18272     | 15836 | .18272      | 15836 | .26549     | 1306 | .26557      | 1306 | 6  |
| 7  | .31662     | 13389 | .31662      | 13389 | .27856     | 1286 | .27864      | 1287 | 7  |
| 8  | .43260     | 11598 | .43260      | 11598 | .29142     | 1268 | .29151      | 1268 | 8  |
| 9  | .53490     | 10230 | .53491      | 10230 | .30410     | 1250 | .30419      | 1250 | 9  |
| 10 | 4.62642    | 9151  | 4.62642     | 9151  | 6.31666    | 1232 | 6.31669     | 1232 | 10 |
| 11 | .70920     | 8278  | .70921      | 8279  | .32892     | 1214 | .32901      | 1215 | 11 |
| 12 | .78478     | 7558  | .78478      | 7557  | .34107     | 1198 | .34116      | 1198 | 12 |
| 13 | .85431     | 6953  | .85431      | 6952  | .35305     | 1182 | .35315      | 1182 | 13 |
| 14 | .91868     | 6437  | .91868      | 6437  | .36487     | 1166 | .36497      | 1166 | 14 |
| 15 | 4.97860    | 5992  | 4.97861     | 5993  | 6.37653    | 1150 | 6.37663     | 1151 | 15 |
| 16 | 5.03466    | 5603  | 5.03466     | 5603  | .38803     | 1135 | .38814      | 1135 | 16 |
| 17 | .08732     | 5266  | .08732      | 5266  | .39938     | 1121 | .39949      | 1121 | 17 |
| 18 | .13696     | 4964  | .13697      | 4964  | .41059     | 1106 | .41070      | 1106 | 18 |
| 19 | .18393     | 4696  | .18393      | 4696  | .42163     | 1093 | .42177      | 1093 | 19 |
| 20 | 5.22848    | 4455  | 5.22849     | 4456  | 6.43258    | 1078 | 6.43270     | 1079 | 20 |
| 21 | .27086     | 4238  | .27087      | 4238  | .44337     | 1066 | .44349      | 1066 | 21 |
| 22 | .31126     | 4040  | .31127      | 4040  | .45403     | 1052 | .45413      | 1053 | 22 |
| 23 | .34987     | 3861  | .34988      | 3861  | .46453     | 1040 | .46468      | 1040 | 23 |
| 24 | .38684     | 3697  | .38685      | 3697  | .47496     | 1028 | .47509      | 1028 | 24 |
| 25 | 5.42230    | 3543  | 5.42231     | 3543  | 6.48524    | 1015 | 6.48537     | 1016 | 25 |
| 26 | .45636     | 3406  | .45638      | 3407  | .49539     | 1004 | .49553      | 1004 | 26 |
| 27 | .48915     | 3278  | .48916      | 3278  | .50544     | 992  | .50557      | 993  | 27 |
| 28 | .52073     | 3158  | .52075      | 3159  | .51536     | 981  | .51550      | 982  | 28 |
| 29 | .55121     | 3048  | .55123      | 3048  | .52518     | 970  | .52532      | 970  | 29 |
| 30 | 5.58066    | 2944  | 5.58068     | 2945  | 6.53488    | 960  | 6.53503     | 960  | 30 |
| 31 | .60914     | 2848  | .60916      | 2848  | .54448     | 949  | .54463      | 950  | 31 |
| 32 | .63672     | 2757  | .63674      | 2758  | .55397     | 939  | .55413      | 939  | 32 |
| 33 | .66344     | 2672  | .66346      | 2672  | .56336     | 929  | .56352      | 929  | 33 |
| 34 | .68937     | 2593  | .68940      | 2593  | .57265     | 919  | .57281      | 919  | 34 |
| 35 | 5.71453    | 2518  | 5.71457     | 2517  | 6.58184    | 909  | 6.58201     | 909  | 35 |
| 36 | .73902     | 2447  | .73904      | 2447  | .59093     | 900  | .59110      | 900  | 36 |
| 37 | .76282     | 2379  | .76284      | 2380  | .59993     | 891  | .60011      | 891  | 37 |
| 38 | .78598     | 2316  | .78601      | 2316  | .60884     | 882  | .60902      | 882  | 38 |
| 39 | .80854     | 2256  | .80857      | 2256  | .61766     | 873  | .61784      | 873  | 39 |
| 40 | 5.83053    | 2199  | 5.83056     | 2199  | 6.62639    | 864  | 6.62657     | 864  | 40 |
| 41 | .85198     | 2145  | .85201      | 2145  | .63503     | 855  | .63522      | 856  | 41 |
| 42 | .87291     | 2093  | .87295      | 2093  | .64359     | 847  | .64378      | 848  | 42 |
| 43 | .89333     | 2044  | .89338      | 2043  | .65206     | 839  | .65226      | 839  | 43 |
| 44 | .91332     | 1996  | .91335      | 1997  | .66043     | 831  | .66063      | 831  | 44 |
| 45 | 5.93284    | 1952  | 5.93288     | 1952  | 6.66876    | 823  | 6.66897     | 823  | 45 |
| 46 | .95193     | 1909  | .95197      | 1909  | .67700     | 815  | .67720      | 816  | 46 |
| 47 | .97061     | 1868  | .97065      | 1868  | .68513     | 808  | .68536      | 808  | 47 |
| 48 | 5.98890    | 1829  | 5.98894     | 1829  | .69323     | 800  | .69345      | 800  | 48 |
| 49 | 6.00680    | 1790  | 6.00685     | 1791  | .70124     | 793  | .70143      | 794  | 49 |
| 50 | 6.02433    | 1755  | 6.02440     | 1755  | 6.70917    | 786  | 6.70939     | 786  | 50 |
| 51 | .04153     | 1720  | .04160      | 1720  | .71703     | 779  | .71723      | 779  | 51 |
| 52 | .05842     | 1686  | .05847      | 1687  | .72482     | 772  | .72505      | 772  | 52 |
| 53 | .07496     | 1654  | .07501      | 1654  | .73254     | 765  | .73277      | 765  | 53 |
| 54 | .09120     | 1623  | .09125      | 1623  | .74019     | 758  | .74043      | 759  | 54 |
| 55 | 6.10714    | 1594  | 6.10719     | 1594  | 6.74777    | 752  | 6.74802     | 752  | 55 |
| 56 | .12279     | 1565  | .12284      | 1565  | .75529     | 745  | .75554      | 746  | 56 |
| 57 | .13816     | 1537  | .13822      | 1537  | .76275     | 739  | .76300      | 739  | 57 |
| 58 | .15327     | 1511  | .15333      | 1511  | .77014     | 733  | .77040      | 733  | 58 |
| 59 | .16811     | 1484  | .16818      | 1485  | .77747     | 726  | .77773      | 727  | 59 |
| 60 | 6.18271    | 1460  | 6.18278     | 1460  | 6.78474    |      | 6.78500     |      | 60 |
| '  | Log. Vers. | D     | Log. Exsec. | D     | Log. Vers. | D    | Log. Exsec. | D    | '  |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

2°

3°

|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    |
|----|------------|-----|-------------|-----|------------|-----|-------------|-----|----|
| 0  | 6.78474    | 721 | 6.78500     | 721 | 7.13687    | 481 | 7.13746     | 481 | 0  |
| 1  | .79195     | 714 | .79221      | 715 | .14168     | 478 | .14228      | 479 | 1  |
| 2  | .79909     | 709 | .79937      | 709 | .14646     | 475 | .14707      | 476 | 2  |
| 3  | .80618     | 703 | .80646      | 703 | .15122     | 473 | .15183      | 474 | 3  |
| 4  | .81322     | 697 | .81350      | 698 | .15593     | 470 | .15657      | 471 | 4  |
| 5  | 6.82019    | 692 | 6.82048     | 692 | 7.16066    | 468 | 7.16129     | 469 | 5  |
| 6  | .82711     | 686 | .82740      | 687 | .16534     | 466 | .16598      | 466 | 6  |
| 7  | .83398     | 681 | .83427      | 682 | .17000     | 463 | .17064      | 464 | 7  |
| 8  | .84079     | 676 | .84109      | 676 | .17463     | 460 | .17528      | 461 | 8  |
| 9  | .84755     | 670 | .84783      | 671 | .17923     | 458 | .17989      | 459 | 9  |
| 10 | 6.85423    | 663 | 6.85457     | 666 | 7.18382    | 455 | 7.18448     | 456 | 10 |
| 11 | .86091     | 660 | .86123      | 660 | .18837     | 453 | .18905      | 454 | 11 |
| 12 | .86751     | 653 | .86783      | 656 | .19291     | 451 | .19359      | 452 | 12 |
| 13 | .87407     | 650 | .87439      | 651 | .19742     | 448 | .19811      | 449 | 13 |
| 14 | .88057     | 646 | .88090      | 646 | .20191     | 446 | .20260      | 447 | 14 |
| 15 | 6.88703    | 641 | 6.88737     | 641 | 7.20637    | 444 | 7.20707     | 445 | 15 |
| 16 | .89344     | 636 | .89378      | 636 | .21081     | 442 | .21152      | 443 | 16 |
| 17 | .89980     | 631 | .90015      | 632 | .21523     | 440 | .21595      | 440 | 17 |
| 18 | .90612     | 627 | .90647      | 628 | .21963     | 437 | .22033      | 438 | 18 |
| 19 | .91239     | 622 | .91275      | 623 | .22400     | 435 | .22473      | 436 | 19 |
| 20 | 6.91862    | 618 | 6.91898     | 618 | 7.22836    | 433 | 7.22909     | 434 | 20 |
| 21 | .92480     | 613 | .92516      | 614 | .23269     | 431 | .23343      | 431 | 21 |
| 22 | .93093     | 609 | .93131      | 610 | .23700     | 429 | .23775      | 429 | 22 |
| 23 | .93703     | 603 | .93741      | 603 | .24129     | 426 | .24204      | 427 | 23 |
| 24 | .94308     | 601 | .94346      | 601 | .24553     | 424 | .24632      | 425 | 24 |
| 25 | 6.94909    | 597 | 6.94948     | 597 | 7.24980    | 422 | 7.25057     | 423 | 25 |
| 26 | .95506     | 592 | .95543      | 593 | .25402     | 420 | .25480      | 421 | 26 |
| 27 | .96099     | 589 | .96139      | 589 | .25823     | 418 | .25902      | 419 | 27 |
| 28 | .96688     | 584 | .96728      | 583 | .26241     | 416 | .26321      | 417 | 28 |
| 29 | .97272     | 581 | .97313      | 581 | .26658     | 414 | .26738      | 415 | 29 |
| 30 | 6.97853    | 577 | 6.97895     | 577 | 7.27072    | 412 | 7.27153     | 413 | 30 |
| 31 | .98430     | 573 | .98472      | 574 | .27485     | 410 | .27567      | 411 | 31 |
| 32 | .99004     | 569 | .99046      | 570 | .27895     | 409 | .27978      | 409 | 32 |
| 33 | 6.99573    | 563 | 6.99616     | 566 | .28304     | 406 | .28387      | 407 | 33 |
| 34 | 7.00139    | 562 | 7.00182     | 563 | .28711     | 405 | .28795      | 405 | 34 |
| 35 | 7.00701    | 558 | 7.00743     | 559 | 7.29116    | 402 | 7.29200     | 404 | 35 |
| 36 | .01259     | 555 | .01304      | 555 | .29518     | 401 | .29604      | 402 | 36 |
| 37 | .01814     | 551 | .01860      | 552 | .29919     | 399 | .30006      | 400 | 37 |
| 38 | .02366     | 548 | .02412      | 548 | .30319     | 397 | .30406      | 398 | 38 |
| 39 | .02914     | 544 | .02960      | 545 | .30716     | 395 | .30804      | 396 | 39 |
| 40 | 7.03458    | 541 | 7.03503     | 541 | 7.31112    | 393 | 7.31201     | 394 | 40 |
| 41 | .03999     | 537 | .04047      | 538 | .31503     | 392 | .31593      | 393 | 41 |
| 42 | .04537     | 534 | .04583      | 535 | .31897     | 390 | .31988      | 391 | 42 |
| 43 | .05071     | 531 | .05120      | 531 | .32288     | 388 | .32379      | 389 | 43 |
| 44 | .05603     | 527 | .05652      | 528 | .32676     | 386 | .32768      | 388 | 44 |
| 45 | 7.06130    | 525 | 7.06180     | 525 | 7.33063    | 385 | 7.33156     | 385 | 45 |
| 46 | .06653     | 521 | .06706      | 522 | .33448     | 383 | .33542      | 384 | 46 |
| 47 | .07177     | 518 | .07228      | 519 | .33831     | 382 | .33926      | 382 | 47 |
| 48 | .07693     | 513 | .07747      | 516 | .34213     | 380 | .34309      | 380 | 48 |
| 49 | .08211     | 512 | .08263      | 513 | .34593     | 378 | .34689      | 379 | 49 |
| 50 | 7.08723    | 509 | 7.08776     | 509 | 7.34971    | 377 | 7.35069     | 377 | 50 |
| 51 | .09232     | 506 | .09286      | 507 | .35348     | 375 | .35446      | 376 | 51 |
| 52 | .09739     | 503 | .09793      | 503 | .35723     | 373 | .35822      | 374 | 52 |
| 53 | .10242     | 500 | .10297      | 501 | .36097     | 371 | .36196      | 373 | 53 |
| 54 | .10743     | 497 | .10798      | 498 | .36468     | 370 | .36569      | 371 | 54 |
| 55 | 7.11240    | 495 | 7.11297     | 495 | 7.36839    | 368 | 7.36940     | 369 | 55 |
| 56 | .11733     | 492 | .11792      | 493 | .37207     | 367 | .37310      | 368 | 56 |
| 57 | .12227     | 489 | .12285      | 490 | .37574     | 366 | .37678      | 366 | 57 |
| 58 | .12716     | 486 | .12775      | 487 | .37940     | 364 | .38044      | 365 | 58 |
| 59 | .13203     | 484 | .13262      | 484 | .38304     | 362 | .38409      | 363 | 59 |
| 60 | 7.13687    |     | 7.13746     |     | 7.38667    |     | 7.38773     |     | 60 |
|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

4°

5°

|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    | P. P.                |
|----|------------|-----|-------------|-----|------------|-----|-------------|-----|----|----------------------|
| 0  | 7.38667    | 361 | 7.38773     | 361 | 7.58039    | 289 | 7.58204     | 290 | 0  |                      |
| 1  | .39028     | 359 | .39134      | 360 | .58328     | 287 | .58494      | 289 | 1  |                      |
| 2  | .39387     | 358 | .39495      | 359 | .58615     | 287 | .58783      | 288 | 2  |                      |
| 3  | .39745     | 356 | .39854      | 357 | .58902     | 286 | .59071      | 287 | 3  | 360 350 340          |
| 4  | .40102     | 355 | .40211      | 356 | .59188     | 285 | .59358      | 286 | 4  | 6 36.0 35.0 34.0     |
| 5  | 7.40457    | 353 | 7.40567     | 354 | 7.59473    | 284 | 7.59645     | 285 | 5  | 7 42.0 40.0 39.6     |
| 6  | .40810     | 352 | .40922      | 353 | .59758     | 283 | .59930      | 284 | 6  | 8 48.0 46.0 45.3     |
| 7  | .41163     | 350 | .41275      | 352 | .60041     | 282 | .60214      | 283 | 7  | 9 54.0 51.5 51.0     |
| 8  | .41513     | 349 | .41627      | 350 | .60323     | 281 | .60498      | 282 | 8  | 10 60.0 58.5 56.6    |
| 9  | .41863     | 348 | .41977      | 349 | .60604     | 280 | .60780      | 281 | 9  | 20 120.0 116.0 113.3 |
| 10 | 7.42211    | 346 | 7.42326     | 347 | 7.60885    | 279 | 7.61062     | 280 | 10 | 30 180.0 175.0 170.0 |
| 11 | .42557     | 345 | .42673      | 346 | .61164     | 279 | .61342      | 280 | 11 | 40 240.0 233.0 226.6 |
| 12 | .42903     | 343 | .43019      | 345 | .61443     | 277 | .61622      | 279 | 12 | 50 300.0 291.6 283.3 |
| 13 | .43246     | 342 | .43364      | 343 | .61721     | 277 | .61901      | 278 | 13 |                      |
| 14 | .43589     | 341 | .43708      | 342 | .61998     | 276 | .62179      | 277 | 14 |                      |
| 15 | 7.43930    | 339 | 7.44050     | 340 | 7.62274    | 275 | 7.62456     | 276 | 15 | 330 320 310          |
| 16 | .44270     | 338 | .44390      | 339 | .62549     | 274 | .62733      | 275 | 16 | 6 33.0 32.0 31.0     |
| 17 | .44608     | 337 | .44730      | 338 | .62823     | 273 | .63008      | 274 | 17 | 7 38.5 37.0 36.1     |
| 18 | .44946     | 335 | .45068      | 337 | .63096     | 272 | .63282      | 273 | 18 | 8 44.0 42.0 41.3     |
| 19 | .45281     | 334 | .45405      | 335 | .63369     | 272 | .63556      | 273 | 19 | 9 49.5 48.0 46.5     |
| 20 | 7.45616    | 333 | 7.45740     | 334 | 7.63641    | 270 | 7.63826     | 272 | 20 | 10 55.0 53.0 51.6    |
| 21 | .45949     | 332 | .46075      | 333 | .63911     | 270 | .64101      | 271 | 21 | 20 110.0 106.0 103.3 |
| 22 | .46281     | 330 | .46407      | 332 | .64181     | 269 | .64372      | 270 | 22 | 30 165.0 160.0 155.0 |
| 23 | .46612     | 329 | .46739      | 330 | .64451     | 268 | .64643      | 269 | 23 | 40 220.0 213.0 206.6 |
| 24 | .46941     | 328 | .47070      | 329 | .64719     | 267 | .64912      | 268 | 24 | 50 275.0 266.6 258.3 |
| 25 | 7.47270    | 327 | 7.47399     | 328 | 7.64986    | 266 | 7.65181     | 267 | 25 |                      |
| 26 | .47597     | 325 | .47727      | 327 | .65253     | 266 | .65449      | 267 | 26 | 300 290 280          |
| 27 | .47922     | 324 | .48054      | 325 | .65519     | 265 | .65716      | 266 | 27 | 6 30.0 29.0 28.0     |
| 28 | .48247     | 323 | .48379      | 324 | .65784     | 264 | .65982      | 265 | 28 | 7 35.0 33.0 32.6     |
| 29 | .48570     | 322 | .48703      | 323 | .66048     | 263 | .66247      | 264 | 29 | 8 40.0 38.0 37.3     |
| 30 | 7.48892    | 321 | 7.49026     | 322 | 7.66311    | 263 | 7.66512     | 264 | 30 | 9 45.0 43.0 42.0     |
| 31 | .49213     | 320 | .49348      | 321 | .66574     | 261 | .66776      | 263 | 31 | 10 50.0 48.0 46.6    |
| 32 | .49533     | 318 | .49669      | 319 | .66836     | 261 | .67039      | 262 | 32 | 20 100.0 96.0 93.3   |
| 33 | .49852     | 317 | .49989      | 318 | .67097     | 260 | .67301      | 261 | 33 | 30 150.0 145.0 140.0 |
| 34 | .50169     | 316 | .50307      | 317 | .67357     | 259 | .67562      | 260 | 34 | 40 200.0 193.0 186.6 |
| 35 | 7.50485    | 315 | 7.50624     | 316 | 7.67617    | 258 | 7.67823     | 259 | 35 | 50 250.0 241.6 233.3 |
| 36 | .50800     | 314 | .50941      | 315 | .67875     | 258 | .68083      | 259 | 36 |                      |
| 37 | .51114     | 313 | .51256      | 314 | .68133     | 257 | .68342      | 258 | 37 | 270 260 250          |
| 38 | .51427     | 311 | .51569      | 313 | .68390     | 256 | .68601      | 257 | 38 | 6 27.0 26.0 25.0     |
| 39 | .51739     | 311 | .51882      | 313 | .68647     | 255 | .68858      | 256 | 39 | 7 31.5 30.0 29.1     |
| 40 | 7.52050    | 309 | 7.52194     | 310 | 7.68902    | 255 | 7.69115     | 256 | 40 | 8 36.0 34.0 33.3     |
| 41 | .52359     | 308 | .52504      | 309 | .69157     | 254 | .69371      | 255 | 41 | 9 40.5 39.0 37.5     |
| 42 | .52667     | 307 | .52814      | 308 | .69411     | 253 | .69627      | 254 | 42 | 10 45.0 43.0 41.6    |
| 43 | .52975     | 306 | .53122      | 307 | .69665     | 252 | .69881      | 253 | 43 | 20 90.0 87.0 83.3    |
| 44 | .53281     | 305 | .53429      | 306 | .69917     | 252 | .70135      | 253 | 44 | 30 135.0 130.0 125.0 |
| 45 | 7.53586    | 304 | 7.53735     | 305 | 7.70169    | 251 | 7.70388     | 252 | 45 | 40 180.0 173.0 166.6 |
| 46 | .53890     | 303 | .54041      | 304 | .70421     | 250 | .70641      | 251 | 46 | 50 225.0 216.6 208.3 |
| 47 | .54193     | 302 | .54345      | 303 | .70671     | 249 | .70893      | 250 | 47 |                      |
| 48 | .54495     | 300 | .54648      | 302 | .70921     | 248 | .71144      | 249 | 48 | 240 230 220          |
| 49 | .54796     | 299 | .54950      | 301 | .71170     | 247 | .71394      | 248 | 49 | 6 24.0 23.0 22.0     |
| 50 | 7.55096    | 297 | 7.55251     | 299 | 7.71418    | 247 | 7.71644     | 248 | 50 | 7 28.0 26.0 25.6     |
| 51 | .55395     | 297 | .55550      | 299 | .71666     | 247 | .71892      | 248 | 51 | 8 32.0 30.0 29.3     |
| 52 | .55692     | 297 | .55849      | 298 | .71913     | 246 | .72141      | 247 | 52 | 9 36.0 34.5 33.0     |
| 53 | .55989     | 295 | .56147      | 296 | .72159     | 245 | .72388      | 246 | 53 | 10 40.0 38.0 36.6    |
| 54 | .56285     | 295 | .56444      | 296 | .72404     | 245 | .72635      | 246 | 54 | 20 80.0 76.0 73.3    |
| 55 | 7.56580    | 293 | 7.56740     | 295 | 7.72640    | 244 | 7.72881     | 245 | 55 | 30 120.0 115.0 110.0 |
| 56 | .56873     | 293 | .57035      | 294 | .72893     | 243 | .73126      | 244 | 56 | 40 160.0 153.0 146.6 |
| 57 | .57166     | 292 | .57329      | 292 | .73137     | 242 | .73371      | 243 | 57 | 50 200.0 191.6 183.3 |
| 58 | .57458     | 290 | .57621      | 292 | .73379     | 242 | .73615      | 243 | 58 |                      |
| 59 | .57749     | 290 | .57913      | 291 | .73621     | 241 | .73859      | 242 | 59 | 210 200 190          |
| 60 | 7.58039    | 290 | 7.58204     | 291 | 7.73863    | 241 | 7.74101     | 242 | 60 | 6 21.0 20.0 19.0     |
|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    | P. P.                |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

6°

7°

|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    | P. P. |
|----|------------|-----|-------------|-----|------------|-----|-------------|-----|----|-------|
| 0  | 7.73863    | 241 | 7.74101     | 242 | 7.87238    | 206 | 7.87563     | 208 | 0  |       |
| 1  | .74104     | 240 | .74343      | 241 | .87444     | 205 | .87771      | 207 | 1  | 180   |
| 2  | .74344     | 239 | .74585      | 241 | .87650     | 205 | .87978      | 207 | 2  | 9     |
| 3  | .74583     | 239 | .74826      | 240 | .87855     | 204 | .88185      | 206 | 3  | 6     |
| 4  | .74822     | 238 | .75066      | 239 | .88060     | 204 | .88391      | 206 | 4  | 8     |
| 5  | 7.75060    | 237 | 7.75305     | 239 | 7.88264    | 204 | 7.88597     | 205 | 5  | 9     |
| 6  | .75297     | 236 | .75544      | 238 | .88468     | 203 | .88803      | 205 | 6  | 10    |
| 7  | .75534     | 236 | .75782      | 237 | .88672     | 203 | .89008      | 205 | 7  | 20    |
| 8  | .75770     | 235 | .76019      | 237 | .88875     | 203 | .89212      | 204 | 8  | 30    |
| 9  | .76006     | 235 | .76256      | 236 | .89077     | 202 | .89416      | 204 | 9  | 40    |
| 10 | 7.76240    | 234 | 7.76492     | 236 | 7.89279    | 202 | 7.89620     | 203 | 10 | 50    |
| 11 | .76475     | 234 | .76728      | 235 | .89481     | 201 | .89823      | 203 | 11 | 6     |
| 12 | .76708     | 233 | .76963      | 235 | .89682     | 201 | .90025      | 202 | 12 | 7     |
| 13 | .76941     | 233 | .77197      | 234 | .89882     | 200 | .90228      | 202 | 13 | 8     |
| 14 | .77173     | 232 | .77431      | 233 | .90082     | 199 | .90429      | 201 | 14 | 9     |
| 15 | 7.77405    | 232 | 7.77664     | 232 | 7.90282    | 199 | 7.90630     | 201 | 15 | 10    |
| 16 | .77636     | 231 | .77897      | 231 | .90481     | 198 | .90831      | 201 | 16 | 20    |
| 17 | .77867     | 230 | .78128      | 231 | .90680     | 198 | .91032      | 199 | 17 | 30    |
| 18 | .78097     | 229 | .78359      | 230 | .90878     | 197 | .91231      | 199 | 18 | 40    |
| 19 | .78326     | 228 | .78590      | 230 | .91076     | 197 | .91431      | 199 | 19 | 50    |
| 20 | 7.78554    | 228 | 7.78820     | 229 | 7.91273    | 197 | 7.91630     | 198 | 20 | 6     |
| 21 | .78783     | 227 | .79050      | 229 | .91470     | 196 | .91828      | 198 | 21 | 7     |
| 22 | .79010     | 227 | .79279      | 228 | .91667     | 196 | .92027      | 197 | 22 | 8     |
| 23 | .79237     | 226 | .79507      | 228 | .91863     | 195 | .92224      | 197 | 23 | 9     |
| 24 | .79463     | 225 | .79735      | 227 | .92058     | 195 | .92421      | 197 | 24 | 10    |
| 25 | 7.79689    | 225 | 7.79962     | 226 | 7.92253    | 195 | 7.92618     | 196 | 25 | 20    |
| 26 | .79914     | 224 | .80188      | 226 | .92448     | 194 | .92815      | 196 | 26 | 30    |
| 27 | .80138     | 224 | .80414      | 225 | .92642     | 194 | .93010      | 195 | 27 | 40    |
| 28 | .80362     | 223 | .80639      | 225 | .92836     | 193 | .93206      | 195 | 28 | 50    |
| 29 | .80586     | 223 | .80864      | 224 | .93029     | 193 | .93401      | 195 | 29 | 6     |
| 30 | 7.80808    | 222 | 7.81088     | 224 | 7.93222    | 192 | 7.93596     | 194 | 30 | 7     |
| 31 | .81031     | 221 | .81312      | 223 | .93415     | 192 | .93790      | 194 | 31 | 8     |
| 32 | .81252     | 221 | .81533      | 222 | .93607     | 191 | .93984      | 193 | 32 | 9     |
| 33 | .81473     | 220 | .81758      | 222 | .93799     | 191 | .94177      | 193 | 33 | 10    |
| 34 | .81694     | 220 | .81980      | 221 | .93990     | 190 | .94370      | 192 | 34 | 20    |
| 35 | 7.81914    | 219 | 7.82201     | 221 | 7.94181    | 190 | 7.94562     | 192 | 35 | 30    |
| 36 | .82133     | 219 | .82422      | 220 | .94371     | 190 | .94754      | 192 | 36 | 40    |
| 37 | .82352     | 218 | .82642      | 219 | .94561     | 189 | .94946      | 191 | 37 | 50    |
| 38 | .82570     | 217 | .82862      | 219 | .94751     | 189 | .95137      | 191 | 38 | 6     |
| 39 | .82788     | 217 | .83081      | 219 | .94940     | 189 | .95328      | 191 | 39 | 7     |
| 40 | 7.83005    | 217 | 7.83300     | 218 | 7.95129    | 188 | 7.95519     | 190 | 40 | 8     |
| 41 | .83222     | 216 | .83518      | 217 | .95317     | 187 | .95709      | 189 | 41 | 9     |
| 42 | .83438     | 215 | .83735      | 217 | .95505     | 188 | .95898      | 189 | 42 | 10    |
| 43 | .83653     | 215 | .83952      | 216 | .95693     | 187 | .96088      | 188 | 43 | 20    |
| 44 | .83868     | 214 | .84169      | 216 | .95880     | 186 | .96276      | 188 | 44 | 30    |
| 45 | 7.84083    | 214 | 7.84385     | 215 | 7.96066    | 186 | 7.96465     | 188 | 45 | 40    |
| 46 | .84297     | 213 | .84600      | 215 | .96253     | 186 | .96653      | 188 | 46 | 50    |
| 47 | .84510     | 213 | .84813      | 214 | .96439     | 185 | .96841      | 187 | 47 | 6     |
| 48 | .84723     | 212 | .85030      | 213 | .96624     | 185 | .97028      | 187 | 48 | 7     |
| 49 | .84935     | 212 | .85243      | 213 | .96809     | 184 | .97215      | 186 | 49 | 8     |
| 50 | 7.85147    | 211 | 7.85457     | 213 | 7.96994    | 184 | 7.97401     | 186 | 50 | 9     |
| 51 | .85359     | 211 | .85670      | 212 | .97178     | 184 | .97587      | 185 | 51 | 10    |
| 52 | .85570     | 210 | .85882      | 211 | .97362     | 183 | .97773      | 185 | 52 | 20    |
| 53 | .85780     | 210 | .86094      | 211 | .97546     | 183 | .97958      | 184 | 53 | 30    |
| 54 | .85990     | 209 | .86305      | 211 | .97729     | 183 | .98143      | 184 | 54 | 40    |
| 55 | 7.86199    | 209 | 7.86516     | 210 | 7.97912    | 182 | 7.98327     | 184 | 55 | 50    |
| 56 | .86408     | 208 | .86726      | 210 | .98094     | 182 | .98512      | 183 | 56 | 6     |
| 57 | .86616     | 208 | .86936      | 209 | .98276     | 182 | .98693      | 183 | 57 | 7     |
| 58 | .86824     | 207 | .87146      | 208 | .98458     | 181 | .98879      | 183 | 58 | 8     |
| 59 | .87031     | 206 | .87354      | 208 | .98639     | 181 | .99062      | 182 | 59 | 9     |
| 60 | 7.87238    |     | 7.87563     |     | 7.98820    |     | 7.99244     |     | 60 |       |
|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    | P. P. |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

8°

9°

| 8° |            |     |             | 9°  |            |     |             |
|----|------------|-----|-------------|-----|------------|-----|-------------|
|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. |
| 0  | 7.98820    | 180 | 7.99244     | 182 | 8.09031    | 160 | 8.09569     |
| 1  | .99000     | 180 | .99427      | 182 | .09192     | 160 | .09732      |
| 2  | .99180     | 179 | .99609      | 181 | .09352     | 160 | .09894      |
| 3  | .99360     | 179 | .99790      | 181 | .09512     | 159 | .10056      |
| 4  | .99539     | 179 | 7.99971     | 180 | .09671     | 159 | .10217      |
| 5  | 7.99718    | 178 | 8.00152     | 180 | 8.09830    | 159 | 8.10378     |
| 6  | 7.99897    | 178 | .00332      | 180 | .09989     | 158 | .10539      |
| 7  | 8.00075    | 177 | .00512      | 180 | .10148     | 158 | .10700      |
| 8  | .00253     | 178 | .00692      | 180 | .10306     | 158 | .10860      |
| 9  | .00431     | 177 | .00871      | 179 | .10464     | 158 | .11020      |
| 10 | 8.00608    | 176 | 8.01050     | 179 | 8.10622    | 157 | 8.11180     |
| 11 | .00784     | 176 | .01229      | 178 | .10779     | 157 | .11340      |
| 12 | .00961     | 176 | .01407      | 178 | .10936     | 157 | .11499      |
| 13 | .01137     | 176 | .01585      | 177 | .11093     | 156 | .11658      |
| 14 | .01313     | 175 | .01763      | 177 | .11250     | 156 | .11816      |
| 15 | 8.01488    | 175 | 8.01940     | 177 | 8.11406    | 156 | 8.11975     |
| 16 | .01663     | 175 | .02117      | 176 | .11562     | 155 | .12133      |
| 17 | .01838     | 174 | .02293      | 176 | .11718     | 155 | .12291      |
| 18 | .02012     | 174 | .02469      | 175 | .11873     | 155 | .12448      |
| 19 | .02186     | 173 | .02645      | 175 | .12029     | 155 | .12605      |
| 20 | 8.02359    | 173 | 8.02820     | 175 | 8.12184    | 154 | 8.12762     |
| 21 | .02533     | 173 | .02995      | 175 | .12338     | 154 | .12919      |
| 22 | .02706     | 172 | .03170      | 174 | .12492     | 154 | .13075      |
| 23 | .02878     | 172 | .03345      | 174 | .12647     | 153 | .13232      |
| 24 | .03050     | 172 | .03519      | 174 | .12800     | 153 | .13387      |
| 25 | 8.03222    | 171 | 8.03692     | 173 | 8.12954    | 153 | 8.13543     |
| 26 | .03394     | 171 | .03866      | 173 | .13107     | 153 | .13698      |
| 27 | .03565     | 171 | .04039      | 173 | .13260     | 152 | .13854      |
| 28 | .03736     | 170 | .04212      | 172 | .13413     | 152 | .14008      |
| 29 | .03906     | 170 | .04384      | 172 | .13565     | 152 | .14163      |
| 30 | 8.04076    | 169 | 8.04556     | 171 | 8.13717    | 152 | 8.14317     |
| 31 | .04246     | 169 | .04728      | 171 | .13869     | 151 | .14471      |
| 32 | .04416     | 169 | .04899      | 171 | .14021     | 151 | .14625      |
| 33 | .04585     | 169 | .05070      | 170 | .14172     | 151 | .14778      |
| 34 | .04754     | 168 | .05241      | 170 | .14323     | 151 | .14932      |
| 35 | 8.04922    | 168 | 8.05411     | 170 | 8.14474    | 150 | 8.15085     |
| 36 | .05090     | 168 | .05581      | 170 | .14625     | 150 | .15237      |
| 37 | .05258     | 167 | .05751      | 169 | .14775     | 150 | .15390      |
| 38 | .05426     | 167 | .05921      | 169 | .14925     | 149 | .15542      |
| 39 | .05593     | 167 | .06090      | 169 | .15075     | 149 | .15694      |
| 40 | 8.05760    | 166 | 8.06259     | 169 | 8.15225    | 149 | 8.15846     |
| 41 | .05926     | 166 | .06427      | 168 | .15374     | 149 | .15997      |
| 42 | .06093     | 166 | .06593      | 168 | .15523     | 149 | .16148      |
| 43 | .06259     | 165 | .06763      | 167 | .15672     | 148 | .16299      |
| 44 | .06424     | 165 | .06931      | 167 | .15820     | 148 | .16450      |
| 45 | 8.06589    | 165 | 8.07098     | 167 | 8.15968    | 148 | 8.16600     |
| 46 | .06754     | 165 | .07265      | 166 | .16116     | 148 | .16750      |
| 47 | .06919     | 164 | .07431      | 166 | .16264     | 147 | .16900      |
| 48 | .07083     | 164 | .07598      | 166 | .16412     | 147 | .17050      |
| 49 | .07247     | 164 | .07764      | 166 | .16559     | 147 | .17199      |
| 50 | 8.07411    | 163 | 8.07929     | 165 | 8.16706    | 147 | 8.17349     |
| 51 | .07575     | 163 | .08095      | 165 | .16852     | 146 | .17497      |
| 52 | .07738     | 162 | .08260      | 164 | .16999     | 146 | .17646      |
| 53 | .07900     | 162 | .08424      | 164 | .17145     | 146 | .17795      |
| 54 | .08063     | 162 | .08589      | 164 | .17291     | 145 | .17943      |
| 55 | 8.08225    | 161 | 8.08753     | 163 | 8.17437    | 145 | 8.18091     |
| 56 | .08387     | 161 | .08917      | 163 | .17582     | 145 | .18238      |
| 57 | .08549     | 161 | .09081      | 163 | .17728     | 145 | .18386      |
| 58 | .08710     | 161 | .09244      | 163 | .17873     | 144 | .18533      |
| 59 | .08871     | 160 | .09407      | 162 | .18017     | 144 | .18680      |
| 60 | 8.09031    |     | 8.09569     |     | 8.18162    |     | 8.18827     |
|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. |
| 0  |            |     |             |     |            |     |             |
| 1  |            |     |             |     |            |     |             |
| 2  |            |     |             |     |            |     |             |
| 3  |            |     |             |     |            |     |             |
| 4  |            |     |             |     |            |     |             |
| 5  |            |     |             |     |            |     |             |
| 6  |            |     |             |     |            |     |             |
| 7  |            |     |             |     |            |     |             |
| 8  |            |     |             |     |            |     |             |
| 9  |            |     |             |     |            |     |             |
| 10 |            |     |             |     |            |     |             |
| 11 |            |     |             |     |            |     |             |
| 12 |            |     |             |     |            |     |             |
| 13 |            |     |             |     |            |     |             |
| 14 |            |     |             |     |            |     |             |
| 15 |            |     |             |     |            |     |             |
| 16 |            |     |             |     |            |     |             |
| 17 |            |     |             |     |            |     |             |
| 18 |            |     |             |     |            |     |             |
| 19 |            |     |             |     |            |     |             |
| 20 |            |     |             |     |            |     |             |
| 21 |            |     |             |     |            |     |             |
| 22 |            |     |             |     |            |     |             |
| 23 |            |     |             |     |            |     |             |
| 24 |            |     |             |     |            |     |             |
| 25 |            |     |             |     |            |     |             |
| 26 |            |     |             |     |            |     |             |
| 27 |            |     |             |     |            |     |             |
| 28 |            |     |             |     |            |     |             |
| 29 |            |     |             |     |            |     |             |
| 30 |            |     |             |     |            |     |             |
| 31 |            |     |             |     |            |     |             |
| 32 |            |     |             |     |            |     |             |
| 33 |            |     |             |     |            |     |             |
| 34 |            |     |             |     |            |     |             |
| 35 |            |     |             |     |            |     |             |
| 36 |            |     |             |     |            |     |             |
| 37 |            |     |             |     |            |     |             |
| 38 |            |     |             |     |            |     |             |
| 39 |            |     |             |     |            |     |             |
| 40 |            |     |             |     |            |     |             |
| 41 |            |     |             |     |            |     |             |
| 42 |            |     |             |     |            |     |             |
| 43 |            |     |             |     |            |     |             |
| 44 |            |     |             |     |            |     |             |
| 45 |            |     |             |     |            |     |             |
| 46 |            |     |             |     |            |     |             |
| 47 |            |     |             |     |            |     |             |
| 48 |            |     |             |     |            |     |             |
| 49 |            |     |             |     |            |     |             |
| 50 |            |     |             |     |            |     |             |
| 51 |            |     |             |     |            |     |             |
| 52 |            |     |             |     |            |     |             |
| 53 |            |     |             |     |            |     |             |
| 54 |            |     |             |     |            |     |             |
| 55 |            |     |             |     |            |     |             |
| 56 |            |     |             |     |            |     |             |
| 57 |            |     |             |     |            |     |             |
| 58 |            |     |             |     |            |     |             |
| 59 |            |     |             |     |            |     |             |
| 60 |            |     |             |     |            |     |             |

10°

11°

|    | Log. Vers. | D   | Log. Exsec. | D   |            | Log. Vers. | D           | Log. Exsec. | D  |       | P. P. |
|----|------------|-----|-------------|-----|------------|------------|-------------|-------------|----|-------|-------|
| 0  | 8.18162    |     | 8.18827     |     | 8.26417    |            | 8.27223     |             | 0  |       |       |
| 1  | .18306     | 144 | .18973      | 146 | .26548     | 131        | .27356      | 133         | 1  |       |       |
| 2  | .18450     | 144 | .19120      | 146 | .26679     | 131        | .27490      | 133         | 2  |       |       |
| 3  | .18594     | 144 | .19266      | 145 | .26810     | 130        | .27623      | 133         | 3  |       |       |
| 4  | .18738     | 143 | .19411      | 145 | .26941     | 130        | .27756      | 133         | 4  |       |       |
| 5  | 8.18881    | 143 | 8.19557     | 145 | 8.27071    | 130        | 8.27889     | 133         | 5  |       |       |
| 6  | .19024     | 142 | .19702      | 145 | .27201     | 130        | .28021      | 132         | 6  |       |       |
| 7  | .19167     | 142 | .19847      | 145 | .27331     | 130        | .28153      | 132         | 7  |       |       |
| 8  | .19309     | 142 | .19992      | 144 | .27461     | 129        | .28286      | 132         | 8  |       |       |
| 9  | .19452     | 142 | .20137      | 144 | .27590     | 129        | .28418      | 132         | 9  |       |       |
| 10 | 8.19594    | 142 | 8.20281     | 144 | 8.27719    | 129        | 8.28550     | 132         | 10 |       |       |
| 11 | .19736     | 142 | .20423      | 144 | .27849     | 128        | .28681      | 131         | 11 |       |       |
| 12 | .19878     | 142 | .20569      | 144 | .27977     | 128        | .28813      | 131         | 12 |       |       |
| 13 | .20019     | 141 | .20713      | 144 | .28106     | 129        | .28944      | 131         | 13 |       |       |
| 14 | .20160     | 141 | .20857      | 143 | .28235     | 128        | .29075      | 131         | 14 |       |       |
| 15 | 8.20301    | 140 | 8.21000     | 143 | 8.28363    | 128        | 8.29206     | 130         | 15 |       |       |
| 16 | .20442     | 140 | .21143      | 143 | .28491     | 128        | .29336      | 130         | 16 |       |       |
| 17 | .20582     | 140 | .21286      | 142 | .28619     | 128        | .29467      | 130         | 17 |       |       |
| 18 | .20723     | 140 | .21428      | 142 | .28747     | 127        | .29597      | 130         | 18 |       |       |
| 19 | .20863     | 140 | .21571      | 142 | .28875     | 127        | .29727      | 130         | 19 |       |       |
| 20 | 8.21003    | 139 | 8.21713     | 142 | 8.29002    | 127        | 8.29857     | 130         | 20 |       |       |
| 21 | .21142     | 139 | .21855      | 141 | .29129     | 127        | .29987      | 129         | 21 |       |       |
| 22 | .21282     | 139 | .21996      | 141 | .29256     | 127        | .30117      | 129         | 22 |       |       |
| 23 | .21421     | 139 | .22138      | 141 | .29383     | 126        | .30246      | 129         | 23 |       |       |
| 24 | .21560     | 138 | .22279      | 141 | .29510     | 126        | .30375      | 129         | 24 |       |       |
| 25 | 8.21698    | 138 | 8.22420     | 140 | 8.29636    | 126        | 8.30504     | 129         | 25 |       |       |
| 26 | .21837     | 138 | .22561      | 140 | .29763     | 126        | .30633      | 128         | 26 |       |       |
| 27 | .21975     | 138 | .22701      | 140 | .29889     | 126        | .30762      | 128         | 27 |       |       |
| 28 | .22113     | 137 | .22842      | 140 | .30015     | 125        | .30890      | 128         | 28 |       |       |
| 29 | .22251     | 138 | .22982      | 140 | .30140     | 125        | .31019      | 128         | 29 |       |       |
| 30 | 8.22389    | 137 | 8.23122     | 140 | 8.30266    | 125        | 8.31147     | 128         | 30 |       |       |
| 31 | .22526     | 137 | .23262      | 139 | .30391     | 125        | .31275      | 127         | 31 |       |       |
| 32 | .22663     | 136 | .23401      | 139 | .30516     | 125        | .31402      | 127         | 32 |       |       |
| 33 | .22800     | 137 | .23540      | 139 | .30642     | 124        | .31530      | 127         | 33 |       |       |
| 34 | .22937     | 136 | .23679      | 139 | .30766     | 124        | .31657      | 127         | 34 |       |       |
| 35 | 8.23073    | 136 | 8.23818     | 138 | 8.30891    | 124        | 8.31785     | 127         | 35 |       |       |
| 36 | .23209     | 136 | .23957      | 138 | .31015     | 124        | .31912      | 127         | 36 |       |       |
| 37 | .23346     | 135 | .24095      | 138 | .31140     | 124        | .32039      | 126         | 37 |       |       |
| 38 | .23481     | 136 | .24234      | 138 | .31264     | 124        | .32165      | 126         | 38 |       |       |
| 39 | .23617     | 135 | .24372      | 137 | .31388     | 123        | .32292      | 126         | 39 |       |       |
| 40 | 8.23752    | 135 | 8.24509     | 138 | 8.31511    | 124        | 8.32418     | 126         | 40 |       |       |
| 41 | .23888     | 135 | .24647      | 137 | .31635     | 123        | .32544      | 126         | 41 |       |       |
| 42 | .24023     | 135 | .24784      | 137 | .31758     | 123        | .32670      | 126         | 42 |       |       |
| 43 | .24158     | 134 | .24922      | 137 | .31882     | 123        | .32796      | 125         | 43 |       |       |
| 44 | .24292     | 134 | .25059      | 136 | .32005     | 123        | .32922      | 125         | 44 |       |       |
| 45 | 8.24426    | 134 | 8.25195     | 136 | 8.32128    | 122        | 8.33047     | 125         | 45 |       |       |
| 46 | .24561     | 134 | .25332      | 136 | .32250     | 122        | .33173      | 125         | 46 |       |       |
| 47 | .24695     | 133 | .25468      | 136 | .32373     | 122        | .33298      | 125         | 47 |       |       |
| 48 | .24828     | 133 | .25604      | 136 | .32495     | 122        | .33423      | 124         | 48 |       |       |
| 49 | .24962     | 133 | .25740      | 136 | .32617     | 122        | .33547      | 125         | 49 |       |       |
| 50 | 8.25095    | 133 | 8.25876     | 135 | 8.32739    | 122        | 8.33672     | 124         | 50 |       |       |
| 51 | .25228     | 133 | .26012      | 135 | .32861     | 121        | .33797      | 124         | 51 |       |       |
| 52 | .25361     | 132 | .26147      | 135 | .32983     | 121        | .33921      | 124         | 52 |       |       |
| 53 | .25494     | 133 | .26282      | 135 | .33104     | 121        | .34045      | 123         | 53 |       |       |
| 54 | .25627     | 132 | .26417      | 134 | .33225     | 121        | .34169      | 124         | 54 |       |       |
| 55 | 8.25759    | 132 | 8.26552     | 134 | 8.33347    | 121        | 8.34293     | 124         | 55 |       |       |
| 56 | .25891     | 132 | .26686      | 134 | .33468     | 120        | .34417      | 123         | 56 |       |       |
| 57 | .26023     | 132 | .26821      | 134 | .33588     | 120        | .34540      | 123         | 57 |       |       |
| 58 | .26155     | 131 | .26955      | 134 | .33709     | 120        | .34663      | 123         | 58 |       |       |
| 59 | .26286     | 131 | .27089      | 134 | .33829     | 120        | .34786      | 123         | 59 |       |       |
| 60 | 8.26417    |     | 8.27223     |     | 8.33950    |            | 8.34909     |             | 60 |       |       |
|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D          | Log. Exsec. | D           |    | P. P. |       |

130

120

|    |       |       |
|----|-------|-------|
| 6  | 13.0  | 12.0  |
| 7  | 15.1  | 14.0  |
| 8  | 17.3  | 16.0  |
| 9  | 19.5  | 18.0  |
| 10 | 21.0  | 20.0  |
| 20 | 43.3  | 40.0  |
| 30 | 65.0  | 60.0  |
| 40 | 86.6  | 80.0  |
| 50 | 108.3 | 100.0 |

4

3

|    |     |     |
|----|-----|-----|
| 6  | 0.4 | 0.3 |
| 7  | 0.5 | 0.4 |
| 8  | 0.6 | 0.5 |
| 9  | 0.7 | 0.6 |
| 10 | 0.7 | 0.6 |
| 20 | 1.5 | 1.1 |
| 30 | 2.2 | 1.7 |
| 40 | 3.0 | 2.3 |
| 50 | 3.7 | 2.9 |

3

2

|    |     |     |
|----|-----|-----|
| 6  | 0.3 | 0.2 |
| 7  | 0.3 | 0.3 |
| 8  | 0.4 | 0.3 |
| 9  | 0.4 | 0.4 |
| 10 | 0.5 | 0.4 |
| 20 | 1.0 | 0.8 |
| 30 | 1.5 | 1.2 |
| 40 | 2.0 | 1.6 |
| 50 | 2.5 | 2.1 |

2

1

|    |     |     |
|----|-----|-----|
| 6  | 0.2 | 0.1 |
| 7  | 0.2 | 0.2 |
| 8  | 0.2 | 0.2 |
| 9  | 0.3 | 0.2 |
| 10 | 0.3 | 0.2 |
| 20 | 0.6 | 0.5 |
| 30 | 1.0 | 0.7 |
| 40 | 1.3 | 1.0 |
| 50 | 1.6 | 1.3 |

1

0

|    |     |     |
|----|-----|-----|
| 6  | 0.1 | 0.0 |
| 7  | 0.1 | 0.0 |
| 8  | 0.1 | 0.0 |
| 9  | 0.1 | 0.1 |
| 10 | 0.1 | 0.1 |
| 20 | 0.3 | 0.1 |
| 30 | 0.5 | 0.2 |
| 40 | 0.6 | 0.3 |
| 50 | 0.8 | 0.4 |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

12°

13°

|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    | P. P. |  |  |
|----|------------|-----|-------------|-----|------------|-----|-------------|-----|----|-------|--|--|
| 0  | 8.33950    | 120 | 8.34909     | 123 | 8.40875    | 110 | 8.42002     | 113 | 0  |       |  |  |
| 1  | .34070     | 120 | .35032      | 122 | .40985     | 110 | .42116      | 113 | 1  |       |  |  |
| 2  | .34190     | 119 | .35155      | 122 | .41096     | 110 | .42229      | 113 | 2  |       |  |  |
| 3  | .34309     | 120 | .35277      | 122 | .41206     | 110 | .42343      | 113 | 3  |       |  |  |
| 4  | .34429     | 119 | .35399      | 122 | .41317     | 110 | .42456      | 113 | 4  |       |  |  |
| 5  | 8.34549    | 119 | 8.35522     | 122 | 8.41427    | 110 | 8.42569     | 113 | 5  |       |  |  |
| 6  | .34668     | 119 | .35644      | 121 | .41537     | 110 | .42682      | 113 | 6  |       |  |  |
| 7  | .34787     | 119 | .35765      | 122 | .41647     | 109 | .42795      | 113 | 7  |       |  |  |
| 8  | .34906     | 119 | .35887      | 121 | .41757     | 110 | .42908      | 112 | 8  |       |  |  |
| 9  | .35025     | 118 | .36009      | 121 | .41867     | 109 | .43021      | 112 | 9  |       |  |  |
| 10 | 8.35143    | 118 | 8.36130     | 121 | 8.41976    | 109 | 8.43133     | 112 | 10 |       |  |  |
| 11 | .35262     | 118 | .36251      | 121 | .42086     | 109 | .43246      | 112 | 11 |       |  |  |
| 12 | .35380     | 118 | .36372      | 120 | .42195     | 109 | .43358      | 112 | 12 |       |  |  |
| 13 | .35498     | 118 | .36493      | 121 | .42304     | 109 | .43470      | 112 | 13 |       |  |  |
| 14 | .35616     | 117 | .36614      | 120 | .42413     | 109 | .43582      | 112 | 14 |       |  |  |
| 15 | 8.35734    | 118 | 8.36734     | 120 | 8.42522    | 108 | 8.43694     | 111 | 15 |       |  |  |
| 16 | .35852     | 117 | .36855      | 120 | .42630     | 109 | .43805      | 111 | 16 |       |  |  |
| 17 | .35969     | 117 | .36975      | 120 | .42739     | 108 | .43917      | 111 | 17 |       |  |  |
| 18 | .36086     | 117 | .37095      | 120 | .42847     | 108 | .44028      | 111 | 18 |       |  |  |
| 19 | .36204     | 117 | .37215      | 120 | .42956     | 108 | .44139      | 111 | 19 |       |  |  |
| 20 | 8.36321    | 116 | 8.37335     | 119 | 8.43064    | 108 | 8.44251     | 111 | 20 |       |  |  |
| 21 | .36437     | 117 | .37454      | 119 | .43172     | 108 | .44362      | 111 | 21 |       |  |  |
| 22 | .36554     | 116 | .37574      | 119 | .43280     | 108 | .44473      | 110 | 22 |       |  |  |
| 23 | .36671     | 116 | .37693      | 119 | .43388     | 107 | .44583      | 110 | 23 |       |  |  |
| 24 | .36787     | 116 | .37812      | 119 | .43495     | 107 | .44694      | 110 | 24 |       |  |  |
| 25 | 8.36903    | 116 | 8.37931     | 118 | 8.43603    | 107 | 8.44804     | 110 | 25 |       |  |  |
| 26 | .37019     | 116 | .38050      | 119 | .43710     | 107 | .44915      | 110 | 26 |       |  |  |
| 27 | .37133     | 115 | .38169      | 118 | .43817     | 107 | .45025      | 110 | 27 |       |  |  |
| 28 | .37251     | 115 | .38287      | 118 | .43924     | 107 | .45135      | 109 | 28 |       |  |  |
| 29 | .37366     | 115 | .38406      | 118 | .44031     | 106 | .45245      | 110 | 29 |       |  |  |
| 30 | 8.37482    | 115 | 8.38524     | 118 | 8.44138    | 107 | 8.45355     | 110 | 30 |       |  |  |
| 31 | .37597     | 115 | .38642      | 118 | .44245     | 106 | .45465      | 109 | 31 |       |  |  |
| 32 | .37712     | 115 | .38760      | 118 | .44351     | 106 | .45574      | 109 | 32 |       |  |  |
| 33 | .37827     | 115 | .38878      | 117 | .44458     | 106 | .45684      | 109 | 33 |       |  |  |
| 34 | .37942     | 114 | .38995      | 117 | .44564     | 106 | .45793      | 109 | 34 |       |  |  |
| 35 | 8.38057    | 114 | 8.39113     | 117 | 8.44670    | 106 | 8.45902     | 109 | 35 |       |  |  |
| 36 | .38171     | 114 | .39230      | 117 | .44776     | 105 | .46011      | 109 | 36 |       |  |  |
| 37 | .38286     | 114 | .39347      | 117 | .44882     | 106 | .46120      | 108 | 37 |       |  |  |
| 38 | .38400     | 114 | .39464      | 117 | .44988     | 105 | .46229      | 109 | 38 |       |  |  |
| 39 | .38514     | 114 | .39581      | 116 | .45093     | 105 | .46338      | 108 | 39 |       |  |  |
| 40 | 8.38628    | 113 | 8.39698     | 116 | 8.45199    | 105 | 8.46446     | 108 | 40 |       |  |  |
| 41 | .38741     | 114 | .39814      | 116 | .45304     | 105 | .46555      | 108 | 41 |       |  |  |
| 42 | .38855     | 113 | .39931      | 116 | .45409     | 105 | .46663      | 108 | 42 |       |  |  |
| 43 | .38969     | 113 | .40047      | 116 | .45514     | 105 | .46771      | 108 | 43 |       |  |  |
| 44 | .39082     | 113 | .40163      | 116 | .45619     | 105 | .46879      | 108 | 44 |       |  |  |
| 45 | 8.39195    | 113 | 8.40279     | 116 | 8.45724    | 104 | 8.46987     | 107 | 45 |       |  |  |
| 46 | .39308     | 113 | .40395      | 115 | .45829     | 105 | .47095      | 108 | 46 |       |  |  |
| 47 | .39421     | 113 | .40511      | 115 | .45934     | 104 | .47203      | 107 | 47 |       |  |  |
| 48 | .39534     | 112 | .40626      | 115 | .46038     | 104 | .47310      | 107 | 48 |       |  |  |
| 49 | .39646     | 112 | .40742      | 115 | .46142     | 104 | .47417      | 107 | 49 |       |  |  |
| 50 | 8.39758    | 112 | 8.40857     | 115 | 8.46247    | 104 | 8.47525     | 107 | 50 |       |  |  |
| 51 | .39871     | 112 | .40972      | 115 | .46351     | 104 | .47632      | 107 | 51 |       |  |  |
| 52 | .39983     | 112 | .41087      | 115 | .46455     | 103 | .47739      | 107 | 52 |       |  |  |
| 53 | .40095     | 112 | .41202      | 114 | .46558     | 104 | .47846      | 106 | 53 |       |  |  |
| 54 | .40207     | 111 | .41317      | 114 | .46662     | 103 | .47953      | 107 | 54 |       |  |  |
| 55 | 8.40318    | 111 | 8.41431     | 114 | 8.46766    | 103 | 8.48060     | 106 | 55 |       |  |  |
| 56 | .40430     | 111 | .41546      | 114 | .46869     | 103 | .48166      | 106 | 56 |       |  |  |
| 57 | .40541     | 111 | .41660      | 114 | .46972     | 103 | .48273      | 106 | 57 |       |  |  |
| 58 | .40652     | 111 | .41774      | 114 | .47076     | 103 | .48379      | 106 | 58 |       |  |  |
| 59 | .40764     | 111 | .41888      | 114 | .47179     | 103 | .48485      | 106 | 59 |       |  |  |
| 60 | 8.40875    |     | 8.42002     |     | 8.47282    |     | 8.48591     |     | 60 |       |  |  |
|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    | P. P. |  |  |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

14°

15°

| 14° |            |     |             | 15° |            |            |             | P. P.       |    |       |  |
|-----|------------|-----|-------------|-----|------------|------------|-------------|-------------|----|-------|--|
|     | Log. Vers. | D   | Log. Exsec. | D   |            | Log. Vers. | D           | Log. Exsec. | D  |       |  |
| 0   | 8.47282    | 102 | 8.48591     | 106 | 8.53242    | 96         | 8.54748     | 99          | 0  |       |  |
| 1   | .47384     | 103 | .48697      | 106 | .53338     | 95         | .54847      | 99          | 1  |       |  |
| 2   | .47487     | 102 | .48803      | 105 | .53434     | 96         | .54946      | 99          | 2  |       |  |
| 3   | .47590     | 102 | .48909      | 105 | .53530     | 95         | .55045      | 99          | 3  |       |  |
| 4   | .47692     | 102 | .49014      | 105 | .53623     | 95         | .55144      | 99          | 4  |       |  |
| 5   | 8.47795    | 102 | 8.49120     | 105 | 8.53721    | 95         | 8.55243     | 99          | 5  |       |  |
| 6   | .47897     | 102 | .49223      | 105 | .53816     | 95         | .55342      | 99          | 6  |       |  |
| 7   | .47999     | 102 | .49331      | 105 | .53911     | 95         | .55441      | 98          | 7  |       |  |
| 8   | .48101     | 102 | .49436      | 105 | .54007     | 95         | .55539      | 98          | 8  |       |  |
| 9   | .48203     | 101 | .49541      | 105 | .54102     | 95         | .55638      | 98          | 9  |       |  |
| 10  | 8.48304    | 101 | 8.49646     | 104 | 8.54197    | 94         | 8.55736     | 98          | 10 |       |  |
| 11  | .48406     | 101 | .49750      | 105 | .54291     | 95         | .55834      | 98          | 11 |       |  |
| 12  | .48507     | 101 | .49853      | 104 | .54386     | 94         | .55933      | 98          | 12 |       |  |
| 13  | .48609     | 101 | .49960      | 104 | .54481     | 94         | .56031      | 98          | 13 |       |  |
| 14  | .48716     | 101 | .50064      | 104 | .54573     | 94         | .56129      | 98          | 14 |       |  |
| 15  | 8.48811    | 101 | 8.50168     | 104 | 8.54670    | 94         | 8.56226     | 98          | 15 |       |  |
| 16  | .48912     | 101 | .50273      | 104 | .54764     | 94         | .56324      | 97          | 16 |       |  |
| 17  | .49013     | 100 | .50377      | 104 | .54858     | 94         | .56422      | 97          | 17 |       |  |
| 18  | .49114     | 101 | .50481      | 104 | .54952     | 94         | .56519      | 97          | 18 |       |  |
| 19  | .49215     | 100 | .50585      | 103 | .55046     | 94         | .56617      | 97          | 19 |       |  |
| 20  | 8.49315    | 100 | 8.50688     | 104 | 8.55140    | 93         | 8.56714     | 97          | 20 |       |  |
| 21  | .49415     | 100 | .50792      | 103 | .55234     | 94         | .56812      | 97          | 21 |       |  |
| 22  | .49516     | 100 | .50896      | 103 | .55328     | 93         | .56909      | 97          | 22 |       |  |
| 23  | .49616     | 100 | .50999      | 103 | .55421     | 93         | .57006      | 97          | 23 |       |  |
| 24  | .49716     | 100 | .51103      | 103 | .55515     | 93         | .57103      | 97          | 24 |       |  |
| 25  | 8.49816    | 100 | 8.51205     | 103 | 8.55608    | 93         | 8.57200     | 97          | 25 |       |  |
| 26  | .49916     | 99  | .51309      | 103 | .55701     | 93         | .57296      | 97          | 26 |       |  |
| 27  | .50015     | 100 | .51412      | 102 | .55795     | 93         | .57393      | 96          | 27 |       |  |
| 28  | .50115     | 99  | .51514      | 103 | .55888     | 93         | .57490      | 96          | 28 |       |  |
| 29  | .50215     | 99  | .51617      | 102 | .55981     | 93         | .57586      | 96          | 29 |       |  |
| 30  | 8.50314    | 99  | 8.51720     | 102 | 8.56074    | 92         | 8.57682     | 96          | 30 |       |  |
| 31  | .50413     | 99  | .51822      | 102 | .56166     | 92         | .57779      | 96          | 31 |       |  |
| 32  | .50513     | 99  | .51925      | 102 | .56259     | 93         | .57875      | 96          | 32 |       |  |
| 33  | .50611     | 99  | .52027      | 102 | .56352     | 92         | .57971      | 96          | 33 |       |  |
| 34  | .50716     | 98  | .52129      | 102 | .56444     | 92         | .58067      | 95          | 34 |       |  |
| 35  | 8.50809    | 99  | 8.52231     | 102 | 8.56536    | 92         | 8.58163     | 95          | 35 |       |  |
| 36  | .50908     | 98  | .52333      | 102 | .56629     | 92         | .58259      | 95          | 36 |       |  |
| 37  | .51006     | 98  | .52435      | 101 | .56721     | 92         | .58354      | 95          | 37 |       |  |
| 38  | .51105     | 98  | .52537      | 101 | .56813     | 92         | .58450      | 95          | 38 |       |  |
| 39  | .51203     | 98  | .52638      | 101 | .56905     | 92         | .58546      | 95          | 39 |       |  |
| 40  | 8.51301    | 98  | 8.52740     | 101 | 8.56997    | 92         | 8.58641     | 95          | 40 |       |  |
| 41  | .51399     | 98  | .52841      | 101 | .57089     | 91         | .58736      | 95          | 41 |       |  |
| 42  | .51497     | 98  | .52943      | 101 | .57180     | 91         | .58832      | 95          | 42 |       |  |
| 43  | .51595     | 97  | .53044      | 101 | .57272     | 91         | .58927      | 95          | 43 |       |  |
| 44  | .51693     | 98  | .53145      | 101 | .57363     | 91         | .59022      | 95          | 44 |       |  |
| 45  | 8.51791    | 97  | 8.53246     | 101 | 8.57455    | 91         | 8.59117     | 94          | 45 |       |  |
| 46  | .51888     | 97  | .53347      | 101 | .57546     | 91         | .59211      | 95          | 46 |       |  |
| 47  | .51986     | 97  | .53448      | 100 | .57637     | 91         | .59306      | 94          | 47 |       |  |
| 48  | .52083     | 97  | .53548      | 100 | .57728     | 91         | .59401      | 94          | 48 |       |  |
| 49  | .52180     | 97  | .53649      | 100 | .57819     | 91         | .59495      | 94          | 49 |       |  |
| 50  | 8.52277    | 97  | 8.53749     | 100 | 8.57916    | 90         | 8.59590     | 94          | 50 |       |  |
| 51  | .52374     | 97  | .53850      | 100 | .58001     | 91         | .59684      | 94          | 51 |       |  |
| 52  | .52471     | 96  | .53950      | 100 | .58092     | 90         | .59779      | 94          | 52 |       |  |
| 53  | .52568     | 97  | .54050      | 100 | .58183     | 90         | .59873      | 94          | 53 |       |  |
| 54  | .52665     | 96  | .54150      | 100 | .58273     | 90         | .59967      | 94          | 54 |       |  |
| 55  | 8.52761    | 96  | 8.54250     | 100 | 8.58363    | 90         | 8.60061     | 94          | 55 |       |  |
| 56  | .52858     | 96  | .54350      | 99  | .58453     | 90         | .60155      | 94          | 56 |       |  |
| 57  | .52954     | 96  | .54449      | 100 | .58544     | 90         | .60249      | 93          | 57 |       |  |
| 58  | .53050     | 96  | .54549      | 99  | .58634     | 90         | .60342      | 94          | 58 |       |  |
| 59  | .53146     | 96  | .54649      | 99  | .58724     | 90         | .60436      | 93          | 59 |       |  |
| 60  | 8.53242    |     | 8.54748     |     | 8.58814    |            | 8.60530     |             | 60 |       |  |
|     | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D          | Log. Exsec. | D           |    | P. P. |  |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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17°

|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |  |  |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|--|--|
| 0  | 8.58814    | 90 | 8.60530     | 93 | 8.64043    | 84 | 8.65984     | 88 | 0  |       |  |  |
| 1  | .58904     | 89 | .60623      | 93 | .64128     | 84 | .66072      | 88 | 1  |       |  |  |
| 2  | .58993     | 90 | .60716      | 93 | .64212     | 84 | .66160      | 88 | 2  |       |  |  |
| 3  | .59083     | 89 | .60810      | 93 | .64296     | 84 | .66248      | 88 | 3  |       |  |  |
| 4  | .59173     | 89 | .60903      | 93 | .64381     | 84 | .66336      | 88 | 4  |       |  |  |
| 5  | 8.59262    | 89 | 8.60996     | 93 | 8.64465    | 84 | 8.66425     | 88 | 5  |       |  |  |
| 6  | .59351     | 89 | .61089      | 93 | .64549     | 84 | .66512      | 88 | 6  |       |  |  |
| 7  | .59441     | 89 | .61182      | 93 | .64633     | 84 | .66600      | 88 | 7  |       |  |  |
| 8  | .59530     | 89 | .61275      | 92 | .64717     | 84 | .66688      | 88 | 8  |       |  |  |
| 9  | .59619     | 89 | .61368      | 93 | .64801     | 84 | .66776      | 87 | 9  |       |  |  |
| 10 | 8.59708    | 89 | 8.61460     | 92 | 8.64884    | 83 | 8.66863     | 87 | 10 |       |  |  |
| 11 | .59797     | 89 | .61553      | 92 | .64968     | 83 | .66951      | 87 | 11 |       |  |  |
| 12 | .59886     | 88 | .61645      | 92 | .65052     | 83 | .67039      | 87 | 12 |       |  |  |
| 13 | .59974     | 89 | .61738      | 92 | .65135     | 83 | .67126      | 87 | 13 |       |  |  |
| 14 | .60063     | 88 | .61830      | 92 | .65218     | 83 | .67213      | 87 | 14 |       |  |  |
| 15 | 8.60152    | 88 | 8.61922     | 92 | 8.65302    | 83 | 8.67301     | 87 | 15 |       |  |  |
| 16 | .60240     | 88 | .62014      | 92 | .65385     | 83 | .67388      | 87 | 16 |       |  |  |
| 17 | .60328     | 88 | .62106      | 92 | .65468     | 83 | .67475      | 87 | 17 |       |  |  |
| 18 | .60417     | 88 | .62198      | 92 | .65551     | 83 | .67562      | 87 | 18 |       |  |  |
| 19 | .60505     | 88 | .62290      | 92 | .65634     | 83 | .67649      | 87 | 19 |       |  |  |
| 20 | 8.60593    | 88 | 8.62382     | 91 | 8.65717    | 83 | 8.67736     | 86 | 20 |       |  |  |
| 21 | .60681     | 88 | .62474      | 91 | .65800     | 82 | .67822      | 86 | 21 |       |  |  |
| 22 | .60769     | 88 | .62565      | 91 | .65883     | 82 | .67909      | 86 | 22 |       |  |  |
| 23 | .60857     | 87 | .62657      | 91 | .65965     | 83 | .67996      | 86 | 23 |       |  |  |
| 24 | .60944     | 87 | .62748      | 91 | .66048     | 82 | .68082      | 86 | 24 |       |  |  |
| 25 | 8.61032    | 87 | 8.62840     | 91 | 8.66131    | 82 | 8.68169     | 86 | 25 |       |  |  |
| 26 | .61119     | 87 | .62931      | 91 | .66213     | 82 | .68255      | 86 | 26 |       |  |  |
| 27 | .61207     | 87 | .63022      | 91 | .66295     | 82 | .68341      | 86 | 27 |       |  |  |
| 28 | .61294     | 87 | .63113      | 91 | .66378     | 82 | .68428      | 86 | 28 |       |  |  |
| 29 | .61381     | 87 | .63204      | 90 | .66460     | 82 | .68514      | 86 | 29 |       |  |  |
| 30 | 8.61469    | 87 | 8.63295     | 91 | 8.66542    | 82 | 8.68600     | 86 | 30 |       |  |  |
| 31 | .61556     | 87 | .63386      | 91 | .66624     | 82 | .68686      | 85 | 31 |       |  |  |
| 32 | .61643     | 87 | .63477      | 90 | .66706     | 82 | .68772      | 86 | 32 |       |  |  |
| 33 | .61730     | 86 | .63567      | 90 | .66788     | 82 | .68858      | 86 | 33 |       |  |  |
| 34 | .61816     | 87 | .63658      | 90 | .66870     | 81 | .68944      | 85 | 34 |       |  |  |
| 35 | 8.61903    | 86 | 8.63748     | 90 | 8.66951    | 81 | 8.69029     | 86 | 35 |       |  |  |
| 36 | .61990     | 86 | .63839      | 90 | .67033     | 82 | .69115      | 85 | 36 |       |  |  |
| 37 | .62076     | 86 | .63929      | 90 | .67115     | 81 | .69201      | 85 | 37 |       |  |  |
| 38 | .62163     | 86 | .64019      | 90 | .67196     | 81 | .69286      | 85 | 38 |       |  |  |
| 39 | .62249     | 86 | .64109      | 90 | .67277     | 81 | .69372      | 85 | 39 |       |  |  |
| 40 | 8.62330    | 86 | 8.64199     | 90 | 8.67359    | 81 | 8.69457     | 85 | 40 |       |  |  |
| 41 | .62422     | 86 | .64289      | 90 | .67440     | 81 | .69542      | 85 | 41 |       |  |  |
| 42 | .62508     | 86 | .64379      | 90 | .67521     | 81 | .69627      | 85 | 42 |       |  |  |
| 43 | .62594     | 86 | .64469      | 89 | .67602     | 81 | .69712      | 85 | 43 |       |  |  |
| 44 | .62680     | 86 | .64559      | 90 | .67683     | 81 | .69798      | 85 | 44 |       |  |  |
| 45 | 8.62766    | 86 | 8.64649     | 89 | 8.67764    | 81 | 8.69883     | 84 | 45 |       |  |  |
| 46 | .62852     | 85 | .64738      | 89 | .67845     | 80 | .69967      | 85 | 46 |       |  |  |
| 47 | .62937     | 85 | .64828      | 89 | .67926     | 81 | .70052      | 85 | 47 |       |  |  |
| 48 | .63023     | 85 | .64917      | 89 | .68007     | 80 | .70137      | 84 | 48 |       |  |  |
| 49 | .63108     | 85 | .65006      | 89 | .68087     | 80 | .70222      | 84 | 49 |       |  |  |
| 50 | 8.63194    | 85 | 8.65096     | 89 | 8.68168    | 80 | 8.70306     | 84 | 50 |       |  |  |
| 51 | .63279     | 85 | .65185      | 89 | .68248     | 80 | .70391      | 84 | 51 |       |  |  |
| 52 | .63364     | 85 | .65274      | 89 | .68329     | 80 | .70475      | 84 | 52 |       |  |  |
| 53 | .63449     | 85 | .65363      | 89 | .68409     | 80 | .70560      | 84 | 53 |       |  |  |
| 54 | .63534     | 85 | .65452      | 89 | .68489     | 80 | .70644      | 84 | 54 |       |  |  |
| 55 | 8.63619    | 85 | 8.65541     | 88 | 8.68569    | 80 | 8.70728     | 84 | 55 |       |  |  |
| 56 | .63704     | 85 | .65629      | 88 | .68650     | 80 | .70813      | 84 | 56 |       |  |  |
| 57 | .63789     | 84 | .65718      | 89 | .68730     | 80 | .70897      | 84 | 57 |       |  |  |
| 58 | .63874     | 85 | .65807      | 88 | .68810     | 79 | .70981      | 84 | 58 |       |  |  |
| 59 | .63959     | 84 | .65895      | 88 | .68889     | 80 | .71065      | 84 | 59 |       |  |  |
| 60 | 8.64043    |    | 8.65984     |    | 8.68969    |    | 8.71149     |    | 60 |       |  |  |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |  |  |

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19°

|    |            |    |             |    |            |    |             |    |    | P. P. |      |      |      |      |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|------|------|------|------|
| '  | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  | '  |       |      |      |      |      |
| 0  | 8.68969    | 79 | 8.71149     | 83 | 8.73625    | 75 | 8.76058     | 79 | 0  |       | 84   | 83   | 82   |      |
| 1  | .69049     | 80 | .71232      | 84 | .73700     | 75 | .76137      | 80 | 1  |       | 6    | 8.4  | 8.3  | 8.2  |
| 2  | .69129     | 79 | .71316      | 83 | .73773     | 75 | .76217      | 79 | 2  |       | 7    | 9.8  | 9.7  | 9.5  |
| 3  | .69208     | 79 | .71400      | 84 | .73851     | 75 | .76297      | 79 | 3  |       | 8    | 11.2 | 11.0 | 10.9 |
| 4  | .69288     | 79 | .71484      | 83 | .73926     | 75 | .76376      | 80 | 4  |       | 9    | 12.6 | 12.4 | 12.3 |
| 5  | 8.69367    | 79 | 8.71567     | 83 | 8.74001    | 75 | 8.76456     | 79 | 5  | 10    | 14.0 | 13.8 | 13.6 |      |
| 6  | .69446     | 79 | .71651      | 83 | .74076     | 75 | .76536      | 79 | 6  | 20    | 28.0 | 27.6 | 27.3 |      |
| 7  | .69526     | 79 | .71734      | 83 | .74151     | 75 | .76615      | 79 | 7  | 30    | 42.0 | 41.5 | 41.0 |      |
| 8  | .69605     | 79 | .71817      | 83 | .74226     | 75 | .76694      | 79 | 8  | 40    | 56.0 | 55.3 | 54.6 |      |
| 9  | .69684     | 79 | .71901      | 83 | .74301     | 75 | .76774      | 79 | 9  | 50    | 70.0 | 69.1 | 68.3 |      |
| 10 | 8.69763    | 79 | 8.71984     | 83 | 8.74376    | 75 | 8.76853     | 79 | 10 |       | 81   | 80   | 79   |      |
| 11 | .69842     | 79 | .72067      | 83 | .74451     | 74 | .76932      | 79 | 11 |       | 6    | 8.1  | 8.0  | 7.9  |
| 12 | .69921     | 79 | .72150      | 83 | .74526     | 75 | .77011      | 79 | 12 |       | 7    | 9.4  | 9.3  | 9.2  |
| 13 | .70000     | 78 | .72233      | 83 | .74600     | 74 | .77090      | 79 | 13 |       | 8    | 10.8 | 10.6 | 10.5 |
| 14 | .70079     | 79 | .72316      | 83 | .74675     | 74 | .77169      | 79 | 14 |       | 9    | 12.1 | 12.0 | 11.8 |
| 15 | 8.70157    | 78 | 8.72399     | 82 | 8.74749    | 74 | 8.77248     | 79 | 15 | 10    | 13.5 | 13.3 | 13.1 |      |
| 16 | .70236     | 78 | .72481      | 82 | .74824     | 74 | .77327      | 79 | 16 | 20    | 27.0 | 26.6 | 26.3 |      |
| 17 | .70314     | 78 | .72564      | 82 | .74898     | 74 | .77406      | 79 | 17 | 30    | 40.5 | 40.0 | 39.5 |      |
| 18 | .70393     | 78 | .72647      | 82 | .74973     | 74 | .77485      | 78 | 18 | 40    | 54.0 | 53.3 | 52.6 |      |
| 19 | .70471     | 78 | .72729      | 82 | .75047     | 74 | .77563      | 78 | 19 | 50    | 67.5 | 66.6 | 65.8 |      |
| 20 | 8.70550    | 78 | 8.72812     | 82 | 8.75121    | 74 | 8.77642     | 78 | 20 |       | 78   | 77   | 76   |      |
| 21 | .70628     | 78 | .72894      | 82 | .75193     | 74 | .77720      | 79 | 21 |       | 6    | 7.8  | 7.7  | 7.6  |
| 22 | .70706     | 78 | .72977      | 82 | .75269     | 74 | .77799      | 79 | 22 |       | 7    | 9.1  | 9.0  | 8.9  |
| 23 | .70784     | 78 | .73059      | 82 | .75343     | 74 | .77877      | 78 | 23 |       | 8    | 10.4 | 10.2 | 10.1 |
| 24 | .70862     | 78 | .73141      | 82 | .75417     | 74 | .77956      | 78 | 24 |       | 9    | 11.7 | 11.5 | 11.4 |
| 25 | 8.70940    | 78 | 8.73223     | 82 | 8.75491    | 73 | 8.78034     | 78 | 25 | 10    | 13.0 | 12.7 | 12.6 |      |
| 26 | .71018     | 78 | .73306      | 82 | .75565     | 73 | .78112      | 78 | 26 | 20    | 26.0 | 25.6 | 25.3 |      |
| 27 | .71096     | 77 | .73388      | 82 | .75639     | 73 | .78191      | 78 | 27 | 30    | 39.0 | 38.5 | 38.0 |      |
| 28 | .71174     | 78 | .73470      | 81 | .75712     | 73 | .78269      | 78 | 28 | 40    | 52.0 | 51.3 | 50.6 |      |
| 29 | .71251     | 77 | .73551      | 82 | .75786     | 73 | .78347      | 78 | 29 | 50    | 65.0 | 64.1 | 63.3 |      |
| 30 | 8.71329    | 77 | 8.73633     | 82 | 8.75860    | 74 | 8.78425     | 78 | 30 |       | 75   | 74   | 73   |      |
| 31 | .71406     | 77 | .73715      | 81 | .75933     | 74 | .78503      | 78 | 31 |       | 6    | 7.5  | 7.4  | 7.3  |
| 32 | .71484     | 77 | .73797      | 81 | .76006     | 74 | .78581      | 78 | 32 |       | 7    | 8.7  | 8.6  | 8.5  |
| 33 | .71561     | 77 | .73878      | 81 | .76080     | 73 | .78659      | 78 | 33 |       | 8    | 10.0 | 9.8  | 9.7  |
| 34 | .71639     | 77 | .73960      | 81 | .76153     | 73 | .78736      | 77 | 34 |       | 9    | 11.2 | 11.1 | 10.9 |
| 35 | 8.71716    | 77 | 8.74041     | 81 | 8.76226    | 73 | 8.78814     | 78 | 35 | 10    | 12.5 | 12.3 | 12.1 |      |
| 36 | .71793     | 77 | .74123      | 81 | .76300     | 73 | .78892      | 77 | 36 | 20    | 25.0 | 24.6 | 24.3 |      |
| 37 | .71870     | 77 | .74204      | 81 | .76373     | 73 | .78969      | 77 | 37 | 30    | 37.5 | 37.0 | 36.5 |      |
| 38 | .71947     | 77 | .74286      | 81 | .76446     | 73 | .79047      | 77 | 38 | 40    | 50.0 | 49.3 | 48.6 |      |
| 39 | .72024     | 77 | .74367      | 81 | .76519     | 73 | .79124      | 77 | 39 | 50    | 62.5 | 61.6 | 60.8 |      |
| 40 | 8.72101    | 77 | 8.74448     | 81 | 8.76592    | 73 | 8.79202     | 77 | 40 |       | 72   | 71   | 70   |      |
| 41 | .72178     | 76 | .74529      | 81 | .76664     | 72 | .79279      | 77 | 41 |       | 6    | 7.2  | 7.1  | 7.0  |
| 42 | .72255     | 77 | .74610      | 81 | .76737     | 73 | .79357      | 77 | 42 |       | 7    | 8.4  | 8.3  | 8.2  |
| 43 | .72331     | 76 | .74691      | 81 | .76810     | 72 | .79434      | 77 | 43 |       | 8    | 9.6  | 9.4  | 9.3  |
| 44 | .72408     | 77 | .74772      | 80 | .76883     | 73 | .79511      | 77 | 44 |       | 9    | 10.8 | 10.6 | 10.5 |
| 45 | 8.72485    | 76 | 8.74853     | 81 | 8.76953    | 72 | 8.79588     | 77 | 45 | 10    | 12.0 | 11.7 | 11.6 |      |
| 46 | .72561     | 76 | .74934      | 80 | .77028     | 72 | .79663      | 77 | 46 | 20    | 24.0 | 23.6 | 23.3 |      |
| 47 | .72637     | 76 | .75014      | 80 | .77100     | 72 | .79742      | 77 | 47 | 30    | 36.0 | 35.5 | 35.2 |      |
| 48 | .72714     | 76 | .75095      | 80 | .77173     | 72 | .79819      | 77 | 48 | 40    | 48.0 | 47.3 | 46.6 |      |
| 49 | .72790     | 76 | .75173      | 80 | .77245     | 72 | .79896      | 76 | 49 | 50    | 60.0 | 59.1 | 58.4 |      |
| 50 | 8.72866    | 76 | 8.75256     | 80 | 8.77317    | 72 | 8.79973     | 77 | 50 |       | 69   | 68   | 67   |      |
| 51 | .72942     | 76 | .75336      | 80 | .77390     | 72 | .80050      | 76 | 51 |       | 6    | 7.0  | 6.9  | 6.8  |
| 52 | .73018     | 76 | .75417      | 80 | .77462     | 72 | .80126      | 77 | 52 |       | 7    | 8.2  | 8.1  | 8.0  |
| 53 | .73094     | 76 | .75497      | 80 | .77534     | 72 | .80203      | 76 | 53 |       | 8    | 9.4  | 9.3  | 9.2  |
| 54 | .73170     | 76 | .75577      | 80 | .77606     | 72 | .80280      | 76 | 54 |       | 9    | 10.6 | 10.4 | 10.3 |
| 55 | 8.73246    | 76 | 8.75658     | 80 | 8.77678    | 72 | 8.80356     | 76 | 55 | 10    | 11.8 | 11.6 | 11.5 |      |
| 56 | .73322     | 73 | .75738      | 80 | .77750     | 72 | .80433      | 76 | 56 | 20    | 23.0 | 22.6 | 22.3 |      |
| 57 | .73398     | 73 | .75818      | 80 | .77822     | 71 | .80509      | 76 | 57 | 30    | 35.0 | 34.5 | 34.2 |      |
| 58 | .73473     | 76 | .75898      | 80 | .77893     | 72 | .80586      | 76 | 58 | 40    | 47.0 | 46.3 | 45.6 |      |
| 59 | .73549     | 73 | .75978      | 80 | .77965     | 71 | .80662      | 76 | 59 | 50    | 59.0 | 58.1 | 57.3 |      |
| 60 | 8.73625    |    | 8.76058     |    | 8.78037    |    | 8.80738     |    | 60 |       |      |      |      |      |
| '  | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  | '  | P. P. |      |      |      |      |

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21°

|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 8.78037    | 71 | 8.80738     | 76 | 8.82229    | 68 | 8.85214     | 73 | 0  |       |
| 1  | .78108     | 71 | .80814      | 76 | .82297     | 68 | .85287      | 73 | 1  |       |
| 2  | .78180     | 71 | .80891      | 76 | .82366     | 68 | .85360      | 73 | 2  |       |
| 3  | .78251     | 71 | .80967      | 76 | .82434     | 68 | .85433      | 73 | 3  |       |
| 4  | .78323     | 71 | .81043      | 76 | .82502     | 67 | .85506      | 73 | 4  |       |
| 5  | 8.78394    | 71 | 8.81119     | 76 | 8.82569    | 68 | 8.85579     | 73 | 5  |       |
| 6  | .78466     | 71 | .81195      | 76 | .82637     | 68 | .85651      | 73 | 6  | 76    |
| 7  | .78537     | 71 | .81271      | 75 | .82705     | 67 | .85724      | 72 | 7  | 75    |
| 8  | .78608     | 71 | .81346      | 76 | .82773     | 68 | .85797      | 72 | 8  | 74    |
| 9  | .78679     | 71 | .81422      | 75 | .82841     | 67 | .85869      | 72 | 9  |       |
| 10 | 8.78750    | 71 | 8.81498     | 75 | 8.82908    | 67 | 8.85942     | 72 | 10 |       |
| 11 | .78821     | 71 | .81573      | 76 | .82976     | 67 | .86014      | 72 | 11 |       |
| 12 | .78892     | 71 | .81649      | 75 | .83043     | 67 | .86087      | 72 | 12 |       |
| 13 | .78963     | 70 | .81725      | 75 | .83111     | 67 | .86159      | 72 | 13 |       |
| 14 | .79034     | 71 | .81800      | 75 | .83178     | 67 | .86231      | 72 | 14 |       |
| 15 | 8.79105    | 70 | 8.81876     | 75 | 8.83246    | 67 | 8.86304     | 72 | 15 |       |
| 16 | .79175     | 71 | .81951      | 75 | .83313     | 67 | .86376      | 72 | 16 |       |
| 17 | .79246     | 70 | .82026      | 75 | .83380     | 67 | .86448      | 72 | 17 |       |
| 18 | .79317     | 70 | .82102      | 75 | .83447     | 67 | .86520      | 72 | 18 |       |
| 19 | .79387     | 70 | .82177      | 75 | .83515     | 67 | .86592      | 72 | 19 |       |
| 20 | 8.79458    | 70 | 8.82252     | 75 | 8.83582    | 67 | 8.86664     | 72 | 20 |       |
| 21 | .79528     | 70 | .82327      | 75 | .83649     | 67 | .86736      | 72 | 21 |       |
| 22 | .79598     | 70 | .82402      | 75 | .83716     | 67 | .86808      | 72 | 22 |       |
| 23 | .79669     | 70 | .82477      | 74 | .83783     | 67 | .86880      | 71 | 23 |       |
| 24 | .79739     | 70 | .82552      | 75 | .83850     | 66 | .86952      | 72 | 24 |       |
| 25 | 8.79809    | 70 | 8.82627     | 75 | 8.83916    | 67 | 8.87024     | 71 | 25 |       |
| 26 | .79879     | 70 | .82702      | 74 | .83983     | 66 | .87095      | 72 | 26 |       |
| 27 | .79949     | 70 | .82776      | 75 | .84050     | 67 | .87167      | 71 | 27 |       |
| 28 | .80019     | 70 | .82851      | 74 | .84117     | 66 | .87239      | 71 | 28 |       |
| 29 | .80089     | 70 | .82926      | 74 | .84183     | 66 | .87310      | 71 | 29 |       |
| 30 | 8.80159    | 70 | 8.83000     | 74 | 8.84250    | 66 | 8.87382     | 71 | 30 |       |
| 31 | .80229     | 69 | .83075      | 74 | .84316     | 66 | .87453      | 71 | 31 |       |
| 32 | .80299     | 70 | .83149      | 74 | .84383     | 66 | .87525      | 71 | 32 |       |
| 33 | .80369     | 69 | .83224      | 74 | .84449     | 66 | .87596      | 71 | 33 |       |
| 34 | .80438     | 69 | .83298      | 74 | .84515     | 66 | .87668      | 71 | 34 |       |
| 35 | 8.80508    | 69 | 8.83373     | 74 | 8.84582    | 66 | 8.87739     | 71 | 35 |       |
| 36 | .80577     | 69 | .83447      | 74 | .84648     | 66 | .87810      | 71 | 36 |       |
| 37 | .80647     | 69 | .83521      | 74 | .84714     | 66 | .87881      | 71 | 37 |       |
| 38 | .80716     | 69 | .83595      | 74 | .84780     | 66 | .87953      | 71 | 38 |       |
| 39 | .80786     | 69 | .83670      | 74 | .84846     | 66 | .88024      | 71 | 39 |       |
| 40 | 8.80855    | 69 | 8.83744     | 74 | 8.84912    | 66 | 8.88095     | 71 | 40 |       |
| 41 | .80924     | 69 | .83818      | 74 | .84978     | 66 | .88166      | 71 | 41 |       |
| 42 | .80993     | 69 | .83892      | 74 | .85044     | 66 | .88237      | 71 | 42 |       |
| 43 | .81063     | 69 | .83966      | 74 | .85110     | 66 | .88308      | 70 | 43 |       |
| 44 | .81132     | 69 | .84039      | 73 | .85176     | 66 | .88378      | 71 | 44 |       |
| 45 | 8.81201    | 69 | 8.84113     | 74 | 8.85242    | 65 | 8.88449     | 71 | 45 |       |
| 46 | .81270     | 69 | .84187      | 73 | .85308     | 66 | .88520      | 71 | 46 |       |
| 47 | .81339     | 69 | .84261      | 74 | .85373     | 65 | .88591      | 70 | 47 |       |
| 48 | .81407     | 68 | .84334      | 73 | .85439     | 65 | .88661      | 70 | 48 |       |
| 49 | .81476     | 69 | .84408      | 73 | .85505     | 66 | .88732      | 71 | 49 |       |
| 50 | 8.81545    | 68 | 8.84481     | 73 | 8.85570    | 65 | 8.88803     | 70 | 50 |       |
| 51 | .81614     | 68 | .84555      | 73 | .85626     | 65 | .88873      | 70 | 51 |       |
| 52 | .81682     | 68 | .84628      | 73 | .85701     | 65 | .88944      | 70 | 52 |       |
| 53 | .81751     | 68 | .84702      | 73 | .85766     | 65 | .89014      | 70 | 53 |       |
| 54 | .81819     | 68 | .84775      | 73 | .85832     | 65 | .89085      | 70 | 54 |       |
| 55 | 8.81888    | 68 | 8.84848     | 73 | 8.85897    | 65 | 8.89155     | 70 | 55 |       |
| 56 | .81956     | 68 | .84922      | 73 | .85962     | 65 | .89225      | 70 | 56 |       |
| 57 | .82025     | 68 | .84995      | 73 | .86027     | 65 | .89295      | 70 | 57 |       |
| 58 | .82093     | 68 | .85068      | 73 | .86092     | 65 | .89366      | 70 | 58 |       |
| 59 | .82161     | 68 | .85141      | 73 | .86158     | 65 | .89436      | 70 | 59 |       |
| 60 | 8.82229    |    | 8.85214     |    | 8.86223    |    | 8.89506     |    | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

22°

23°

|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 8.86223    | 64 | 8.89506     | 70 | 8.90034    | 62 | 8.93031     | 67 | 0  |       |
| 1  | .86287     | 65 | .89576      | 70 | .90096     | 62 | .93099      | 67 | 1  |       |
| 2  | .86352     | 65 | .89646      | 70 | .90158     | 62 | .93166      | 67 | 2  |       |
| 3  | .86417     | 65 | .89716      | 69 | .90220     | 62 | .93233      | 67 | 3  |       |
| 4  | .86483     | 64 | .89786      | 70 | .90282     | 62 | .93301      | 67 | 4  |       |
| 5  | 8.86547    | 65 | 8.89856     | 70 | 8.90344    | 62 | 8.93368     | 67 | 5  |       |
| 6  | .86612     | 64 | .89926      | 69 | .90406     | 61 | .94035      | 67 | 6  |       |
| 7  | .86676     | 64 | .89995      | 70 | .90467     | 62 | .94102      | 67 | 7  |       |
| 8  | .86741     | 64 | .90065      | 69 | .90529     | 61 | .94170      | 67 | 8  |       |
| 9  | .86805     | 64 | .90135      | 70 | .90591     | 61 | .94237      | 67 | 9  |       |
| 10 | 8.86870    | 64 | 8.90205     | 69 | 8.90652    | 62 | 8.94304     | 67 | 10 |       |
| 11 | .86934     | 64 | .90274      | 69 | .90714     | 61 | .94371      | 67 | 11 |       |
| 12 | .86999     | 64 | .90344      | 69 | .90776     | 61 | .94438      | 67 | 12 |       |
| 13 | .87063     | 64 | .90413      | 69 | .90837     | 61 | .94505      | 67 | 13 |       |
| 14 | .87127     | 64 | .90483      | 69 | .90899     | 61 | .94572      | 67 | 14 |       |
| 15 | 8.87192    | 64 | 8.90552     | 69 | 8.90960    | 61 | 8.94638     | 67 | 15 |       |
| 16 | .87256     | 64 | .90622      | 69 | .91021     | 61 | .94705      | 67 | 16 |       |
| 17 | .87320     | 64 | .90691      | 69 | .91083     | 61 | .94772      | 67 | 17 |       |
| 18 | .87384     | 64 | .90760      | 69 | .91144     | 61 | .94839      | 67 | 18 |       |
| 19 | .87448     | 64 | .90830      | 69 | .91205     | 61 | .94905      | 67 | 19 |       |
| 20 | 8.87512    | 64 | 8.90899     | 69 | 8.91267    | 61 | 8.94972     | 67 | 20 |       |
| 21 | .87576     | 64 | .90968      | 69 | .91328     | 61 | .95039      | 67 | 21 |       |
| 22 | .87640     | 64 | .91037      | 69 | .91389     | 61 | .95105      | 67 | 22 |       |
| 23 | .87704     | 63 | .91106      | 69 | .91450     | 61 | .95172      | 67 | 23 |       |
| 24 | .87768     | 64 | .91175      | 69 | .91511     | 61 | .95238      | 67 | 24 |       |
| 25 | 8.87832    | 63 | 8.91244     | 69 | 8.91572    | 61 | 8.95305     | 67 | 25 |       |
| 26 | .87895     | 64 | .91313      | 68 | .91633     | 61 | .95371      | 67 | 26 |       |
| 27 | .87959     | 63 | .91382      | 69 | .91694     | 61 | .95437      | 67 | 27 |       |
| 28 | .88023     | 63 | .91451      | 69 | .91755     | 60 | .95504      | 67 | 28 |       |
| 29 | .88086     | 63 | .91520      | 68 | .91815     | 61 | .95570      | 67 | 29 |       |
| 30 | 8.88150    | 63 | 8.91588     | 69 | 8.91876    | 60 | 8.95636     | 67 | 30 |       |
| 31 | .88213     | 63 | .91657      | 68 | .91937     | 60 | .95703      | 67 | 31 |       |
| 32 | .88277     | 63 | .91726      | 68 | .91997     | 61 | .95769      | 67 | 32 |       |
| 33 | .88340     | 63 | .91794      | 68 | .92058     | 60 | .95835      | 67 | 33 |       |
| 34 | .88404     | 63 | .91863      | 69 | .92119     | 60 | .95901      | 67 | 34 |       |
| 35 | 8.88467    | 63 | 8.91932     | 68 | 8.92179    | 60 | 8.95967     | 67 | 35 |       |
| 36 | .88530     | 63 | .92000      | 68 | .92240     | 60 | .96033      | 67 | 36 |       |
| 37 | .88593     | 63 | .92068      | 68 | .92300     | 60 | .96099      | 67 | 37 |       |
| 38 | .88656     | 63 | .92137      | 68 | .92361     | 60 | .96165      | 67 | 38 |       |
| 39 | .88720     | 63 | .92205      | 68 | .92421     | 60 | .96231      | 67 | 39 |       |
| 40 | 8.88783    | 63 | 8.92274     | 68 | 8.92487    | 60 | 8.96297     | 67 | 40 |       |
| 41 | .88846     | 63 | .92342      | 68 | .92542     | 60 | .96362      | 67 | 41 |       |
| 42 | .88909     | 62 | .92410      | 68 | .92602     | 60 | .96428      | 67 | 42 |       |
| 43 | .88971     | 63 | .92478      | 68 | .92662     | 60 | .96494      | 67 | 43 |       |
| 44 | .89034     | 63 | .92546      | 68 | .92722     | 60 | .96560      | 67 | 44 |       |
| 45 | 8.89097    | 62 | 8.92615     | 68 | 8.92782    | 60 | 8.96625     | 67 | 45 |       |
| 46 | .89160     | 63 | .92683      | 68 | .92842     | 60 | .96691      | 67 | 46 |       |
| 47 | .89223     | 62 | .92751      | 68 | .92902     | 60 | .96757      | 67 | 47 |       |
| 48 | .89285     | 62 | .92819      | 68 | .92962     | 60 | .96822      | 67 | 48 |       |
| 49 | .89348     | 63 | .92887      | 68 | .93022     | 60 | .96888      | 67 | 49 |       |
| 50 | 8.89411    | 62 | 8.92955     | 67 | 8.93082    | 59 | 8.96953     | 67 | 50 |       |
| 51 | .89473     | 62 | .93022      | 68 | .93142     | 60 | .97018      | 67 | 51 |       |
| 52 | .89536     | 62 | .93090      | 67 | .93202     | 59 | .97084      | 67 | 52 |       |
| 53 | .89598     | 62 | .93158      | 68 | .93261     | 60 | .97149      | 67 | 53 |       |
| 54 | .89660     | 62 | .93226      | 67 | .93321     | 59 | .97214      | 67 | 54 |       |
| 55 | 8.89723    | 62 | 8.93293     | 68 | 8.93381    | 59 | 8.97280     | 67 | 55 |       |
| 56 | .89785     | 62 | .93361      | 67 | .93440     | 60 | .97345      | 67 | 56 |       |
| 57 | .89847     | 62 | .93429      | 67 | .93500     | 59 | .97410      | 67 | 57 |       |
| 58 | .89910     | 62 | .93496      | 67 | .93560     | 59 | .97475      | 67 | 58 |       |
| 59 | .89972     | 62 | .93564      | 67 | .93619     | 59 | .97540      | 67 | 59 |       |
| 60 | 8.90034    |    | 8.93631     |    | 8.93679    |    | 8.97606     |    | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

24°

25°

| 24°        |         |             |         | 25°        |         |             |         |    |    |       |      |
|------------|---------|-------------|---------|------------|---------|-------------|---------|----|----|-------|------|
| Log. Vers. | D       | Log. Exsec. | D       | Log. Vers. | D       | Log. Exsec. | D       |    |    | P. P. |      |
| 0          | 8.93679 |             | 8.97606 | 65         | 8.97170 |             | 9.01443 | 62 | 0  |       |      |
| 1          | .93738  | 59          | .97671  | 65         | .97227  | 57          | .01505  | 63 | 1  |       |      |
| 2          | .93797  | 59          | .97736  | 65         | .97284  | 56          | .01568  | 62 | 2  |       |      |
| 3          | .93857  | 59          | .97801  | 64         | .97341  | 57          | .01631  | 63 | 3  |       |      |
| 4          | .93916  | 59          | .97863  | 64         | .97398  | 57          | .01694  | 63 | 4  |       |      |
| 5          | 8.93973 | 59          | 8.97930 | 65         | 8.97455 | 57          | 9.01756 | 62 | 5  | 65    | 64   |
| 6          | .94034  | 59          | .97995  | 64         | .97511  | 56          | .01819  | 62 | 6  | 6.5   | 6.4  |
| 7          | .94094  | 59          | .98060  | 65         | .97568  | 57          | .01882  | 62 | 7  | 7.6   | 7.4  |
| 8          | .94153  | 59          | .98125  | 65         | .97625  | 56          | .01944  | 62 | 8  | 8.5   | 8.4  |
| 9          | .94212  | 59          | .98190  | 65         | .97681  | 56          | .02007  | 63 | 9  | 9.6   | 9.4  |
| 10         | 8.94271 | 59          | 8.98254 | 64         | 8.97738 | 56          | 9.02070 | 62 | 10 | 10.6  | 10.5 |
| 11         | .94330  | 59          | .98319  | 64         | .97795  | 57          | .02132  | 62 | 11 | 21.5  | 21.0 |
| 12         | .94389  | 59          | .98383  | 64         | .97851  | 56          | .02195  | 62 | 12 | 32.5  | 31.5 |
| 13         | .94448  | 59          | .98448  | 65         | .97908  | 56          | .02257  | 62 | 13 | 43.5  | 42.0 |
| 14         | .94506  | 58          | .98513  | 64         | .97964  | 56          | .02319  | 62 | 14 | 54.1  | 52.5 |
| 15         | 8.94565 | 59          | 8.98577 | 64         | 8.98020 | 56          | 9.02382 | 62 | 15 |       |      |
| 16         | .94624  | 59          | .98642  | 64         | .98077  | 56          | .02444  | 62 | 16 | 62    | 61   |
| 17         | .94683  | 58          | .98706  | 64         | .98133  | 56          | .02506  | 62 | 17 | 6.2   | 6.0  |
| 18         | .94742  | 59          | .98770  | 64         | .98190  | 56          | .02569  | 62 | 18 | 7.1   | 7.0  |
| 19         | .94800  | 58          | .98835  | 64         | .98246  | 56          | .02631  | 62 | 19 | 8.1   | 8.0  |
| 20         | 8.94859 | 58          | 8.98899 | 64         | 8.98302 | 56          | 9.02693 | 62 | 20 | 9.1   | 9.0  |
| 21         | .94917  | 58          | .98963  | 64         | .98358  | 56          | .02755  | 62 | 21 | 10.1  | 10.0 |
| 22         | .94976  | 58          | .99028  | 64         | .98414  | 56          | .02817  | 62 | 22 | 20.6  | 20.0 |
| 23         | .95034  | 58          | .99092  | 64         | .98470  | 56          | .02880  | 62 | 23 | 31.0  | 30.0 |
| 24         | .95093  | 58          | .99156  | 64         | .98527  | 56          | .02942  | 62 | 24 | 41.6  | 40.0 |
| 25         | 8.95151 | 58          | 8.99220 | 64         | 8.98583 | 56          | 9.03004 | 62 | 25 | 51.6  | 50.0 |
| 26         | .95210  | 58          | .99284  | 64         | .98639  | 56          | .03066  | 62 | 26 |       |      |
| 27         | .95268  | 58          | .99348  | 64         | .98695  | 55          | .03128  | 62 | 27 | 59    | 58   |
| 28         | .95326  | 58          | .99412  | 64         | .98750  | 55          | .03190  | 62 | 28 | 5.9   | 5.8  |
| 29         | .95384  | 58          | .99476  | 64         | .98806  | 56          | .03252  | 62 | 29 | 6.9   | 6.7  |
| 30         | 8.95443 | 58          | 8.99540 | 64         | 8.98862 | 56          | 9.03313 | 61 | 30 | 7.7   | 7.6  |
| 31         | .95501  | 58          | .99604  | 64         | .98918  | 56          | .03375  | 62 | 31 | 8.7   | 8.5  |
| 32         | .95559  | 58          | .99668  | 64         | .98974  | 55          | .03437  | 62 | 32 | 9.6   | 9.5  |
| 33         | .95617  | 58          | .99732  | 64         | .99030  | 56          | .03499  | 62 | 33 | 19.6  | 19.0 |
| 34         | .95675  | 58          | .99796  | 63         | .99085  | 55          | .03561  | 62 | 34 | 29.5  | 28.5 |
| 35         | 8.95733 | 58          | 8.99860 | 64         | 8.99141 | 55          | 9.03622 | 61 | 35 | 39.3  | 38.0 |
| 36         | .95791  | 58          | .99923  | 64         | .99197  | 56          | .03684  | 62 | 36 | 49.1  | 47.5 |
| 37         | .95849  | 57          | 8.99987 | 64         | .99252  | 55          | .03746  | 61 | 37 |       |      |
| 38         | .95907  | 58          | 9.00051 | 63         | .99308  | 55          | .03807  | 61 | 38 | 56    | 55   |
| 39         | .95965  | 58          | .00114  | 64         | .99363  | 55          | .03869  | 61 | 39 | 5.6   | 5.4  |
| 40         | 8.96023 | 58          | 9.00178 | 63         | 8.99419 | 55          | 9.03930 | 61 | 40 | 6.5   | 6.3  |
| 41         | .96080  | 57          | .00242  | 63         | .99474  | 55          | .03992  | 61 | 41 | 7.4   | 7.2  |
| 42         | .96138  | 57          | .00305  | 63         | .99529  | 55          | .04053  | 61 | 42 | 8.4   | 8.1  |
| 43         | .96196  | 58          | .00369  | 63         | .99585  | 55          | .04115  | 61 | 43 | 9.3   | 9.0  |
| 44         | .96253  | 57          | .00432  | 63         | .99640  | 55          | .04176  | 61 | 44 | 10.3  | 9.6  |
| 45         | 8.96311 | 57          | 9.00495 | 63         | 8.99695 | 55          | 9.04238 | 61 | 45 | 18.6  | 18.0 |
| 46         | .96368  | 57          | .00559  | 63         | .99751  | 55          | .04299  | 61 | 46 | 28.0  | 27.0 |
| 47         | .96426  | 57          | .00622  | 63         | .99806  | 55          | .04360  | 61 | 47 | 37.3  | 36.0 |
| 48         | .96483  | 57          | .00686  | 63         | .99861  | 55          | .04421  | 61 | 48 | 46.6  | 45.0 |
| 49         | .96541  | 57          | .00749  | 63         | .99916  | 55          | .04483  | 61 | 49 |       |      |
| 50         | 8.96598 | 57          | 9.00812 | 63         | 8.99971 | 55          | 9.04544 | 61 | 50 |       |      |
| 51         | .96656  | 57          | .00875  | 63         | 9.00026 | 55          | .04605  | 61 | 51 | 6     | 0.6  |
| 52         | .96713  | 57          | .00938  | 63         | .00081  | 55          | .04666  | 61 | 52 | 7     | 0.6  |
| 53         | .96770  | 57          | .01002  | 63         | .00136  | 55          | .04727  | 61 | 53 | 8     | 0.6  |
| 54         | .96827  | 57          | .01065  | 63         | .00191  | 55          | .04788  | 61 | 54 | 9     | 0.1  |
| 55         | 8.96885 | 57          | 9.01128 | 63         | 9.00246 | 55          | 9.04850 | 61 | 55 | 10    | 0.1  |
| 56         | .96942  | 57          | .01191  | 63         | .00301  | 55          | .04911  | 61 | 56 | 20    | 0.1  |
| 57         | .96999  | 57          | .01254  | 63         | .00356  | 55          | .04972  | 61 | 57 | 30    | 0.2  |
| 58         | .97056  | 57          | .01317  | 63         | .00411  | 54          | .05033  | 60 | 58 | 40    | 0.3  |
| 59         | .97113  | 57          | .01380  | 63         | .00466  | 55          | .05093  | 61 | 59 | 50    | 0.4  |
| 60         | 8.97170 | 57          | 9.01443 | 63         | 9.00520 | 54          | 9.05154 | 61 | 60 |       |      |
| Log. Vers. | D       | Log. Exsec. | D       | Log. Vers. | D       | Log. Exsec. | D       |    |    | P. P. |      |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

26°

27°

|    | Log. Vers. | D  | Log. Exsec. | D  |            | Log. Vers. | D           | Log. Exsec. | D  |       | P. P. |
|----|------------|----|-------------|----|------------|------------|-------------|-------------|----|-------|-------|
| 0  | 9.00520    | 55 | 9.05154     | 61 | 9.03740    | 53         | 9.08752     | 59          | 0  |       |       |
| 1  | .00575     | 54 | .05215      | 61 | .03792     | 52         | .08811      | 59          | 1  |       |       |
| 2  | .00630     | 54 | .05276      | 60 | .03845     | 52         | .08870      | 59          | 2  |       |       |
| 3  | .00684     | 54 | .05337      | 61 | .03898     | 53         | .08929      | 59          | 3  |       |       |
| 4  | .00739     | 54 | .05398      | 60 | .03950     | 52         | .08988      | 59          | 4  |       |       |
| 5  | 9.00794    | 55 | 9.05458     | 61 | 9.04002    | 52         | 9.09047     | 59          | 5  | 61    | 60    |
| 6  | .00848     | 54 | .05519      | 60 | .04055     | 52         | .09106      | 59          | 6  | 6.1   | 6.0   |
| 7  | .00903     | 54 | .05580      | 60 | .04107     | 53         | .09164      | 58          | 7  | 7.1   | 7.0   |
| 8  | .00957     | 54 | .05640      | 60 | .04160     | 53         | .09223      | 59          | 8  | 8.1   | 8.0   |
| 9  | .01011     | 54 | .05701      | 60 | .04212     | 52         | .09282      | 59          | 9  | 9.1   | 9.0   |
| 10 | 9.01066    | 54 | 9.05762     | 61 | 9.04264    | 52         | 9.09341     | 59          | 10 | 10.1  | 10.0  |
| 11 | .01120     | 54 | .05823      | 60 | .04317     | 52         | .09400      | 58          | 11 | 20    | 20.0  |
| 12 | .01174     | 54 | .05883      | 60 | .04369     | 52         | .09458      | 59          | 12 | 30    | 30.0  |
| 13 | .01229     | 54 | .05943      | 60 | .04421     | 52         | .09517      | 58          | 13 | 40    | 40.0  |
| 14 | .01283     | 54 | .06004      | 60 | .04473     | 52         | .09576      | 58          | 14 | 50    | 50.0  |
| 15 | 9.01337    | 54 | 9.06064     | 60 | 9.04525    | 52         | 9.09634     | 58          | 15 |       |       |
| 16 | .01391     | 54 | .06124      | 60 | .04577     | 53         | .09693      | 59          | 16 | 58    | 57    |
| 17 | .01445     | 54 | .06185      | 60 | .04630     | 52         | .09752      | 58          | 17 | 6     | 5.8   |
| 18 | .01499     | 54 | .06245      | 60 | .04682     | 52         | .09810      | 58          | 18 | 7     | 6.7   |
| 19 | .01551     | 54 | .06305      | 60 | .04734     | 52         | .09869      | 58          | 19 | 8     | 7.7   |
| 20 | 9.01605    | 54 | 9.06366     | 60 | 9.04786    | 52         | 9.09927     | 58          | 20 | 9     | 8.7   |
| 21 | .01662     | 54 | .06426      | 60 | .04837     | 51         | .09986      | 58          | 21 | 10    | 9.6   |
| 22 | .01715     | 53 | .06486      | 60 | .04889     | 52         | .10044      | 58          | 22 | 20    | 19.3  |
| 23 | .01769     | 54 | .06546      | 60 | .04941     | 52         | .10102      | 58          | 23 | 30    | 29.0  |
| 24 | .01823     | 54 | .06606      | 60 | .04993     | 52         | .10161      | 58          | 24 | 40    | 38.0  |
| 25 | 9.01877    | 54 | 9.06667     | 60 | 9.05045    | 51         | 9.10219     | 58          | 25 | 50    | 48.3  |
| 26 | .01931     | 53 | .06727      | 60 | .05097     | 52         | .10278      | 58          | 26 |       |       |
| 27 | .01985     | 53 | .06787      | 60 | .05148     | 52         | .10336      | 58          | 27 | 55    | 54    |
| 28 | .02038     | 54 | .06847      | 60 | .05200     | 52         | .10394      | 58          | 28 | 6     | 5.5   |
| 29 | .02092     | 54 | .06907      | 60 | .05252     | 52         | .10452      | 58          | 29 | 7     | 6.4   |
| 30 | 9.02146    | 53 | 9.06967     | 60 | 9.05303    | 51         | 9.10511     | 58          | 30 | 8     | 7.3   |
| 31 | .02199     | 53 | .07027      | 60 | .05355     | 52         | .10569      | 58          | 31 | 9     | 8.2   |
| 32 | .02253     | 54 | .07087      | 60 | .05407     | 51         | .10627      | 58          | 32 | 10    | 9.1   |
| 33 | .02307     | 53 | .07146      | 60 | .05458     | 51         | .10685      | 58          | 33 | 20    | 18.0  |
| 34 | .02360     | 53 | .07206      | 60 | .05510     | 51         | .10743      | 58          | 34 | 30    | 27.0  |
| 35 | 9.02414    | 53 | 9.07266     | 60 | 9.05561    | 51         | 9.10801     | 58          | 35 | 40    | 36.0  |
| 36 | .02467     | 53 | .07326      | 59 | .05613     | 51         | .10859      | 58          | 36 | 50    | 45.8  |
| 37 | .02521     | 53 | .07386      | 60 | .05664     | 51         | .10917      | 58          | 37 |       |       |
| 38 | .02574     | 53 | .07445      | 59 | .05715     | 51         | .10975      | 58          | 38 | 53    | 52    |
| 39 | .02627     | 53 | .07505      | 60 | .05767     | 51         | .11033      | 58          | 39 | 6     | 5.3   |
| 40 | 9.02681    | 53 | 9.07565     | 59 | 9.05818    | 51         | 9.11091     | 58          | 40 | 7     | 6.2   |
| 41 | .02734     | 53 | .07624      | 59 | .05869     | 51         | .11149      | 58          | 41 | 8     | 7.0   |
| 42 | .02787     | 53 | .07684      | 59 | .05921     | 51         | .11207      | 57          | 42 | 9     | 7.9   |
| 43 | .02840     | 53 | .07743      | 59 | .05972     | 51         | .11265      | 58          | 43 | 10    | 8.8   |
| 44 | .02894     | 53 | .07803      | 60 | .06023     | 51         | .11323      | 58          | 44 | 20    | 17.6  |
| 45 | 9.02947    | 53 | 9.07863     | 59 | 9.06074    | 51         | 9.11380     | 57          | 45 | 30    | 26.5  |
| 46 | .03000     | 53 | .07922      | 59 | .06125     | 51         | .11438      | 58          | 46 | 40    | 35.4  |
| 47 | .03053     | 53 | .07981      | 59 | .06176     | 51         | .11496      | 57          | 47 | 50    | 44.1  |
| 48 | .03106     | 53 | .08041      | 59 | .06227     | 51         | .11554      | 57          | 48 |       |       |
| 49 | .03159     | 53 | .08100      | 59 | .06279     | 51         | .11611      | 57          | 49 |       |       |
| 50 | 9.03212    | 53 | 9.08160     | 59 | 9.06330    | 51         | 9.11669     | 58          | 50 | 51    | 50    |
| 51 | .03265     | 53 | .08219      | 59 | .06380     | 50         | .11727      | 57          | 51 | 6     | 5.1   |
| 52 | .03318     | 53 | .08278      | 59 | .06431     | 51         | .11784      | 57          | 52 | 7     | 6.0   |
| 53 | .03371     | 52 | .08338      | 59 | .06482     | 51         | .11842      | 57          | 53 | 8     | 6.8   |
| 54 | .03423     | 52 | .08397      | 59 | .06533     | 51         | .11899      | 57          | 54 | 9     | 7.6   |
| 55 | 9.03476    | 53 | 9.08456     | 59 | 9.06584    | 51         | 9.11957     | 58          | 55 | 10    | 8.5   |
| 56 | .03529     | 53 | .08515      | 59 | .06635     | 50         | .12015      | 57          | 56 | 20    | 17.0  |
| 57 | .03582     | 52 | .08574      | 59 | .06686     | 51         | .12072      | 57          | 57 | 30    | 25.8  |
| 58 | .03634     | 53 | .08634      | 59 | .06736     | 51         | .12129      | 57          | 58 | 40    | 34.0  |
| 59 | .03687     | 52 | .08693      | 59 | .06787     | 50         | .12187      | 57          | 59 | 50    | 42.5  |
| 60 | 9.03740    |    | 9.08752     |    | 9.06838    |            | 9.12244     |             | 60 |       |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D          | Log. Exsec. | D           |    | P. P. |       |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |  |  |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|--|--|
| 0  | 9.06838    | 50 | 9.12244     | 57 | 9.09823    | 49 | 9.15641     | 56 | 0  |       |  |  |
| 1  | .06888     | 51 | .12302      | 57 | .09872     | 48 | .15697      | 55 | 1  |       |  |  |
| 2  | .06939     | 50 | .12359      | 57 | .09920     | 49 | .15752      | 56 | 2  |       |  |  |
| 3  | .06990     | 50 | .12416      | 57 | .09969     | 48 | .15808      | 55 | 3  |       |  |  |
| 4  | .07040     | 50 | .12474      | 57 | .10018     | 48 | .15864      | 55 | 4  |       |  |  |
| 5  | 9.07091    | 50 | 9.12531     | 57 | 9.10067    | 49 | 9.15920     | 56 | 5  |       |  |  |
| 6  | .07141     | 50 | .12588      | 57 | .10113     | 48 | .15975      | 55 | 6  |       |  |  |
| 7  | .07192     | 50 | .12645      | 57 | .10164     | 48 | .16031      | 56 | 7  |       |  |  |
| 8  | .07242     | 50 | .12703      | 57 | .10213     | 49 | .16087      | 55 | 8  |       |  |  |
| 9  | .07293     | 50 | .12760      | 57 | .10261     | 48 | .16142      | 55 | 9  |       |  |  |
| 10 | 9.07343    | 50 | 9.12817     | 57 | 9.10310    | 48 | 9.16198     | 56 | 10 |       |  |  |
| 11 | .07393     | 50 | .12874      | 57 | .10358     | 48 | .16254      | 55 | 11 |       |  |  |
| 12 | .07444     | 50 | .12931      | 57 | .10407     | 48 | .16309      | 55 | 12 |       |  |  |
| 13 | .07494     | 50 | .12988      | 57 | .10455     | 48 | .16365      | 55 | 13 |       |  |  |
| 14 | .07544     | 50 | .13045      | 57 | .10504     | 48 | .16420      | 55 | 14 |       |  |  |
| 15 | 9.07594    | 50 | 9.13102     | 57 | 9.10552    | 48 | 9.16476     | 55 | 15 |       |  |  |
| 16 | .07644     | 50 | .13159      | 57 | .10601     | 48 | .16531      | 55 | 16 |       |  |  |
| 17 | .07695     | 50 | .13216      | 57 | .10649     | 48 | .16587      | 55 | 17 |       |  |  |
| 18 | .07745     | 50 | .13273      | 57 | .10697     | 48 | .16642      | 55 | 18 |       |  |  |
| 19 | .07795     | 50 | .13330      | 57 | .10746     | 48 | .16698      | 55 | 19 |       |  |  |
| 20 | 9.07845    | 50 | 9.13387     | 57 | 9.10794    | 48 | 9.16753     | 55 | 20 |       |  |  |
| 21 | .07895     | 50 | .13444      | 57 | .10842     | 48 | .16808      | 55 | 21 |       |  |  |
| 22 | .07945     | 50 | .13500      | 57 | .10890     | 48 | .16864      | 55 | 22 |       |  |  |
| 23 | .07995     | 50 | .13557      | 57 | .10939     | 48 | .16919      | 55 | 23 |       |  |  |
| 24 | .08045     | 50 | .13614      | 57 | .10987     | 48 | .16974      | 55 | 24 |       |  |  |
| 25 | 9.08095    | 50 | 9.13671     | 57 | 9.11035    | 48 | 9.17029     | 55 | 25 |       |  |  |
| 26 | .08145     | 50 | .13727      | 57 | .11083     | 48 | .17085      | 55 | 26 |       |  |  |
| 27 | .08195     | 50 | .13784      | 57 | .11131     | 48 | .17140      | 55 | 27 |       |  |  |
| 28 | .08244     | 49 | .13841      | 56 | .11179     | 48 | .17195      | 55 | 28 |       |  |  |
| 29 | .08294     | 50 | .13897      | 56 | .11227     | 48 | .17250      | 55 | 29 |       |  |  |
| 30 | 9.08344    | 49 | 9.13954     | 57 | 9.11275    | 48 | 9.17305     | 55 | 30 |       |  |  |
| 31 | .08394     | 50 | .14011      | 56 | .11323     | 48 | .17361      | 55 | 31 |       |  |  |
| 32 | .08443     | 49 | .14067      | 56 | .11371     | 47 | .17416      | 55 | 32 |       |  |  |
| 33 | .08493     | 50 | .14124      | 56 | .11419     | 48 | .17471      | 55 | 33 |       |  |  |
| 34 | .08543     | 49 | .14180      | 56 | .11467     | 48 | .17526      | 55 | 34 |       |  |  |
| 35 | 9.08592    | 49 | 9.14237     | 56 | 9.11515    | 47 | 9.17581     | 55 | 35 |       |  |  |
| 36 | .08642     | 49 | .14293      | 56 | .11562     | 47 | .17636      | 55 | 36 |       |  |  |
| 37 | .08691     | 49 | .14350      | 56 | .11610     | 48 | .17691      | 55 | 37 |       |  |  |
| 38 | .08741     | 49 | .14406      | 56 | .11658     | 48 | .17746      | 55 | 38 |       |  |  |
| 39 | .08790     | 49 | .14462      | 56 | .11706     | 47 | .17801      | 55 | 39 |       |  |  |
| 40 | 9.08840    | 49 | 9.14519     | 56 | 9.11754    | 47 | 9.17856     | 55 | 40 |       |  |  |
| 41 | .08889     | 49 | .14575      | 56 | .11801     | 47 | .17910      | 54 | 41 |       |  |  |
| 42 | .08939     | 49 | .14631      | 56 | .11849     | 47 | .17965      | 55 | 42 |       |  |  |
| 43 | .08988     | 49 | .14688      | 56 | .11897     | 48 | .18020      | 55 | 43 |       |  |  |
| 44 | .09037     | 49 | .14744      | 56 | .11944     | 47 | .18075      | 54 | 44 |       |  |  |
| 45 | 9.09087    | 49 | 9.14800     | 56 | 9.11992    | 47 | 9.18130     | 55 | 45 |       |  |  |
| 46 | .09136     | 49 | .14856      | 56 | .12039     | 47 | .18185      | 55 | 46 |       |  |  |
| 47 | .09185     | 49 | .14913      | 56 | .12087     | 47 | .18239      | 54 | 47 |       |  |  |
| 48 | .09234     | 49 | .14969      | 56 | .12134     | 47 | .18294      | 55 | 48 |       |  |  |
| 49 | .09284     | 49 | .15025      | 56 | .12182     | 47 | .18349      | 54 | 49 |       |  |  |
| 50 | 9.09333    | 49 | 9.15081     | 56 | 9.12229    | 47 | 9.18403     | 55 | 50 |       |  |  |
| 51 | .09382     | 49 | .15137      | 56 | .12277     | 47 | .18458      | 54 | 51 |       |  |  |
| 52 | .09431     | 49 | .15193      | 56 | .12324     | 47 | .18513      | 54 | 52 |       |  |  |
| 53 | .09480     | 49 | .15249      | 56 | .12371     | 47 | .18567      | 54 | 53 |       |  |  |
| 54 | .09529     | 49 | .15305      | 56 | .12419     | 47 | .18622      | 54 | 54 |       |  |  |
| 55 | 9.09578    | 49 | 9.15361     | 56 | 9.12466    | 47 | 9.18676     | 54 | 55 |       |  |  |
| 56 | .09627     | 49 | .15417      | 56 | .12513     | 47 | .18731      | 54 | 56 |       |  |  |
| 57 | .09676     | 49 | .15473      | 56 | .12560     | 47 | .18786      | 55 | 57 |       |  |  |
| 58 | .09725     | 48 | .15529      | 56 | .12608     | 47 | .18840      | 54 | 58 |       |  |  |
| 59 | .09774     | 49 | .15585      | 55 | .12655     | 47 | .18894      | 54 | 59 |       |  |  |
| 60 | 9.09823    | 49 | 9.15641     | 56 | 9.12702    | 47 | 9.18949     | 54 | 60 |       |  |  |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |  |  |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.12702    |    | 9.18949     |    | 9.15483    |    | 9.22176     |    | 0  |       |
| 1  | .12749     | 47 | .19003      | 54 | .15528     | 45 | .22229      | 53 | 1  |       |
| 2  | .12796     | 47 | .19058      | 54 | .15574     | 45 | .22282      | 53 | 2  |       |
| 3  | .12843     | 47 | .19112      | 54 | .15619     | 45 | .22335      | 53 | 3  |       |
| 4  | .12890     | 47 | .19167      | 54 | .15665     | 45 | .22388      | 53 | 4  |       |
| 5  | 9.12937    | 47 | 9.19221     | 54 | 9.15710    | 45 | 9.22441     | 53 | 5  | 54    |
| 6  | .12984     | 47 | .19273      | 54 | .15755     | 45 | .22494      | 53 | 6  | 54    |
| 7  | .13031     | 47 | .19329      | 54 | .15801     | 45 | .22547      | 53 | 7  | 53    |
| 8  | .13078     | 47 | .19384      | 54 | .15846     | 45 | .22600      | 53 | 8  | 52    |
| 9  | .13125     | 47 | .19438      | 54 | .15891     | 45 | .22653      | 53 | 9  | 51    |
| 10 | 9.13172    | 47 | 9.19492     | 54 | 9.15937    | 45 | 9.22706     | 53 | 10 |       |
| 11 | .13219     | 46 | .19546      | 54 | .15982     | 45 | .22759      | 53 | 11 |       |
| 12 | .13266     | 47 | .19601      | 54 | .16027     | 45 | .22812      | 53 | 12 |       |
| 13 | .13313     | 47 | .19655      | 54 | .16073     | 45 | .22865      | 53 | 13 |       |
| 14 | .13359     | 46 | .19709      | 54 | .16118     | 45 | .22918      | 53 | 14 |       |
| 15 | 9.13406    | 47 | 9.19763     | 54 | 9.16163    | 45 | 9.22971     | 53 | 15 |       |
| 16 | .13453     | 46 | .19817      | 54 | .16208     | 45 | .23024      | 53 | 16 | 53    |
| 17 | .13500     | 47 | .19871      | 54 | .16253     | 45 | .23076      | 52 | 17 | 52    |
| 18 | .13546     | 46 | .19925      | 54 | .16298     | 45 | .23129      | 53 | 18 | 51    |
| 19 | .13593     | 46 | .19979      | 54 | .16343     | 45 | .23182      | 52 | 19 | 50    |
| 20 | 9.13639    | 46 | 9.20033     | 54 | 9.16388    | 45 | 9.23235     | 53 | 20 |       |
| 21 | .13686     | 47 | .20087      | 54 | .16434     | 45 | .23287      | 52 | 21 |       |
| 22 | .13733     | 46 | .20141      | 54 | .16479     | 45 | .23340      | 53 | 22 |       |
| 23 | .13779     | 46 | .20195      | 54 | .16523     | 45 | .23393      | 53 | 23 |       |
| 24 | .13826     | 46 | .20249      | 54 | .16568     | 45 | .23446      | 53 | 24 |       |
| 25 | 9.13872    | 46 | 9.20303     | 53 | 9.16613    | 45 | 9.23498     | 52 | 25 |       |
| 26 | .13919     | 46 | .20357      | 54 | .16658     | 45 | .23551      | 52 | 26 |       |
| 27 | .13965     | 46 | .20411      | 54 | .16703     | 45 | .23603      | 52 | 27 | 47    |
| 28 | .14011     | 46 | .20465      | 54 | .16748     | 45 | .23656      | 52 | 28 | 47    |
| 29 | .14058     | 46 | .20518      | 53 | .16793     | 44 | .23709      | 53 | 29 | 46    |
| 30 | 9.14104    | 46 | 9.20572     | 54 | 9.16838    | 45 | 9.23761     | 52 | 30 |       |
| 31 | .14151     | 46 | .20626      | 53 | .16882     | 44 | .23814      | 52 | 31 |       |
| 32 | .14197     | 46 | .20680      | 54 | .16927     | 45 | .23866      | 52 | 32 |       |
| 33 | .14243     | 46 | .20733      | 53 | .16972     | 44 | .23919      | 52 | 33 |       |
| 34 | .14289     | 46 | .20787      | 54 | .17017     | 45 | .23971      | 52 | 34 |       |
| 35 | 9.14336    | 46 | 9.20841     | 53 | 9.17061    | 44 | 9.24024     | 53 | 35 |       |
| 36 | .14382     | 46 | .20894      | 53 | .17106     | 44 | .24076      | 52 | 36 |       |
| 37 | .14428     | 46 | .20948      | 54 | .17151     | 45 | .24128      | 52 | 37 |       |
| 38 | .14474     | 46 | .21002      | 53 | .17195     | 44 | .24181      | 52 | 38 |       |
| 39 | .14520     | 46 | .21055      | 52 | .17240     | 44 | .24233      | 52 | 39 | 46    |
| 40 | 9.14566    | 46 | 9.21109     | 53 | 9.17284    | 44 | 9.24285     | 52 | 40 | 45    |
| 41 | .14612     | 46 | .21162      | 53 | .17329     | 44 | .24338      | 52 | 41 |       |
| 42 | .14658     | 46 | .21216      | 53 | .17373     | 44 | .24390      | 52 | 42 |       |
| 43 | .14704     | 46 | .21269      | 53 | .17418     | 44 | .24442      | 52 | 43 |       |
| 44 | .14750     | 46 | .21323      | 53 | .17462     | 44 | .24495      | 52 | 44 |       |
| 45 | 9.14796    | 46 | 9.21376     | 53 | 9.17507    | 44 | 9.24547     | 52 | 45 |       |
| 46 | .14842     | 46 | .21430      | 53 | .17551     | 44 | .24599      | 52 | 46 |       |
| 47 | .14888     | 46 | .21483      | 53 | .17596     | 44 | .24651      | 52 | 47 |       |
| 48 | .14934     | 45 | .21537      | 53 | .17640     | 44 | .24704      | 52 | 48 |       |
| 49 | .14980     | 45 | .21590      | 53 | .17684     | 44 | .24756      | 52 | 49 |       |
| 50 | 9.15026    | 46 | 9.21643     | 53 | 9.17729    | 44 | 9.24808     | 52 | 50 | 44    |
| 51 | .15071     | 45 | .21697      | 53 | .17773     | 44 | .24860      | 52 | 51 | 44    |
| 52 | .15117     | 46 | .21750      | 53 | .17817     | 44 | .24912      | 52 | 52 |       |
| 53 | .15163     | 46 | .21803      | 53 | .17861     | 44 | .24964      | 52 | 53 |       |
| 54 | .15209     | 45 | .21857      | 53 | .17906     | 44 | .25016      | 52 | 54 |       |
| 55 | 9.15254    | 45 | 9.21910     | 53 | 9.17950    | 44 | 9.25068     | 52 | 55 |       |
| 56 | .15300     | 46 | .21963      | 53 | .17994     | 44 | .25120      | 52 | 56 |       |
| 57 | .15346     | 45 | .22016      | 53 | .18038     | 44 | .25172      | 52 | 57 |       |
| 58 | .15391     | 45 | .22070      | 53 | .18082     | 44 | .25224      | 52 | 58 |       |
| 59 | .15437     | 46 | .22123      | 53 | .18126     | 44 | .25276      | 52 | 59 |       |
| 60 | 9.15483    |    | 9.22176     |    | 9.18170    |    | 9.25328     |    | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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| 32° |            |    |             | 33° |         |            |         | P. P.       |    |      |       |
|-----|------------|----|-------------|-----|---------|------------|---------|-------------|----|------|-------|
|     | Log. Vers. | D  | Log. Exsec. | D   |         | Log. Vers. | D       | Log. Exsec. | D  |      | P. P. |
| 0   | 9.18170    | 44 | 9.25328     | 52  | 9.20771 | 42         | 9.28412 | 51          | 0  |      |       |
| 1   | .18214     | 44 | .25380      | 52  | .20814  | 42         | .28463  | 51          | 1  |      |       |
| 2   | .18258     | 44 | .25432      | 52  | .20856  | 42         | .28514  | 51          | 2  |      |       |
| 3   | .18302     | 44 | .25484      | 51  | .20899  | 42         | .28564  | 50          | 3  |      |       |
| 4   | .18346     | 44 | .25536      | 51  | .20942  | 43         | .28615  | 51          | 4  |      |       |
| 5   | 9.18390    | 44 | 9.25588     | 52  | 9.20984 | 42         | 9.28666 | 51          | 5  | 52   | 51    |
| 6   | .18434     | 44 | .25640      | 52  | .21027  | 42         | .28717  | 50          | 6  | 5.2  | 5.1   |
| 7   | .18478     | 44 | .25692      | 52  | .21069  | 42         | .28768  | 51          | 7  | 6.6  | 6.0   |
| 8   | .18522     | 44 | .25743      | 51  | .21112  | 42         | .28818  | 50          | 8  | 6.9  | 6.8   |
| 9   | .18566     | 43 | .25795      | 52  | .21154  | 42         | .28869  | 50          | 9  | 7.8  | 7.7   |
| 10  | 9.18610    | 44 | 9.25847     | 51  | 9.21196 | 42         | 9.28920 | 51          | 10 | 8.6  | 8.6   |
| 11  | .18654     | 44 | .25899      | 51  | .21239  | 42         | .28970  | 50          | 11 | 17.3 | 17.1  |
| 12  | .18697     | 43 | .25950      | 52  | .21281  | 42         | .29021  | 51          | 12 | 26.0 | 25.7  |
| 13  | .18741     | 44 | .26002      | 51  | .21324  | 42         | .29072  | 50          | 13 | 34.6 | 34.3  |
| 14  | .18785     | 43 | .26054      | 51  | .21366  | 42         | .29122  | 50          | 14 | 43.3 | 42.9  |
| 15  | 9.18829    | 44 | 9.26105     | 51  | 9.21408 | 42         | 9.29173 | 51          | 15 |      |       |
| 16  | .18872     | 43 | .26157      | 52  | .21451  | 42         | .29223  | 50          | 16 | 5.6  | 5.0   |
| 17  | .18916     | 43 | .26209      | 51  | .21493  | 42         | .29274  | 50          | 17 | 5.9  | 5.8   |
| 18  | .18959     | 43 | .26260      | 51  | .21535  | 42         | .29324  | 50          | 18 | 6.7  | 6.6   |
| 19  | .19003     | 44 | .26312      | 51  | .21577  | 42         | .29375  | 51          | 19 | 7.6  | 7.5   |
| 20  | 9.19047    | 43 | 9.26364     | 52  | 9.21620 | 42         | 9.29426 | 50          | 20 | 8.4  | 8.3   |
| 21  | .19090     | 43 | .26415      | 51  | .21662  | 42         | .29476  | 50          | 21 | 16.8 | 16.6  |
| 22  | .19134     | 43 | .26467      | 51  | .21704  | 42         | .29527  | 50          | 22 | 25.2 | 25.0  |
| 23  | .19177     | 43 | .26518      | 51  | .21746  | 42         | .29577  | 50          | 23 | 33.6 | 33.3  |
| 24  | .19221     | 43 | .26570      | 51  | .21788  | 42         | .29627  | 50          | 24 | 42.1 | 41.6  |
| 25  | 9.19264    | 43 | 9.26621     | 51  | 9.21830 | 42         | 9.29678 | 50          | 25 |      |       |
| 26  | .19308     | 43 | .26673      | 51  | .21872  | 42         | .29728  | 50          | 26 |      |       |
| 27  | .19351     | 43 | .26724      | 51  | .21914  | 42         | .29779  | 50          | 27 | 44   | 43    |
| 28  | .19395     | 43 | .26776      | 51  | .21956  | 42         | .29829  | 50          | 28 | 4.4  | 4.3   |
| 29  | .19438     | 43 | .26827      | 51  | .21998  | 42         | .29879  | 50          | 29 | 5.1  | 5.1   |
| 30  | 9.19481    | 43 | 9.26878     | 51  | 9.22040 | 42         | 9.29930 | 50          | 30 | 5.8  | 5.8   |
| 31  | .19525     | 43 | .26930      | 51  | .22082  | 42         | .29980  | 50          | 31 | 6.6  | 6.5   |
| 32  | .19568     | 43 | .26981      | 51  | .22124  | 42         | .30030  | 50          | 32 | 7.3  | 7.2   |
| 33  | .19611     | 43 | .27032      | 51  | .22166  | 42         | .30081  | 50          | 33 | 14.6 | 14.5  |
| 34  | .19654     | 43 | .27084      | 51  | .22208  | 42         | .30131  | 50          | 34 | 22.0 | 21.7  |
| 35  | 9.19698    | 43 | 9.27135     | 51  | 9.22250 | 42         | 9.30181 | 50          | 35 | 29.3 | 29.0  |
| 36  | .19741     | 43 | .27186      | 51  | .22292  | 41         | .30231  | 50          | 36 | 36.6 | 36.2  |
| 37  | .19784     | 43 | .27238      | 51  | .22334  | 42         | .30282  | 50          | 37 |      |       |
| 38  | .19827     | 43 | .27289      | 51  | .22376  | 41         | .30332  | 50          | 38 |      |       |
| 39  | .19870     | 43 | .27340      | 51  | .22417  | 42         | .30382  | 50          | 39 | 42   | 42    |
| 40  | 9.19914    | 43 | 9.27391     | 51  | 9.22459 | 41         | 9.30432 | 50          | 40 | 4.2  | 4.2   |
| 41  | .19957     | 43 | .27443      | 51  | .22501  | 42         | .30482  | 50          | 41 | 4.9  | 4.9   |
| 42  | .20000     | 43 | .27494      | 51  | .22543  | 41         | .30533  | 50          | 42 | 5.6  | 5.6   |
| 43  | .20043     | 43 | .27545      | 51  | .22584  | 41         | .30583  | 50          | 43 | 6.4  | 6.3   |
| 44  | .20086     | 43 | .27596      | 51  | .22626  | 42         | .30633  | 50          | 44 | 7.1  | 7.0   |
| 45  | 9.20129    | 43 | 9.27647     | 51  | 9.22668 | 41         | 9.30683 | 50          | 45 | 14.1 | 14.0  |
| 46  | .20172     | 43 | .27698      | 51  | .22709  | 41         | .30733  | 50          | 46 | 21.2 | 21.0  |
| 47  | .20215     | 43 | .27749      | 51  | .22751  | 41         | .30783  | 50          | 47 | 28.3 | 28.0  |
| 48  | .20258     | 43 | .27800      | 51  | .22792  | 41         | .30833  | 50          | 48 | 35.4 | 35.0  |
| 49  | .20301     | 42 | .27852      | 51  | .22834  | 41         | .30883  | 50          | 49 |      |       |
| 50  | 9.20343    | 43 | 9.27903     | 51  | 9.22876 | 41         | 9.30933 | 50          | 50 |      |       |
| 51  | .20386     | 43 | .27954      | 51  | .22917  | 41         | .30983  | 50          | 51 | 41   |       |
| 52  | .20429     | 43 | .28005      | 51  | .22959  | 41         | .31033  | 50          | 52 | 6    | 4.8   |
| 53  | .20472     | 43 | .28056      | 51  | .23000  | 41         | .31083  | 50          | 53 | 7    | 5.4   |
| 54  | .20515     | 42 | .28107      | 51  | .23042  | 41         | .31133  | 50          | 54 | 8    | 6.1   |
| 55  | 9.20558    | 43 | 9.28157     | 50  | 9.23083 | 41         | 9.31183 | 49          | 55 | 9    | 6.8   |
| 56  | .20600     | 42 | .28208      | 51  | .23124  | 41         | .31233  | 50          | 56 | 10   | 13.6  |
| 57  | .20643     | 43 | .28259      | 51  | .23166  | 41         | .31283  | 50          | 57 | 20   | 20.5  |
| 58  | .20686     | 42 | .28310      | 51  | .23207  | 41         | .31333  | 50          | 58 | 30   | 27.3  |
| 59  | .20728     | 43 | .28361      | 50  | .23248  | 41         | .31383  | 49          | 59 | 40   | 34.1  |
| 60  | 9.20771    |    | 9.28412     |     | 9.23290 |            | 9.31432 |             | 60 | 50   |       |
|     | Log. Vers. | D  | Log. Exsec. | D   |         | Log. Vers. | D       | Log. Exsec. | D  |      | P. P. |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P.             |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------------------|
| 0  | 9.23290    | 41 | 9.31432     | 50 | 9.25731    | 40 | 9.34395     | 49 | 0  |                   |
| 1  | .23331     | 41 | .31482      | 50 | .25771     | 40 | .34444      | 48 | 1  |                   |
| 2  | .23372     | 41 | .31532      | 49 | .25811     | 40 | .34492      | 49 | 2  |                   |
| 3  | .23414     | 41 | .31582      | 50 | .25851     | 40 | .34541      | 49 | 3  | 50 49 49          |
| 4  | .23455     | 41 | .31632      | 49 | .25891     | 40 | .34590      | 49 | 4  | 6 5 0 4.9 1.0     |
| 5  | 9.23496    | 41 | 9.31681     | 50 | 9.25931    | 40 | 9.34639     | 48 | 5  | 7 6 5 5.5 5.2     |
| 6  | .23537     | 41 | .31731      | 49 | .25971     | 40 | .34688      | 48 | 6  | 8 6 0 6.6 6.1     |
| 7  | .23579     | 41 | .31781      | 50 | .26011     | 40 | .34737      | 49 | 7  | 9 7 5 7.4 7.2     |
| 8  | .23620     | 41 | .31831      | 49 | .26051     | 39 | .34785      | 48 | 8  | 10 8 3 8.2 8.1    |
| 9  | .23661     | 41 | .31880      | 50 | .26091     | 40 | .34834      | 49 | 9  | 20 16.1 16.5 16.3 |
| 10 | 9.23702    | 41 | 9.31930     | 49 | 9.26131    | 40 | 9.34883     | 48 | 10 | 30 25.2 24.7 24.5 |
| 11 | .23743     | 41 | .31980      | 49 | .26171     | 39 | .34932      | 48 | 11 | 40 33.3 33.0 32.9 |
| 12 | .23784     | 41 | .32029      | 49 | .26210     | 40 | .34980      | 49 | 12 | 50 41.0 41.2 41.5 |
| 13 | .23825     | 41 | .32079      | 50 | .26250     | 40 | .35029      | 48 | 13 |                   |
| 14 | .23866     | 41 | .32129      | 49 | .26290     | 39 | .35078      | 49 | 14 | 48 48             |
| 15 | 9.23907    | 41 | 9.32178     | 49 | 9.26330    | 40 | 9.35127     | 48 | 15 | 6 4.8 4.8         |
| 16 | .23948     | 41 | .32228      | 49 | .26370     | 39 | .35175      | 48 | 16 | 7 5.0 5.6         |
| 17 | .23989     | 41 | .32277      | 49 | .26409     | 40 | .35224      | 49 | 17 | 8 5.4 6.4         |
| 18 | .24030     | 41 | .32327      | 50 | .26449     | 39 | .35273      | 48 | 18 | 9 6.2 7.2         |
| 19 | .24071     | 41 | .32377      | 49 | .26489     | 39 | .35321      | 48 | 19 | 10 8.1 8.0        |
| 20 | 9.24112    | 41 | 9.32426     | 49 | 9.26528    | 40 | 9.35370     | 48 | 20 | 20 16.1 16.0      |
| 21 | .24153     | 41 | .32476      | 49 | .26568     | 39 | .35419      | 49 | 21 | 30 24.2 24.0      |
| 22 | .24194     | 41 | .32525      | 49 | .26608     | 39 | .35467      | 48 | 22 | 40 32.3 32.0      |
| 23 | .24235     | 41 | .32575      | 49 | .26647     | 39 | .35516      | 48 | 23 | 50 40.4 40.0      |
| 24 | .24275     | 41 | .32624      | 49 | .26687     | 39 | .35564      | 48 | 24 |                   |
| 25 | 9.24316    | 41 | 9.32673     | 49 | 9.26726    | 40 | 9.35613     | 48 | 25 | 41 41             |
| 26 | .24357     | 41 | .32723      | 49 | .26766     | 39 | .35661      | 48 | 26 | 6 4.1 4.1         |
| 27 | .24398     | 41 | .32772      | 49 | .26806     | 39 | .35710      | 48 | 27 | 7 4.8 4.8         |
| 28 | .24438     | 41 | .32822      | 49 | .26845     | 39 | .35758      | 48 | 28 | 8 5.5 5.4         |
| 29 | .24479     | 41 | .32871      | 49 | .26885     | 39 | .35807      | 48 | 29 | 9 6.2 6.1         |
| 30 | 9.24520    | 41 | 9.32920     | 49 | 9.26924    | 39 | 9.35855     | 48 | 30 | 10 6.0 6.8        |
| 31 | .24561     | 41 | .32970      | 49 | .26964     | 39 | .35904      | 48 | 31 | 20 13.8 13.6      |
| 32 | .24601     | 41 | .33019      | 49 | .27003     | 39 | .35952      | 48 | 32 | 30 20.7 20.5      |
| 33 | .24642     | 41 | .33069      | 49 | .27042     | 39 | .36001      | 48 | 33 | 40 27.6 27.2      |
| 34 | .24682     | 41 | .33118      | 49 | .27082     | 39 | .36049      | 48 | 34 | 50 34.6 34.1      |
| 35 | 9.24723    | 41 | 9.33167     | 49 | 9.27121    | 39 | 9.36098     | 48 | 35 |                   |
| 36 | .24764     | 41 | .33216      | 49 | .27161     | 39 | .36146      | 48 | 36 | 40 4.0 4.0        |
| 37 | .24804     | 41 | .33266      | 49 | .27200     | 39 | .36194      | 48 | 37 | 7 4.7 4.6         |
| 38 | .24845     | 41 | .33315      | 49 | .27239     | 39 | .36243      | 48 | 38 | 8 5.4 5.3         |
| 39 | .24885     | 41 | .33364      | 49 | .27278     | 39 | .36291      | 48 | 39 | 9 6.1 6.0         |
| 40 | 9.24926    | 41 | 9.33413     | 49 | 9.27318    | 39 | 9.36340     | 48 | 40 | 10 6.7 6.6        |
| 41 | .24966     | 41 | .33463      | 49 | .27357     | 39 | .36388      | 48 | 41 | 20 13.5 13.3      |
| 42 | .25007     | 41 | .33512      | 49 | .27396     | 39 | .36436      | 48 | 42 | 30 20.2 20.0      |
| 43 | .25047     | 41 | .33561      | 49 | .27435     | 39 | .36484      | 48 | 43 | 40 27.0 26.6      |
| 44 | .25087     | 41 | .33610      | 49 | .27475     | 39 | .36533      | 48 | 44 | 50 33.7 33.3      |
| 45 | 9.25128    | 41 | 9.33659     | 49 | 9.27514    | 39 | 9.36581     | 48 | 45 |                   |
| 46 | .25168     | 41 | .33708      | 49 | .27553     | 39 | .36629      | 48 | 46 | 39 39             |
| 47 | .25209     | 41 | .33758      | 49 | .27592     | 39 | .36678      | 48 | 47 | 6 3.9 3.9         |
| 48 | .25249     | 41 | .33807      | 49 | .27631     | 39 | .36726      | 48 | 48 | 7 4.6 4.5         |
| 49 | .25289     | 41 | .33856      | 49 | .27670     | 39 | .36774      | 48 | 49 | 8 5.2 5.2         |
| 50 | 9.25329    | 41 | 9.33905     | 49 | 9.27709    | 39 | 9.36822     | 48 | 50 | 9 5.0 5.8         |
| 51 | .25370     | 41 | .33954      | 49 | .27749     | 39 | .36870      | 48 | 51 | 10 6.6 6.5        |
| 52 | .25410     | 41 | .34003      | 49 | .27788     | 39 | .36919      | 48 | 52 | 20 13.1 13.0      |
| 53 | .25450     | 41 | .34052      | 49 | .27827     | 39 | .36967      | 48 | 53 | 30 19.7 19.5      |
| 54 | .25490     | 41 | .34101      | 49 | .27866     | 39 | .37015      | 48 | 54 | 40 26.3 26.0      |
| 55 | 9.25531    | 41 | 9.34150     | 49 | 9.27905    | 39 | 9.37063     | 48 | 55 | 50 32.9 32.5      |
| 56 | .25571     | 41 | .34199      | 49 | .27944     | 39 | .37111      | 48 | 56 |                   |
| 57 | .25611     | 41 | .34248      | 49 | .27982     | 39 | .37159      | 48 | 57 | 38 3.8 3.8        |
| 58 | .25651     | 41 | .34297      | 49 | .28021     | 39 | .37207      | 48 | 58 | 6 4.5 4.5         |
| 59 | .25691     | 41 | .34346      | 49 | .28060     | 39 | .37255      | 48 | 59 | 7 5.1 5.1         |
| 60 | 9.25731    | 41 | 9.34395     | 49 | 9.28099    | 39 | 9.37303     | 48 | 60 | 8 5.8 5.8         |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P.             |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.28099    | 39 | 9.37303     | 48 | 9.30398    | 37 | 9.40163     | 47 | 0  |       |
| 1  | .28138     | 38 | .37352      | 48 | .30436     | 38 | .40210      | 47 | 1  |       |
| 2  | .28177     | 39 | .37400      | 48 | .30474     | 37 | .40258      | 47 | 2  |       |
| 3  | .28216     | 39 | .37448      | 48 | .30511     | 37 | .40305      | 47 | 3  | 48    |
| 4  | .28255     | 38 | .37496      | 48 | .30549     | 37 | .40352      | 47 | 4  | 48    |
| 5  | 9.28293    | 38 | 9.37544     | 48 | 9.30587    | 38 | 9.40399     | 47 | 5  | 4.8   |
| 6  | .28332     | 39 | .37592      | 48 | .30624     | 37 | .40447      | 47 | 6  | 5.6   |
| 7  | .28371     | 38 | .37640      | 48 | .30662     | 37 | .40494      | 47 | 7  | 6.4   |
| 8  | .28410     | 39 | .37687      | 47 | .30700     | 38 | .40541      | 47 | 8  | 7.2   |
| 9  | .28448     | 38 | .37735      | 48 | .30737     | 37 | .40588      | 47 | 9  | 8.0   |
| 10 | 9.28487    | 39 | 9.37783     | 48 | 9.30775    | 37 | 9.40635     | 47 | 10 | 16.1  |
| 11 | .28526     | 38 | .37831      | 48 | .30812     | 37 | .40682      | 47 | 11 | 16.0  |
| 12 | .28564     | 38 | .37879      | 48 | .30850     | 37 | .40730      | 47 | 12 | 24.2  |
| 13 | .28603     | 39 | .37927      | 48 | .30887     | 37 | .40777      | 47 | 13 | 24.0  |
| 14 | .28642     | 38 | .37975      | 47 | .30925     | 37 | .40824      | 47 | 14 | 32.3  |
| 15 | 9.28680    | 38 | 9.38023     | 48 | 9.30963    | 37 | 9.40871     | 47 | 15 | 32.0  |
| 16 | .28719     | 38 | .38071      | 48 | .31000     | 37 | .40918      | 47 | 16 | 40.4  |
| 17 | .28757     | 38 | .38119      | 48 | .31037     | 37 | .40965      | 47 | 17 |       |
| 18 | .28796     | 38 | .38166      | 47 | .31075     | 37 | .41012      | 47 | 18 | 47    |
| 19 | .28835     | 39 | .38214      | 48 | .31112     | 37 | .41059      | 47 | 19 | 47    |
| 20 | 9.28873    | 38 | 9.38262     | 47 | 9.31150    | 37 | 9.41106     | 47 | 20 | 4.7   |
| 21 | .28912     | 38 | .38310      | 48 | .31187     | 37 | .41153      | 47 | 21 | 5.4   |
| 22 | .28950     | 38 | .38357      | 47 | .31224     | 37 | .41200      | 47 | 22 | 5.5   |
| 23 | .28988     | 38 | .38405      | 48 | .31262     | 37 | .41247      | 47 | 23 | 6.2   |
| 24 | .29027     | 38 | .38453      | 47 | .31299     | 37 | .41294      | 47 | 24 | 7.6   |
| 25 | 9.29065    | 38 | 9.38501     | 48 | 9.31336    | 37 | 9.41341     | 47 | 25 | 7.6   |
| 26 | .29104     | 38 | .38548      | 47 | .31374     | 37 | .41388      | 47 | 26 | 7.8   |
| 27 | .29142     | 38 | .38596      | 48 | .31411     | 37 | .41435      | 47 | 27 | 15.5  |
| 28 | .29180     | 38 | .38644      | 47 | .31448     | 37 | .41482      | 47 | 28 | 23.2  |
| 29 | .29219     | 38 | .38692      | 48 | .31485     | 37 | .41529      | 47 | 29 | 31.0  |
| 30 | 9.29257    | 38 | 9.38739     | 47 | 9.31523    | 37 | 9.41576     | 46 | 30 | 38.7  |
| 31 | .29295     | 38 | .38787      | 47 | .31560     | 37 | .41623      | 47 | 31 |       |
| 32 | .29334     | 38 | .38834      | 48 | .31597     | 37 | .41670      | 47 | 32 | 39    |
| 33 | .29372     | 38 | .38882      | 47 | .31634     | 37 | .41717      | 47 | 33 | 38    |
| 34 | .29410     | 38 | .38930      | 47 | .31671     | 37 | .41763      | 46 | 34 | 3.8   |
| 35 | 9.29448    | 38 | 9.38977     | 47 | 9.31708    | 37 | 9.41810     | 47 | 35 | 4.5   |
| 36 | .29487     | 38 | .39025      | 47 | .31746     | 37 | .41857      | 47 | 36 | 5.1   |
| 37 | .29525     | 38 | .39072      | 47 | .31783     | 37 | .41904      | 46 | 37 | 5.8   |
| 38 | .29563     | 38 | .39120      | 48 | .31820     | 37 | .41951      | 47 | 38 | 6.4   |
| 39 | .29601     | 38 | .39168      | 47 | .31857     | 37 | .41998      | 47 | 39 | 12.8  |
| 40 | 9.29639    | 38 | 9.39215     | 47 | 9.31894    | 37 | 9.42044     | 46 | 40 | 19.2  |
| 41 | .29677     | 38 | .39263      | 48 | .31931     | 37 | .42091      | 47 | 41 | 25.6  |
| 42 | .29715     | 38 | .39310      | 47 | .31968     | 37 | .42138      | 46 | 42 | 32.1  |
| 43 | .29754     | 38 | .39358      | 47 | .32005     | 37 | .42185      | 47 | 43 |       |
| 44 | .29792     | 38 | .39405      | 47 | .32042     | 37 | .42231      | 46 | 44 | 37    |
| 45 | 9.29830    | 38 | 9.39453     | 47 | 9.32079    | 37 | 9.42278     | 47 | 45 | 3.7   |
| 46 | .29868     | 38 | .39500      | 47 | .32116     | 37 | .42325      | 46 | 46 | 4.4   |
| 47 | .29906     | 38 | .39548      | 47 | .32153     | 37 | .42372      | 47 | 47 | 5.0   |
| 48 | .29944     | 38 | .39595      | 47 | .32190     | 37 | .42418      | 46 | 48 | 5.6   |
| 49 | .29982     | 38 | .39642      | 47 | .32227     | 37 | .42465      | 47 | 49 | 6.2   |
| 50 | 9.30020    | 38 | 9.39690     | 47 | 9.32263    | 36 | 9.42512     | 46 | 50 | 12.5  |
| 51 | .30057     | 37 | .39737      | 47 | .32300     | 37 | .42558      | 46 | 51 | 18.7  |
| 52 | .30095     | 38 | .39785      | 47 | .32337     | 37 | .42605      | 47 | 52 | 25.0  |
| 53 | .30133     | 38 | .39832      | 47 | .32374     | 36 | .42652      | 46 | 53 | 31.2  |
| 54 | .30171     | 38 | .39879      | 47 | .32411     | 37 | .42698      | 46 | 54 |       |
| 55 | 9.30209    | 38 | 9.39927     | 47 | 9.32447    | 36 | 9.42745     | 46 | 55 | 3.7   |
| 56 | .30247     | 37 | .39974      | 47 | .32484     | 37 | .42792      | 47 | 56 | 4.2   |
| 57 | .30285     | 38 | .40021      | 47 | .32521     | 36 | .42838      | 46 | 57 | 4.8   |
| 58 | .30323     | 37 | .40069      | 47 | .32558     | 37 | .42885      | 46 | 58 | 5.5   |
| 59 | .30360     | 38 | .40116      | 47 | .32594     | 36 | .42931      | 46 | 59 | 6.1   |
| 60 | 9.30398    | 38 | 9.40163     | 47 | 9.32631    | 37 | 9.42978     | 46 | 60 | 12.1  |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

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| 38° |            |    |             | 39° |    |            |    | P. P.       |    |    |       |
|-----|------------|----|-------------|-----|----|------------|----|-------------|----|----|-------|
| '   | Log. Vers. | D  | Log. Exsec. | D   | '  | Log. Vers. | D  | Log. Exsec. | D  | '  | P. P. |
| 0   | 9.32631    | 36 | 9.42978     | 46  | 0  | 9.34802    | 35 | 9.45752     | 45 | 0  |       |
| 1   | .32668     | 36 | .43024      | 47  | 1  | .34837     | 36 | .45797      | 46 | 1  |       |
| 2   | .32704     | 37 | .43071      | 46  | 2  | .34873     | 35 | .45843      | 46 | 2  |       |
| 3   | .32741     | 36 | .43118      | 46  | 3  | .34909     | 35 | .45889      | 46 | 3  | 47    |
| 4   | .32778     | 36 | .43164      | 46  | 4  | .34944     | 35 | .45935      | 46 | 4  | 46    |
| 5   | 9.32814    | 36 | 9.43211     | 46  | 5  | 9.34980    | 35 | 9.45981     | 45 | 5  | 6     |
| 6   | .32851     | 36 | .43257      | 46  | 6  | .35016     | 36 | .46027      | 46 | 6  | 4.7   |
| 7   | .32888     | 37 | .43304      | 46  | 7  | .35051     | 35 | .46073      | 46 | 7  | 4.7   |
| 8   | .32924     | 36 | .43350      | 46  | 8  | .35087     | 35 | .46118      | 45 | 8  | 5.5   |
| 9   | .32961     | 36 | .43396      | 46  | 9  | .35122     | 35 | .46164      | 46 | 9  | 6.2   |
| 10  | 9.32997    | 36 | 9.43443     | 46  | 10 | 9.35158    | 35 | 9.46210     | 45 | 10 | 7.0   |
| 11  | .33034     | 36 | .43489      | 46  | 11 | .35193     | 35 | .46256      | 45 | 11 | 7.7   |
| 12  | .33070     | 36 | .43536      | 46  | 12 | .35229     | 35 | .46302      | 46 | 12 | 15.5  |
| 13  | .33107     | 36 | .43582      | 46  | 13 | .35264     | 35 | .46347      | 45 | 13 | 23.2  |
| 14  | .33143     | 36 | .43629      | 46  | 14 | .35300     | 35 | .46393      | 46 | 14 | 31.0  |
| 15  | 9.33180    | 36 | 9.43675     | 46  | 15 | 9.35335    | 35 | 9.46439     | 45 | 15 | 38.7  |
| 16  | .33216     | 36 | .43721      | 46  | 16 | .35370     | 35 | .46485      | 45 | 16 | 6     |
| 17  | .33252     | 36 | .43768      | 46  | 17 | .35406     | 35 | .46530      | 45 | 17 | 4.6   |
| 18  | .33289     | 36 | .43814      | 46  | 18 | .35441     | 35 | .46576      | 46 | 18 | 5.3   |
| 19  | .33325     | 36 | .43861      | 46  | 19 | .35477     | 35 | .46622      | 45 | 19 | 6.0   |
| 20  | 9.33361    | 36 | 9.43907     | 46  | 20 | 9.35512    | 35 | 9.46668     | 46 | 20 | 6.7   |
| 21  | .33398     | 36 | .43953      | 46  | 21 | .35547     | 35 | .46713      | 45 | 21 | 7.5   |
| 22  | .33434     | 36 | .43999      | 46  | 22 | .35583     | 35 | .46759      | 45 | 22 | 15.0  |
| 23  | .33470     | 36 | .44046      | 46  | 23 | .35618     | 35 | .46805      | 45 | 23 | 22.5  |
| 24  | .33507     | 36 | .44092      | 46  | 24 | .35653     | 35 | .46850      | 45 | 24 | 30.0  |
| 25  | 9.33543    | 36 | 9.44138     | 46  | 25 | 9.35689    | 35 | 9.46896     | 45 | 25 | 37.5  |
| 26  | .33579     | 36 | .44185      | 46  | 26 | .35724     | 35 | .46942      | 46 | 26 | 6     |
| 27  | .33615     | 36 | .44231      | 46  | 27 | .35759     | 35 | .46987      | 45 | 27 | 4.5   |
| 28  | .33652     | 36 | .44277      | 46  | 28 | .35794     | 35 | .47033      | 45 | 28 | 5.2   |
| 29  | .33688     | 36 | .44323      | 46  | 29 | .35829     | 35 | .47078      | 45 | 29 | 6.0   |
| 30  | 9.33724    | 36 | 9.44370     | 46  | 30 | 9.35865    | 35 | 9.47124     | 46 | 30 | 6.7   |
| 31  | .33760     | 36 | .44416      | 46  | 31 | .35900     | 35 | .47170      | 45 | 31 | 7.5   |
| 32  | .33796     | 36 | .44462      | 46  | 32 | .35935     | 35 | .47215      | 45 | 32 | 15.0  |
| 33  | .33833     | 36 | .44508      | 46  | 33 | .35970     | 35 | .47261      | 45 | 33 | 22.5  |
| 34  | .33869     | 36 | .44554      | 46  | 34 | .36005     | 35 | .47306      | 45 | 34 | 30.0  |
| 35  | 9.33905    | 36 | 9.44601     | 46  | 35 | 9.36040    | 35 | 9.47352     | 46 | 35 | 37.5  |
| 36  | .33941     | 36 | .44647      | 46  | 36 | .36076     | 35 | .47398      | 45 | 36 | 6     |
| 37  | .33977     | 36 | .44693      | 46  | 37 | .36111     | 35 | .47443      | 45 | 37 | 4.3   |
| 38  | .34013     | 36 | .44739      | 46  | 38 | .36146     | 35 | .47489      | 45 | 38 | 4.8   |
| 39  | .34049     | 36 | .44785      | 46  | 39 | .36181     | 35 | .47534      | 45 | 39 | 5.5   |
| 40  | 9.34085    | 36 | 9.44831     | 46  | 40 | 9.36216    | 35 | 9.47580     | 45 | 40 | 6.1   |
| 41  | .34121     | 36 | .44877      | 46  | 41 | .36251     | 35 | .47625      | 45 | 41 | 12.3  |
| 42  | .34157     | 36 | .44924      | 46  | 42 | .36286     | 35 | .47671      | 45 | 42 | 18.5  |
| 43  | .34193     | 36 | .44970      | 46  | 43 | .36321     | 35 | .47716      | 45 | 43 | 24.6  |
| 44  | .34229     | 36 | .45016      | 46  | 44 | .36356     | 35 | .47762      | 45 | 44 | 30.8  |
| 45  | 9.34265    | 36 | 9.45062     | 46  | 45 | 9.36391    | 35 | 9.47807     | 45 | 45 | 6     |
| 46  | .34301     | 36 | .45108      | 46  | 46 | .36426     | 35 | .47853      | 45 | 46 | 3.6   |
| 47  | .34337     | 36 | .45154      | 46  | 47 | .36461     | 35 | .47898      | 45 | 47 | 4.2   |
| 48  | .34373     | 35 | .45200      | 46  | 48 | .36495     | 34 | .47943      | 45 | 48 | 4.8   |
| 49  | .34408     | 35 | .45246      | 46  | 49 | .36530     | 35 | .47989      | 45 | 49 | 5.5   |
| 50  | 9.34444    | 36 | 9.45292     | 46  | 50 | 9.36565    | 35 | 9.48034     | 45 | 50 | 5.9   |
| 51  | .34480     | 35 | .45338      | 46  | 51 | .36600     | 35 | .48080      | 45 | 51 | 12.0  |
| 52  | .34516     | 36 | .45384      | 46  | 52 | .36635     | 34 | .48125      | 45 | 52 | 18.0  |
| 53  | .34552     | 35 | .45430      | 46  | 53 | .36670     | 35 | .48170      | 45 | 53 | 24.0  |
| 54  | .34587     | 35 | .45476      | 46  | 54 | .36705     | 35 | .48216      | 45 | 54 | 30.0  |
| 55  | 9.34623    | 36 | 9.45522     | 46  | 55 | 9.36739    | 34 | 9.48261     | 45 | 55 | 6     |
| 56  | .34659     | 35 | .45568      | 46  | 56 | .36774     | 35 | .48306      | 45 | 56 | 3.6   |
| 57  | .34695     | 36 | .45614      | 46  | 57 | .36809     | 34 | .48352      | 45 | 57 | 4.2   |
| 58  | .34730     | 35 | .45660      | 46  | 58 | .36844     | 35 | .48397      | 45 | 58 | 4.8   |
| 59  | .34766     | 36 | .45706      | 46  | 59 | .36878     | 34 | .48443      | 45 | 59 | 5.5   |
| 60  | 9.34802    | 35 | 9.45752     | 45  | 60 | 9.36913    | 35 | 9.48488     | 45 | 60 | 5.9   |
| 38° |            |    |             | 39° |    |            |    | P. P.       |    |    |       |
| '   | Log. Vers. | D  | Log. Exsec. | D   | '  | Log. Vers. | D  | Log. Exsec. | D  | '  | P. P. |
| 0   | 9.32631    | 36 | 9.42978     | 46  | 0  | 9.34802    | 35 | 9.45752     | 45 | 0  |       |
| 1   | .32668     | 36 | .43024      | 46  | 1  | .34837     | 35 | .45797      | 45 | 1  |       |
| 2   | .32704     | 36 | .43071      | 47  | 2  | .34873     | 36 | .45843      | 46 | 2  |       |
| 3   | .32741     | 37 | .43118      | 46  | 3  | .34909     | 35 | .45889      | 46 | 3  | 47    |
| 4   | .32778     | 36 | .43164      | 46  | 4  | .34944     | 35 | .45935      | 46 | 4  | 46    |
| 5   | 9.32814    | 36 | 9.43211     | 46  | 5  | 9.34980    | 35 | 9.45981     | 45 | 5  | 6     |
| 6   | .32851     | 36 | .43257      | 46  | 6  | .35016     | 36 | .46027      | 46 | 6  | 4.7   |
| 7   | .32888     | 37 | .43304      | 46  | 7  | .35051     | 35 | .46073      | 46 | 7  | 4.7   |
| 8   | .32924     | 36 | .43350      | 46  | 8  | .35087     | 35 | .46118      | 45 | 8  | 5.5   |
| 9   | .32961     | 36 | .43396      | 46  | 9  | .35122     | 35 | .46164      | 46 | 9  | 6.2   |
| 10  | 9.32997    | 36 | 9.43443     | 46  | 10 | 9.35158    | 35 | 9.46210     | 45 | 10 | 7.0   |
| 11  | .33034     | 36 | .43489      | 46  | 11 | .35193     | 35 | .46256      | 45 | 11 | 7.7   |
| 12  | .33070     | 36 | .43536      | 46  | 12 | .35229     | 35 | .46302      | 46 | 12 | 15.5  |
| 13  | .33107     | 36 | .43582      | 46  | 13 | .35264     | 35 | .46347      | 45 | 13 | 23.2  |
| 14  | .33143     | 36 | .43629      | 46  | 14 | .35300     | 35 | .46393      | 46 | 14 | 31.0  |
| 15  | 9.33180    | 36 | 9.43675     | 46  | 15 | 9.35335    | 35 | 9.46439     | 45 | 15 | 38.7  |
| 16  | .33216     | 36 | .43721      | 46  | 16 | .35370     | 35 | .46485      | 45 | 16 | 6     |
| 17  | .33252     | 36 | .43768      | 46  | 17 | .35406     | 35 | .46530      | 45 | 17 | 4.6   |
| 18  | .33289     | 36 | .43814      | 46  | 18 | .35441     | 35 | .46576      | 46 | 18 | 5.3   |
| 19  | .33325     | 36 | .43861      | 46  | 19 | .35477     | 35 | .46622      | 45 | 19 | 6.0   |
| 20  | 9.33361    | 36 | 9.43907     | 46  | 20 | 9.35512    | 35 | 9.46668     | 46 | 20 | 6.7   |
| 21  | .33398     | 36 | .43953      | 46  | 21 | .35547     | 35 | .46713      | 45 | 21 | 7.5   |
| 22  | .33434     | 36 | .43999      | 46  | 22 | .35583     | 35 | .46759      | 45 | 22 | 15.0  |
| 23  | .33470     | 36 | .44046      | 46  | 23 | .35618     | 35 | .46805      | 45 | 23 | 22.5  |
| 24  | .33507     | 36 | .44092      | 46  | 24 | .35653     | 35 | .46850      | 45 | 24 | 30.0  |
| 25  | 9.33543    | 36 | 9.44138     | 46  | 25 | 9.35689    | 35 | 9.46896     | 46 | 25 | 37.5  |
| 26  | .33579     | 36 | .44185      | 46  | 26 | .35724     | 35 | .46942      | 45 | 26 | 6     |
| 27  | .33615     | 36 | .44231      | 46  | 27 | .35759     | 35 | .46987      | 45 | 27 | 4.5   |
| 28  | .33652     | 36 | .44277      | 46  | 28 | .35794     | 35 | .47033      | 45 | 28 | 5.2   |
| 29  | .33688     | 36 | .44323      | 46  | 29 | .35829     | 35 | .47078      | 45 | 29 | 6.0   |
| 30  | 9.33724    | 36 | 9.44370     | 46  | 30 | 9.35865    | 35 | 9.47124     | 46 | 30 | 6.7   |
| 31  | .33760     | 36 | .44416      | 46  | 31 | .35900     | 35 | .47170      | 45 | 31 | 7.5   |
| 32  | .33796     | 36 | .44462      | 46  | 32 | .35935     | 35 | .47215      | 45 | 32 | 15.0  |
| 33  | .33833     | 36 | .44508      | 46  | 33 | .35970     | 35 | .47261      | 45 | 33 | 22.5  |
| 34  | .33869     | 36 | .44554      | 46  | 34 | .36005     | 35 | .47306      | 45 | 34 | 30.0  |
| 35  | 9.33905    | 36 | 9.44601     | 46  | 35 | 9.36040    | 35 | 9.47352     | 46 | 35 | 37.5  |
| 36  | .33941     | 36 | .44647      | 46  | 36 | .36076     |    |             |    |    |       |

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|    | Log. Vers. | D  | Log. Exsec. | D  |            | Log. Vers. | D           | Log. Exsec. | D    |      | P. P. |
|----|------------|----|-------------|----|------------|------------|-------------|-------------|------|------|-------|
| 0  | 9.36913    | 34 | 9.48488     | 45 | 9.38968    | 9.51190    | 45          | 0           |      |      |       |
| 1  | .36948     | 34 | .48533      | 45 | .39002     | .51235     | 45          | 1           |      |      |       |
| 2  | .36982     | 34 | .48578      | 45 | .39035     | .51279     | 45          | 2           |      |      |       |
| 3  | .37017     | 34 | .48624      | 45 | .39069     | .51324     | 45          | 3           |      |      |       |
| 4  | .37052     | 34 | .48669      | 45 | .39103     | .51369     | 45          | 4           |      |      |       |
| 5  | 9.37086    | 34 | 9.48714     | 45 | 9.39137    | 9.51414    | 45          | 5           | 6    | 45   | 45    |
| 6  | .37121     | 34 | .48759      | 45 | .39170     | .51458     | 45          | 6           | 7    | 4.5  | 4.5   |
| 7  | .37156     | 34 | .48805      | 45 | .39204     | .51503     | 45          | 7           | 8    | 5.3  | 5.2   |
| 8  | .37190     | 34 | .48850      | 45 | .39238     | .51548     | 45          | 8           | 9    | 6.0  | 6.0   |
| 9  | .37225     | 34 | .48895      | 45 | .39271     | .51592     | 45          | 9           | 10   | 6.8  | 6.7   |
| 10 | 9.37259    | 34 | 9.48940     | 45 | 9.39305    | 9.51637    | 45          | 10          | 20   | 7.6  | 7.5   |
| 11 | .37294     | 34 | .48986      | 45 | .39339     | .51682     | 45          | 20          | 30   | 15.1 | 15.0  |
| 12 | .37328     | 34 | .49031      | 45 | .39372     | .51726     | 45          | 30          | 40   | 22.7 | 22.5  |
| 13 | .37363     | 34 | .49076      | 45 | .39406     | .51771     | 45          | 40          | 50   | 30.3 | 30.0  |
| 14 | .37397     | 34 | .49121      | 45 | .39439     | .51816     | 45          | 50          |      | 37.9 | 37.5  |
| 15 | 9.37432    | 34 | 9.49166     | 45 | 9.39473    | 9.51860    | 45          |             |      |      |       |
| 16 | .37466     | 34 | .49211      | 45 | .39507     | .51905     | 45          | 6           | 44   | 44   | 44    |
| 17 | .37501     | 34 | .49257      | 45 | .39540     | .51950     | 45          | 7           | 4.4  | 4.4  | 4.4   |
| 18 | .37535     | 34 | .49302      | 45 | .39574     | .51994     | 45          | 8           | 5.2  | 5.1  | 5.1   |
| 19 | .37570     | 34 | .49347      | 45 | .39607     | .52039     | 45          | 9           | 5.9  | 5.8  | 5.8   |
| 20 | 9.37604    | 34 | 9.49392     | 45 | 9.39641    | 9.52084    | 45          | 10          | 6.7  | 6.6  | 6.6   |
| 21 | .37639     | 34 | .49437      | 45 | .39674     | .52128     | 45          | 20          | 7.4  | 7.3  | 7.3   |
| 22 | .37673     | 34 | .49482      | 45 | .39708     | .52173     | 45          | 30          | 14.8 | 14.6 | 14.6  |
| 23 | .37707     | 34 | .49527      | 45 | .39741     | .52217     | 45          | 40          | 22.2 | 22.0 | 22.0  |
| 24 | .37742     | 34 | .49572      | 45 | .39774     | .52262     | 45          | 50          | 29.6 | 29.3 | 29.3  |
| 25 | 9.37776    | 34 | 9.49618     | 45 | 9.39808    | 9.52306    | 45          |             |      |      |       |
| 26 | .37810     | 34 | .49663      | 45 | .39841     | .52351     | 45          | 6           | 35   | 34   | 34    |
| 27 | .37845     | 34 | .49708      | 45 | .39875     | .52396     | 45          | 7           | 3.5  | 3.4  | 3.4   |
| 28 | .37879     | 34 | .49753      | 45 | .39908     | .52440     | 45          | 8           | 4.1  | 4.0  | 4.0   |
| 29 | .37913     | 34 | .49798      | 45 | .39941     | .52485     | 45          | 9           | 4.6  | 4.6  | 4.6   |
| 30 | 9.37947    | 34 | 9.49843     | 45 | 9.39975    | 9.52529    | 45          | 10          | 5.2  | 5.2  | 5.2   |
| 31 | .37982     | 34 | .49888      | 45 | .40008     | .52574     | 45          | 20          | 5.8  | 5.7  | 5.7   |
| 32 | .38016     | 34 | .49933      | 45 | .40041     | .52618     | 45          | 30          | 11.6 | 11.5 | 11.5  |
| 33 | .38050     | 34 | .49978      | 45 | .40075     | .52663     | 45          | 40          | 17.5 | 17.2 | 17.2  |
| 34 | .38084     | 34 | .50023      | 45 | .40108     | .52707     | 45          | 50          | 23.3 | 23.0 | 23.0  |
| 35 | 9.38118    | 34 | 9.50068     | 45 | 9.40141    | 9.52752    | 45          |             |      |      |       |
| 36 | .38153     | 34 | .50113      | 45 | .40175     | .52796     | 45          | 6           | 34   | 33   | 33    |
| 37 | .38187     | 34 | .50158      | 45 | .40208     | .52841     | 45          | 7           | 3.4  | 3.3  | 3.3   |
| 38 | .38221     | 34 | .50203      | 45 | .40241     | .52885     | 45          | 8           | 3.9  | 3.9  | 3.9   |
| 39 | .38255     | 34 | .50248      | 45 | .40274     | .52930     | 45          | 9           | 4.5  | 4.4  | 4.4   |
| 40 | 9.38289    | 34 | 9.50293     | 45 | 9.40307    | 9.52974    | 45          | 10          | 5.1  | 5.0  | 5.0   |
| 41 | .38323     | 34 | .50338      | 45 | .40341     | .53018     | 45          | 20          | 5.6  | 5.6  | 5.6   |
| 42 | .38357     | 34 | .50383      | 45 | .40374     | .53063     | 45          | 30          | 11.3 | 11.1 | 11.1  |
| 43 | .38391     | 34 | .50427      | 45 | .40407     | .53107     | 45          | 40          | 17.0 | 16.7 | 16.7  |
| 44 | .38425     | 34 | .50472      | 45 | .40440     | .53152     | 45          | 50          | 22.6 | 22.3 | 22.3  |
| 45 | 9.38459    | 34 | 9.50517     | 45 | 9.40473    | 9.53196    | 45          |             |      |      |       |
| 46 | .38493     | 34 | .50562      | 45 | .40506     | .53240     | 45          | 6           | 33   | 33   | 33    |
| 47 | .38527     | 34 | .50607      | 45 | .40540     | .53285     | 45          | 7           | 3.4  | 3.3  | 3.3   |
| 48 | .38561     | 34 | .50652      | 45 | .40573     | .53329     | 45          | 8           | 3.9  | 3.9  | 3.9   |
| 49 | .38595     | 34 | .50697      | 45 | .40606     | .53374     | 45          | 9           | 4.5  | 4.4  | 4.4   |
| 50 | 9.38629    | 34 | 9.50742     | 45 | 9.40639    | 9.53418    | 45          | 10          | 5.1  | 5.0  | 5.0   |
| 51 | .38663     | 34 | .50787      | 45 | .40672     | .53462     | 45          | 20          | 5.6  | 5.6  | 5.6   |
| 52 | .38697     | 34 | .50831      | 45 | .40705     | .53507     | 45          | 30          | 11.3 | 11.1 | 11.1  |
| 53 | .38731     | 34 | .50876      | 45 | .40738     | .53551     | 45          | 40          | 17.0 | 16.7 | 16.7  |
| 54 | .38765     | 34 | .50921      | 45 | .40771     | .53595     | 45          | 50          | 22.6 | 22.3 | 22.3  |
| 55 | 9.38799    | 34 | 9.50966     | 45 | 9.40804    | 9.53640    | 45          |             |      |      |       |
| 56 | .38833     | 34 | .51011      | 45 | .40837     | .53684     | 45          | 6           | 33   | 33   | 33    |
| 57 | .38866     | 34 | .51055      | 45 | .40870     | .53728     | 45          | 7           | 3.4  | 3.3  | 3.3   |
| 58 | .38900     | 34 | .51100      | 45 | .40903     | .53773     | 45          | 8           | 3.9  | 3.8  | 3.8   |
| 59 | .38934     | 34 | .51145      | 45 | .40936     | .53817     | 45          | 9           | 4.5  | 4.4  | 4.4   |
| 60 | 9.38968    | 34 | 9.51190     | 45 | 9.40969    | 9.53861    | 45          | 10          | 5.1  | 4.9  | 4.9   |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D          | Log. Exsec. | D           |      |      | P. P. |



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|    | Log. Vers. | D  | Log. Exsec. | D  |         | Log. Vers. | D       | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|---------|------------|---------|-------------|----|----|-------|
| 0  | 9.40969    | 32 | 9.53861     | 44 | 9.42918 | 32         | 9.56505 | 44          | 0  |    |       |
| 1  | .41001     | 33 | .53906      | 44 | .42950  | 32         | .56549  | 44          | 1  |    |       |
| 2  | .41034     | 33 | .53950      | 44 | .42982  | 32         | .56593  | 44          | 2  |    |       |
| 3  | .41067     | 33 | .53994      | 44 | .43014  | 32         | .56637  | 44          | 3  |    |       |
| 4  | .41100     | 33 | .54038      | 44 | .43046  | 32         | .56680  | 44          | 4  |    |       |
| 5  | 9.41133    | 32 | 9.54083     | 44 | 9.43078 | 32         | 9.56724 | 44          | 5  | 6  | 44    |
| 6  | .41166     | 33 | .54127      | 44 | .43110  | 32         | .56768  | 44          | 6  | 7  | 44    |
| 7  | .41199     | 33 | .54171      | 44 | .43142  | 31         | .56812  | 44          | 7  | 8  | 44    |
| 8  | .41231     | 32 | .54215      | 44 | .43174  | 32         | .56856  | 44          | 8  | 9  | 44    |
| 9  | .41264     | 33 | .54259      | 44 | .43206  | 32         | .56899  | 44          | 9  | 10 | 44    |
| 10 | 9.41297    | 32 | 9.54304     | 44 | 9.43238 | 32         | 9.56943 | 44          | 10 | 20 | 44    |
| 11 | .41330     | 33 | .54348      | 44 | .43270  | 32         | .56987  | 44          | 11 | 30 | 44    |
| 12 | .41362     | 32 | .54392      | 44 | .43302  | 32         | .57031  | 44          | 12 | 40 | 44    |
| 13 | .41395     | 33 | .54436      | 44 | .43334  | 32         | .57075  | 44          | 13 | 50 | 44    |
| 14 | .41428     | 32 | .54480      | 44 | .43366  | 31         | .57118  | 44          | 14 |    |       |
| 15 | 9.41461    | 33 | 9.54525     | 44 | 9.43397 | 32         | 9.57162 | 44          | 15 |    |       |
| 16 | .41493     | 32 | .54569      | 44 | .43429  | 32         | .57206  | 44          | 16 | 6  | 43    |
| 17 | .41526     | 33 | .54613      | 44 | .43461  | 32         | .57250  | 44          | 17 | 7  | 43    |
| 18 | .41559     | 32 | .54657      | 44 | .43493  | 31         | .57293  | 44          | 18 | 8  | 43    |
| 19 | .41591     | 33 | .54701      | 44 | .43525  | 32         | .57337  | 44          | 19 | 9  | 43    |
| 20 | 9.41624    | 32 | 9.54745     | 44 | 9.43557 | 32         | 9.57381 | 44          | 20 | 10 | 43    |
| 21 | .41657     | 33 | .54790      | 44 | .43588  | 31         | .57424  | 44          | 21 | 20 | 43    |
| 22 | .41689     | 32 | .54834      | 44 | .43620  | 32         | .57468  | 44          | 22 | 30 | 43    |
| 23 | .41722     | 33 | .54878      | 44 | .43652  | 31         | .57512  | 44          | 23 | 40 | 43    |
| 24 | .41754     | 32 | .54922      | 44 | .43684  | 32         | .57556  | 44          | 24 | 50 | 43    |
| 25 | 9.41787    | 33 | 9.54966     | 44 | 9.43715 | 31         | 9.57599 | 44          | 25 |    |       |
| 26 | .41819     | 32 | .55010      | 44 | .43747  | 32         | .57643  | 44          | 26 |    |       |
| 27 | .41852     | 33 | .55054      | 44 | .43779  | 31         | .57687  | 44          | 27 |    |       |
| 28 | .41885     | 32 | .55098      | 44 | .43810  | 31         | .57730  | 44          | 28 | 6  | 33    |
| 29 | .41917     | 33 | .55142      | 44 | .43842  | 32         | .57774  | 44          | 29 | 7  | 32    |
| 30 | 9.41950    | 32 | 9.55186     | 44 | 9.43874 | 31         | 9.57818 | 44          | 30 | 8  | 32    |
| 31 | .41982     | 33 | .55230      | 44 | .43906  | 32         | .57861  | 44          | 31 | 9  | 32    |
| 32 | .42014     | 32 | .55275      | 44 | .43937  | 31         | .57905  | 44          | 32 | 10 | 32    |
| 33 | .42047     | 33 | .55319      | 44 | .43969  | 31         | .57949  | 44          | 33 | 20 | 32    |
| 34 | .42079     | 32 | .55363      | 44 | .44000  | 31         | .57992  | 44          | 34 | 30 | 32    |
| 35 | 9.42112    | 33 | 9.55407     | 44 | 9.44032 | 32         | 9.58036 | 44          | 35 | 40 | 32    |
| 36 | .42144     | 32 | .55451      | 44 | .44064  | 31         | .58079  | 44          | 36 | 50 | 32    |
| 37 | .42177     | 33 | .55495      | 44 | .44095  | 31         | .58123  | 44          | 37 |    |       |
| 38 | .42209     | 32 | .55539      | 44 | .44127  | 31         | .58167  | 44          | 38 |    |       |
| 39 | .42241     | 33 | .55583      | 44 | .44158  | 31         | .58210  | 44          | 39 | 6  | 32    |
| 40 | 9.42274    | 32 | 9.55627     | 44 | 9.44190 | 31         | 9.58254 | 44          | 40 | 7  | 31    |
| 41 | .42306     | 33 | .55671      | 44 | .44221  | 31         | .58297  | 44          | 41 | 8  | 31    |
| 42 | .42338     | 32 | .55715      | 44 | .44253  | 31         | .58341  | 44          | 42 | 9  | 31    |
| 43 | .42371     | 33 | .55759      | 44 | .44284  | 31         | .58385  | 44          | 43 | 10 | 31    |
| 44 | .42403     | 32 | .55803      | 44 | .44316  | 31         | .58428  | 44          | 44 | 20 | 31    |
| 45 | 9.42435    | 33 | 9.55847     | 44 | 9.44347 | 31         | 9.58472 | 44          | 45 | 30 | 31    |
| 46 | .42467     | 32 | .55890      | 44 | .44379  | 31         | .58515  | 44          | 46 | 40 | 31    |
| 47 | .42500     | 33 | .55934      | 44 | .44410  | 31         | .58559  | 44          | 47 | 50 | 31    |
| 48 | .42532     | 32 | .55978      | 44 | .44442  | 31         | .58602  | 44          | 48 |    |       |
| 49 | .42564     | 33 | .56022      | 44 | .44473  | 31         | .58646  | 44          | 49 |    |       |
| 50 | 9.42596    | 32 | 9.56066     | 44 | 9.44504 | 31         | 9.58689 | 44          | 50 |    |       |
| 51 | .42629     | 33 | .56110      | 44 | .44536  | 31         | .58733  | 44          | 51 | 6  | 31    |
| 52 | .42661     | 32 | .56154      | 44 | .44567  | 31         | .58776  | 44          | 52 | 7  | 31    |
| 53 | .42693     | 33 | .56198      | 44 | .44599  | 31         | .58820  | 44          | 53 | 8  | 31    |
| 54 | .42725     | 32 | .56242      | 44 | .44630  | 31         | .58864  | 44          | 54 | 9  | 31    |
| 55 | 9.42757    | 33 | 9.56286     | 44 | 9.44661 | 31         | 9.58907 | 44          | 55 | 10 | 31    |
| 56 | .42789     | 32 | .56330      | 44 | .44693  | 31         | .58951  | 44          | 56 | 20 | 31    |
| 57 | .42822     | 33 | .56374      | 44 | .44724  | 31         | .58994  | 44          | 57 | 30 | 31    |
| 58 | .42854     | 32 | .56417      | 44 | .44755  | 31         | .59037  | 44          | 58 | 40 | 31    |
| 59 | .42886     | 33 | .56461      | 44 | .44787  | 31         | .59081  | 44          | 59 | 50 | 31    |
| 60 | 9.42918    | 32 | 9.56505     | 44 | 9.44818 | 31         | 9.59124 | 44          | 60 |    |       |
|    | Log. Vers. | D  | Log. Exsec. | D  |         | Log. Vers. | D       | Log. Exsec. | D  |    | P. P. |

44°

45°

|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.44818    | 31 | 9.59124     | 43 | 9.46671    | 30 | 9.61722     | 43 | 0  |       |
| 1  | .44849     | 31 | .59168      | 43 | .46701     | 30 | .61763      | 43 | 1  |       |
| 2  | .44880     | 31 | .59211      | 43 | .46732     | 30 | .61808      | 43 | 2  |       |
| 3  | .44912     | 31 | .59255      | 43 | .46762     | 30 | .61852      | 43 | 3  |       |
| 4  | .44943     | 31 | .59298      | 43 | .46793     | 30 | .61895      | 43 | 4  |       |
| 5  | 9.44974    | 31 | 9.59342     | 43 | 9.46823    | 30 | 9.61938     | 43 | 5  | 43    |
| 6  | .45005     | 31 | .59385      | 43 | .46853     | 30 | .61981      | 43 | 6  | 4.3   |
| 7  | .45036     | 31 | .59429      | 43 | .46884     | 30 | .62024      | 43 | 7  | 5.1   |
| 8  | .45068     | 31 | .59472      | 43 | .46914     | 30 | .62067      | 43 | 8  | 5.8   |
| 9  | .45099     | 31 | .59515      | 43 | .46945     | 30 | .62110      | 43 | 9  | 6.5   |
| 10 | 9.45130    | 31 | 9.59559     | 43 | 9.46975    | 30 | 9.62153     | 43 | 10 | 7.2   |
| 11 | .45161     | 31 | .59602      | 43 | .47005     | 30 | .62196      | 43 | 11 | 7.1   |
| 12 | .45192     | 31 | .59646      | 43 | .47036     | 30 | .62239      | 43 | 12 | 14.3  |
| 13 | .45223     | 31 | .59689      | 43 | .47066     | 30 | .62282      | 43 | 13 | 21.5  |
| 14 | .45254     | 31 | .59732      | 43 | .47096     | 30 | .62326      | 43 | 14 | 28.6  |
| 15 | 9.45285    | 31 | 9.59776     | 43 | 9.47127    | 30 | 9.62369     | 43 | 15 | 35.8  |
| 16 | .45316     | 31 | .59819      | 43 | .47157     | 30 | .62412      | 43 | 16 |       |
| 17 | .45348     | 31 | .59863      | 43 | .47187     | 30 | .62455      | 43 | 17 | 4.2   |
| 18 | .45379     | 31 | .59906      | 43 | .47218     | 30 | .62498      | 43 | 18 | 4.9   |
| 19 | .45410     | 31 | .59949      | 43 | .47248     | 30 | .62541      | 43 | 19 | 5.6   |
| 20 | 9.45441    | 31 | 9.59993     | 43 | 9.47278    | 30 | 9.62584     | 43 | 20 | 6.4   |
| 21 | .45472     | 31 | .60036      | 43 | .47308     | 30 | .62627      | 43 | 21 | 7.1   |
| 22 | .45503     | 31 | .60079      | 43 | .47339     | 30 | .62670      | 43 | 22 | 14.1  |
| 23 | .45534     | 31 | .60123      | 43 | .47369     | 30 | .62713      | 43 | 23 | 21.2  |
| 24 | .45565     | 31 | .60166      | 43 | .47399     | 30 | .62756      | 43 | 24 | 28.3  |
| 25 | 9.45595    | 30 | 9.60209     | 43 | 9.47429    | 30 | 9.62799     | 43 | 25 | 35.4  |
| 26 | .45626     | 31 | .60253      | 43 | .47459     | 30 | .62842      | 43 | 26 |       |
| 27 | .45657     | 31 | .60296      | 43 | .47490     | 30 | .62885      | 43 | 27 | 31    |
| 28 | .45688     | 31 | .60339      | 43 | .47520     | 30 | .62928      | 43 | 28 | 3.1   |
| 29 | .45719     | 31 | .60383      | 43 | .47550     | 30 | .62971      | 43 | 29 | 3.6   |
| 30 | 9.45750    | 31 | 9.60426     | 43 | 9.47580    | 30 | 9.63014     | 43 | 30 | 4.1   |
| 31 | .45781     | 30 | .60469      | 43 | .47610     | 30 | .63057      | 43 | 31 | 4.6   |
| 32 | .45812     | 31 | .60512      | 43 | .47640     | 30 | .63100      | 43 | 32 | 5.1   |
| 33 | .45843     | 31 | .60556      | 43 | .47670     | 30 | .63143      | 43 | 33 | 10.3  |
| 34 | .45873     | 30 | .60599      | 43 | .47700     | 30 | .63186      | 43 | 34 | 15.5  |
| 35 | 9.45904    | 31 | 9.60642     | 43 | 9.47731    | 30 | 9.63229     | 43 | 35 | 20.6  |
| 36 | .45935     | 31 | .60685      | 43 | .47761     | 30 | .63272      | 43 | 36 | 25.8  |
| 37 | .45966     | 31 | .60729      | 43 | .47791     | 30 | .63315      | 43 | 37 |       |
| 38 | .45997     | 31 | .60772      | 43 | .47821     | 30 | .63358      | 43 | 38 | 30    |
| 39 | .46027     | 30 | .60815      | 43 | .47851     | 30 | .63401      | 43 | 39 | 3.0   |
| 40 | 9.46058    | 31 | 9.60858     | 43 | 9.47881    | 30 | 9.63443     | 42 | 40 | 3.5   |
| 41 | .46089     | 30 | .60902      | 43 | .47911     | 30 | .63486      | 43 | 41 | 4.0   |
| 42 | .46120     | 31 | .60945      | 43 | .47941     | 30 | .63529      | 43 | 42 | 4.5   |
| 43 | .46150     | 30 | .60988      | 43 | .47971     | 30 | .63572      | 43 | 43 | 5.0   |
| 44 | .46181     | 31 | .61031      | 43 | .48001     | 30 | .63615      | 43 | 44 | 10.0  |
| 45 | 9.46212    | 30 | 9.61075     | 43 | 9.48031    | 30 | 9.63658     | 43 | 45 | 15.0  |
| 46 | .46242     | 30 | .61118      | 43 | .48061     | 30 | .63701      | 42 | 46 | 20.0  |
| 47 | .46273     | 31 | .61161      | 43 | .48090     | 29 | .63744      | 43 | 47 | 25.4  |
| 48 | .46304     | 30 | .61204      | 43 | .48120     | 30 | .63787      | 43 | 48 |       |
| 49 | .46334     | 30 | .61247      | 43 | .48150     | 30 | .63830      | 43 | 49 | 29    |
| 50 | 9.46365    | 31 | 9.61291     | 43 | 9.48180    | 30 | 9.63873     | 43 | 50 | 2.9   |
| 51 | .46396     | 30 | .61334      | 43 | .48210     | 30 | .63915      | 42 | 51 | 3.4   |
| 52 | .46426     | 30 | .61377      | 43 | .48240     | 29 | .63958      | 43 | 52 | 3.9   |
| 53 | .46457     | 30 | .61420      | 43 | .48270     | 30 | .64001      | 43 | 53 | 4.4   |
| 54 | .46487     | 30 | .61463      | 43 | .48300     | 30 | .64044      | 43 | 54 | 4.9   |
| 55 | 9.46518    | 31 | 9.61506     | 43 | 9.48329    | 29 | 9.64087     | 42 | 55 | 9.8   |
| 56 | .46549     | 30 | .61550      | 43 | .48359     | 30 | .64130      | 43 | 56 | 14.7  |
| 57 | .46579     | 30 | .61593      | 43 | .48389     | 30 | .64173      | 43 | 57 | 19.6  |
| 58 | .46610     | 30 | .61636      | 43 | .48419     | 30 | .64216      | 43 | 58 | 24.6  |
| 59 | .46640     | 30 | .61679      | 43 | .48449     | 29 | .64258      | 42 | 59 |       |
| 60 | 9.46671    | 30 | 9.61722     | 43 | 9.48478    | 29 | 9.64301     | 43 | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

46°

47°

| 46°        |         |             |         | 47°        |         |             |         | P. P. |    |  |  |
|------------|---------|-------------|---------|------------|---------|-------------|---------|-------|----|--|--|
| Log. Vers. | D       | Log. Exsec. | D       | Log. Vers. | D       | Log. Exsec. | D       |       |    |  |  |
| 0          | 9.48478 | 30          | 9.64301 | 43         | 9.50243 | 29          | 9.66864 | 42    | 0  |  |  |
| 1          | .48508  | 29          | .64344  | 42         | .50272  | 29          | .66907  | 43    | 1  |  |  |
| 2          | .48538  | 30          | .64387  | 43         | .50301  | 29          | .66950  | 42    | 2  |  |  |
| 3          | .48568  | 29          | .64430  | 43         | .50330  | 29          | .66992  | 42    | 3  |  |  |
| 4          | .48597  | 30          | .64473  | 42         | .50359  | 29          | .67035  | 42    | 4  |  |  |
| 5          | 9.48627 | 29          | 9.64515 | 43         | 9.50388 | 29          | 9.67077 | 42    | 5  |  |  |
| 6          | .48657  | 29          | .64558  | 43         | .50417  | 29          | .67120  | 42    | 6  |  |  |
| 7          | .48686  | 30          | .64601  | 42         | .50446  | 29          | .67162  | 43    | 7  |  |  |
| 8          | .48716  | 29          | .64644  | 43         | .50475  | 29          | .67205  | 42    | 8  |  |  |
| 9          | .48746  | 29          | .64687  | 42         | .50504  | 29          | .67248  | 42    | 9  |  |  |
| 10         | 9.48775 | 30          | 9.64729 | 43         | 9.50533 | 29          | 9.67290 | 42    | 10 |  |  |
| 11         | .48805  | 29          | .64772  | 43         | .50562  | 29          | .67333  | 42    | 11 |  |  |
| 12         | .48835  | 29          | .64815  | 43         | .50591  | 28          | .67375  | 42    | 12 |  |  |
| 13         | .48864  | 29          | .64858  | 42         | .50619  | 29          | .67418  | 42    | 13 |  |  |
| 14         | .48894  | 29          | .64901  | 43         | .50648  | 29          | .67460  | 42    | 14 |  |  |
| 15         | 9.48923 | 30          | 9.64943 | 43         | 9.50677 | 29          | 9.67503 | 43    | 15 |  |  |
| 16         | .48953  | 29          | .64986  | 42         | .50706  | 28          | .67546  | 42    | 16 |  |  |
| 17         | .48983  | 29          | .65029  | 43         | .50735  | 29          | .67588  | 42    | 17 |  |  |
| 18         | .49012  | 29          | .65072  | 42         | .50764  | 29          | .67631  | 42    | 18 |  |  |
| 19         | .49042  | 29          | .65114  | 43         | .50793  | 28          | .67673  | 42    | 19 |  |  |
| 20         | 9.49071 | 30          | 9.65157 | 42         | 9.50821 | 29          | 9.67716 | 42    | 20 |  |  |
| 21         | .49101  | 29          | .65200  | 43         | .50850  | 29          | .67758  | 42    | 21 |  |  |
| 22         | .49130  | 29          | .65243  | 43         | .50879  | 28          | .67801  | 42    | 22 |  |  |
| 23         | .49160  | 29          | .65285  | 43         | .50908  | 29          | .67843  | 42    | 23 |  |  |
| 24         | .49189  | 29          | .65328  | 42         | .50937  | 28          | .67886  | 42    | 24 |  |  |
| 25         | 9.49219 | 30          | 9.65371 | 43         | 9.50965 | 29          | 9.67928 | 42    | 25 |  |  |
| 26         | .49248  | 29          | .65414  | 42         | .50994  | 28          | .67971  | 42    | 26 |  |  |
| 27         | .49278  | 29          | .65456  | 43         | .51023  | 29          | .68013  | 42    | 27 |  |  |
| 28         | .49307  | 29          | .65499  | 42         | .51052  | 28          | .68056  | 42    | 28 |  |  |
| 29         | .49336  | 29          | .65542  | 43         | .51080  | 29          | .68098  | 42    | 29 |  |  |
| 30         | 9.49366 | 30          | 9.65585 | 42         | 9.51109 | 28          | 9.68141 | 42    | 30 |  |  |
| 31         | .49395  | 29          | .65627  | 43         | .51138  | 29          | .68183  | 42    | 31 |  |  |
| 32         | .49425  | 29          | .65670  | 42         | .51167  | 28          | .68226  | 42    | 32 |  |  |
| 33         | .49454  | 29          | .65713  | 42         | .51195  | 28          | .68268  | 42    | 33 |  |  |
| 34         | .49483  | 29          | .65755  | 43         | .51224  | 29          | .68311  | 42    | 34 |  |  |
| 35         | 9.49513 | 30          | 9.65798 | 42         | 9.51253 | 28          | 9.68353 | 42    | 35 |  |  |
| 36         | .49542  | 29          | .65841  | 43         | .51281  | 28          | .68396  | 42    | 36 |  |  |
| 37         | .49571  | 29          | .65884  | 42         | .51310  | 28          | .68438  | 42    | 37 |  |  |
| 38         | .49601  | 29          | .65926  | 42         | .51338  | 29          | .68481  | 42    | 38 |  |  |
| 39         | .49630  | 29          | .65969  | 43         | .51367  | 28          | .68523  | 42    | 39 |  |  |
| 40         | 9.49659 | 30          | 9.66012 | 42         | 9.51396 | 29          | 9.68566 | 42    | 40 |  |  |
| 41         | .49689  | 29          | .66054  | 42         | .51424  | 28          | .68608  | 42    | 41 |  |  |
| 42         | .49718  | 29          | .66097  | 43         | .51453  | 28          | .68651  | 42    | 42 |  |  |
| 43         | .49747  | 29          | .66140  | 42         | .51481  | 28          | .68693  | 42    | 43 |  |  |
| 44         | .49776  | 29          | .66182  | 42         | .51510  | 29          | .68735  | 42    | 44 |  |  |
| 45         | 9.49806 | 30          | 9.66225 | 43         | 9.51539 | 28          | 9.68778 | 42    | 45 |  |  |
| 46         | .49835  | 29          | .66268  | 42         | .51567  | 28          | .68820  | 42    | 46 |  |  |
| 47         | .49864  | 29          | .66310  | 42         | .51596  | 28          | .68863  | 42    | 47 |  |  |
| 48         | .49893  | 29          | .66353  | 43         | .51624  | 28          | .68905  | 42    | 48 |  |  |
| 49         | .49922  | 29          | .66396  | 42         | .51653  | 28          | .68948  | 42    | 49 |  |  |
| 50         | 9.49952 | 30          | 9.66438 | 42         | 9.51681 | 28          | 9.68990 | 42    | 50 |  |  |
| 51         | .49981  | 29          | .66481  | 42         | .51710  | 28          | .69033  | 42    | 51 |  |  |
| 52         | .50010  | 29          | .66523  | 42         | .51738  | 28          | .69075  | 42    | 52 |  |  |
| 53         | .50039  | 29          | .66566  | 43         | .51767  | 28          | .69117  | 42    | 53 |  |  |
| 54         | .50068  | 29          | .66609  | 42         | .51795  | 28          | .69160  | 42    | 54 |  |  |
| 55         | 9.50097 | 30          | 9.66651 | 42         | 9.51823 | 28          | 9.69202 | 42    | 55 |  |  |
| 56         | .50126  | 29          | .66694  | 43         | .51852  | 28          | .69245  | 42    | 56 |  |  |
| 57         | .50155  | 29          | .66737  | 42         | .51880  | 28          | .69287  | 42    | 57 |  |  |
| 58         | .50185  | 29          | .66779  | 42         | .51909  | 28          | .69330  | 42    | 58 |  |  |
| 59         | .50214  | 29          | .66822  | 42         | .51937  | 28          | .69372  | 42    | 59 |  |  |
| 60         | 9.50243 | 30          | 9.66864 | 42         | 9.51965 | 28          | 9.69414 | 42    | 60 |  |  |
| Log. Vers. | D       | Log. Exsec. | D       | Log. Vers. | D       | Log. Exsec. | D       | P. P. |    |  |  |

48°

49°

| 48° |            |    |             | 49° |            |    |             |    |    |  |  | P. P. |  |
|-----|------------|----|-------------|-----|------------|----|-------------|----|----|--|--|-------|--|
| '   | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D  | Log. Exsec. | D  | '  |  |  |       |  |
| 0   | 9.51965    | 28 | 9.69414     | 42  | 9.53648    | 27 | 9.71954     | 42 | 0  |  |  |       |  |
| 1   | .51994     | 28 | .69457      | 42  | .53676     | 27 | .71996      | 42 | 1  |  |  |       |  |
| 2   | .52022     | 28 | .69499      | 42  | .53704     | 27 | .72038      | 42 | 2  |  |  |       |  |
| 3   | .52050     | 28 | .69542      | 42  | .53731     | 27 | .72081      | 42 | 3  |  |  |       |  |
| 4   | .52079     | 28 | .69584      | 42  | .53759     | 27 | .72123      | 42 | 4  |  |  |       |  |
| 5   | 9.52107    | 28 | 9.69626     | 42  | 9.53787    | 27 | 9.72165     | 42 | 5  |  |  |       |  |
| 6   | .52135     | 28 | .69669      | 42  | .53814     | 27 | .72207      | 42 | 6  |  |  |       |  |
| 7   | .52164     | 28 | .69711      | 42  | .53842     | 27 | .72250      | 42 | 7  |  |  |       |  |
| 8   | .52192     | 28 | .69753      | 42  | .53870     | 27 | .72292      | 42 | 8  |  |  |       |  |
| 9   | .52220     | 28 | .69796      | 42  | .53897     | 27 | .72334      | 42 | 9  |  |  |       |  |
| 10  | 9.52249    | 28 | 9.69838     | 42  | 9.53925    | 27 | 9.72376     | 42 | 10 |  |  |       |  |
| 11  | .52277     | 28 | .69881      | 42  | .53952     | 27 | .72419      | 42 | 11 |  |  |       |  |
| 12  | .52305     | 28 | .69923      | 42  | .53980     | 27 | .72461      | 42 | 12 |  |  |       |  |
| 13  | .52333     | 28 | .69965      | 42  | .54008     | 27 | .72503      | 42 | 13 |  |  |       |  |
| 14  | .52362     | 28 | .70008      | 42  | .54035     | 27 | .72545      | 42 | 14 |  |  |       |  |
| 15  | 9.52390    | 28 | 9.70050     | 42  | 9.54063    | 27 | 9.72587     | 42 | 15 |  |  |       |  |
| 16  | .52418     | 28 | .70092      | 42  | .54090     | 27 | .72630      | 42 | 16 |  |  |       |  |
| 17  | .52446     | 28 | .70135      | 42  | .54118     | 27 | .72672      | 42 | 17 |  |  |       |  |
| 18  | .52474     | 28 | .70177      | 42  | .54145     | 27 | .72714      | 42 | 18 |  |  |       |  |
| 19  | .52503     | 28 | .70220      | 42  | .54173     | 27 | .72756      | 42 | 19 |  |  |       |  |
| 20  | 9.52531    | 28 | 9.70262     | 42  | 9.54200    | 27 | 9.72799     | 42 | 20 |  |  |       |  |
| 21  | .52559     | 28 | .70304      | 42  | .54228     | 27 | .72841      | 42 | 21 |  |  |       |  |
| 22  | .52587     | 28 | .70347      | 42  | .54255     | 27 | .72883      | 42 | 22 |  |  |       |  |
| 23  | .52615     | 28 | .70389      | 42  | .54283     | 27 | .72925      | 42 | 23 |  |  |       |  |
| 24  | .52643     | 28 | .70431      | 42  | .54310     | 27 | .72967      | 42 | 24 |  |  |       |  |
| 25  | 9.52671    | 28 | 9.70474     | 42  | 9.54338    | 27 | 9.73010     | 42 | 25 |  |  |       |  |
| 26  | .52699     | 28 | .70516      | 42  | .54365     | 27 | .73052      | 42 | 26 |  |  |       |  |
| 27  | .52727     | 28 | .70558      | 42  | .54393     | 27 | .73094      | 42 | 27 |  |  |       |  |
| 28  | .52756     | 28 | .70601      | 42  | .54420     | 27 | .73136      | 42 | 28 |  |  |       |  |
| 29  | .52784     | 28 | .70643      | 42  | .54448     | 27 | .73178      | 42 | 29 |  |  |       |  |
| 30  | 9.52812    | 28 | 9.70685     | 42  | 9.54475    | 27 | 9.73221     | 42 | 30 |  |  |       |  |
| 31  | .52840     | 28 | .70728      | 42  | .54502     | 27 | .73263      | 42 | 31 |  |  |       |  |
| 32  | .52868     | 28 | .70770      | 42  | .54530     | 27 | .73305      | 42 | 32 |  |  |       |  |
| 33  | .52896     | 28 | .70812      | 42  | .54557     | 27 | .73347      | 42 | 33 |  |  |       |  |
| 34  | .52924     | 28 | .70854      | 42  | .54585     | 27 | .73389      | 42 | 34 |  |  |       |  |
| 35  | 9.52952    | 28 | 9.70897     | 42  | 9.54612    | 27 | 9.73431     | 42 | 35 |  |  |       |  |
| 36  | .52980     | 28 | .70939      | 42  | .54639     | 27 | .73474      | 42 | 36 |  |  |       |  |
| 37  | .53008     | 28 | .70981      | 42  | .54667     | 27 | .73516      | 42 | 37 |  |  |       |  |
| 38  | .53036     | 28 | .71024      | 42  | .54694     | 27 | .73558      | 42 | 38 |  |  |       |  |
| 39  | .53064     | 28 | .71066      | 42  | .54721     | 27 | .73600      | 42 | 39 |  |  |       |  |
| 40  | 9.53092    | 28 | 9.71108     | 42  | 9.54748    | 27 | 9.73642     | 42 | 40 |  |  |       |  |
| 41  | .53120     | 27 | .71151      | 42  | .54776     | 27 | .73685      | 42 | 41 |  |  |       |  |
| 42  | .53147     | 28 | .71193      | 42  | .54803     | 27 | .73727      | 42 | 42 |  |  |       |  |
| 43  | .53175     | 28 | .71235      | 42  | .54830     | 27 | .73769      | 42 | 43 |  |  |       |  |
| 44  | .53203     | 28 | .71278      | 42  | .54858     | 27 | .73811      | 42 | 44 |  |  |       |  |
| 45  | 9.53231    | 27 | 9.71320     | 42  | 9.54885    | 27 | 9.73853     | 42 | 45 |  |  |       |  |
| 46  | .53259     | 28 | .71362      | 42  | .54912     | 27 | .73895      | 42 | 46 |  |  |       |  |
| 47  | .53287     | 28 | .71404      | 42  | .54939     | 27 | .73938      | 42 | 47 |  |  |       |  |
| 48  | .53315     | 28 | .71447      | 42  | .54967     | 27 | .73980      | 42 | 48 |  |  |       |  |
| 49  | .53343     | 27 | .71489      | 42  | .54994     | 27 | .74022      | 42 | 49 |  |  |       |  |
| 50  | 9.53370    | 28 | 9.71531     | 42  | 9.55021    | 27 | 9.74064     | 42 | 50 |  |  |       |  |
| 51  | .53398     | 28 | .71573      | 42  | .55048     | 27 | .74106      | 42 | 51 |  |  |       |  |
| 52  | .53426     | 27 | .71616      | 42  | .55075     | 27 | .74148      | 42 | 52 |  |  |       |  |
| 53  | .53454     | 28 | .71658      | 42  | .55103     | 27 | .74191      | 42 | 53 |  |  |       |  |
| 54  | .53482     | 27 | .71700      | 42  | .55130     | 27 | .74233      | 42 | 54 |  |  |       |  |
| 55  | 9.53509    | 28 | 9.71743     | 42  | 9.55157    | 27 | 9.74275     | 42 | 55 |  |  |       |  |
| 56  | .53537     | 27 | .71785      | 42  | .55184     | 27 | .74317      | 42 | 56 |  |  |       |  |
| 57  | .53565     | 28 | .71827      | 42  | .55211     | 27 | .74359      | 42 | 57 |  |  |       |  |
| 58  | .53593     | 27 | .71869      | 42  | .55238     | 27 | .74401      | 42 | 58 |  |  |       |  |
| 59  | .53620     | 28 | .71912      | 42  | .55265     | 27 | .74444      | 42 | 59 |  |  |       |  |
| 60  | 9.53648    |    | 9.71954     |     | 9.55292    |    | 9.74486     |    | 60 |  |  |       |  |
| '   | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D  | Log. Exsec. | D  | '  |  |  | P. P. |  |

|    |      |      |    |
|----|------|------|----|
|    |      | 42   | 42 |
| 6  | 4.2  | 4.2  |    |
| 7  | 4.9  | 4.9  |    |
| 8  | 5.6  | 5.6  |    |
| 9  | 6.4  | 6.3  |    |
| 10 | 7.1  | 7.0  |    |
| 20 | 14.1 | 14.0 |    |
| 30 | 21.2 | 21.0 |    |
| 40 | 28.3 | 28.0 |    |
| 50 | 35.4 | 35.0 |    |

|    |      |      |    |
|----|------|------|----|
|    |      | 28   | 28 |
| 6  | 2.8  | 2.8  |    |
| 7  | 3.3  | 3.2  |    |
| 8  | 3.8  | 3.7  |    |
| 9  | 4.3  | 4.2  |    |
| 10 | 4.7  | 4.6  |    |
| 20 | 9.5  | 9.3  |    |
| 30 | 14.2 | 14.0 |    |
| 40 | 19.0 | 18.6 |    |
| 50 | 23.7 | 23.3 |    |

|    |      |      |    |
|----|------|------|----|
|    |      | 27   | 27 |
| 6  | 2.7  | 2.7  |    |
| 7  | 3.2  | 3.1  |    |
| 8  | 3.6  | 3.6  |    |
| 9  | 4.1  | 4.0  |    |
| 10 | 4.6  | 4.5  |    |
| 20 | 9.1  | 9.0  |    |
| 30 | 13.7 | 13.5 |    |
| 40 | 18.3 | 18.0 |    |
| 50 | 22.9 | 22.5 |    |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    |            |    |             |    |            |    |             |    | P. P. |       |    |
|----|------------|----|-------------|----|------------|----|-------------|----|-------|-------|----|
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |       |       |    |
| 0  | 9.55292    | 27 | 9.74486     | 42 | 9.56900    | 26 | 9.77012     | 42 | 0     |       |    |
| 1  | .55319     | 27 | .74528      | 42 | .56926     | 26 | .77055      | 42 | 1     |       |    |
| 2  | .55347     | 27 | .74570      | 42 | .56953     | 26 | .77097      | 42 | 2     |       |    |
| 3  | .55374     | 27 | .74612      | 42 | .56979     | 26 | .77139      | 42 | 3     |       |    |
| 4  | .55401     | 27 | .74654      | 42 | .57005     | 26 | .77181      | 42 | 4     |       |    |
| 5  | 9.55428    | 27 | 9.74696     | 42 | 9.57032    | 26 | 9.77223     | 42 | 5     |       |    |
| 6  | .55455     | 27 | .74739      | 42 | .57058     | 26 | .77265      | 42 | 6     |       |    |
| 7  | .55482     | 27 | .74781      | 42 | .57085     | 26 | .77307      | 42 | 7     |       |    |
| 8  | .55509     | 27 | .74823      | 42 | .57111     | 26 | .77349      | 42 | 8     |       |    |
| 9  | .55536     | 27 | .74865      | 42 | .57138     | 26 | .77391      | 42 | 9     | 6     | 42 |
| 10 | 9.55563    | 27 | 9.74907     | 42 | 9.57164    | 26 | 9.77433     | 42 | 10    | 7     | 42 |
| 11 | .55590     | 27 | .74949      | 42 | .57190     | 26 | .77475      | 42 | 11    | 8     | 42 |
| 12 | .55617     | 27 | .74991      | 42 | .57217     | 26 | .77517      | 42 | 12    | 9     | 42 |
| 13 | .55644     | 27 | .75033      | 42 | .57243     | 26 | .77560      | 42 | 13    | 10    | 42 |
| 14 | .55671     | 27 | .75076      | 42 | .57269     | 26 | .77602      | 42 | 14    | 20    | 42 |
| 15 | 9.55698    | 27 | 9.75118     | 42 | 9.57296    | 26 | 9.77644     | 42 | 15    | 30    | 42 |
| 16 | .55725     | 26 | .75160      | 42 | .57322     | 26 | .77686      | 42 | 16    | 40    | 42 |
| 17 | .55751     | 27 | .75202      | 42 | .57348     | 26 | .77728      | 42 | 17    | 50    | 42 |
| 18 | .55778     | 27 | .75244      | 42 | .57375     | 26 | .77770      | 42 | 18    |       |    |
| 19 | .55805     | 27 | .75286      | 42 | .57401     | 26 | .77812      | 42 | 19    |       |    |
| 20 | 9.55832    | 27 | 9.75328     | 42 | 9.57427    | 26 | 9.77854     | 42 | 20    |       |    |
| 21 | .55859     | 26 | .75370      | 42 | .57454     | 26 | .77896      | 42 | 21    | 6     | 27 |
| 22 | .55886     | 27 | .75413      | 42 | .57480     | 26 | .77938      | 42 | 22    | 7     | 27 |
| 23 | .55913     | 27 | .75455      | 42 | .57506     | 26 | .77980      | 42 | 23    | 8     | 27 |
| 24 | .55940     | 26 | .75497      | 42 | .57532     | 26 | .78022      | 42 | 24    | 9     | 27 |
| 25 | 9.55966    | 27 | 9.75539     | 42 | 9.57559    | 26 | 9.78064     | 42 | 25    | 10    | 27 |
| 26 | .55993     | 27 | .75581      | 42 | .57585     | 26 | .78107      | 42 | 26    | 20    | 27 |
| 27 | .56020     | 26 | .75623      | 42 | .57611     | 26 | .78149      | 42 | 27    | 30    | 27 |
| 28 | .56047     | 27 | .75665      | 42 | .57637     | 26 | .78191      | 42 | 28    | 40    | 27 |
| 29 | .56074     | 27 | .75707      | 42 | .57664     | 26 | .78233      | 42 | 29    | 50    | 27 |
| 30 | 9.56101    | 26 | 9.75750     | 42 | 9.57690    | 26 | 9.78275     | 42 | 30    |       |    |
| 31 | .56127     | 27 | .75792      | 42 | .57716     | 26 | .78317      | 42 | 31    |       |    |
| 32 | .56154     | 26 | .75834      | 42 | .57742     | 26 | .78359      | 42 | 32    |       |    |
| 33 | .56181     | 27 | .75876      | 42 | .57768     | 26 | .78401      | 42 | 33    |       |    |
| 34 | .56208     | 26 | .75918      | 42 | .57794     | 26 | .78443      | 42 | 34    | 6     | 26 |
| 35 | 9.56234    | 27 | 9.75960     | 42 | 9.57821    | 26 | 9.78485     | 42 | 35    | 7     | 26 |
| 36 | .56261     | 26 | .76002      | 42 | .57847     | 26 | .78527      | 42 | 36    | 8     | 26 |
| 37 | .56288     | 27 | .76044      | 42 | .57873     | 26 | .78569      | 42 | 37    | 9     | 26 |
| 38 | .56315     | 27 | .76086      | 42 | .57899     | 26 | .78611      | 42 | 38    | 10    | 26 |
| 39 | .56341     | 26 | .76128      | 42 | .57925     | 26 | .78653      | 42 | 39    | 20    | 26 |
| 40 | 9.56368    | 27 | 9.76171     | 42 | 9.57951    | 26 | 9.78696     | 42 | 40    | 30    | 26 |
| 41 | .56395     | 26 | .76213      | 42 | .57977     | 26 | .78738      | 42 | 41    | 40    | 26 |
| 42 | .56421     | 27 | .76255      | 42 | .58003     | 26 | .78780      | 42 | 42    | 50    | 26 |
| 43 | .56448     | 26 | .76297      | 42 | .58029     | 26 | .78822      | 42 | 43    |       |    |
| 44 | .56475     | 26 | .76339      | 42 | .58055     | 26 | .78864      | 42 | 44    |       |    |
| 45 | 9.56501    | 26 | 9.76381     | 42 | 9.58082    | 26 | 9.78906     | 42 | 45    |       |    |
| 46 | .56528     | 26 | .76423      | 42 | .58108     | 26 | .78948      | 42 | 46    |       |    |
| 47 | .56554     | 27 | .76465      | 42 | .58134     | 26 | .78990      | 42 | 47    | 6     | 25 |
| 48 | .56581     | 26 | .76507      | 42 | .58160     | 26 | .79032      | 42 | 48    | 7     | 25 |
| 49 | .56608     | 26 | .76549      | 42 | .58186     | 26 | .79074      | 42 | 49    | 8     | 25 |
| 50 | 9.56634    | 26 | 9.76592     | 42 | 9.58212    | 26 | 9.79116     | 42 | 50    | 9     | 25 |
| 51 | .56661     | 26 | .76634      | 42 | .58238     | 26 | .79158      | 42 | 51    | 10    | 25 |
| 52 | .56687     | 26 | .76676      | 42 | .58264     | 26 | .79200      | 42 | 52    | 20    | 25 |
| 53 | .56714     | 27 | .76718      | 42 | .58290     | 26 | .79242      | 42 | 53    | 30    | 25 |
| 54 | .56741     | 26 | .76760      | 42 | .58316     | 26 | .79285      | 42 | 54    | 40    | 25 |
| 55 | 9.56767    | 26 | 9.76802     | 42 | 9.58342    | 26 | 9.79327     | 42 | 55    | 50    | 25 |
| 56 | .56794     | 26 | .76844      | 42 | .58367     | 26 | .79369      | 42 | 56    |       |    |
| 57 | .56820     | 26 | .76886      | 42 | .58393     | 26 | .79411      | 42 | 57    |       |    |
| 58 | .56847     | 26 | .76928      | 42 | .58419     | 26 | .79453      | 42 | 58    |       |    |
| 59 | .56873     | 26 | .76970      | 42 | .58445     | 26 | .79495      | 42 | 59    |       |    |
| 60 | 9.56900    |    | 9.77012     |    | 9.58471    |    | 9.79537     |    | 60    |       |    |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |       | P. P. |    |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.58471    | 26 | 9.79537     | 42 | 9.60008    | 25 | 9.82062     | 42 | 0  |       |
| 1  | .58497     | 25 | .79579      | 42 | .60034     | 25 | .82104      | 42 | 1  |       |
| 2  | .58523     | 26 | .79621      | 42 | .60059     | 25 | .82146      | 42 | 2  |       |
| 3  | .58549     | 26 | .79663      | 42 | .60084     | 25 | .82188      | 42 | 3  |       |
| 4  | .58575     | 26 | .79705      | 42 | .60110     | 25 | .82230      | 42 | 4  |       |
| 5  | 9.58601    | 25 | 9.79747     | 42 | 9.60135    | 25 | 9.82272     | 42 | 5  |       |
| 6  | .58626     | 26 | .79789      | 42 | .60160     | 25 | .82315      | 42 | 6  |       |
| 7  | .58652     | 26 | .79831      | 42 | .60185     | 25 | .82357      | 42 | 7  |       |
| 8  | .58678     | 25 | .79874      | 42 | .60211     | 25 | .82399      | 42 | 8  |       |
| 9  | .58704     | 26 | .79916      | 42 | .60236     | 25 | .82441      | 42 | 9  |       |
| 10 | 9.58730    | 25 | 9.79958     | 42 | 9.60261    | 25 | 9.82483     | 42 | 10 |       |
| 11 | .58755     | 26 | .80000      | 42 | .60286     | 25 | .82525      | 42 | 11 |       |
| 12 | .58781     | 26 | .80042      | 42 | .60312     | 25 | .82567      | 42 | 12 |       |
| 13 | .58807     | 25 | .80084      | 42 | .60337     | 25 | .82609      | 42 | 13 |       |
| 14 | .58833     | 26 | .80126      | 42 | .60362     | 25 | .82651      | 42 | 14 |       |
| 15 | 9.58859    | 25 | 9.80168     | 42 | 9.60387    | 25 | 9.82694     | 42 | 15 | 42    |
| 16 | .58884     | 26 | .80210      | 42 | .60412     | 25 | .82736      | 42 | 16 | 4.2   |
| 17 | .58910     | 25 | .80252      | 42 | .60438     | 25 | .82778      | 42 | 17 | 4.9   |
| 18 | .58936     | 26 | .80294      | 42 | .60463     | 25 | .82820      | 42 | 18 | 5.6   |
| 19 | .58962     | 25 | .80336      | 42 | .60488     | 25 | .82862      | 42 | 19 | 6.4   |
| 20 | 9.58987    | 26 | 9.80378     | 42 | 9.60513    | 25 | 9.82904     | 42 | 20 | 7.1   |
| 21 | .59013     | 25 | .80420      | 42 | .60538     | 25 | .82946      | 42 | 21 | 14.1  |
| 22 | .59039     | 26 | .80463      | 42 | .60563     | 25 | .82988      | 42 | 22 | 14.0  |
| 23 | .59064     | 25 | .80505      | 42 | .60589     | 25 | .83031      | 42 | 23 | 21.2  |
| 24 | .59090     | 26 | .80547      | 42 | .60614     | 25 | .83073      | 42 | 24 | 28.3  |
| 25 | 9.59116    | 25 | 9.80589     | 42 | 9.60639    | 25 | 9.83115     | 42 | 25 | 35.4  |
| 26 | .59141     | 26 | .80631      | 42 | .60664     | 25 | .83157      | 42 | 26 |       |
| 27 | .59167     | 25 | .80673      | 42 | .60689     | 25 | .83199      | 42 | 27 | 2.6   |
| 28 | .59193     | 26 | .80715      | 42 | .60714     | 25 | .83241      | 42 | 28 | 3.0   |
| 29 | .59218     | 25 | .80757      | 42 | .60739     | 25 | .83283      | 42 | 29 | 3.4   |
| 30 | 9.59244    | 26 | 9.80799     | 42 | 9.60764    | 25 | 9.83325     | 42 | 30 | 3.9   |
| 31 | .59270     | 25 | .80841      | 42 | .60789     | 25 | .83368      | 42 | 31 | 4.2   |
| 32 | .59295     | 26 | .80883      | 42 | .60814     | 25 | .83410      | 42 | 32 | 8.6   |
| 33 | .59321     | 25 | .80925      | 42 | .60839     | 25 | .83452      | 42 | 33 | 13.0  |
| 34 | .59346     | 26 | .80968      | 42 | .60864     | 25 | .83494      | 42 | 34 | 17.0  |
| 35 | 9.59372    | 25 | 9.81010     | 42 | 9.60889    | 25 | 9.83536     | 42 | 35 | 21.2  |
| 36 | .59397     | 26 | .81052      | 42 | .60914     | 25 | .83578      | 42 | 36 |       |
| 37 | .59423     | 25 | .81094      | 42 | .60939     | 25 | .83620      | 42 | 37 | 2.5   |
| 38 | .59449     | 26 | .81136      | 42 | .60964     | 25 | .83663      | 42 | 38 | 3.0   |
| 39 | .59474     | 25 | .81178      | 42 | .60989     | 25 | .83705      | 42 | 39 | 3.4   |
| 40 | 9.59500    | 26 | 9.81220     | 42 | 9.61014    | 25 | 9.83747     | 42 | 40 | 3.9   |
| 41 | .59525     | 25 | .81262      | 42 | .61039     | 25 | .83789      | 42 | 41 | 4.2   |
| 42 | .59551     | 26 | .81304      | 42 | .61064     | 25 | .83831      | 42 | 42 | 8.1   |
| 43 | .59576     | 25 | .81346      | 42 | .61089     | 25 | .83873      | 42 | 43 | 12.2  |
| 44 | .59602     | 26 | .81388      | 42 | .61114     | 25 | .83916      | 42 | 44 | 16.3  |
| 45 | 9.59627    | 25 | 9.81430     | 42 | 9.61139    | 25 | 9.83958     | 42 | 45 | 20.4  |
| 46 | .59653     | 26 | .81473      | 42 | .61164     | 25 | .84000      | 42 | 46 |       |
| 47 | .59678     | 25 | .81515      | 42 | .61189     | 24 | .84042      | 42 | 47 | 2.4   |
| 48 | .59704     | 26 | .81557      | 42 | .61214     | 25 | .84084      | 42 | 48 | 2.8   |
| 49 | .59729     | 25 | .81599      | 42 | .61239     | 25 | .84126      | 42 | 49 | 3.2   |
| 50 | 9.59754    | 26 | 9.81641     | 42 | 9.61264    | 25 | 9.84168     | 42 | 50 | 3.7   |
| 51 | .59780     | 25 | .81683      | 42 | .61289     | 24 | .84211      | 42 | 51 | 4.1   |
| 52 | .59805     | 26 | .81725      | 42 | .61313     | 25 | .84253      | 42 | 52 | 8.3   |
| 53 | .59831     | 25 | .81767      | 42 | .61338     | 25 | .84295      | 42 | 53 | 12.5  |
| 54 | .59856     | 26 | .81809      | 42 | .61363     | 25 | .84337      | 42 | 54 | 16.6  |
| 55 | 9.59881    | 25 | 9.81851     | 42 | 9.61388    | 24 | 9.84379     | 42 | 55 | 20.8  |
| 56 | .59907     | 26 | .81894      | 42 | .61413     | 25 | .84422      | 42 | 56 |       |
| 57 | .59932     | 25 | .81936      | 42 | .61438     | 24 | .84464      | 42 | 57 | 2.4   |
| 58 | .59958     | 26 | .81978      | 42 | .61462     | 25 | .84506      | 42 | 58 | 2.8   |
| 59 | .59983     | 25 | .82020      | 42 | .61487     | 25 | .84548      | 42 | 59 | 3.2   |
| 60 | 9.60008    | 26 | 9.82062     | 42 | 9.61512    | 25 | 9.84590     | 42 | 60 | 3.7   |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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55°

|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.61512    | 24 | 9.84590     | 42 | 9.62984    | 24 | 9.87125     | 42 | 0  |       |
| 1  | .61537     | 25 | .84632      | 42 | .63008     | 24 | .87167      | 42 | 1  |       |
| 2  | .61562     | 24 | .84675      | 42 | .63032     | 24 | .87209      | 42 | 2  |       |
| 3  | .61586     | 25 | .84717      | 42 | .63057     | 24 | .87252      | 42 | 3  |       |
| 4  | .61611     | 24 | .84759      | 42 | .63081     | 24 | .87294      | 42 | 4  |       |
| 5  | 9.61636    | 24 | 9.84801     | 42 | 9.63103    | 24 | 9.87336     | 42 | 5  |       |
| 6  | .61661     | 25 | .84843      | 42 | .63129     | 24 | .87379      | 42 | 6  |       |
| 7  | .61685     | 24 | .84886      | 42 | .63154     | 24 | .87421      | 42 | 7  |       |
| 8  | .61710     | 25 | .84928      | 42 | .63178     | 24 | .87463      | 42 | 8  |       |
| 9  | .61735     | 24 | .84970      | 42 | .63202     | 24 | .87506      | 42 | 9  |       |
| 10 | 9.61760    | 25 | 9.85012     | 42 | 9.63226    | 24 | 9.87548     | 42 | 10 |       |
| 11 | .61784     | 24 | .85054      | 42 | .63250     | 24 | .87590      | 42 | 11 |       |
| 12 | .61809     | 25 | .85097      | 42 | .63274     | 24 | .87633      | 42 | 12 |       |
| 13 | .61834     | 24 | .85139      | 42 | .63299     | 24 | .87675      | 42 | 13 |       |
| 14 | .61858     | 24 | .85181      | 42 | .63323     | 24 | .87717      | 42 | 14 |       |
| 15 | 9.61883    | 25 | 9.85223     | 42 | 9.63347    | 24 | 9.87760     | 42 | 15 |       |
| 16 | .61908     | 24 | .85265      | 42 | .63371     | 24 | .87802      | 42 | 16 |       |
| 17 | .61932     | 24 | .85308      | 42 | .63395     | 24 | .87844      | 42 | 17 |       |
| 18 | .61957     | 25 | .85350      | 42 | .63419     | 24 | .87887      | 42 | 18 |       |
| 19 | .61982     | 24 | .85392      | 42 | .63443     | 24 | .87929      | 42 | 19 |       |
| 20 | 9.62006    | 24 | 9.85434     | 42 | 9.63468    | 24 | 9.87971     | 42 | 20 |       |
| 21 | .62031     | 24 | .85476      | 42 | .63492     | 24 | .88014      | 42 | 21 |       |
| 22 | .62055     | 25 | .85519      | 42 | .63516     | 24 | .88056      | 42 | 22 |       |
| 23 | .62080     | 24 | .85561      | 42 | .63540     | 24 | .88099      | 42 | 23 |       |
| 24 | .62105     | 24 | .85603      | 42 | .63564     | 24 | .88141      | 42 | 24 |       |
| 25 | 9.62129    | 24 | 9.85645     | 42 | 9.63588    | 24 | 9.88183     | 42 | 25 |       |
| 26 | .62154     | 24 | .85688      | 42 | .63612     | 24 | .88226      | 42 | 26 |       |
| 27 | .62178     | 24 | .85730      | 42 | .63636     | 24 | .88268      | 42 | 27 |       |
| 28 | .62203     | 24 | .85772      | 42 | .63660     | 24 | .88310      | 42 | 28 |       |
| 29 | .62227     | 24 | .85814      | 42 | .63684     | 24 | .88353      | 42 | 29 |       |
| 30 | 9.62252    | 24 | 9.85857     | 42 | 9.63708    | 24 | 9.88395     | 42 | 30 |       |
| 31 | .62276     | 24 | .85899      | 42 | .63732     | 24 | .88438      | 42 | 31 |       |
| 32 | .62301     | 24 | .85941      | 42 | .63756     | 24 | .88480      | 42 | 32 |       |
| 33 | .62325     | 24 | .85983      | 42 | .63780     | 24 | .88522      | 42 | 33 |       |
| 34 | .62350     | 24 | .86026      | 42 | .63804     | 24 | .88565      | 42 | 34 |       |
| 35 | 9.62374    | 24 | 9.86068     | 42 | 9.63828    | 24 | 9.88607     | 42 | 35 |       |
| 36 | .62399     | 24 | .86110      | 42 | .63852     | 24 | .88650      | 42 | 36 |       |
| 37 | .62423     | 24 | .86152      | 42 | .63876     | 24 | .88692      | 42 | 37 |       |
| 38 | .62448     | 24 | .86195      | 42 | .63900     | 24 | .88734      | 42 | 38 |       |
| 39 | .62472     | 24 | .86237      | 42 | .63924     | 24 | .88777      | 42 | 39 |       |
| 40 | 9.62497    | 24 | 9.86279     | 42 | 9.63948    | 24 | 9.88819     | 42 | 40 |       |
| 41 | .62521     | 24 | .86321      | 42 | .63972     | 24 | .88862      | 42 | 41 |       |
| 42 | .62546     | 24 | .86364      | 42 | .63996     | 23 | .88904      | 42 | 42 |       |
| 43 | .62570     | 24 | .86406      | 42 | .64019     | 24 | .88947      | 42 | 43 |       |
| 44 | .62594     | 24 | .86448      | 42 | .64043     | 24 | .88989      | 42 | 44 |       |
| 45 | 9.62619    | 24 | 9.86490     | 42 | 9.64067    | 24 | 9.89031     | 42 | 45 |       |
| 46 | .62643     | 24 | .86533      | 42 | .64091     | 24 | .89074      | 42 | 46 |       |
| 47 | .62668     | 24 | .86575      | 42 | .64115     | 23 | .89116      | 42 | 47 |       |
| 48 | .62692     | 24 | .86617      | 42 | .64139     | 24 | .89159      | 42 | 48 |       |
| 49 | .62716     | 24 | .86659      | 42 | .64163     | 24 | .89201      | 42 | 49 |       |
| 50 | 9.62741    | 24 | 9.86702     | 42 | 9.64187    | 23 | 9.89244     | 42 | 50 |       |
| 51 | .62765     | 24 | .86744      | 42 | .64210     | 24 | .89286      | 42 | 51 |       |
| 52 | .62789     | 24 | .86786      | 42 | .64234     | 24 | .89329      | 42 | 52 |       |
| 53 | .62814     | 24 | .86829      | 42 | .64258     | 23 | .89371      | 42 | 53 |       |
| 54 | .62838     | 24 | .86871      | 42 | .64282     | 23 | .89414      | 42 | 54 |       |
| 55 | 9.62862    | 24 | 9.86913     | 42 | 9.64306    | 24 | 9.89456     | 42 | 55 |       |
| 56 | .62887     | 24 | .86956      | 42 | .64330     | 23 | .89499      | 42 | 56 |       |
| 57 | .62911     | 24 | .86998      | 42 | .64353     | 24 | .89541      | 42 | 57 |       |
| 58 | .62935     | 24 | .87040      | 42 | .64377     | 23 | .89583      | 42 | 58 |       |
| 59 | .62960     | 24 | .87082      | 42 | .64401     | 24 | .89626      | 42 | 59 |       |
| 60 | 9.62984    |    | 9.87125     |    | 9.64425    |    | 9.89668     |    | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

|    |      |      |
|----|------|------|
| 6  | 42   | 42   |
| 7  | 4.2  | 4.2  |
| 8  | 5.6  | 5.6  |
| 9  | 6.4  | 6.4  |
| 10 | 7.1  | 7.0  |
| 20 | 14.1 | 14.0 |
| 30 | 21.2 | 21.0 |
| 40 | 28.3 | 28.0 |
| 50 | 35.4 | 35.0 |

|    |      |      |
|----|------|------|
| 6  | 25   | 24   |
| 7  | 2.5  | 2.4  |
| 8  | 2.9  | 2.8  |
| 9  | 3.3  | 3.2  |
| 10 | 3.7  | 3.7  |
| 20 | 4.1  | 4.1  |
| 30 | 8.3  | 8.1  |
| 40 | 12.5 | 12.2 |
| 50 | 16.6 | 16.3 |

|    |      |      |
|----|------|------|
| 6  | 24   | 23   |
| 7  | 2.4  | 2.3  |
| 8  | 2.8  | 2.7  |
| 9  | 3.2  | 3.1  |
| 10 | 3.6  | 3.5  |
| 20 | 4.0  | 3.9  |
| 30 | 8.0  | 7.8  |
| 40 | 12.0 | 11.7 |
| 50 | 16.0 | 15.6 |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

56°

57°

|    |  | Log. Vers. | D  |    |  | Log. Exsec. | D  |    |  | Log. Vers. | D  |    |  | Log. Exsec. | D  |
|----|--|------------|----|----|--|-------------|----|----|--|------------|----|----|--|-------------|----|
| 0  |  | 9.64425    |    | 23 |  | 9.89668     | 42 | 23 |  | 9.65835    |    | 23 |  | 9.92224     | 43 |
| 1  |  | .64448     | 23 |    |  | .89711      | 42 |    |  | .65859     | 23 |    |  | .92267      | 42 |
| 2  |  | .64472     | 23 |    |  | .89753      | 42 |    |  | .65882     | 23 |    |  | .92310      | 43 |
| 3  |  | .64496     | 24 |    |  | .89796      | 42 |    |  | .65905     | 23 |    |  | .92353      | 42 |
| 4  |  | .64520     | 23 |    |  | .89838      | 42 |    |  | .65928     | 23 |    |  | .92395      | 43 |
| 5  |  | 9.64543    | 24 |    |  | 9.89881     | 42 |    |  | 9.65952    | 23 |    |  | 9.92438     | 42 |
| 6  |  | .64567     | 23 |    |  | .89923      | 42 |    |  | .65975     | 23 |    |  | .92481      | 43 |
| 7  |  | .64591     | 23 |    |  | .89966      | 42 |    |  | .65998     | 23 |    |  | .92524      | 42 |
| 8  |  | .64614     | 24 |    |  | .90008      | 42 |    |  | .66021     | 23 |    |  | .92566      | 43 |
| 9  |  | .64638     | 23 |    |  | .90051      | 42 |    |  | .66044     | 23 |    |  | .92609      | 42 |
| 10 |  | 9.64662    | 23 |    |  | 9.90094     | 43 |    |  | 9.66068    | 23 |    |  | 9.92652     | 43 |
| 11 |  | .64685     | 24 |    |  | .90136      | 42 |    |  | .66091     | 23 |    |  | .92695      | 42 |
| 12 |  | .64709     | 23 |    |  | .90179      | 42 |    |  | .66114     | 23 |    |  | .92737      | 43 |
| 13 |  | .64733     | 23 |    |  | .90221      | 42 |    |  | .66137     | 23 |    |  | .92780      | 42 |
| 14 |  | .64756     | 24 |    |  | .90264      | 42 |    |  | .66160     | 23 |    |  | .92823      | 43 |
| 15 |  | 9.64780    | 23 |    |  | 9.90306     | 42 |    |  | 9.66183    | 23 |    |  | 9.92866     | 43 |
| 16 |  | .64804     | 23 |    |  | .90349      | 42 |    |  | .66207     | 23 |    |  | .92909      | 42 |
| 17 |  | .64827     | 23 |    |  | .90391      | 42 |    |  | .66230     | 23 |    |  | .92951      | 43 |
| 18 |  | .64851     | 24 |    |  | .90434      | 42 |    |  | .66253     | 23 |    |  | .92994      | 42 |
| 19 |  | .64875     | 23 |    |  | .90476      | 42 |    |  | .66276     | 23 |    |  | .93037      | 43 |
| 20 |  | 9.64898    | 23 |    |  | 9.90519     | 42 |    |  | 9.66299    | 23 |    |  | 9.93080     | 43 |
| 21 |  | .64922     | 23 |    |  | .90561      | 42 |    |  | .66322     | 23 |    |  | .93123      | 42 |
| 22 |  | .64945     | 23 |    |  | .90604      | 43 |    |  | .66345     | 23 |    |  | .93165      | 43 |
| 23 |  | .64969     | 23 |    |  | .90647      | 42 |    |  | .66368     | 23 |    |  | .93208      | 43 |
| 24 |  | .64992     | 24 |    |  | .90689      | 42 |    |  | .66391     | 23 |    |  | .93251      | 42 |
| 25 |  | 9.65016    | 23 |    |  | 9.90732     | 42 |    |  | 9.66415    | 23 |    |  | 9.93294     | 43 |
| 26 |  | .65040     | 23 |    |  | .90774      | 42 |    |  | .66438     | 23 |    |  | .93337      | 43 |
| 27 |  | .65063     | 23 |    |  | .90817      | 43 |    |  | .66461     | 23 |    |  | .93380      | 42 |
| 28 |  | .65087     | 23 |    |  | .90860      | 42 |    |  | .66484     | 23 |    |  | .93422      | 43 |
| 29 |  | .65110     | 23 |    |  | .90902      | 42 |    |  | .66507     | 23 |    |  | .93465      | 43 |
| 30 |  | 9.65134    | 23 |    |  | 9.90945     | 42 |    |  | 9.66530    | 23 |    |  | 9.93508     | 42 |
| 31 |  | .65157     | 23 |    |  | .90987      | 42 |    |  | .66553     | 23 |    |  | .93551      | 43 |
| 32 |  | .65181     | 23 |    |  | .91030      | 43 |    |  | .66576     | 23 |    |  | .93594      | 43 |
| 33 |  | .65204     | 23 |    |  | .91073      | 42 |    |  | .66599     | 23 |    |  | .93637      | 43 |
| 34 |  | .65228     | 23 |    |  | .91115      | 42 |    |  | .66622     | 23 |    |  | .93680      | 42 |
| 35 |  | 9.65251    | 23 |    |  | 9.91158     | 42 |    |  | 9.66645    | 23 |    |  | 9.93722     | 43 |
| 36 |  | .65275     | 23 |    |  | .91200      | 42 |    |  | .66668     | 23 |    |  | .93765      | 43 |
| 37 |  | .65298     | 23 |    |  | .91243      | 42 |    |  | .66691     | 23 |    |  | .93808      | 43 |
| 38 |  | .65321     | 23 |    |  | .91286      | 43 |    |  | .66714     | 23 |    |  | .93851      | 43 |
| 39 |  | .65345     | 23 |    |  | .91328      | 42 |    |  | .66737     | 23 |    |  | .93894      | 42 |
| 40 |  | 9.65368    | 23 |    |  | 9.91371     | 43 |    |  | 9.66760    | 23 |    |  | 9.93937     | 43 |
| 41 |  | .65392     | 23 |    |  | .91414      | 42 |    |  | .66783     | 23 |    |  | .93980      | 43 |
| 42 |  | .65415     | 23 |    |  | .91456      | 42 |    |  | .66805     | 23 |    |  | .94023      | 43 |
| 43 |  | .65439     | 23 |    |  | .91499      | 42 |    |  | .66828     | 23 |    |  | .94066      | 43 |
| 44 |  | .65462     | 23 |    |  | .91541      | 43 |    |  | .66851     | 23 |    |  | .94109      | 42 |
| 45 |  | 9.65485    | 23 |    |  | 9.91584     | 42 |    |  | 9.66874    | 23 |    |  | 9.94151     | 43 |
| 46 |  | .65509     | 23 |    |  | .91627      | 42 |    |  | .66897     | 23 |    |  | .94194      | 43 |
| 47 |  | .65532     | 23 |    |  | .91669      | 43 |    |  | .66920     | 22 |    |  | .94237      | 43 |
| 48 |  | .65556     | 23 |    |  | .91712      | 42 |    |  | .66943     | 23 |    |  | .94280      | 43 |
| 49 |  | .65579     | 23 |    |  | .91755      | 42 |    |  | .66966     | 23 |    |  | .94323      | 43 |
| 50 |  | 9.65602    | 23 |    |  | 9.91797     | 42 |    |  | 9.66989    | 23 |    |  | 9.94366     | 43 |
| 51 |  | .65626     | 23 |    |  | .91840      | 43 |    |  | .67012     | 22 |    |  | .94409      | 43 |
| 52 |  | .65649     | 23 |    |  | .91883      | 42 |    |  | .67034     | 23 |    |  | .94452      | 43 |
| 53 |  | .65672     | 23 |    |  | .91926      | 43 |    |  | .67057     | 23 |    |  | .94495      | 43 |
| 54 |  | .65696     | 23 |    |  | .91968      | 42 |    |  | .67080     | 23 |    |  | .94538      | 43 |
| 55 |  | 9.65719    | 23 |    |  | 9.92011     | 42 |    |  | 9.67103    | 22 |    |  | 9.94581     | 43 |
| 56 |  | .65742     | 23 |    |  | .92054      | 43 |    |  | .67126     | 23 |    |  | .94624      | 43 |
| 57 |  | .65765     | 23 |    |  | .92096      | 42 |    |  | .67149     | 23 |    |  | .94667      | 43 |
| 58 |  | .65789     | 23 |    |  | .92139      | 43 |    |  | .67171     | 22 |    |  | .94710      | 43 |
| 59 |  | .65812     | 23 |    |  | .92182      | 42 |    |  | .67194     | 23 |    |  | .94753      | 43 |
| 60 |  | 9.65835    | 23 |    |  | 9.92224     | 42 |    |  | 9.67217    | 22 |    |  | 9.94796     | 43 |
|    |  | Log. Vers. | D  |    |  | Log. Exsec. | D  |    |  | Log. Vers. | D  |    |  | Log. Exsec. | D  |

| P. P. |      |      |
|-------|------|------|
| 43    | 42   |      |
| 6     | 4.3  | 4.2  |
| 7     | 5.0  | 4.9  |
| 8     | 5.7  | 5.6  |
| 9     | 6.4  | 6.4  |
| 10    | 7.1  | 7.1  |
| 20    | 14.3 | 14.1 |
| 30    | 21.5 | 21.2 |
| 40    | 28.6 | 28.3 |
| 50    | 35.8 | 35.4 |

| P. P. |      |      |
|-------|------|------|
| 24    | 23   |      |
| 6     | 2.4  | 2.3  |
| 7     | 2.8  | 2.7  |
| 8     | 3.2  | 3.1  |
| 9     | 3.6  | 3.5  |
| 10    | 4.0  | 3.9  |
| 20    | 8.0  | 7.8  |
| 30    | 12.0 | 11.7 |
| 40    | 16.0 | 15.6 |
| 50    | 20.0 | 19.6 |

| P. P. |      |      |
|-------|------|------|
| 23    | 22   |      |
| 6     | 2.3  | 2.2  |
| 7     | 2.7  | 2.6  |
| 8     | 3.0  | 3.0  |
| 9     | 3.4  | 3.4  |
| 10    | 3.8  | 3.7  |
| 20    | 7.6  | 7.5  |
| 30    | 11.5 | 11.2 |
| 40    | 15.3 | 15.0 |
| 50    | 19.1 | 18.7 |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    |            |    |             |    |    |            |    |             |    |    |  | P. P. |  |  |  |
|----|------------|----|-------------|----|----|------------|----|-------------|----|----|--|-------|--|--|--|
|    | Log. Vers. | D  | Log. Exsec. | D  |    | Log. Vers. | D  | Log. Exsec. | D  |    |  |       |  |  |  |
| 0  | 9.67217    | 23 | 9.94796     | 43 | 0  | 9.68571    | 22 | 9.97387     | 43 | 0  |  |       |  |  |  |
| 1  | .67240     | 23 | .94839      | 43 | 1  | .68593     | 22 | .97430      | 43 | 1  |  |       |  |  |  |
| 2  | .67263     | 22 | .94882      | 43 | 2  | .68615     | 22 | .97473      | 43 | 2  |  |       |  |  |  |
| 3  | .67285     | 23 | .94925      | 43 | 3  | .68637     | 22 | .97517      | 43 | 3  |  |       |  |  |  |
| 4  | .67308     | 22 | .94968      | 43 | 4  | .68660     | 22 | .97560      | 43 | 4  |  |       |  |  |  |
| 5  | 9.67331    | 23 | 9.95011     | 43 | 5  | 9.68682    | 22 | 9.97603     | 43 | 5  |  |       |  |  |  |
| 6  | .67354     | 22 | .95054      | 43 | 6  | .68704     | 22 | .97647      | 43 | 6  |  |       |  |  |  |
| 7  | .67376     | 23 | .95097      | 43 | 7  | .68727     | 22 | .97690      | 43 | 7  |  |       |  |  |  |
| 8  | .67399     | 22 | .95140      | 43 | 8  | .68749     | 22 | .97734      | 43 | 8  |  |       |  |  |  |
| 9  | .67422     | 23 | .95183      | 43 | 9  | .68771     | 22 | .97777      | 43 | 9  |  |       |  |  |  |
| 10 | 9.67445    | 22 | 9.95226     | 43 | 10 | 9.68793    | 22 | 9.97820     | 43 | 10 |  |       |  |  |  |
| 11 | .67467     | 22 | .95269      | 43 | 11 | .68816     | 22 | .97864      | 43 | 11 |  |       |  |  |  |
| 12 | .67490     | 23 | .95313      | 43 | 12 | .68838     | 22 | .97907      | 43 | 12 |  |       |  |  |  |
| 13 | .67513     | 22 | .95356      | 43 | 13 | .68860     | 22 | .97951      | 43 | 13 |  |       |  |  |  |
| 14 | .67535     | 22 | .95399      | 43 | 14 | .68882     | 22 | .97994      | 43 | 14 |  |       |  |  |  |
| 15 | 9.67558    | 23 | 9.95442     | 43 | 15 | 9.68905    | 22 | 9.98038     | 43 | 15 |  |       |  |  |  |
| 16 | .67581     | 22 | .95485      | 43 | 16 | .68927     | 22 | .98081      | 43 | 16 |  |       |  |  |  |
| 17 | .67603     | 22 | .95528      | 43 | 17 | .68949     | 22 | .98125      | 43 | 17 |  |       |  |  |  |
| 18 | .67626     | 23 | .95571      | 43 | 18 | .68971     | 22 | .98168      | 43 | 18 |  |       |  |  |  |
| 19 | .67649     | 22 | .95614      | 43 | 19 | .68993     | 22 | .98211      | 43 | 19 |  |       |  |  |  |
| 20 | 9.67671    | 22 | 9.95657     | 43 | 20 | 9.69016    | 22 | 9.98255     | 43 | 20 |  |       |  |  |  |
| 21 | .67694     | 23 | .95700      | 43 | 21 | .69038     | 22 | .98298      | 43 | 21 |  |       |  |  |  |
| 22 | .67717     | 22 | .95744      | 43 | 22 | .69060     | 22 | .98342      | 43 | 22 |  |       |  |  |  |
| 23 | .67739     | 22 | .95787      | 43 | 23 | .69082     | 22 | .98385      | 43 | 23 |  |       |  |  |  |
| 24 | .67762     | 23 | .95830      | 43 | 24 | .69104     | 22 | .98429      | 43 | 24 |  |       |  |  |  |
| 25 | 9.67784    | 22 | 9.95873     | 43 | 25 | 9.69126    | 22 | 9.98472     | 43 | 25 |  |       |  |  |  |
| 26 | .67807     | 23 | .95916      | 43 | 26 | .69149     | 22 | .98516      | 43 | 26 |  |       |  |  |  |
| 27 | .67830     | 22 | .95959      | 43 | 27 | .69171     | 22 | .98559      | 44 | 27 |  |       |  |  |  |
| 28 | .67852     | 22 | .96002      | 43 | 28 | .69193     | 22 | .98603      | 43 | 28 |  |       |  |  |  |
| 29 | .67875     | 22 | .96046      | 43 | 29 | .69215     | 22 | .98647      | 43 | 29 |  |       |  |  |  |
| 30 | 9.67897    | 22 | 9.96089     | 43 | 30 | 9.69237    | 22 | 9.98690     | 43 | 30 |  |       |  |  |  |
| 31 | .67920     | 22 | .96132      | 43 | 31 | .69259     | 22 | .98734      | 43 | 31 |  |       |  |  |  |
| 32 | .67942     | 23 | .96175      | 43 | 32 | .69281     | 22 | .98777      | 43 | 32 |  |       |  |  |  |
| 33 | .67965     | 22 | .96218      | 43 | 33 | .69303     | 22 | .98821      | 43 | 33 |  |       |  |  |  |
| 34 | .67987     | 22 | .96261      | 43 | 34 | .69325     | 22 | .98864      | 43 | 34 |  |       |  |  |  |
| 35 | 9.68010    | 22 | 9.96305     | 43 | 35 | 9.69347    | 22 | 9.98908     | 43 | 35 |  |       |  |  |  |
| 36 | .68032     | 22 | .96348      | 43 | 36 | .69369     | 22 | .98952      | 44 | 36 |  |       |  |  |  |
| 37 | .68055     | 22 | .96391      | 43 | 37 | .69392     | 22 | .98995      | 43 | 37 |  |       |  |  |  |
| 38 | .68077     | 22 | .96434      | 43 | 38 | .69414     | 22 | .99039      | 43 | 38 |  |       |  |  |  |
| 39 | .68100     | 22 | .96478      | 43 | 39 | .69436     | 22 | .99082      | 43 | 39 |  |       |  |  |  |
| 40 | 9.68122    | 22 | 9.96521     | 43 | 40 | 9.69458    | 22 | 9.99126     | 43 | 40 |  |       |  |  |  |
| 41 | .68145     | 22 | .96564      | 43 | 41 | .69480     | 22 | .99170      | 43 | 41 |  |       |  |  |  |
| 42 | .68167     | 22 | .96607      | 43 | 42 | .69502     | 22 | .99213      | 43 | 42 |  |       |  |  |  |
| 43 | .68190     | 22 | .96650      | 43 | 43 | .69524     | 22 | .99257      | 43 | 43 |  |       |  |  |  |
| 44 | .68212     | 22 | .96694      | 43 | 44 | .69546     | 22 | .99300      | 43 | 44 |  |       |  |  |  |
| 45 | 9.68235    | 22 | 9.96737     | 43 | 45 | 9.69568    | 22 | 9.99344     | 43 | 45 |  |       |  |  |  |
| 46 | .68257     | 22 | .96780      | 43 | 46 | .69590     | 22 | .99388      | 43 | 46 |  |       |  |  |  |
| 47 | .68280     | 22 | .96824      | 43 | 47 | .69612     | 22 | .99431      | 43 | 47 |  |       |  |  |  |
| 48 | .68302     | 22 | .96867      | 43 | 48 | .69634     | 22 | .99475      | 44 | 48 |  |       |  |  |  |
| 49 | .68324     | 22 | .96910      | 43 | 49 | .69656     | 22 | .99519      | 43 | 49 |  |       |  |  |  |
| 50 | 9.68347    | 22 | 9.96953     | 43 | 50 | 9.69678    | 22 | 9.99562     | 43 | 50 |  |       |  |  |  |
| 51 | .68369     | 22 | .96997      | 43 | 51 | .69700     | 22 | .99606      | 43 | 51 |  |       |  |  |  |
| 52 | .68392     | 22 | .97040      | 43 | 52 | .69721     | 21 | .99650      | 43 | 52 |  |       |  |  |  |
| 53 | .68414     | 22 | .97083      | 43 | 53 | .69743     | 22 | .99694      | 43 | 53 |  |       |  |  |  |
| 54 | .68436     | 22 | .97127      | 43 | 54 | .69765     | 22 | .99737      | 43 | 54 |  |       |  |  |  |
| 55 | 9.68459    | 22 | 9.97170     | 43 | 55 | 9.69787    | 22 | 9.99781     | 43 | 55 |  |       |  |  |  |
| 56 | .68481     | 22 | .97213      | 43 | 56 | .69809     | 22 | .99825      | 43 | 56 |  |       |  |  |  |
| 57 | .68503     | 22 | .97257      | 43 | 57 | .69831     | 21 | .99868      | 43 | 57 |  |       |  |  |  |
| 58 | .68526     | 22 | .97300      | 43 | 58 | .69853     | 22 | .99912      | 43 | 58 |  |       |  |  |  |
| 59 | .68548     | 22 | .97343      | 43 | 59 | .69875     | 22 | 9.99956     | 44 | 59 |  |       |  |  |  |
| 60 | 9.68571    |    | 9.97387     |    | 60 | 9.69897    |    | 10 00000    |    | 60 |  |       |  |  |  |
|    | Log. Vers. | D  | Log. Exsec. | D  |    | Log. Vers. | D  | Log. Exsec. | D  |    |  | P. P. |  |  |  |

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| Log. Vers. |         |    |          | Log. Exsec. |         |    |          | P. P. |    |  |  |
|------------|---------|----|----------|-------------|---------|----|----------|-------|----|--|--|
|            | D       |    |          |             | D       |    |          |       | D  |  |  |
| 0          | 9.69897 | 22 | 10.00000 | 44          | 9.71197 | 21 | 10.02639 | 44    | 0  |  |  |
| 1          | .69919  | 21 | .00044   | 43          | .71218  | 21 | .02684   | 44    | 1  |  |  |
| 2          | .69940  | 22 | .00087   | 44          | .71239  | 21 | .02728   | 44    | 2  |  |  |
| 3          | .69962  | 22 | .00131   | 43          | .71261  | 21 | .02772   | 44    | 3  |  |  |
| 4          | .69984  | 22 | .00175   | 43          | .71282  | 21 | .02816   | 44    | 4  |  |  |
| 5          | 9.70006 | 21 | 10.00219 | 44          | 9.71304 | 21 | 10.02861 | 44    | 5  |  |  |
| 6          | .70028  | 22 | .00262   | 43          | .71325  | 21 | .02905   | 44    | 6  |  |  |
| 7          | .70050  | 22 | .00306   | 44          | .71346  | 21 | .02949   | 44    | 7  |  |  |
| 8          | .70072  | 21 | .00350   | 44          | .71368  | 21 | .02994   | 44    | 8  |  |  |
| 9          | .70093  | 21 | .00394   | 43          | .71389  | 21 | .03038   | 44    | 9  |  |  |
| 10         | 9.70115 | 22 | 10.00438 | 44          | 9.71411 | 21 | 10.03082 | 44    | 10 |  |  |
| 11         | .70137  | 21 | .00482   | 44          | .71432  | 21 | .03127   | 44    | 11 |  |  |
| 12         | .70159  | 22 | .00525   | 43          | .71453  | 21 | .03171   | 44    | 12 |  |  |
| 13         | .70181  | 21 | .00569   | 44          | .71475  | 21 | .03215   | 44    | 13 |  |  |
| 14         | .70202  | 21 | .00613   | 44          | .71496  | 21 | .03260   | 44    | 14 |  |  |
| 15         | 9.70224 | 22 | 10.00657 | 44          | 9.71517 | 21 | 10.03304 | 44    | 15 |  |  |
| 16         | .70246  | 21 | .00701   | 43          | .71539  | 21 | .03348   | 44    | 16 |  |  |
| 17         | .70268  | 22 | .00745   | 44          | .71560  | 21 | .03393   | 44    | 17 |  |  |
| 18         | .70289  | 21 | .00789   | 44          | .71581  | 21 | .03437   | 44    | 18 |  |  |
| 19         | .70311  | 22 | .00833   | 44          | .71603  | 21 | .03481   | 44    | 19 |  |  |
| 20         | 9.70333 | 21 | 10.00876 | 43          | 9.71624 | 21 | 10.03526 | 44    | 20 |  |  |
| 21         | .70355  | 22 | .00920   | 44          | .71645  | 21 | .03570   | 44    | 21 |  |  |
| 22         | .70376  | 21 | .00964   | 44          | .71667  | 21 | .03615   | 44    | 22 |  |  |
| 23         | .70398  | 22 | .01008   | 44          | .71688  | 21 | .03659   | 44    | 23 |  |  |
| 24         | .70420  | 21 | .01052   | 44          | .71709  | 21 | .03704   | 44    | 24 |  |  |
| 25         | 9.70441 | 22 | 10.01096 | 44          | 9.71730 | 21 | 10.03748 | 44    | 25 |  |  |
| 26         | .70463  | 21 | .01140   | 44          | .71752  | 21 | .03793   | 44    | 26 |  |  |
| 27         | .70485  | 22 | .01184   | 44          | .71773  | 21 | .03837   | 44    | 27 |  |  |
| 28         | .70507  | 21 | .01228   | 44          | .71794  | 21 | .03881   | 44    | 28 |  |  |
| 29         | .70528  | 21 | .01272   | 44          | .71815  | 21 | .03926   | 44    | 29 |  |  |
| 30         | 9.70550 | 22 | 10.01316 | 44          | 9.71837 | 21 | 10.03970 | 44    | 30 |  |  |
| 31         | .70572  | 21 | .01360   | 44          | .71858  | 21 | .04015   | 44    | 31 |  |  |
| 32         | .70593  | 22 | .01404   | 44          | .71879  | 21 | .04059   | 44    | 32 |  |  |
| 33         | .70615  | 21 | .01448   | 44          | .71900  | 21 | .04104   | 44    | 33 |  |  |
| 34         | .70636  | 22 | .01492   | 44          | .71922  | 21 | .04149   | 44    | 34 |  |  |
| 35         | 9.70658 | 21 | 10.01536 | 44          | 9.71943 | 21 | 10.04193 | 44    | 35 |  |  |
| 36         | .70680  | 22 | .01580   | 44          | .71964  | 21 | .04238   | 44    | 36 |  |  |
| 37         | .70701  | 21 | .01624   | 44          | .71985  | 21 | .04282   | 44    | 37 |  |  |
| 38         | .70723  | 22 | .01668   | 44          | .72006  | 21 | .04327   | 44    | 38 |  |  |
| 39         | .70745  | 21 | .01712   | 44          | .72028  | 21 | .04371   | 44    | 39 |  |  |
| 40         | 9.70766 | 22 | 10.01756 | 44          | 9.72049 | 21 | 10.04416 | 44    | 40 |  |  |
| 41         | .70788  | 21 | .01800   | 44          | .72070  | 21 | .04461   | 44    | 41 |  |  |
| 42         | .70809  | 22 | .01844   | 44          | .72091  | 21 | .04505   | 44    | 42 |  |  |
| 43         | .70831  | 21 | .01889   | 44          | .72112  | 21 | .04550   | 44    | 43 |  |  |
| 44         | .70852  | 22 | .01933   | 44          | .72133  | 21 | .04594   | 44    | 44 |  |  |
| 45         | 9.70874 | 21 | 10.01977 | 44          | 9.72154 | 21 | 10.04639 | 44    | 45 |  |  |
| 46         | .70896  | 22 | .02021   | 44          | .72176  | 21 | .04684   | 44    | 46 |  |  |
| 47         | .70917  | 21 | .02065   | 44          | .72197  | 21 | .04728   | 44    | 47 |  |  |
| 48         | .70939  | 22 | .02109   | 44          | .72218  | 21 | .04773   | 44    | 48 |  |  |
| 49         | .70960  | 21 | .02153   | 44          | .72239  | 21 | .04818   | 44    | 49 |  |  |
| 50         | 9.70982 | 22 | 10.02197 | 44          | 9.72260 | 21 | 10.04862 | 44    | 50 |  |  |
| 51         | .71003  | 21 | .02242   | 44          | .72281  | 21 | .04907   | 44    | 51 |  |  |
| 52         | .71025  | 22 | .02286   | 44          | .72302  | 21 | .04952   | 44    | 52 |  |  |
| 53         | .71046  | 21 | .02330   | 44          | .72323  | 21 | .04996   | 44    | 53 |  |  |
| 54         | .71068  | 22 | .02374   | 44          | .72344  | 21 | .05041   | 44    | 54 |  |  |
| 55         | 9.71089 | 21 | 10.02418 | 44          | 9.72365 | 21 | 10.05086 | 44    | 55 |  |  |
| 56         | .71111  | 22 | .02463   | 44          | .72386  | 21 | .05131   | 44    | 56 |  |  |
| 57         | .71132  | 21 | .02507   | 44          | .72408  | 21 | .05175   | 44    | 57 |  |  |
| 58         | .71154  | 22 | .02551   | 44          | .72429  | 21 | .05220   | 44    | 58 |  |  |
| 59         | .71175  | 21 | .02595   | 44          | .72450  | 21 | .05265   | 44    | 59 |  |  |
| 60         | 9.71197 | 22 | 10.02639 | 44          | 9.72471 | 21 | 10.05310 | 44    | 60 |  |  |
| Log. Vers. |         |    |          | Log. Exsec. |         |    |          | P. P. |    |  |  |

|    |      |      |
|----|------|------|
| 6  | 4.5  | 44   |
| 7  | 5.2  | 5.2  |
| 8  | 6.0  | 5.9  |
| 9  | 6.7  | 6.7  |
| 10 | 7.5  | 7.4  |
| 20 | 15.0 | 14.8 |
| 30 | 22.5 | 22.2 |
| 40 | 30.0 | 29.6 |
| 50 | 37.5 | 37.1 |

|    |      |      |
|----|------|------|
| 6  | 4.4  | 4.4  |
| 7  | 5.1  | 5.1  |
| 8  | 5.8  | 5.8  |
| 9  | 6.6  | 6.5  |
| 10 | 7.3  | 7.2  |
| 20 | 14.6 | 14.5 |
| 30 | 22.0 | 21.7 |
| 40 | 29.3 | 29.0 |
| 50 | 36.6 | 36.2 |

|    |      |      |
|----|------|------|
| 6  | 2.2  | 2.1  |
| 7  | 2.5  | 2.5  |
| 8  | 2.9  | 2.8  |
| 9  | 3.3  | 3.2  |
| 10 | 3.6  | 3.6  |
| 20 | 7.3  | 7.1  |
| 30 | 11.0 | 10.7 |
| 40 | 14.6 | 14.3 |
| 50 | 18.3 | 17.9 |

|    |      |      |
|----|------|------|
| 6  | 2.1  | 2.1  |
| 7  | 2.4  | 2.4  |
| 8  | 2.8  | 2.8  |
| 9  | 3.1  | 3.1  |
| 10 | 3.5  | 3.5  |
| 20 | 7.0  | 7.0  |
| 30 | 10.5 | 10.5 |
| 40 | 14.0 | 14.0 |
| 50 | 17.5 | 17.5 |

|    |      |      |
|----|------|------|
| 6  | 45   | 44   |
| 7  | 4.5  | 4.4  |
| 8  | 5.2  | 5.2  |
| 9  | 6.0  | 5.9  |
| 10 | 6.7  | 6.7  |
| 10 | 7.5  | 7.4  |
| 20 | 15.0 | 14.8 |
| 30 | 22.5 | 22.2 |
| 40 | 30.0 | 29.6 |
| 50 | 37.5 | 37.1 |

|    |      |      |
|----|------|------|
| 6  | 44   | 43   |
| 7  | 4.4  | 4.3  |
| 8  | 5.1  | 5.1  |
| 9  | 5.8  | 5.8  |
| 10 | 6.6  | 6.5  |
| 10 | 7.3  | 7.2  |
| 20 | 14.6 | 14.5 |
| 30 | 22.0 | 21.7 |
| 40 | 29.3 | 29.0 |
| 50 | 36.6 | 36.2 |

|    |      |      |
|----|------|------|
| 6  | 22   | 21   |
| 7  | 2.2  | 2.1  |
| 8  | 2.5  | 2.5  |
| 9  | 2.9  | 2.8  |
| 10 | 3.3  | 3.2  |
| 10 | 3.6  | 3.6  |
| 20 | 7.3  | 7.1  |
| 30 | 11.0 | 10.7 |
| 40 | 14.6 | 14.3 |
| 50 | 18.3 | 17.9 |

|    |      |
|----|------|
| 6  | 21   |
| 7  | 2.1  |
| 8  | 2.4  |
| 9  | 2.8  |
| 10 | 3.1  |
| 10 | 3.5  |
| 20 | 7.0  |
| 30 | 10.5 |
| 40 | 14.0 |
| 50 | 17.5 |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

62°

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| 62°        |            |    |             | 63°         |            |    |             | P. P. |    |       |  |
|------------|------------|----|-------------|-------------|------------|----|-------------|-------|----|-------|--|
| '          | Log. Vers. | D  | Log. Exsec. | D           | Log. Vers. | D  | Log. Exsec. | D     | '  | P. P. |  |
| 0          | 9.72471    | 21 | 10.05310    | 44          | 9.73720    | 20 | 10.08015    | 45    | 0  |       |  |
| 1          | .72492     | 21 | .05354      | 45          | .73740     | 20 | .08061      | 45    | 1  |       |  |
| 2          | .72513     | 21 | .05399      | 45          | .73761     | 21 | .08106      | 45    | 2  |       |  |
| 3          | .72534     | 21 | .05444      | 44          | .73782     | 20 | .08151      | 45    | 3  |       |  |
| 4          | .72555     | 21 | .05489      | 45          | .73802     | 20 | .08197      | 45    | 4  |       |  |
| 5          | 9.72576    | 21 | 10.05534    | 45          | 9.73823    | 20 | 10.08242    | 45    | 5  |       |  |
| 6          | .72597     | 21 | .05579      | 45          | .73843     | 20 | .08288      | 45    | 6  |       |  |
| 7          | .72618     | 21 | .05623      | 44          | .73864     | 20 | .08333      | 45    | 7  |       |  |
| 8          | .72639     | 21 | .05668      | 45          | .73884     | 21 | .08379      | 45    | 8  |       |  |
| 9          | .72660     | 21 | .05713      | 45          | .73905     | 21 | .08424      | 45    | 9  |       |  |
| 10         | 9.72681    | 20 | 10.05758    | 45          | 9.73926    | 20 | 10.08470    | 45    | 10 |       |  |
| 11         | .72701     | 21 | .05803      | 44          | .73946     | 20 | .08515      | 45    | 11 |       |  |
| 12         | .72722     | 21 | .05848      | 45          | .73967     | 20 | .08561      | 45    | 12 |       |  |
| 13         | .72743     | 21 | .05893      | 45          | .73987     | 20 | .08606      | 45    | 13 |       |  |
| 14         | .72764     | 21 | .05938      | 45          | .74008     | 20 | .08652      | 45    | 14 |       |  |
| 15         | 9.72785    | 21 | 10.05983    | 45          | 9.74028    | 20 | 10.08697    | 45    | 15 |       |  |
| 16         | .72806     | 21 | .06028      | 45          | .74049     | 20 | .08743      | 45    | 16 |       |  |
| 17         | .72827     | 20 | .06072      | 44          | .74069     | 20 | .08789      | 45    | 17 |       |  |
| 18         | .72848     | 21 | .06117      | 45          | .74090     | 20 | .08834      | 45    | 18 |       |  |
| 19         | .72869     | 21 | .06162      | 45          | .74110     | 20 | .08880      | 45    | 19 |       |  |
| 20         | 9.72890    | 21 | 10.06207    | 45          | 9.74131    | 20 | 10.08926    | 46    | 20 |       |  |
| 21         | .72911     | 21 | .06252      | 45          | .74151     | 20 | .08971      | 45    | 21 |       |  |
| 22         | .72931     | 20 | .06297      | 45          | .74172     | 20 | .09017      | 45    | 22 |       |  |
| 23         | .72952     | 21 | .06342      | 45          | .74192     | 20 | .09062      | 45    | 23 |       |  |
| 24         | .72973     | 21 | .06387      | 45          | .74213     | 20 | .09108      | 46    | 24 |       |  |
| 25         | 9.72994    | 20 | 10.06432    | 45          | 9.74233    | 20 | 10.09154    | 45    | 25 |       |  |
| 26         | .73015     | 21 | .06477      | 45          | .74254     | 20 | .09200      | 46    | 26 |       |  |
| 27         | .73036     | 21 | .06522      | 45          | .74274     | 20 | .09245      | 45    | 27 |       |  |
| 28         | .73057     | 20 | .06568      | 45          | .74294     | 20 | .09291      | 46    | 28 |       |  |
| 29         | .73077     | 21 | .06613      | 45          | .74315     | 20 | .09337      | 45    | 29 |       |  |
| 30         | 9.73098    | 21 | 10.06658    | 45          | 9.74335    | 20 | 10.09382    | 45    | 30 |       |  |
| 31         | .73119     | 20 | .06703      | 45          | .74356     | 20 | .09428      | 46    | 31 |       |  |
| 32         | .73140     | 21 | .06748      | 45          | .74376     | 20 | .09474      | 46    | 32 |       |  |
| 33         | .73161     | 20 | .06793      | 45          | .74396     | 20 | .09520      | 45    | 33 |       |  |
| 34         | .73181     | 21 | .06838      | 45          | .74417     | 20 | .09566      | 46    | 34 |       |  |
| 35         | 9.73202    | 21 | 10.06883    | 45          | 9.74437    | 20 | 10.09611    | 45    | 35 |       |  |
| 36         | .73223     | 20 | .06928      | 45          | .74458     | 20 | .09657      | 46    | 36 |       |  |
| 37         | .73244     | 21 | .06974      | 45          | .74478     | 20 | .09703      | 45    | 37 |       |  |
| 38         | .73265     | 20 | .07019      | 45          | .74498     | 20 | .09749      | 45    | 38 |       |  |
| 39         | .73285     | 21 | .07064      | 45          | .74519     | 20 | .09795      | 46    | 39 |       |  |
| 40         | 9.73306    | 20 | 10.07109    | 45          | 9.74539    | 20 | 10.09841    | 46    | 40 |       |  |
| 41         | .73327     | 21 | .07154      | 45          | .74559     | 20 | .09886      | 45    | 41 |       |  |
| 42         | .73348     | 20 | .07200      | 45          | .74580     | 20 | .09932      | 46    | 42 |       |  |
| 43         | .73368     | 21 | .07245      | 45          | .74600     | 20 | .09978      | 46    | 43 |       |  |
| 44         | .73389     | 20 | .07290      | 45          | .74620     | 20 | .10024      | 46    | 44 |       |  |
| 45         | 9.73410    | 20 | 10.07335    | 45          | 9.74641    | 20 | 10.10070    | 46    | 45 |       |  |
| 46         | .73430     | 21 | .07380      | 45          | .74661     | 20 | .10116      | 46    | 46 |       |  |
| 47         | .73451     | 20 | .07426      | 45          | .74681     | 20 | .10162      | 45    | 47 |       |  |
| 48         | .73472     | 21 | .07471      | 45          | .74702     | 20 | .10208      | 46    | 48 |       |  |
| 49         | .73493     | 20 | .07516      | 45          | .74722     | 20 | .10254      | 46    | 49 |       |  |
| 50         | 9.73513    | 20 | 10.07562    | 45          | 9.74742    | 20 | 10.10300    | 46    | 50 |       |  |
| 51         | .73534     | 21 | .07607      | 45          | .74762     | 20 | .10346      | 46    | 51 |       |  |
| 52         | .73555     | 20 | .07652      | 45          | .74783     | 20 | .10392      | 46    | 52 |       |  |
| 53         | .73575     | 20 | .07697      | 45          | .74803     | 20 | .10438      | 46    | 53 |       |  |
| 54         | .73596     | 21 | .07743      | 45          | .74823     | 20 | .10484      | 46    | 54 |       |  |
| 55         | 9.73617    | 20 | 10.07788    | 45          | 9.74844    | 20 | 10.10530    | 46    | 55 |       |  |
| 56         | .73637     | 20 | .07834      | 45          | .74864     | 20 | .10576      | 46    | 56 |       |  |
| 57         | .73658     | 21 | .07879      | 45          | .74884     | 20 | .10622      | 46    | 57 |       |  |
| 58         | .73679     | 20 | .07924      | 45          | .74904     | 20 | .10668      | 46    | 58 |       |  |
| 59         | .73699     | 20 | .07970      | 45          | .74924     | 20 | .10714      | 46    | 59 |       |  |
| 60         | 9.73720    |    | 10.08015    |             | 9.74945    |    | 10.10760    |       | 60 |       |  |
| Log. Vers. |            |    |             | Log. Exsec. |            |    |             | P. P. |    |       |  |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.74945    | 20 | 10.10760    | 46 | 9.76146    | 19 | 10.13551    | 47 | 0  |       |
| 1  | .74965     | 20 | .10807      | 46 | .76166     | 20 | .13598      | 47 | 1  |       |
| 2  | .74985     | 20 | .10853      | 46 | .76186     | 20 | .13645      | 47 | 2  |       |
| 3  | .75005     | 20 | .10899      | 46 | .76206     | 19 | .13692      | 47 | 3  |       |
| 4  | .75026     | 20 | .10945      | 46 | .76225     | 20 | .13739      | 47 | 4  |       |
| 5  | 9.75046    | 20 | 10.10991    | 46 | 9.76245    | 19 | 10.13786    | 47 | 5  |       |
| 6  | .75066     | 20 | .11037      | 46 | .76265     | 20 | .13833      | 47 | 6  |       |
| 7  | .75086     | 20 | .11084      | 46 | .76285     | 19 | .13880      | 47 | 7  |       |
| 8  | .75106     | 20 | .11130      | 46 | .76304     | 20 | .13927      | 47 | 8  |       |
| 9  | .75126     | 20 | .11176      | 46 | .76324     | 20 | .13974      | 47 | 9  |       |
| 10 | 9.75147    | 20 | 10.11222    | 46 | 9.76344    | 20 | 10.14021    | 47 | 10 |       |
| 11 | .75167     | 20 | .11269      | 46 | .76364     | 19 | .14068      | 47 | 11 |       |
| 12 | .75187     | 20 | .11315      | 46 | .76384     | 20 | .14115      | 47 | 12 |       |
| 13 | .75207     | 20 | .11361      | 46 | .76403     | 19 | .14162      | 47 | 13 |       |
| 14 | .75227     | 20 | .11407      | 46 | .76423     | 20 | .14210      | 47 | 14 |       |
| 15 | 9.75247    | 20 | 10.11454    | 46 | 9.76443    | 19 | 10.14257    | 47 | 15 |       |
| 16 | .75267     | 20 | .11500      | 46 | .76463     | 20 | .14304      | 47 | 16 |       |
| 17 | .75287     | 20 | .11546      | 46 | .76482     | 19 | .14351      | 47 | 17 |       |
| 18 | .75308     | 20 | .11593      | 46 | .76502     | 20 | .14398      | 47 | 18 |       |
| 19 | .75328     | 20 | .11639      | 46 | .76522     | 20 | .14445      | 47 | 19 |       |
| 20 | 9.75348    | 20 | 10.11685    | 46 | 9.76541    | 19 | 10.14493    | 47 | 20 |       |
| 21 | .75368     | 20 | .11732      | 46 | .76561     | 20 | .14540      | 47 | 21 |       |
| 22 | .75388     | 20 | .11778      | 46 | .76581     | 19 | .14587      | 47 | 22 |       |
| 23 | .75408     | 20 | .11825      | 46 | .76600     | 20 | .14634      | 47 | 23 |       |
| 24 | .75428     | 20 | .11871      | 46 | .76620     | 20 | .14682      | 47 | 24 |       |
| 25 | 9.75448    | 20 | 10.11917    | 46 | 9.76640    | 19 | 10.14729    | 47 | 25 |       |
| 26 | .75468     | 20 | .11964      | 46 | .76659     | 20 | .14776      | 47 | 26 |       |
| 27 | .75488     | 20 | .12010      | 46 | .76679     | 20 | .14823      | 47 | 27 |       |
| 28 | .75508     | 20 | .12057      | 46 | .76699     | 19 | .14871      | 47 | 28 |       |
| 29 | .75528     | 20 | .12103      | 46 | .76718     | 20 | .14918      | 47 | 29 |       |
| 30 | 9.75548    | 20 | 10.12150    | 46 | 9.76738    | 20 | 10.14965    | 47 | 30 |       |
| 31 | .75568     | 20 | .12196      | 46 | .76758     | 19 | .15013      | 47 | 31 |       |
| 32 | .75588     | 20 | .12243      | 46 | .76777     | 20 | .15060      | 47 | 32 |       |
| 33 | .75608     | 20 | .12289      | 46 | .76797     | 19 | .15108      | 47 | 33 |       |
| 34 | .75628     | 20 | .12336      | 46 | .76817     | 20 | .15155      | 47 | 34 |       |
| 35 | 9.75648    | 20 | 10.12383    | 47 | 9.76836    | 19 | 10.15202    | 47 | 35 |       |
| 36 | .75668     | 20 | .12429      | 46 | .76856     | 20 | .15250      | 47 | 36 |       |
| 37 | .75688     | 20 | .12476      | 46 | .76875     | 19 | .15297      | 47 | 37 |       |
| 38 | .75708     | 20 | .12522      | 46 | .76895     | 20 | .15345      | 47 | 38 |       |
| 39 | .75728     | 20 | .12569      | 46 | .76915     | 19 | .15392      | 47 | 39 |       |
| 40 | 9.75748    | 20 | 10.12616    | 47 | 9.76934    | 20 | 10.15440    | 47 | 40 |       |
| 41 | .75768     | 20 | .12662      | 46 | .76954     | 19 | .15487      | 47 | 41 |       |
| 42 | .75788     | 20 | .12709      | 46 | .76973     | 20 | .15535      | 47 | 42 |       |
| 43 | .75808     | 20 | .12756      | 47 | .76993     | 19 | .15582      | 47 | 43 |       |
| 44 | .75828     | 19 | .12802      | 46 | .77012     | 20 | .15630      | 47 | 44 |       |
| 45 | 9.75848    | 20 | 10.12849    | 46 | 9.77032    | 20 | 10.15678    | 48 | 45 |       |
| 46 | .75868     | 20 | .12896      | 47 | .77052     | 19 | .15725      | 47 | 46 |       |
| 47 | .75888     | 20 | .12942      | 46 | .77071     | 20 | .15773      | 47 | 47 |       |
| 48 | .75908     | 20 | .12989      | 47 | .77091     | 19 | .15820      | 47 | 48 |       |
| 49 | .75928     | 20 | .13036      | 46 | .77110     | 20 | .15868      | 48 | 49 |       |
| 50 | 9.75947    | 19 | 10.13083    | 47 | 9.77130    | 19 | 10.15916    | 47 | 50 |       |
| 51 | .75967     | 20 | .13130      | 47 | .77149     | 20 | .15963      | 48 | 51 |       |
| 52 | .75987     | 20 | .13176      | 46 | .77169     | 19 | .16011      | 47 | 52 |       |
| 53 | .76007     | 20 | .13223      | 47 | .77188     | 20 | .16059      | 47 | 53 |       |
| 54 | .76027     | 19 | .13270      | 46 | .77208     | 19 | .16106      | 47 | 54 |       |
| 55 | 9.76047    | 20 | 10.13317    | 47 | 9.77227    | 20 | 10.16154    | 48 | 55 |       |
| 56 | .76067     | 20 | .13364      | 47 | .77247     | 19 | .16202      | 47 | 56 |       |
| 57 | .76087     | 20 | .13411      | 47 | .77266     | 20 | .16250      | 48 | 57 |       |
| 58 | .76106     | 19 | .13457      | 46 | .77286     | 19 | .16298      | 48 | 58 |       |
| 59 | .76126     | 20 | .13504      | 47 | .77305     | 20 | .16345      | 47 | 59 |       |
| 60 | 9.76146    | 20 | 10.13551    | 47 | 9.77325    | 19 | 10.16393    | 48 | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

|    |      |      |
|----|------|------|
| 6  | 4.8  | 4.7  |
| 7  | 5.6  | 5.5  |
| 8  | 6.4  | 6.3  |
| 9  | 7.2  | 7.1  |
| 10 | 8.0  | 7.9  |
| 20 | 16.0 | 15.8 |
| 30 | 24.0 | 23.7 |
| 40 | 32.0 | 31.6 |
| 50 | 40.0 | 39.6 |

|    |      |      |
|----|------|------|
| 6  | 4.7  | 4.6  |
| 7  | 5.5  | 5.4  |
| 8  | 6.2  | 6.2  |
| 9  | 7.0  | 7.0  |
| 10 | 7.8  | 7.7  |
| 20 | 15.6 | 15.5 |
| 30 | 23.5 | 23.2 |
| 40 | 31.4 | 31.0 |
| 50 | 39.1 | 38.7 |

|    |      |      |
|----|------|------|
| 6  | 4.6  | 4.6  |
| 7  | 5.3  | 5.3  |
| 8  | 6.1  | 6.1  |
| 9  | 6.9  | 6.9  |
| 10 | 7.6  | 7.6  |
| 20 | 15.3 | 15.3 |
| 30 | 23.0 | 23.0 |
| 40 | 30.6 | 30.6 |
| 50 | 38.3 | 38.3 |

|    |      |      |
|----|------|------|
| 6  | 2.6  | 2.0  |
| 7  | 2.4  | 2.3  |
| 8  | 2.7  | 2.6  |
| 9  | 3.1  | 3.0  |
| 10 | 3.4  | 3.3  |
| 20 | 6.8  | 6.6  |
| 30 | 10.2 | 10.0 |
| 40 | 13.6 | 13.3 |
| 50 | 17.1 | 16.6 |

|    |      |      |
|----|------|------|
| 6  | 1.9  | 1.9  |
| 7  | 2.3  | 2.3  |
| 8  | 2.6  | 2.6  |
| 9  | 2.9  | 2.9  |
| 10 | 3.2  | 3.2  |
| 20 | 6.5  | 6.5  |
| 30 | 9.7  | 9.7  |
| 40 | 13.0 | 13.0 |
| 50 | 16.2 | 16.2 |

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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.77325    |    | 10.16393    |    | 9.78481    |    | 10.19293    |    | 0  |       |
| 1  | .77344     | 19 | .16441      | 48 | .78500     | 19 | .19342      | 49 | 1  |       |
| 2  | .77363     | 19 | .16489      | 47 | .78519     | 19 | .19391      | 49 | 2  |       |
| 3  | .77383     | 19 | .16537      | 48 | .78538     | 19 | .19439      | 48 | 3  |       |
| 4  | .77402     | 19 | .16585      | 48 | .78557     | 19 | .19488      | 49 | 4  |       |
| 5  | 9.77422    | 19 | 10.16633    | 48 | 9.78576    | 19 | 10.19537    | 49 | 5  | 50    |
| 6  | .77441     | 19 | .16680      | 47 | .78595     | 19 | .19586      | 49 | 6  | 5.0   |
| 7  | .77461     | 19 | .16728      | 48 | .78614     | 19 | .19635      | 49 | 7  | 5.8   |
| 8  | .77480     | 19 | .16776      | 48 | .78633     | 19 | .19684      | 49 | 8  | 6.0   |
| 9  | .77499     | 19 | .16824      | 48 | .78652     | 19 | .19733      | 49 | 9  | 7.5   |
| 10 | 9.77519    | 19 | 10.16872    | 48 | 9.78671    | 19 | 10.19782    | 49 | 10 | 8.2   |
| 11 | .77538     | 19 | .16920      | 48 | .78690     | 19 | .19831      | 49 | 11 | 16.5  |
| 12 | .77557     | 19 | .16968      | 48 | .78709     | 19 | .19880      | 49 | 12 | 24.7  |
| 13 | .77577     | 19 | .17016      | 48 | .78728     | 19 | .19929      | 49 | 13 | 31.0  |
| 14 | .77596     | 19 | .17064      | 48 | .78747     | 19 | .19979      | 49 | 14 | 41.2  |
| 15 | 9.77616    | 19 | 10.17112    | 48 | 9.78766    | 19 | 10.20028    | 49 | 15 |       |
| 16 | .77635     | 19 | .17160      | 48 | .78785     | 19 | .20077      | 49 | 16 | 49    |
| 17 | .77654     | 19 | .17209      | 48 | .78804     | 19 | .20126      | 49 | 17 | 4.9   |
| 18 | .77674     | 19 | .17257      | 48 | .78823     | 19 | .20175      | 49 | 18 | 5.7   |
| 19 | .77693     | 19 | .17305      | 48 | .78842     | 19 | .20224      | 49 | 19 | 6.4   |
| 20 | 9.77712    | 19 | 10.17353    | 48 | 9.78861    | 19 | 10.20273    | 49 | 20 | 7.3   |
| 21 | .77732     | 19 | .17401      | 48 | .78880     | 19 | .20323      | 49 | 21 | 8.1   |
| 22 | .77751     | 19 | .17449      | 48 | .78899     | 19 | .20372      | 49 | 22 | 10.3  |
| 23 | .77770     | 19 | .17498      | 48 | .78918     | 19 | .20421      | 49 | 23 | 16.1  |
| 24 | .77790     | 19 | .17546      | 48 | .78937     | 18 | .20470      | 49 | 24 | 24.2  |
| 25 | 9.77809    | 19 | 10.17594    | 48 | 9.78956    | 19 | 10.20520    | 49 | 25 | 37.3  |
| 26 | .77828     | 19 | .17642      | 48 | .78975     | 19 | .20569      | 49 | 26 | 40.4  |
| 27 | .77847     | 19 | .17690      | 48 | .78994     | 19 | .20618      | 49 | 27 |       |
| 28 | .77867     | 19 | .17739      | 48 | .79013     | 19 | .20668      | 49 | 28 | 48    |
| 29 | .77886     | 19 | .17787      | 48 | .79032     | 19 | .20717      | 49 | 29 | 4.7   |
| 30 | 9.77905    | 19 | 10.17835    | 48 | 9.79051    | 19 | 10.20767    | 49 | 30 | 5.6   |
| 31 | .77925     | 19 | .17884      | 48 | .79069     | 18 | .20816      | 49 | 31 | 6.4   |
| 32 | .77944     | 19 | .17932      | 48 | .79088     | 19 | .20865      | 49 | 32 | 7.2   |
| 33 | .77963     | 19 | .17980      | 48 | .79107     | 19 | .20915      | 49 | 33 | 8.0   |
| 34 | .77982     | 19 | .18029      | 48 | .79126     | 19 | .20964      | 49 | 34 | 16.0  |
| 35 | 9.78002    | 19 | 10.18077    | 48 | 9.79145    | 19 | 10.21014    | 49 | 35 | 23.7  |
| 36 | .78021     | 19 | .18126      | 48 | .79164     | 18 | .21063      | 49 | 36 | 31.6  |
| 37 | .78040     | 19 | .18174      | 48 | .79183     | 19 | .21113      | 49 | 37 | 39.6  |
| 38 | .78059     | 19 | .18222      | 48 | .79202     | 19 | .21162      | 50 | 38 |       |
| 39 | .78078     | 19 | .18271      | 48 | .79220     | 18 | .21212      | 49 | 39 | 19    |
| 40 | 9.78098    | 19 | 10.18319    | 48 | 9.79239    | 19 | 10.21262    | 49 | 40 | 1.9   |
| 41 | .78117     | 19 | .18368      | 48 | .79258     | 19 | .21311      | 49 | 41 | 2.3   |
| 42 | .78136     | 19 | .18416      | 48 | .79277     | 18 | .21361      | 49 | 42 | 2.6   |
| 43 | .78155     | 19 | .18465      | 49 | .79296     | 19 | .21410      | 49 | 43 | 2.9   |
| 44 | .78174     | 19 | .18514      | 48 | .79315     | 19 | .21460      | 50 | 44 | 3.2   |
| 45 | 9.78194    | 19 | 10.18562    | 48 | 9.79333    | 18 | 10.21510    | 49 | 45 | 3.5   |
| 46 | .78213     | 19 | .18611      | 48 | .79352     | 19 | .21560      | 50 | 46 | 6.5   |
| 47 | .78232     | 19 | .18659      | 48 | .79371     | 19 | .21609      | 49 | 47 | 9.5   |
| 48 | .78251     | 19 | .18708      | 48 | .79390     | 18 | .21659      | 50 | 48 | 13.0  |
| 49 | .78270     | 19 | .18757      | 49 | .79409     | 19 | .21709      | 49 | 49 | 16.2  |
| 50 | 9.78289    | 19 | 10.18805    | 48 | 9.79427    | 18 | 10.21759    | 50 | 50 |       |
| 51 | .78309     | 19 | .18854      | 48 | .79446     | 19 | .21808      | 49 | 51 | 18    |
| 52 | .78328     | 19 | .18903      | 49 | .79465     | 19 | .21858      | 50 | 52 | 1.8   |
| 53 | .78347     | 19 | .18951      | 48 | .79484     | 18 | .21908      | 50 | 53 | 2.4   |
| 54 | .78366     | 19 | .19000      | 49 | .79503     | 19 | .21958      | 49 | 54 | 2.8   |
| 55 | 9.78385    | 19 | 10.19049    | 48 | 9.79521    | 18 | 10.22008    | 50 | 55 | 3.1   |
| 56 | .78404     | 19 | .19098      | 49 | .79540     | 19 | .22058      | 50 | 56 | 6.1   |
| 57 | .78423     | 19 | .19146      | 48 | .79559     | 18 | .22108      | 50 | 57 | 9.2   |
| 58 | .78442     | 19 | .19195      | 49 | .79578     | 18 | .22158      | 50 | 58 | 12.3  |
| 59 | .78462     | 19 | .19244      | 48 | .79596     | 19 | .22208      | 50 | 59 | 15.4  |
| 60 | 9.78481    |    | 10.19293    |    | 9.79615    |    | 10.22258    |    | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.79613    | 18 | 10.22258    | 50 | 9.80728    | 18 | 10.25293    | 51 | 0  |       |
| 1  | .79634     | 19 | .22308      | 50 | .80747     | 18 | .25347      | 51 | 1  |       |
| 2  | .79653     | 18 | .22358      | 50 | .80763     | 18 | .25398      | 51 | 2  |       |
| 3  | .79671     | 18 | .22408      | 50 | .80783     | 18 | .25449      | 51 | 3  |       |
| 4  | .79690     | 18 | .22458      | 50 | .80802     | 18 | .25501      | 51 | 4  |       |
| 5  | 9.79709    | 19 | 10.22508    | 50 | 9.80820    | 18 | 10.25552    | 51 | 5  |       |
| 6  | .79727     | 18 | .22558      | 50 | .80839     | 18 | .25604      | 51 | 6  |       |
| 7  | .79746     | 18 | .22608      | 50 | .80857     | 18 | .25655      | 51 | 7  |       |
| 8  | .79765     | 18 | .22658      | 50 | .80873     | 18 | .25707      | 51 | 8  |       |
| 9  | .79783     | 18 | .22708      | 50 | .80894     | 18 | .25758      | 51 | 9  |       |
| 10 | 9.79802    | 19 | 10.22759    | 50 | 9.80912    | 18 | 10.25810    | 51 | 10 |       |
| 11 | .79821     | 18 | .22809      | 50 | .80930     | 18 | .25861      | 51 | 11 |       |
| 12 | .79839     | 18 | .22859      | 50 | .80949     | 18 | .25913      | 51 | 12 |       |
| 13 | .79858     | 18 | .22909      | 50 | .80967     | 18 | .25964      | 51 | 13 |       |
| 14 | .79877     | 18 | .22960      | 50 | .80983     | 18 | .26016      | 51 | 14 |       |
| 15 | 9.79893    | 18 | 10.23010    | 50 | 9.81003    | 18 | 10.26067    | 51 | 15 |       |
| 16 | .79914     | 18 | .23060      | 50 | .81022     | 18 | .26119      | 52 | 16 |       |
| 17 | .79933     | 19 | .23110      | 50 | .81040     | 18 | .26171      | 51 | 17 |       |
| 18 | .79951     | 18 | .23161      | 50 | .81058     | 18 | .26222      | 51 | 18 |       |
| 19 | .79970     | 18 | .23211      | 50 | .81077     | 18 | .26274      | 52 | 19 |       |
| 20 | 9.79988    | 18 | 10.23262    | 50 | 9.81095    | 18 | 10.26326    | 51 | 20 |       |
| 21 | .80007     | 19 | .23312      | 50 | .81113     | 18 | .26378      | 52 | 21 |       |
| 22 | .80026     | 18 | .23362      | 50 | .81131     | 18 | .26429      | 51 | 22 |       |
| 23 | .80044     | 18 | .23413      | 50 | .81150     | 18 | .26481      | 52 | 23 |       |
| 24 | .80063     | 18 | .23463      | 50 | .81168     | 18 | .26533      | 52 | 24 |       |
| 25 | 9.80081    | 18 | 10.23514    | 50 | 9.81186    | 18 | 10.26583    | 52 | 25 |       |
| 26 | .80100     | 19 | .23564      | 50 | .81204     | 18 | .26637      | 51 | 26 |       |
| 27 | .80119     | 18 | .23615      | 50 | .81223     | 18 | .26689      | 52 | 27 |       |
| 28 | .80137     | 18 | .23666      | 50 | .81241     | 18 | .26741      | 52 | 28 |       |
| 29 | .80156     | 18 | .23716      | 50 | .81259     | 18 | .26793      | 52 | 29 |       |
| 30 | 9.80174    | 18 | 10.23767    | 50 | 9.81277    | 18 | 10.26845    | 52 | 30 |       |
| 31 | .80193     | 18 | .23817      | 50 | .81293     | 18 | .26897      | 52 | 31 |       |
| 32 | .80211     | 18 | .23868      | 51 | .81314     | 18 | .26949      | 52 | 32 |       |
| 33 | .80230     | 18 | .23919      | 50 | .81332     | 18 | .27001      | 52 | 33 |       |
| 34 | .80248     | 18 | .23969      | 50 | .81350     | 18 | .27053      | 52 | 34 |       |
| 35 | 9.80267    | 19 | 10.24020    | 51 | 9.81368    | 18 | 10.27105    | 52 | 35 |       |
| 36 | .80286     | 18 | .24071      | 50 | .81386     | 18 | .27157      | 52 | 36 |       |
| 37 | .80304     | 18 | .24122      | 51 | .81405     | 18 | .27209      | 52 | 37 |       |
| 38 | .80323     | 18 | .24172      | 50 | .81423     | 18 | .27261      | 52 | 38 |       |
| 39 | .80341     | 18 | .24223      | 51 | .81441     | 18 | .27314      | 52 | 39 |       |
| 40 | 9.80360    | 18 | 10.24274    | 51 | 9.81459    | 18 | 10.27366    | 52 | 40 |       |
| 41 | .80378     | 18 | .24325      | 51 | .81477     | 18 | .27418      | 52 | 41 |       |
| 42 | .80397     | 18 | .24376      | 51 | .81493     | 18 | .27470      | 52 | 42 |       |
| 43 | .80413     | 18 | .24427      | 51 | .81513     | 18 | .27523      | 52 | 43 |       |
| 44 | .80434     | 18 | .24478      | 51 | .81532     | 18 | .27573      | 52 | 44 |       |
| 45 | 9.80452    | 18 | 10.24529    | 51 | 9.81550    | 18 | 10.22627    | 52 | 45 |       |
| 46 | .80470     | 18 | .24580      | 51 | .81568     | 18 | .27680      | 52 | 46 |       |
| 47 | .80489     | 18 | .24631      | 51 | .81586     | 18 | .27732      | 52 | 47 |       |
| 48 | .80507     | 18 | .24682      | 51 | .81604     | 18 | .27785      | 52 | 48 |       |
| 49 | .80526     | 18 | .24733      | 51 | .81622     | 18 | .27837      | 52 | 49 |       |
| 50 | 9.80544    | 18 | 10.24784    | 51 | 9.81640    | 18 | 10.27890    | 52 | 50 |       |
| 51 | .80563     | 18 | .24835      | 51 | .81658     | 18 | .27942      | 52 | 51 |       |
| 52 | .80587     | 18 | .24886      | 51 | .81676     | 18 | .27995      | 52 | 52 |       |
| 53 | .80600     | 18 | .24937      | 51 | .81695     | 18 | .28047      | 52 | 53 |       |
| 54 | .80618     | 18 | .24988      | 51 | .81713     | 18 | .28100      | 52 | 54 |       |
| 55 | 9.80636    | 18 | 10.25039    | 51 | 9.81731    | 18 | 10.28152    | 52 | 55 |       |
| 56 | .80655     | 18 | .25090      | 51 | .81749     | 18 | .28203      | 52 | 56 |       |
| 57 | .80673     | 18 | .25142      | 51 | .81767     | 18 | .28258      | 52 | 57 |       |
| 58 | .80692     | 18 | .25193      | 51 | .81785     | 18 | .28310      | 52 | 58 |       |
| 59 | .80710     | 18 | .25244      | 51 | .81803     | 18 | .28363      | 52 | 59 |       |
| 60 | 9.80728    | 18 | 10.25295    | 51 | 9.81821    | 18 | 10.28416    | 52 | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

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6 5.3 5.2  
7 6.2 6.1  
8 7.0 7.0  
9 7.6 7.0  
10 8.0 8.7  
20 17.6 17.5  
30 26.5 26.2  
40 35.0 35.0  
50 44.1 43.7

52 51  
6 5.2 5.1  
7 6.0 6.0  
8 6.6 6.8  
9 7.8 7.7  
10 8.6 8.6  
20 17.3 17.1  
30 26.0 25.7  
40 34.6 34.3  
50 43.3 42.9

51 50  
6 5.1 5.0  
7 5.9 5.0  
8 6.8 6.7  
9 7.6 7.6  
10 8.5 8.4  
20 17.0 16.8  
30 25.5 25.2  
40 34.0 33.6  
50 42.5 42.1

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6 5.0 5.0  
7 5.8 5.8  
8 6.6 6.6  
9 7.5 7.5  
10 8.3 8.3  
20 16.6 16.6  
30 25.0 25.0  
40 33.3 33.3  
50 41.6 41.6

19 18  
6 1.9 1.8  
7 2.2 2.1  
8 2.5 2.4  
9 2.8 2.8  
10 3.1 3.1  
20 6.3 6.1  
30 9.5 9.2  
40 12.6 12.3  
50 15.8 15.4

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6 1.8 1.8  
7 2.1 2.1  
8 2.4 2.4  
9 2.7 2.7  
10 3.0 3.0  
20 6.0 6.0  
30 9.0 9.0  
40 12.0 12.0  
50 15.0 15.0

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

70°

71°

|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------|
| 0  | 9.81821    | 18 | 10.28416    | 53 | 9.82894    | 17 | 10.31629    | 54 | 0  |       |
| 1  | .81839     | 18 | .28469      | 52 | .82911     | 17 | .31684      | 54 | 1  |       |
| 2  | .81857     | 18 | .28521      | 53 | .82929     | 17 | .31738      | 54 | 2  |       |
| 3  | .81875     | 18 | .28574      | 53 | .82947     | 18 | .31793      | 54 | 3  | 56    |
| 4  | .81893     | 18 | .28627      | 53 | .82964     | 17 | .31847      | 54 | 4  | 56    |
| 5  | 9.81911    | 18 | 10.28680    | 52 | 9.82982    | 18 | 10.31902    | 54 | 5  | 56    |
| 6  | .81929     | 18 | .28733      | 53 | .83000     | 17 | .31956      | 54 | 6  | 56    |
| 7  | .81947     | 18 | .28786      | 53 | .83017     | 17 | .32011      | 54 | 7  | 56    |
| 8  | .81965     | 18 | .28839      | 53 | .83035     | 18 | .32066      | 54 | 8  | 56    |
| 9  | .81983     | 18 | .28892      | 53 | .83053     | 17 | .32120      | 54 | 9  | 56    |
| 10 | 9.82001    | 18 | 10.28945    | 53 | 9.83070    | 17 | 10.32175    | 54 | 10 |       |
| 11 | .82019     | 18 | .28998      | 53 | .83088     | 18 | .32230      | 54 | 11 |       |
| 12 | .82037     | 18 | .29051      | 53 | .83106     | 17 | .32284      | 54 | 12 |       |
| 13 | .82055     | 18 | .29104      | 53 | .83123     | 17 | .32339      | 54 | 13 | 55    |
| 14 | .82073     | 18 | .29157      | 53 | .83141     | 17 | .32394      | 54 | 14 | 55    |
| 15 | 9.82091    | 17 | 10.29210    | 53 | 9.83159    | 18 | 10.32449    | 54 | 15 |       |
| 16 | .82109     | 18 | .29263      | 53 | .83176     | 17 | .32504      | 54 | 16 |       |
| 17 | .82127     | 18 | .29316      | 53 | .83194     | 17 | .32558      | 54 | 17 |       |
| 18 | .82145     | 18 | .29370      | 53 | .83211     | 17 | .32613      | 54 | 18 |       |
| 19 | .82163     | 18 | .29423      | 53 | .83229     | 18 | .32668      | 54 | 19 |       |
| 20 | 9.82181    | 18 | 10.29476    | 53 | 9.83247    | 17 | 10.32723    | 54 | 20 |       |
| 21 | .82199     | 18 | .29529      | 53 | .83264     | 17 | .32778      | 54 | 21 |       |
| 22 | .82217     | 18 | .29583      | 53 | .83282     | 17 | .32833      | 54 | 22 |       |
| 23 | .82235     | 17 | .29636      | 53 | .83299     | 18 | .32888      | 54 | 23 | 54    |
| 24 | .82252     | 18 | .29689      | 53 | .83317     | 17 | .32944      | 54 | 24 | 54    |
| 25 | 9.82270    | 18 | 10.29743    | 53 | 9.83335    | 17 | 10.32999    | 54 | 25 |       |
| 26 | .82288     | 18 | .29796      | 53 | .83352     | 17 | .33054      | 54 | 26 |       |
| 27 | .82306     | 18 | .29850      | 53 | .83370     | 17 | .33109      | 54 | 27 |       |
| 28 | .82324     | 17 | .29903      | 53 | .83387     | 17 | .33164      | 54 | 28 |       |
| 29 | .82342     | 18 | .29957      | 53 | .83405     | 17 | .33220      | 54 | 29 |       |
| 30 | 9.82360    | 18 | 10.30010    | 53 | 9.83422    | 17 | 10.33275    | 54 | 30 |       |
| 31 | .82378     | 18 | .30064      | 53 | .83440     | 18 | .33330      | 54 | 31 |       |
| 32 | .82396     | 17 | .30117      | 54 | .83458     | 17 | .33385      | 54 | 32 | 53    |
| 33 | .82413     | 18 | .30171      | 54 | .83475     | 17 | .33441      | 54 | 33 | 53    |
| 34 | .82431     | 18 | .30225      | 53 | .83493     | 17 | .33496      | 54 | 34 | 53    |
| 35 | 9.82449    | 17 | 10.30278    | 54 | 9.83510    | 17 | 10.33552    | 54 | 35 |       |
| 36 | .82467     | 18 | .30332      | 54 | .83528     | 17 | .33607      | 54 | 36 |       |
| 37 | .82485     | 18 | .30386      | 53 | .83545     | 17 | .33663      | 54 | 37 |       |
| 38 | .82503     | 17 | .30440      | 54 | .83563     | 17 | .33718      | 54 | 38 |       |
| 39 | .82520     | 18 | .30493      | 53 | .83580     | 17 | .33774      | 54 | 39 |       |
| 40 | 9.82538    | 18 | 10.30547    | 54 | 9.83598    | 17 | 10.33829    | 54 | 40 |       |
| 41 | .82556     | 17 | .30601      | 53 | .83615     | 17 | .33885      | 54 | 41 |       |
| 42 | .82574     | 18 | .30655      | 54 | .83633     | 17 | .33941      | 54 | 42 | 52    |
| 43 | .82592     | 17 | .30709      | 54 | .83650     | 17 | .33996      | 54 | 43 | 52    |
| 44 | .82609     | 18 | .30763      | 54 | .83668     | 17 | .34052      | 54 | 44 | 52    |
| 45 | 9.82627    | 18 | 10.30817    | 54 | 9.83685    | 17 | 10.34108    | 54 | 45 |       |
| 46 | .82645     | 17 | .30871      | 54 | .83703     | 17 | .34164      | 54 | 46 |       |
| 47 | .82663     | 18 | .30925      | 54 | .83720     | 17 | .34220      | 54 | 47 |       |
| 48 | .82681     | 17 | .30979      | 54 | .83737     | 17 | .34275      | 54 | 48 |       |
| 49 | .82698     | 18 | .31033      | 54 | .83755     | 17 | .34331      | 54 | 49 |       |
| 50 | 9.82716    | 17 | 10.31087    | 54 | 9.83772    | 17 | 10.34387    | 54 | 50 |       |
| 51 | .82734     | 18 | .31141      | 54 | .83790     | 17 | .34443      | 54 | 51 | 18    |
| 52 | .82752     | 17 | .31195      | 54 | .83807     | 17 | .34499      | 54 | 52 | 17    |
| 53 | .82769     | 18 | .31249      | 54 | .83825     | 17 | .34555      | 54 | 53 | 17    |
| 54 | .82787     | 17 | .31303      | 54 | .83842     | 17 | .34611      | 54 | 54 | 17    |
| 55 | 9.82805    | 17 | 10.31358    | 54 | 9.83859    | 17 | 10.34667    | 54 | 55 |       |
| 56 | .82823     | 17 | .31412      | 54 | .83877     | 17 | .34723      | 54 | 56 |       |
| 57 | .82840     | 18 | .31466      | 54 | .83894     | 17 | .34780      | 54 | 57 |       |
| 58 | .82858     | 17 | .31521      | 54 | .83912     | 17 | .34836      | 54 | 58 |       |
| 59 | .82876     | 18 | .31575      | 54 | .83929     | 17 | .34892      | 54 | 59 |       |
| 60 | 9.82894    |    | 10.31629    |    | 9.83946    |    | 10.34948    |    | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P. |

72°

73°

| 72° |            |    |             | 73° |         |            |          | P. P.       |    |    |           |
|-----|------------|----|-------------|-----|---------|------------|----------|-------------|----|----|-----------|
|     | Log. Vers. | D  | Log. Exsec. | D   |         | Log. Vers. | D        | Log. Exsec. | D  |    |           |
| 0   | 9.83946    | 17 | 10.34948    | 56  | 9.84980 | 17         | 10.38387 | 58          | 0  |    |           |
| 1   | .83964     | 17 | .35005      | 56  | .84997  | 17         | .38445   | 58          | 1  |    |           |
| 2   | .83981     | 17 | .35061      | 56  | .85014  | 17         | .38504   | 58          | 2  |    |           |
| 3   | .83999     | 17 | .35117      | 56  | .85031  | 17         | .38562   | 58          | 3  |    | 61 66     |
| 4   | .84016     | 17 | .35174      | 56  | .85049  | 17         | .38621   | 58          | 4  | 6  | 6.1 6.6   |
|     |            | 17 |             | 56  |         | 17         |          | 58          |    | 7  | 7.1 7.6   |
| 5   | 9.84033    | 17 | 10.35230    | 56  | 9.85066 | 17         | 10.38679 | 58          | 5  | 8  | 8.1 8.6   |
| 6   | .84051     | 17 | .35286      | 56  | .85083  | 17         | .38738   | 58          | 6  | 9  | 9.1 9.1   |
| 7   | .84068     | 17 | .35343      | 56  | .85100  | 17         | .38796   | 58          | 7  | 10 | 10.1 10.1 |
| 8   | .84085     | 17 | .35399      | 56  | .85117  | 17         | .38855   | 59          | 8  | 20 | 20.3 20.1 |
| 9   | .84103     | 17 | .35456      | 57  | .85134  | 17         | .38914   | 58          | 9  | 30 | 30.5 30.2 |
|     |            | 17 |             | 56  |         | 17         |          | 59          |    | 40 | 40.6 40.3 |
| 10  | 9.84120    | 17 | 10.35513    | 56  | 9.85151 | 17         | 10.38973 | 59          | 10 | 50 | 50.8 50.4 |
| 11  | .84137     | 17 | .35569      | 56  | .85168  | 17         | .39031   | 58          | 11 |    |           |
| 12  | .84155     | 17 | .35626      | 56  | .85185  | 17         | .39090   | 59          | 12 |    |           |
| 13  | .84172     | 17 | .35683      | 57  | .85202  | 17         | .39149   | 59          | 13 |    | 60 59     |
| 14  | .84189     | 17 | .35739      | 56  | .85219  | 17         | .39208   | 58          | 14 | 6  | 6.0 5.9   |
|     |            | 17 |             | 57  |         | 17         |          | 59          |    | 7  | 7.0 6.6   |
| 15  | 9.84207    | 17 | 10.35796    | 56  | 9.85236 | 17         | 10.39267 | 59          | 15 | 8  | 8.0 7.9   |
| 16  | .84224     | 17 | .35853      | 56  | .85253  | 17         | .39326   | 59          | 16 | 9  | 9.0 8.9   |
| 17  | .84241     | 17 | .35910      | 57  | .85270  | 17         | .39385   | 59          | 17 | 10 | 10.0 9.9  |
| 18  | .84259     | 17 | .35967      | 57  | .85287  | 17         | .39444   | 59          | 18 | 20 | 20.0 19.8 |
| 19  | .84276     | 17 | .36023      | 56  | .85304  | 17         | .39503   | 59          | 19 | 30 | 30.0 29.7 |
|     |            | 17 |             | 57  |         | 17         |          | 59          |    | 40 | 40.0 39.6 |
| 20  | 9.84293    | 17 | 10.36080    | 57  | 9.85321 | 17         | 10.39562 | 59          | 20 | 50 | 50.0 49.6 |
| 21  | .84310     | 17 | .36137      | 57  | .85338  | 17         | .39621   | 59          | 21 |    |           |
| 22  | .84328     | 17 | .36194      | 57  | .85355  | 17         | .39681   | 59          | 22 |    |           |
| 23  | .84345     | 17 | .36251      | 57  | .85372  | 17         | .39740   | 59          | 23 | 6  | 5.9 5.8   |
| 24  | .84362     | 17 | .36308      | 57  | .85389  | 17         | .39799   | 59          | 24 | 7  | 6.9 6.8   |
|     |            | 17 |             | 57  |         | 16         |          | 59          |    | 8  | 7.9 7.8   |
| 25  | 9.84380    | 17 | 10.36366    | 57  | 9.85405 | 17         | 10.39859 | 59          | 25 | 9  | 8.9 8.8   |
| 26  | .84397     | 17 | .36423      | 57  | .85422  | 17         | .39918   | 59          | 26 | 10 | 9.9 9.7   |
| 27  | .84414     | 17 | .36480      | 57  | .85439  | 17         | .39977   | 59          | 27 | 20 | 19.6 19.5 |
| 28  | .84431     | 17 | .36537      | 57  | .85456  | 17         | .40037   | 59          | 28 | 30 | 29.5 29.2 |
| 29  | .84449     | 17 | .36594      | 57  | .85473  | 17         | .40096   | 59          | 29 | 40 | 39.3 39.0 |
|     |            | 17 |             | 57  |         | 17         |          | 59          |    | 50 | 49.1 48.7 |
| 30  | 9.84466    | 17 | 10.36652    | 57  | 9.85490 | 17         | 10.40156 | 60          | 30 |    |           |
| 31  | .84483     | 17 | .36709      | 57  | .85507  | 16         | .40216   | 59          | 31 |    |           |
| 32  | .84500     | 17 | .36766      | 57  | .85524  | 17         | .40275   | 59          | 32 | 6  | 5.8 5.7   |
| 33  | .84517     | 17 | .36824      | 57  | .85541  | 17         | .40335   | 60          | 33 | 7  | 6.7 6.7   |
| 34  | .84535     | 17 | .36881      | 57  | .85558  | 17         | .40395   | 60          | 34 | 8  | 7.7 7.6   |
|     |            | 17 |             | 57  |         | 17         |          | 59          |    | 9  | 8.7 8.6   |
| 35  | 9.84552    | 17 | 10.36938    | 57  | 9.85575 | 17         | 10.40454 | 60          | 35 | 10 | 9.6 9.6   |
| 36  | .84569     | 17 | .36996      | 57  | .85592  | 17         | .40514   | 60          | 36 | 20 | 19.3 19.1 |
| 37  | .84586     | 17 | .37054      | 58  | .85608  | 16         | .40574   | 59          | 37 | 30 | 29.0 28.7 |
| 38  | .84603     | 17 | .37111      | 57  | .85625  | 17         | .40634   | 60          | 38 | 40 | 38.6 38.3 |
| 39  | .84620     | 17 | .37169      | 57  | .85642  | 17         | .40694   | 60          | 39 | 50 | 48.3 47.9 |
|     |            | 17 |             | 57  |         | 17         |          | 60          |    |    |           |
| 40  | 9.84638    | 17 | 10.37226    | 57  | 9.85659 | 16         | 10.40754 | 60          | 40 |    |           |
| 41  | .84655     | 17 | .37284      | 57  | .85676  | 17         | .40814   | 60          | 41 |    |           |
| 42  | .84672     | 17 | .37342      | 58  | .85693  | 17         | .40874   | 60          | 42 | 6  | 5.7 5.6   |
| 43  | .84689     | 17 | .37399      | 57  | .85710  | 16         | .40934   | 60          | 43 | 7  | 6.6 6.6   |
| 44  | .84706     | 17 | .37457      | 58  | .85726  | 16         | .40994   | 60          | 44 | 8  | 7.6 7.5   |
|     |            | 17 |             | 58  |         | 17         |          | 60          |    | 9  | 8.5 8.5   |
| 45  | 9.84724    | 17 | 10.37513    | 57  | 9.85743 | 17         | 10.41054 | 60          | 45 | 10 | 9.5 9.4   |
| 46  | .84741     | 17 | .37573      | 57  | .85760  | 16         | .41114   | 60          | 46 | 20 | 10.0 18.8 |
| 47  | .84758     | 17 | .37631      | 58  | .85777  | 17         | .41174   | 60          | 47 | 30 | 28.5 28.2 |
| 48  | .84775     | 17 | .37689      | 58  | .85794  | 17         | .41235   | 60          | 48 | 40 | 38.0 37.6 |
| 49  | .84792     | 17 | .37747      | 58  | .85811  | 17         | .41295   | 60          | 49 | 50 | 47.5 47.1 |
|     |            | 17 |             | 58  |         | 16         |          | 60          |    |    |           |
| 50  | 9.84809    | 17 | 10.37805    | 58  | 9.85829 | 17         | 10.41355 | 60          | 50 |    |           |
| 51  | .84826     | 17 | .37863      | 58  | .85844  | 17         | .41416   | 60          | 51 |    |           |
| 52  | .84844     | 17 | .37921      | 58  | .85861  | 17         | .41476   | 60          | 52 | 17 | 1.7 1.6   |
| 53  | .84861     | 17 | .37979      | 58  | .85878  | 16         | .41537   | 60          | 53 | 6  | 2.0 2.0   |
| 54  | .84878     | 17 | .38037      | 58  | .85895  | 17         | .41597   | 60          | 54 | 7  | 2.3 2.2   |
|     |            | 17 |             | 58  |         | 17         |          | 60          |    | 8  | 2.6 2.5   |
| 55  | 9.84895    | 17 | 10.38095    | 58  | 9.85911 | 16         | 10.41658 | 61          | 55 | 9  | 2.9 2.7   |
| 56  | .84912     | 17 | .38153      | 58  | .85928  | 17         | .41719   | 60          | 56 | 10 | 2.0 2.0   |
| 57  | .84929     | 17 | .38212      | 58  | .85945  | 16         | .41779   | 60          | 57 | 20 | 5.8 5.5   |
| 58  | .84946     | 17 | .38270      | 58  | .85962  | 17         | .41840   | 61          | 58 | 30 | 8.7 8.2   |
| 59  | .84963     | 17 | .38328      | 58  | .85979  | 17         | .41901   | 61          | 59 | 40 | 11.6 11.0 |
|     |            | 17 |             | 58  |         | 16         |          | 61          |    | 50 | 14.6 13.7 |
| 60  | 9.84980    | 17 | 10.38387    | 58  | 9.85995 | 17         | 10.41962 | 61          | 60 |    |           |
|     |            | 17 |             | 58  |         | 17         |          | 61          |    |    |           |
|     | Log. Vers. | D  | Log. Exsec. | D   |         | Log. Vers. | D        | Log. Exsec. | D  |    | P. P.     |



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75°

|    | Log. Vers. | D  | Log. Exsec. | D  |            | Log. Vers. | D           | Log. Exsec. | D |    | P. P.             |
|----|------------|----|-------------|----|------------|------------|-------------|-------------|---|----|-------------------|
| 0  | 9.85993    |    | 10.41962    | 60 | 9.86992    |            | 10.45693    | 63          |   | 0  |                   |
| 1  | .86012     | 17 | .42022      | 61 | .87009     | 16         | .45756      | 63          |   | 1  |                   |
| 2  | .86029     | 16 | .42083      | 61 | .87023     | 16         | .45820      | 64          |   | 2  |                   |
| 3  | .86046     | 17 | .42144      | 61 | .87042     | 16         | .45884      | 63          |   | 3  | 67 66 66          |
| 4  | .86062     | 16 | .42205      | 61 | .87058     | 16         | .45947      | 64          |   | 4  | 6 6.7 6.6 6 6     |
| 5  | 9.86079    | 17 | 10.42266    | 61 | 9.87074    | 16         | 10.46011    | 64          |   | 5  | 7 7.8 7.7 7.7     |
| 6  | .86096     | 16 | .42327      | 61 | .87091     | 16         | .46073      | 64          |   | 6  | 8 8.9 8.8 8.8     |
| 7  | .86113     | 17 | .42388      | 61 | .87107     | 16         | .46139      | 64          |   | 7  | 9 10.0 10.0 9.9   |
| 8  | .86129     | 16 | .42450      | 61 | .87124     | 16         | .46203      | 64          |   | 8  | 10 11.1 11.1 11.0 |
| 9  | .86146     | 17 | .42511      | 61 | .87140     | 16         | .46267      | 64          |   | 9  | 20 22.3 22.1 22.0 |
| 10 | 9.86163    | 16 | 10.42572    | 61 | 9.87157    | 16         | 10.46331    | 64          |   | 10 | 30 33.5 33.2 33.0 |
| 11 | .86179     | 16 | .42633      | 61 | .87173     | 16         | .46395      | 64          |   | 11 | 40 44.6 44.3 44.0 |
| 12 | .86196     | 17 | .42695      | 61 | .87189     | 16         | .46460      | 64          |   | 12 | 50 55.8 55.4 55.0 |
| 13 | .86213     | 16 | .42756      | 61 | .87206     | 16         | .46524      | 64          |   | 13 |                   |
| 14 | .86230     | 17 | .42817      | 61 | .87222     | 16         | .46588      | 64          |   | 14 | 65 65 64          |
| 15 | 9.86246    | 16 | 10.42879    | 61 | 9.87239    | 16         | 10.46652    | 64          |   | 15 | 6 6.3 6.5 6.4     |
| 16 | .86263     | 16 | .42940      | 61 | .87255     | 16         | .46717      | 64          |   | 16 | 7 7.6 7.6 7.5     |
| 17 | .86280     | 17 | .43002      | 61 | .87271     | 16         | .46781      | 64          |   | 17 | 8 8.7 8.6 8.6     |
| 18 | .86296     | 16 | .43063      | 62 | .87288     | 16         | .46846      | 64          |   | 18 | 9 9.8 9.7 9.7     |
| 19 | .86313     | 17 | .43125      | 61 | .87304     | 16         | .46910      | 64          |   | 19 | 10 10.9 10.8 10.7 |
| 20 | 9.86330    | 16 | 10.43187    | 62 | 9.87320    | 16         | 10.46975    | 65          |   | 20 | 20 21.8 21.6 21.5 |
| 21 | .86346     | 16 | .43249      | 61 | .87337     | 16         | .47040      | 64          |   | 21 | 30 32.7 32.5 32.2 |
| 22 | .86363     | 17 | .43310      | 62 | .87353     | 16         | .47104      | 65          |   | 22 | 40 43.6 43.3 43.0 |
| 23 | .86380     | 16 | .43372      | 61 | .87370     | 16         | .47169      | 64          |   | 23 | 50 54.6 54.1 53.7 |
| 24 | .86396     | 16 | .43434      | 62 | .87386     | 16         | .47234      | 65          |   | 24 |                   |
| 25 | 9.86413    | 17 | 10.43496    | 62 | 9.87402    | 16         | 10.47299    | 65          |   | 25 | 64 63 63          |
| 26 | .86430     | 16 | .43558      | 62 | .87419     | 16         | .47364      | 65          |   | 26 | 6 6.4 6.3 6.3     |
| 27 | .86446     | 16 | .43620      | 62 | .87435     | 16         | .47429      | 65          |   | 27 | 7 7.4 7.4 7.3     |
| 28 | .86463     | 16 | .43682      | 62 | .87451     | 16         | .47494      | 65          |   | 28 | 8 8.5 8.4 8.4     |
| 29 | .86479     | 16 | .43744      | 62 | .87468     | 16         | .47559      | 65          |   | 29 | 9 9.6 9.5 9.4     |
| 30 | 9.86496    | 17 | 10.43806    | 62 | 9.87484    | 16         | 10.47624    | 65          |   | 30 | 10 10.6 10.6 10.5 |
| 31 | .86513     | 16 | .43868      | 62 | .87500     | 16         | .47689      | 65          |   | 31 | 20 21.3 21.1 21.0 |
| 32 | .86529     | 16 | .43931      | 62 | .87516     | 16         | .47754      | 65          |   | 32 | 30 32.0 31.7 31.5 |
| 33 | .86546     | 16 | .43993      | 62 | .87533     | 16         | .47820      | 65          |   | 33 | 40 42.6 42.3 42.0 |
| 34 | .86562     | 16 | .44055      | 62 | .87549     | 16         | .47885      | 65          |   | 34 | 50 53.3 52.9 52.5 |
| 35 | 9.86579    | 17 | 10.44118    | 62 | 9.87565    | 16         | 10.47950    | 65          |   | 35 |                   |
| 36 | .86596     | 16 | .44180      | 62 | .87582     | 16         | .48016      | 65          |   | 36 | 62 62 61          |
| 37 | .86612     | 16 | .44242      | 62 | .87598     | 16         | .48081      | 65          |   | 37 | 6 6.2 6.2 6.1     |
| 38 | .86629     | 16 | .44305      | 63 | .87614     | 16         | .48147      | 66          |   | 38 | 7 7.3 7.2 7.2     |
| 39 | .86645     | 16 | .44368      | 62 | .87631     | 16         | .48213      | 65          |   | 39 | 8 8.3 8.2 8.2     |
| 40 | 9.86662    | 16 | 10.44430    | 62 | 9.87647    | 16         | 10.48278    | 65          |   | 40 | 9 9.4 9.3 9.2     |
| 41 | .86678     | 16 | .44493      | 63 | .87653     | 16         | .48344      | 66          |   | 41 | 10 10.4 10.3 10.2 |
| 42 | .86693     | 17 | .44556      | 62 | .87679     | 16         | .48410      | 66          |   | 42 | 20 20.8 20.6 20.5 |
| 43 | .86712     | 16 | .44618      | 62 | .87696     | 16         | .48476      | 66          |   | 43 | 30 31.2 31.0 30.7 |
| 44 | .86728     | 16 | .44681      | 63 | .87712     | 16         | .48542      | 66          |   | 44 | 40 41.6 41.3 41.0 |
| 45 | 9.86745    | 16 | 10.44744    | 62 | 9.87728    | 16         | 10.48607    | 65          |   | 45 | 50 52.1 51.6 51.2 |
| 46 | .86761     | 16 | .44807      | 63 | .87744     | 16         | .48674      | 66          |   | 46 |                   |
| 47 | .86778     | 16 | .44870      | 63 | .87761     | 16         | .48740      | 66          |   | 47 | 61 61 60          |
| 48 | .86794     | 16 | .44933      | 63 | .87777     | 16         | .48806      | 66          |   | 48 | 6 6.1 6.1 6.0     |
| 49 | .86811     | 16 | .44996      | 63 | .87793     | 16         | .48872      | 66          |   | 49 | 7 7.1 7.1 7.0     |
| 50 | 9.86827    | 16 | 10.45059    | 63 | 9.87809    | 16         | 10.48938    | 66          |   | 50 | 8 8.1 8.1 8.0     |
| 51 | .86844     | 16 | .45122      | 63 | .87825     | 16         | .49004      | 66          |   | 51 | 9 9.1 9.1 9.1     |
| 52 | .86860     | 16 | .45185      | 63 | .87842     | 16         | .49071      | 66          |   | 52 | 10 10.1 10.1 10.1 |
| 53 | .86877     | 16 | .45248      | 63 | .87858     | 16         | .49137      | 66          |   | 53 | 20 20.3 20.1 20.1 |
| 54 | .86893     | 16 | .45312      | 63 | .87874     | 16         | .49204      | 66          |   | 54 | 30 30.5 30.2 30.2 |
| 55 | 9.86910    | 16 | 10.45375    | 63 | 9.87890    | 16         | 10.49270    | 66          |   | 55 | 40 40.6 40.3 40.3 |
| 56 | .86926     | 16 | .45439      | 63 | .87906     | 16         | .49337      | 66          |   | 56 | 50 50.8 50.4 50.4 |
| 57 | .86943     | 16 | .45502      | 63 | .87923     | 16         | .49403      | 67          |   | 57 |                   |
| 58 | .86959     | 16 | .45565      | 63 | .87939     | 16         | .49470      | 66          |   | 58 | 17 1.7 1.6 1.6    |
| 59 | .86976     | 16 | .45629      | 64 | .87955     | 16         | .49537      | 67          |   | 59 | 7 2.0 1.9 1.8     |
| 60 | 9.86992    | 16 | 10.45693    |    | 9.87971    | 16         | 10.49604    | 67          |   | 60 | 8 2.2 2.2 2.1     |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D          | Log. Exsec. | D           |   |    | P. P.             |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

76°

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| 76°        |         |             |    | 77°        |          |             |    | P. P. |  |  |  |
|------------|---------|-------------|----|------------|----------|-------------|----|-------|--|--|--|
| Log. Vers. | D       | Log. Exsec. | D  | Log. Vers. | D        | Log. Exsec. | D  |       |  |  |  |
| 0          | 9.87971 | 10.49604    | 66 | 9.88933    | 10.53724 | 70          | 0  |       |  |  |  |
| 1          | .87987  | .49670      | 67 | .88949     | .53794   | 71          | 1  |       |  |  |  |
| 2          | .88003  | .49737      | 67 | .88964     | .53865   | 71          | 2  |       |  |  |  |
| 3          | .88020  | .49804      | 67 | .88980     | .53936   | 71          | 3  |       |  |  |  |
| 4          | .88036  | .49871      | 67 | .88996     | .54007   | 71          | 4  |       |  |  |  |
| 5          | 9.88052 | 10.49939    | 67 | 9.89012    | 10.54078 | 71          | 5  |       |  |  |  |
| 6          | .88068  | .50006      | 67 | .89028     | .54149   | 71          | 6  |       |  |  |  |
| 7          | .88084  | .50073      | 67 | .89044     | .54220   | 71          | 7  |       |  |  |  |
| 8          | .88100  | .50140      | 67 | .89060     | .54291   | 71          | 8  |       |  |  |  |
| 9          | .88116  | .50208      | 67 | .89075     | .54362   | 71          | 9  |       |  |  |  |
| 10         | 9.88133 | 10.50275    | 67 | 9.89091    | 10.54433 | 71          | 10 |       |  |  |  |
| 11         | .88149  | .50342      | 67 | .89107     | .54505   | 71          | 11 |       |  |  |  |
| 12         | .88165  | .50410      | 67 | .89123     | .54576   | 71          | 12 |       |  |  |  |
| 13         | .88181  | .50477      | 68 | .89139     | .54647   | 72          | 13 |       |  |  |  |
| 14         | .88197  | .50545      | 67 | .89155     | .54719   | 71          | 14 |       |  |  |  |
| 15         | 9.88213 | 10.50613    | 68 | 9.89170    | 10.54791 | 71          | 15 |       |  |  |  |
| 16         | .88229  | .50681      | 67 | .89186     | .54862   | 72          | 16 |       |  |  |  |
| 17         | .88245  | .50748      | 68 | .89202     | .54934   | 71          | 17 |       |  |  |  |
| 18         | .88261  | .50816      | 68 | .89218     | .55006   | 72          | 18 |       |  |  |  |
| 19         | .88277  | .50884      | 68 | .89234     | .55078   | 72          | 19 |       |  |  |  |
| 20         | 9.88294 | 10.50952    | 68 | 9.89249    | 10.55150 | 72          | 20 |       |  |  |  |
| 21         | .88310  | .51020      | 68 | .89265     | .55222   | 72          | 21 |       |  |  |  |
| 22         | .88326  | .51088      | 68 | .89281     | .55294   | 72          | 22 |       |  |  |  |
| 23         | .88342  | .51157      | 68 | .89297     | .55366   | 72          | 23 |       |  |  |  |
| 24         | .88358  | .51225      | 68 | .89312     | .55438   | 72          | 24 |       |  |  |  |
| 25         | 9.88374 | 10.51293    | 68 | 9.89328    | 10.55511 | 72          | 25 |       |  |  |  |
| 26         | .88390  | .51361      | 68 | .89344     | .55583   | 72          | 26 |       |  |  |  |
| 27         | .88406  | .51430      | 68 | .89360     | .55655   | 73          | 27 |       |  |  |  |
| 28         | .88422  | .51498      | 68 | .89376     | .55728   | 72          | 28 |       |  |  |  |
| 29         | .88438  | .51567      | 69 | .89391     | .55801   | 72          | 29 |       |  |  |  |
| 30         | 9.88454 | 10.51636    | 68 | 9.89407    | 10.55873 | 73          | 30 |       |  |  |  |
| 31         | .88470  | .51704      | 68 | .89423     | .55946   | 72          | 31 |       |  |  |  |
| 32         | .88486  | .51773      | 69 | .89438     | .56019   | 73          | 32 |       |  |  |  |
| 33         | .88502  | .51842      | 69 | .89454     | .56092   | 73          | 33 |       |  |  |  |
| 34         | .88518  | .51911      | 69 | .89470     | .56165   | 73          | 34 |       |  |  |  |
| 35         | 9.88534 | 10.51980    | 69 | 9.89486    | 10.56238 | 73          | 35 |       |  |  |  |
| 36         | .88550  | .52049      | 69 | .89501     | .56311   | 73          | 36 |       |  |  |  |
| 37         | .88566  | .52118      | 69 | .89517     | .56384   | 73          | 37 |       |  |  |  |
| 38         | .88582  | .52187      | 69 | .89533     | .56457   | 73          | 38 |       |  |  |  |
| 39         | .88598  | .52256      | 69 | .89548     | .56531   | 73          | 39 |       |  |  |  |
| 40         | 9.88614 | 10.52325    | 69 | 9.89564    | 10.56604 | 73          | 40 |       |  |  |  |
| 41         | .88630  | .52394      | 69 | .89580     | .56678   | 73          | 41 |       |  |  |  |
| 42         | .88646  | .52464      | 69 | .89596     | .56751   | 74          | 42 |       |  |  |  |
| 43         | .88662  | .52533      | 69 | .89611     | .56825   | 73          | 43 |       |  |  |  |
| 44         | .88678  | .52603      | 69 | .89627     | .56899   | 74          | 44 |       |  |  |  |
| 45         | 9.88694 | 10.52672    | 70 | 9.89643    | 10.56973 | 74          | 45 |       |  |  |  |
| 46         | .88710  | .52742      | 69 | .89658     | .57047   | 73          | 46 |       |  |  |  |
| 47         | .88726  | .52812      | 69 | .89674     | .57120   | 74          | 47 |       |  |  |  |
| 48         | .88742  | .52881      | 70 | .89690     | .57195   | 74          | 48 |       |  |  |  |
| 49         | .88758  | .52951      | 70 | .89705     | .57269   | 74          | 49 |       |  |  |  |
| 50         | 9.88774 | 10.53021    | 70 | 9.89721    | 10.57343 | 74          | 50 |       |  |  |  |
| 51         | .88790  | .53091      | 70 | .89737     | .57417   | 74          | 51 |       |  |  |  |
| 52         | .88805  | .53161      | 70 | .89752     | .57491   | 74          | 52 |       |  |  |  |
| 53         | .88821  | .53231      | 70 | .89768     | .57566   | 74          | 53 |       |  |  |  |
| 54         | .88837  | .53301      | 70 | .89783     | .57640   | 75          | 54 |       |  |  |  |
| 55         | 9.88853 | 10.53372    | 70 | 9.89799    | 10.57715 | 75          | 55 |       |  |  |  |
| 56         | .88869  | .53442      | 70 | .89815     | .57790   | 74          | 56 |       |  |  |  |
| 57         | .88885  | .53512      | 70 | .89830     | .57864   | 75          | 57 |       |  |  |  |
| 58         | .88901  | .53583      | 70 | .89846     | .57939   | 75          | 58 |       |  |  |  |
| 59         | .88917  | .53653      | 70 | .89862     | .58014   | 75          | 59 |       |  |  |  |
| 60         | 9.88933 | 10.53724    | 70 | 9.89877    | 10.58089 | 75          | 60 |       |  |  |  |
| Log. Vers. | D       | Log. Exsec. | D  | Log. Vers. | D        | Log. Exsec. | D  | P. P. |  |  |  |

| 75 74 73 |      |      |      |
|----------|------|------|------|
| 6        | 7.5  | 7.4  | 7.3  |
| 7        | 8.7  | 8.2  | 8.5  |
| 8        | 10.0 | 9.8  | 9.7  |
| 9        | 11.2 | 11.1 | 10.9 |
| 10       | 12.5 | 12.3 | 12.1 |
| 20       | 25.0 | 24.6 | 24.3 |
| 30       | 37.5 | 37.0 | 36.5 |
| 40       | 50.0 | 49.3 | 48.6 |
| 50       | 62.5 | 61.0 | 60.8 |

| 72 71 70 |      |      |      |
|----------|------|------|------|
| 6        | 7.2  | 7.1  | 7.0  |
| 7        | 8.4  | 8.3  | 8.2  |
| 8        | 9.6  | 9.4  | 9.4  |
| 9        | 10.8 | 10.6 | 10.6 |
| 10       | 12.0 | 11.8 | 11.7 |
| 20       | 24.0 | 23.6 | 23.3 |
| 30       | 36.0 | 35.5 | 35.2 |
| 40       | 48.0 | 47.3 | 47.0 |
| 50       | 60.0 | 59.1 | 58.7 |

| 69 68 67 |      |      |      |
|----------|------|------|------|
| 6        | 6.9  | 6.8  | 6.7  |
| 7        | 8.0  | 7.9  | 7.8  |
| 8        | 9.2  | 9.0  | 8.9  |
| 9        | 10.3 | 10.2 | 10.0 |
| 10       | 11.5 | 11.3 | 11.1 |
| 20       | 23.0 | 22.6 | 22.3 |
| 30       | 34.5 | 34.0 | 33.5 |
| 40       | 46.0 | 45.3 | 44.6 |
| 50       | 57.5 | 56.6 | 55.8 |

| 66 65 |      |      |      |
|-------|------|------|------|
| 6     | 6.6  | 6.5  | 6.4  |
| 7     | 7.7  | 7.6  | 7.5  |
| 8     | 8.8  | 8.7  | 8.6  |
| 9     | 9.9  | 9.8  | 9.7  |
| 10    | 11.0 | 10.9 | 10.8 |
| 20    | 22.0 | 21.8 | 21.6 |
| 30    | 33.0 | 32.7 | 32.4 |
| 40    | 44.0 | 43.6 | 43.2 |
| 50    | 55.0 | 54.4 | 53.8 |

| 16 15 14 |      |      |      |
|----------|------|------|------|
| 6        | 1.6  | 1.6  | 1.5  |
| 7        | 1.9  | 1.8  | 1.8  |
| 8        | 2.2  | 2.1  | 2.0  |
| 9        | 2.5  | 2.4  | 2.3  |
| 10       | 2.7  | 2.6  | 2.6  |
| 20       | 5.5  | 5.3  | 5.1  |
| 30       | 8.2  | 8.0  | 7.7  |
| 40       | 11.0 | 10.6 | 10.3 |
| 50       | 13.7 | 13.3 | 12.9 |



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|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P.             |
|----|------------|----|-------------|----|------------|----|-------------|----|----|-------------------|
| 0  | 9.89877    | 13 | 10.58086    | 75 | 9.90805    | 13 | 10.62745    | 86 | 0  |                   |
| 1  | .89893     | 13 | .58164      | 75 | .90820     | 13 | .62825      | 86 | 1  |                   |
| 2  | .89908     | 13 | .58239      | 75 | .90835     | 13 | .62906      | 86 | 2  |                   |
| 3  | .89924     | 13 | .58315      | 75 | .90851     | 13 | .62986      | 81 | 3  | 86 85 84          |
| 4  | .89939     | 13 | .58390      | 75 | .90866     | 13 | .63067      | 81 | 4  | 6 8.6 8.5 8.4     |
| 5  | 9.89955    | 16 | 10.58463    | 75 | 9.90881    | 13 | 10.63148    | 81 | 5  | 7 10.0 9.5 9.8    |
| 6  | .89971     | 13 | .58541      | 75 | .90897     | 13 | .63229      | 81 | 6  | 8 11.4 11.3 11.2  |
| 7  | .89986     | 13 | .58616      | 76 | .90912     | 13 | .63310      | 81 | 7  | 9 12.0 12.7 12.6  |
| 8  | .90002     | 13 | .58692      | 75 | .90927     | 13 | .63391      | 81 | 8  | 10 14.3 14.1 14.0 |
| 9  | .90017     | 13 | .58768      | 75 | .90943     | 13 | .63472      | 81 | 9  | 20 28.6 28.3 28.0 |
| 10 | 9.90033    | 13 | 10.58844    | 76 | 9.90958    | 13 | 10.63553    | 81 | 10 | 30 43.0 42.5 42.0 |
| 11 | .90048     | 13 | .58920      | 75 | .90973     | 13 | .63634      | 81 | 11 | 40 57.3 56.7 56.0 |
| 12 | .90064     | 13 | .58995      | 75 | .90988     | 13 | .63716      | 81 | 12 | 50 71.6 70.8 70.0 |
| 13 | .90080     | 16 | .59072      | 76 | .91004     | 13 | .63797      | 81 | 13 |                   |
| 14 | .90095     | 13 | .59148      | 76 | .91019     | 13 | .63879      | 82 | 14 | 83 82 81          |
| 15 | 9.90111    | 13 | 10.59224    | 76 | 9.91034    | 13 | 10.63961    | 82 | 15 | 6 8.3 8.2 8.1     |
| 16 | .90126     | 13 | .59300      | 76 | .91049     | 13 | .64043      | 82 | 16 | 7 9.7 9.5 9.4     |
| 17 | .90142     | 13 | .59377      | 76 | .91065     | 13 | .64125      | 82 | 17 | 8 11.0 10.8 10.8  |
| 18 | .90157     | 13 | .59453      | 76 | .91080     | 13 | .64207      | 82 | 18 | 9 12.4 12.2 12.1  |
| 19 | .90173     | 13 | .59530      | 76 | .91095     | 13 | .64289      | 82 | 19 | 10 13.8 13.5 13.5 |
| 20 | 9.90188    | 13 | 10.59606    | 77 | 9.91110    | 13 | 10.64371    | 82 | 20 | 20 27.6 27.3 27.0 |
| 21 | .90204     | 13 | .59683      | 76 | .91126     | 13 | .64453      | 82 | 21 | 30 41.5 41.0 40.5 |
| 22 | .90219     | 13 | .59760      | 77 | .91141     | 13 | .64536      | 82 | 22 | 40 55.3 54.7 54.0 |
| 23 | .90235     | 13 | .59837      | 77 | .91156     | 13 | .64618      | 83 | 23 | 50 69.1 68.3 67.5 |
| 24 | .90250     | 13 | .59914      | 77 | .91171     | 13 | .64701      | 82 | 24 |                   |
| 25 | 9.90266    | 13 | 10.59991    | 77 | 9.91187    | 13 | 10.64784    | 83 | 25 | 80 79 78          |
| 26 | .90281     | 13 | .60068      | 77 | .91202     | 13 | .64867      | 83 | 26 | 6 8.0 7.9 7.8     |
| 27 | .90297     | 13 | .60145      | 77 | .91217     | 13 | .64950      | 83 | 27 | 7 9.3 9.2 9.1     |
| 28 | .90312     | 13 | .60223      | 77 | .91232     | 13 | .65033      | 83 | 28 | 8 10.6 10.4 10.4  |
| 29 | .90328     | 13 | .60300      | 77 | .91247     | 13 | .65116      | 83 | 29 | 9 12.0 11.8 11.7  |
| 30 | 9.90343    | 13 | 10.60378    | 77 | 9.91263    | 13 | 10.65199    | 83 | 30 | 10 13.3 13.1 13.0 |
| 31 | .90359     | 13 | .60455      | 77 | .91278     | 13 | .65283      | 83 | 31 | 20 26.6 26.3 26.0 |
| 32 | .90374     | 13 | .60533      | 77 | .91293     | 13 | .65366      | 83 | 32 | 30 40.0 39.5 39.0 |
| 33 | .90389     | 13 | .60611      | 78 | .91308     | 13 | .65450      | 84 | 33 | 40 53.3 52.6 52.0 |
| 34 | .90405     | 13 | .60688      | 77 | .91323     | 13 | .65534      | 84 | 34 | 50 66.6 65.8 65.0 |
| 35 | 9.90420    | 13 | 10.60766    | 78 | 9.91338    | 13 | 10.65617    | 84 | 35 |                   |
| 36 | .90436     | 13 | .60844      | 78 | .91354     | 13 | .65701      | 84 | 36 | 77 76 75          |
| 37 | .90451     | 13 | .60923      | 78 | .91369     | 13 | .65785      | 84 | 37 | 6 7.7 7.6 7.5     |
| 38 | .90467     | 13 | .61001      | 78 | .91384     | 13 | .65870      | 84 | 38 | 7 9.0 8.8 8.7     |
| 39 | .90482     | 13 | .61079      | 78 | .91399     | 13 | .65954      | 84 | 39 | 8 10.2 10.1 10.0  |
| 40 | 9.90497    | 13 | 10.61158    | 78 | 9.91414    | 13 | 10.66038    | 84 | 40 | 9 11.5 11.4 11.2  |
| 41 | .90513     | 13 | .61236      | 78 | .91429     | 13 | .66123      | 84 | 41 | 10 12.8 12.6 12.5 |
| 42 | .90528     | 13 | .61315      | 78 | .91445     | 13 | .66207      | 84 | 42 | 20 25.6 25.3 25.0 |
| 43 | .90544     | 13 | .61393      | 78 | .91460     | 13 | .66292      | 85 | 43 | 30 38.7 38.0 37.5 |
| 44 | .90559     | 13 | .61472      | 79 | .91475     | 13 | .66377      | 85 | 44 | 40 51.3 50.6 50.0 |
| 45 | 9.90574    | 13 | 10.61551    | 78 | 9.91490    | 13 | 10.66462    | 85 | 45 | 50 64.1 63.3 62.5 |
| 46 | .90590     | 13 | .61630      | 79 | .91505     | 13 | .66547      | 85 | 46 |                   |
| 47 | .90605     | 13 | .61709      | 79 | .91520     | 13 | .66632      | 85 | 47 | 0                 |
| 48 | .90621     | 13 | .61788      | 79 | .91535     | 13 | .66717      | 85 | 48 | 6 0.0 0.0 0.0     |
| 49 | .90636     | 13 | .61867      | 79 | .91550     | 13 | .66803      | 85 | 49 | 7 0.0 0.0 0.0     |
| 50 | 9.90651    | 13 | 10.61947    | 79 | 9.91565    | 13 | 10.66888    | 85 | 50 | 8 0.1 0.1 0.1     |
| 51 | .90667     | 13 | .62026      | 79 | .91581     | 13 | .66974      | 85 | 51 | 9 0.1 0.1 0.1     |
| 52 | .90682     | 13 | .62105      | 80 | .91596     | 13 | .67059      | 86 | 52 | 10 0.1 0.1 0.1    |
| 53 | .90697     | 13 | .62185      | 79 | .91611     | 13 | .67145      | 86 | 53 | 20 0.2 0.2 0.2    |
| 54 | .90713     | 13 | .62265      | 80 | .91626     | 13 | .67231      | 86 | 54 | 30 0.3 0.3 0.3    |
| 55 | 9.90728    | 13 | 10.62345    | 79 | 9.91641    | 13 | 10.67317    | 86 | 55 | 40 0.4 0.4 0.4    |
| 56 | .90744     | 13 | .62424      | 80 | .91656     | 13 | .67403      | 86 | 56 |                   |
| 57 | .90759     | 13 | .62504      | 80 | .91671     | 13 | .67490      | 86 | 57 | 16 1.5 1.5 1.5    |
| 58 | .90774     | 13 | .62585      | 80 | .91686     | 13 | .67576      | 86 | 58 | 7 1.8 1.8 1.7     |
| 59 | .90790     | 13 | .62665      | 80 | .91701     | 13 | .67663      | 86 | 59 | 8 2.1 2.0 2.0     |
| 60 | 9.90805    | 15 | 10.62745    |    | 9.91716    |    | 10.67749    |    | 60 | 9 2.4 2.3 2.2     |
|    | Log. Vers. | D  | Log. Exsec. | D  | Log. Vers. | D  | Log. Exsec. | D  |    | P. P.             |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    |         | Log. Vers. | D        | Log. Exsec. | D       |    |          | Log. Vers. | D  | Log. Exsec. | D |  |  | P. P. |  |
|----|---------|------------|----------|-------------|---------|----|----------|------------|----|-------------|---|--|--|-------|--|
| 0  | 9.91716 | 15         | 10.67749 | 86          | 9.92612 | 14 | 10.73178 | 95         | 0  |             |   |  |  |       |  |
| 1  | .91731  | 15         | .67836   | 87          | .92626  | 15 | .73273   | 94         | 1  |             |   |  |  |       |  |
| 2  | .91746  | 15         | .67923   | 87          | .92641  | 14 | .73368   | 95         | 2  |             |   |  |  |       |  |
| 3  | .91761  | 15         | .68010   | 87          | .92656  | 15 | .73463   | 95         | 3  |             |   |  |  |       |  |
| 4  | .91776  | 15         | .68097   | 87          | .92671  | 15 | .73558   | 95         | 4  |             |   |  |  |       |  |
| 5  | 9.91791 | 15         | 10.68184 | 87          | 9.92686 | 15 | 10.73653 | 95         | 5  |             |   |  |  |       |  |
| 6  | .91807  | 15         | .68272   | 87          | .92700  | 14 | .73748   | 95         | 6  |             |   |  |  |       |  |
| 7  | .91822  | 15         | .68359   | 87          | .92715  | 15 | .73844   | 96         | 7  |             |   |  |  |       |  |
| 8  | .91837  | 15         | .68447   | 87          | .92730  | 14 | .73940   | 95         | 8  |             |   |  |  |       |  |
| 9  | .91852  | 15         | .68534   | 88          | .92745  | 15 | .74035   | 96         | 9  |             |   |  |  |       |  |
| 10 | 9.91867 | 15         | 10.68622 | 88          | 9.92759 | 14 | 10.74131 | 96         | 10 |             |   |  |  |       |  |
| 11 | .91882  | 15         | .68710   | 88          | .92774  | 15 | .74227   | 96         | 11 |             |   |  |  |       |  |
| 12 | .91897  | 15         | .68798   | 88          | .92789  | 14 | .74324   | 96         | 12 |             |   |  |  |       |  |
| 13 | .91912  | 15         | .68886   | 88          | .92804  | 15 | .74420   | 96         | 13 |             |   |  |  |       |  |
| 14 | .91927  | 15         | .68975   | 88          | .92818  | 14 | .74517   | 96         | 14 |             |   |  |  |       |  |
| 15 | 9.91942 | 15         | 10.69063 | 88          | 9.92833 | 15 | 10.74613 | 97         | 15 |             |   |  |  |       |  |
| 16 | .91957  | 15         | .69152   | 88          | .92848  | 14 | .74710   | 97         | 16 |             |   |  |  |       |  |
| 17 | .91972  | 15         | .69240   | 88          | .92862  | 14 | .74807   | 97         | 17 |             |   |  |  |       |  |
| 18 | .91987  | 15         | .69329   | 89          | .92877  | 15 | .74905   | 97         | 18 |             |   |  |  |       |  |
| 19 | .92002  | 15         | .69418   | 89          | .92892  | 14 | .75002   | 97         | 19 |             |   |  |  |       |  |
| 20 | 9.92016 | 14         | 10.69507 | 89          | 9.92907 | 15 | 10.75099 | 97         | 20 |             |   |  |  |       |  |
| 21 | .92031  | 15         | .69596   | 89          | .92921  | 14 | .75197   | 98         | 21 |             |   |  |  |       |  |
| 22 | .92046  | 15         | .69686   | 89          | .92936  | 15 | .75295   | 98         | 22 |             |   |  |  |       |  |
| 23 | .92061  | 15         | .69775   | 89          | .92951  | 14 | .75393   | 98         | 23 |             |   |  |  |       |  |
| 24 | .92076  | 15         | .69865   | 90          | .92965  | 15 | .75491   | 98         | 24 |             |   |  |  |       |  |
| 25 | 9.92091 | 15         | 10.69955 | 89          | 9.92980 | 14 | 10.75589 | 98         | 25 |             |   |  |  |       |  |
| 26 | .92106  | 15         | .70044   | 90          | .92995  | 14 | .75688   | 98         | 26 |             |   |  |  |       |  |
| 27 | .92121  | 15         | .70134   | 90          | .93009  | 14 | .75786   | 98         | 27 |             |   |  |  |       |  |
| 28 | .92136  | 15         | .70224   | 90          | .93024  | 15 | .75885   | 99         | 28 |             |   |  |  |       |  |
| 29 | .92151  | 14         | .70315   | 90          | .93039  | 14 | .75984   | 99         | 29 |             |   |  |  |       |  |
| 30 | 9.92166 | 15         | 10.70405 | 90          | 9.93053 | 14 | 10.76083 | 99         | 30 |             |   |  |  |       |  |
| 31 | .92181  | 15         | .70495   | 91          | .93068  | 15 | .76182   | 99         | 31 |             |   |  |  |       |  |
| 32 | .92196  | 15         | .70586   | 91          | .93083  | 14 | .76282   | 100        | 32 |             |   |  |  |       |  |
| 33 | .92211  | 15         | .70677   | 91          | .93097  | 15 | .76382   | 99         | 33 |             |   |  |  |       |  |
| 34 | .92226  | 15         | .70768   | 91          | .93112  | 14 | .76481   | 99         | 34 |             |   |  |  |       |  |
| 35 | 9.92240 | 14         | 10.70859 | 91          | 9.93127 | 14 | 10.76581 | 100        | 35 |             |   |  |  |       |  |
| 36 | .92255  | 15         | .70950   | 91          | .93141  | 14 | .76681   | 100        | 36 |             |   |  |  |       |  |
| 37 | .92270  | 15         | .71041   | 91          | .93156  | 14 | .76782   | 100        | 37 |             |   |  |  |       |  |
| 38 | .92285  | 15         | .71133   | 91          | .93171  | 15 | .76882   | 100        | 38 |             |   |  |  |       |  |
| 39 | .92300  | 14         | .71224   | 91          | .93185  | 14 | .76983   | 100        | 39 |             |   |  |  |       |  |
| 40 | 9.92315 | 15         | 10.71316 | 92          | 9.93200 | 14 | 10.77083 | 101        | 40 |             |   |  |  |       |  |
| 41 | .92330  | 15         | .71408   | 92          | .93214  | 15 | .77184   | 101        | 41 |             |   |  |  |       |  |
| 42 | .92345  | 15         | .71500   | 92          | .93229  | 14 | .77286   | 101        | 42 |             |   |  |  |       |  |
| 43 | .92360  | 14         | .71592   | 92          | .93244  | 14 | .77387   | 101        | 43 |             |   |  |  |       |  |
| 44 | .92374  | 15         | .71684   | 92          | .93258  | 14 | .77488   | 101        | 44 |             |   |  |  |       |  |
| 45 | 9.92389 | 15         | 10.71776 | 92          | 9.93273 | 14 | 10.77590 | 102        | 45 |             |   |  |  |       |  |
| 46 | .92404  | 14         | .71869   | 92          | .93287  | 15 | .77692   | 102        | 46 |             |   |  |  |       |  |
| 47 | .92419  | 15         | .71961   | 93          | .93302  | 14 | .77794   | 102        | 47 |             |   |  |  |       |  |
| 48 | .92434  | 15         | .72054   | 92          | .93317  | 14 | .77896   | 102        | 48 |             |   |  |  |       |  |
| 49 | .92449  | 14         | .72147   | 93          | .93331  | 14 | .77998   | 102        | 49 |             |   |  |  |       |  |
| 50 | 9.92463 | 15         | 10.72240 | 93          | 9.93346 | 14 | 10.78101 | 102        | 50 |             |   |  |  |       |  |
| 51 | .92478  | 15         | .72333   | 93          | .93360  | 14 | .78203   | 103        | 51 |             |   |  |  |       |  |
| 52 | .92493  | 14         | .72427   | 93          | .93375  | 15 | .78306   | 103        | 52 |             |   |  |  |       |  |
| 53 | .92508  | 15         | .72520   | 93          | .93389  | 14 | .78409   | 103        | 53 |             |   |  |  |       |  |
| 54 | .92523  | 15         | .72614   | 93          | .93404  | 14 | .78513   | 103        | 54 |             |   |  |  |       |  |
| 55 | 9.92538 | 14         | 10.72707 | 94          | 9.93419 | 14 | 10.78616 | 104        | 55 |             |   |  |  |       |  |
| 56 | .92552  | 15         | .72801   | 94          | .93433  | 14 | .78720   | 103        | 56 |             |   |  |  |       |  |
| 57 | .92567  | 15         | .72895   | 94          | .93448  | 14 | .78823   | 104        | 57 |             |   |  |  |       |  |
| 58 | .92582  | 14         | .72990   | 94          | .93462  | 14 | .78927   | 104        | 58 |             |   |  |  |       |  |
| 59 | .92597  | 15         | .73084   | 94          | .93477  | 14 | .79031   | 104        | 59 |             |   |  |  |       |  |
| 60 | 9.92612 |            | 10.73178 |             | 9.93491 |    | 10.79136 |            | 60 |             |   |  |  |       |  |
|    |         | Log. Vers. | D        | Log. Exsec. | D       |    |          | Log. Vers. | D  | Log. Exsec. | D |  |  | P. P. |  |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D  | Log. Exsec. | D   |    | P. P. |
|----|------------|----|-------------|-----|------------|----|-------------|-----|----|-------|
| 0  | 9.93491    | 14 | 10.79136    | 104 | 9.94356    | 14 | 10.85766    | 117 | 0  |       |
| 1  | .93506     | 14 | .79240      | 105 | .94370     | 14 | .85884      | 117 | 1  |       |
| 2  | .93520     | 14 | .79345      | 104 | .94384     | 14 | .86001      | 117 | 2  |       |
| 3  | .93535     | 14 | .79450      | 105 | .94398     | 14 | .86119      | 118 | 3  |       |
| 4  | .93549     | 14 | .79555      | 103 | .94413     | 14 | .86237      | 118 | 4  |       |
| 5  | 9.93564    | 14 | 10.79660    | 105 | 9.94427    | 14 | 10.86355    | 118 | 5  |       |
| 6  | .93578     | 14 | .79766      | 105 | .94441     | 14 | .86474      | 118 | 6  |       |
| 7  | .93593     | 14 | .79871      | 106 | .94456     | 14 | .86592      | 119 | 7  |       |
| 8  | .93607     | 14 | .79977      | 106 | .94470     | 14 | .86711      | 119 | 8  |       |
| 9  | .93622     | 14 | .80083      | 106 | .94484     | 14 | .86831      | 119 | 9  |       |
| 10 | 9.93636    | 14 | 10.80189    | 106 | 9.94498    | 14 | 10.86950    | 120 | 10 |       |
| 11 | .93651     | 14 | .80296      | 106 | .94512     | 14 | .87070      | 120 | 11 |       |
| 12 | .93665     | 14 | .80402      | 107 | .94527     | 14 | .87190      | 120 | 12 |       |
| 13 | .93680     | 14 | .80509      | 107 | .94541     | 14 | .87310      | 120 | 13 |       |
| 14 | .93694     | 14 | .80616      | 107 | .94555     | 14 | .87431      | 121 | 14 |       |
| 15 | 9.93709    | 14 | 10.80723    | 107 | 9.94569    | 14 | 10.87552    | 121 | 15 |       |
| 16 | .93723     | 14 | .80831      | 107 | .94584     | 14 | .87673      | 121 | 16 |       |
| 17 | .93738     | 14 | .80938      | 108 | .94598     | 14 | .87794      | 121 | 17 |       |
| 18 | .93752     | 14 | .81046      | 108 | .94612     | 14 | .87916      | 122 | 18 |       |
| 19 | .93767     | 14 | .81154      | 108 | .94626     | 14 | .88038      | 122 | 19 |       |
| 20 | 9.93781    | 14 | 10.81262    | 108 | 9.94640    | 14 | 10.88160    | 122 | 20 |       |
| 21 | .93796     | 14 | .81371      | 108 | .94655     | 14 | .88282      | 122 | 21 |       |
| 22 | .93810     | 14 | .81479      | 109 | .94669     | 14 | .88405      | 123 | 22 |       |
| 23 | .93824     | 14 | .81588      | 109 | .94683     | 14 | .88528      | 123 | 23 |       |
| 24 | .93839     | 14 | .81697      | 109 | .94697     | 14 | .88651      | 124 | 24 |       |
| 25 | 9.93853    | 14 | 10.81806    | 109 | 9.94711    | 14 | 10.88775    | 124 | 25 |       |
| 26 | .93868     | 14 | .81916      | 109 | .94726     | 14 | .88898      | 124 | 26 |       |
| 27 | .93882     | 14 | .82025      | 110 | .94740     | 14 | .89022      | 124 | 27 |       |
| 28 | .93897     | 14 | .82135      | 110 | .94754     | 14 | .89147      | 124 | 28 |       |
| 29 | .93911     | 14 | .82245      | 110 | .94768     | 14 | .89271      | 125 | 29 |       |
| 30 | 9.93925    | 14 | 10.82356    | 110 | 9.94782    | 14 | 10.89396    | 125 | 30 |       |
| 31 | .93940     | 14 | .82466      | 110 | .94796     | 14 | .89521      | 125 | 31 |       |
| 32 | .93954     | 14 | .82577      | 111 | .94810     | 14 | .89647      | 126 | 32 |       |
| 33 | .93969     | 14 | .82688      | 111 | .94825     | 14 | .89773      | 126 | 33 |       |
| 34 | .93983     | 14 | .82799      | 111 | .94839     | 14 | .89899      | 126 | 34 |       |
| 35 | 9.93997    | 14 | 10.82910    | 111 | 9.94853    | 14 | 10.90025    | 126 | 35 |       |
| 36 | .94012     | 14 | .83022      | 111 | .94867     | 14 | .90152      | 127 | 36 |       |
| 37 | .94026     | 14 | .83133      | 112 | .94881     | 14 | .90279      | 127 | 37 |       |
| 38 | .94041     | 14 | .83245      | 112 | .94895     | 14 | .90406      | 127 | 38 |       |
| 39 | .94055     | 14 | .83358      | 112 | .94909     | 14 | .90533      | 128 | 39 |       |
| 40 | 9.94069    | 14 | 10.83470    | 112 | 9.94923    | 14 | 10.90661    | 127 | 40 |       |
| 41 | .94084     | 14 | .83583      | 112 | .94938     | 14 | .90789      | 128 | 41 |       |
| 42 | .94098     | 14 | .83695      | 113 | .94952     | 14 | .90917      | 129 | 42 |       |
| 43 | .94112     | 14 | .83809      | 113 | .94966     | 14 | .91046      | 129 | 43 |       |
| 44 | .94127     | 14 | .83922      | 113 | .94980     | 14 | .91175      | 129 | 44 |       |
| 45 | 9.94141    | 14 | 10.84035    | 114 | 9.94994    | 14 | 10.91304    | 130 | 45 |       |
| 46 | .94155     | 14 | .84149      | 114 | .95008     | 14 | .91434      | 129 | 46 |       |
| 47 | .94170     | 14 | .84263      | 114 | .95022     | 14 | .91564      | 130 | 47 |       |
| 48 | .94184     | 14 | .84377      | 114 | .95036     | 14 | .91694      | 130 | 48 |       |
| 49 | .94198     | 14 | .84492      | 115 | .95050     | 14 | .91825      | 131 | 49 |       |
| 50 | 9.94213    | 14 | 10.84607    | 114 | 9.95064    | 14 | 10.91956    | 131 | 50 |       |
| 51 | .94227     | 14 | .84721      | 115 | .95078     | 14 | .92087      | 131 | 51 |       |
| 52 | .94241     | 14 | .84837      | 115 | .95093     | 14 | .92218      | 131 | 52 |       |
| 53 | .94256     | 14 | .84952      | 116 | .95107     | 14 | .92350      | 132 | 53 |       |
| 54 | .94270     | 14 | .85068      | 115 | .95121     | 14 | .92482      | 132 | 54 |       |
| 55 | 9.94284    | 14 | 10.85183    | 116 | 9.95135    | 14 | 10.92614    | 133 | 55 |       |
| 56 | .94299     | 14 | .85299      | 116 | .95149     | 14 | .92747      | 133 | 56 |       |
| 57 | .94313     | 14 | .85416      | 116 | .95163     | 14 | .92880      | 133 | 57 |       |
| 58 | .94327     | 14 | .85533      | 117 | .95177     | 14 | .93014      | 133 | 58 |       |
| 59 | .94341     | 14 | .85649      | 117 | .95191     | 14 | .93147      | 134 | 59 |       |
| 60 | 9.94356    |    | 10.85766    |     | 9.95205    |    | 10.93281    |     | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D  | Log. Exsec. | D   |    | P. P. |

130 120

|    |       |       |
|----|-------|-------|
| 6  | 13.0  | 12.0  |
| 7  | 15.1  | 14.0  |
| 8  | 17.3  | 16.0  |
| 9  | 19.5  | 18.0  |
| 10 | 21.7  | 20.0  |
| 20 | 43.3  | 40.0  |
| 30 | 65.0  | 60.0  |
| 40 | 86.6  | 80.0  |
| 50 | 108.3 | 100.0 |

110 100

|    |      |      |
|----|------|------|
| 6  | 11.0 | 10.0 |
| 7  | 12.6 | 11.6 |
| 8  | 14.6 | 13.3 |
| 9  | 16.5 | 15.0 |
| 10 | 18.3 | 16.6 |
| 20 | 36.6 | 33.3 |
| 30 | 55.0 | 50.0 |
| 40 | 73.3 | 66.6 |
| 50 | 91.6 | 83.3 |

3 2

|    |     |     |
|----|-----|-----|
| 6  | 0.3 | 0.2 |
| 7  | 0.3 | 0.2 |
| 8  | 0.4 | 0.2 |
| 9  | 0.4 | 0.3 |
| 10 | 0.5 | 0.3 |
| 20 | 1.0 | 0.6 |
| 30 | 1.5 | 1.0 |
| 40 | 2.0 | 1.3 |
| 50 | 2.5 | 1.6 |

1 0

|    |     |     |
|----|-----|-----|
| 6  | 0.1 | 0.0 |
| 7  | 0.1 | 0.0 |
| 8  | 0.1 | 0.0 |
| 9  | 0.1 | 0.1 |
| 10 | 0.1 | 0.1 |
| 20 | 0.3 | 0.1 |
| 30 | 0.5 | 0.2 |
| 40 | 0.7 | 0.3 |
| 50 | 0.8 | 0.4 |

14 14

|    |      |      |
|----|------|------|
| 6  | 1.4  | 1.4  |
| 7  | 1.7  | 1.6  |
| 8  | 1.9  | 1.8  |
| 9  | 2.2  | 2.1  |
| 10 | 2.4  | 2.3  |
| 20 | 4.8  | 4.6  |
| 30 | 7.2  | 7.0  |
| 40 | 9.6  | 9.3  |
| 50 | 12.1 | 11.6 |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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| 84° |            |    |             | 85° |            |            |             | P. P.       |    |       |  |
|-----|------------|----|-------------|-----|------------|------------|-------------|-------------|----|-------|--|
|     | Log. Vers. | D  | Log. Exsec. | D   |            | Log. Vers. | D           | Log. Exsec. | D  |       |  |
| 0   | 9.95205    | 14 | 10.93281    | 134 | 9.96039    | 14         | 11.02010    | 158         | 0  |       |  |
| 1   | .95219     | 14 | .93416      | 135 | .96053     | 13         | .02168      | 159         | 1  |       |  |
| 2   | .95233     | 14 | .93551      | 135 | .96067     | 14         | .02327      | 159         | 2  |       |  |
| 3   | .95247     | 14 | .93686      | 135 | .96081     | 14         | .02487      | 159         | 3  |       |  |
| 4   | .95261     | 14 | .93821      | 135 | .96095     | 14         | .02646      | 160         | 4  |       |  |
| 5   | 9.95275    | 14 | 10.93957    | 135 | 9.96108    | 13         | 11.02807    | 161         | 5  |       |  |
| 6   | .95289     | 14 | .94093      | 136 | .96122     | 14         | .02968      | 161         | 6  |       |  |
| 7   | .95303     | 14 | .94229      | 136 | .96136     | 13         | .03129      | 161         | 7  |       |  |
| 8   | .95317     | 14 | .94366      | 137 | .96150     | 14         | .03291      | 162         | 8  |       |  |
| 9   | .95331     | 14 | .94503      | 137 | .96163     | 13         | .03453      | 163         | 9  |       |  |
| 10  | 9.95345    | 14 | 10.94641    | 137 | 9.96177    | 14         | 11.03616    | 163         | 10 |       |  |
| 11  | .95359     | 13 | .94778      | 138 | .96191     | 14         | .03780      | 164         | 11 |       |  |
| 12  | .95373     | 14 | .94917      | 138 | .96205     | 13         | .03944      | 164         | 12 |       |  |
| 13  | .95387     | 14 | .95055      | 139 | .96218     | 14         | .04108      | 165         | 13 |       |  |
| 14  | .95401     | 14 | .95194      | 139 | .96232     | 13         | .04273      | 165         | 14 |       |  |
| 15  | 9.95415    | 14 | 10.95333    | 139 | 9.96246    | 13         | 11.04438    | 166         | 15 |       |  |
| 16  | .95429     | 14 | .95473      | 140 | .96259     | 14         | .04604      | 167         | 16 |       |  |
| 17  | .95443     | 14 | .95613      | 140 | .96273     | 13         | .04771      | 167         | 17 |       |  |
| 18  | .95457     | 14 | .95753      | 140 | .96287     | 14         | .04938      | 167         | 18 |       |  |
| 19  | .95471     | 14 | .95894      | 141 | .96301     | 13         | .05106      | 168         | 19 |       |  |
| 20  | 9.95485    | 14 | 10.96035    | 141 | 9.96314    | 14         | 11.05274    | 169         | 20 |       |  |
| 21  | .95499     | 14 | .96176      | 142 | .96328     | 13         | .05443      | 169         | 21 |       |  |
| 22  | .95513     | 14 | .96318      | 142 | .96342     | 13         | .05612      | 169         | 22 |       |  |
| 23  | .95527     | 13 | .96461      | 142 | .96355     | 14         | .05782      | 170         | 23 |       |  |
| 24  | .95540     | 14 | .96603      | 143 | .96369     | 13         | .05952      | 171         | 24 |       |  |
| 25  | 9.95554    | 14 | 10.96746    | 143 | 9.96383    | 14         | 11.06123    | 171         | 25 |       |  |
| 26  | .95568     | 14 | .96889      | 144 | .96397     | 13         | .06295      | 172         | 26 |       |  |
| 27  | .95582     | 14 | .97033      | 144 | .96410     | 13         | .06467      | 173         | 27 |       |  |
| 28  | .95596     | 14 | .97177      | 144 | .96424     | 14         | .06640      | 173         | 28 |       |  |
| 29  | .95610     | 14 | .97322      | 145 | .96438     | 13         | .06813      | 174         | 29 |       |  |
| 30  | 9.95624    | 13 | 10.97467    | 145 | 9.96451    | 13         | 11.06987    | 174         | 30 |       |  |
| 31  | .95638     | 14 | .97612      | 145 | .96465     | 14         | .07161      | 175         | 31 |       |  |
| 32  | .95652     | 14 | .97758      | 146 | .96479     | 13         | .07336      | 176         | 32 |       |  |
| 33  | .95666     | 14 | .97904      | 146 | .96493     | 13         | .07512      | 176         | 33 |       |  |
| 34  | .95680     | 13 | .98050      | 147 | .96506     | 13         | .07688      | 177         | 34 |       |  |
| 35  | 9.95693    | 14 | 10.98197    | 147 | 9.96519    | 14         | 11.07865    | 177         | 35 |       |  |
| 36  | .95707     | 14 | .98345      | 147 | .96533     | 13         | .08043      | 178         | 36 |       |  |
| 37  | .95721     | 14 | .98492      | 148 | .96547     | 13         | .08221      | 179         | 37 |       |  |
| 38  | .95735     | 14 | .98640      | 149 | .96560     | 14         | .08400      | 179         | 38 |       |  |
| 39  | .95749     | 13 | .98789      | 149 | .96574     | 13         | .08579      | 180         | 39 |       |  |
| 40  | 9.95763    | 14 | 10.98938    | 149 | 9.96588    | 13         | 11.08759    | 180         | 40 |       |  |
| 41  | .95777     | 14 | .99087      | 150 | .96601     | 13         | .08940      | 181         | 41 |       |  |
| 42  | .95791     | 13 | .99237      | 150 | .96615     | 14         | .09121      | 182         | 42 |       |  |
| 43  | .95804     | 14 | .99387      | 151 | .96629     | 13         | .09303      | 182         | 43 |       |  |
| 44  | .95818     | 14 | .99538      | 151 | .96642     | 13         | .09486      | 183         | 44 |       |  |
| 45  | 9.95832    | 14 | 10.99689    | 151 | 9.96656    | 13         | 11.09669    | 184         | 45 |       |  |
| 46  | .95846     | 13 | .99841      | 152 | .96669     | 13         | .09853      | 185         | 46 |       |  |
| 47  | .95860     | 14 | 10.99993    | 152 | .96683     | 14         | .10038      | 185         | 47 |       |  |
| 48  | .95874     | 14 | 11.00145    | 153 | .96697     | 13         | .10223      | 186         | 48 |       |  |
| 49  | .95888     | 13 | .00298      | 153 | .96710     | 13         | .10409      | 186         | 49 |       |  |
| 50  | 9.95901    | 14 | 11.00451    | 154 | 9.96724    | 13         | 11.10595    | 187         | 50 |       |  |
| 51  | .95915     | 13 | .00605      | 154 | .96737     | 13         | .10783      | 188         | 51 |       |  |
| 52  | .95929     | 14 | .00759      | 155 | .96751     | 13         | .10971      | 189         | 52 |       |  |
| 53  | .95943     | 14 | .00914      | 155 | .96764     | 14         | .11160      | 189         | 53 |       |  |
| 54  | .95957     | 13 | .01069      | 155 | .96778     | 13         | .11349      | 190         | 54 |       |  |
| 55  | 9.95970    | 14 | 11.01225    | 156 | 9.96792    | 13         | 11.11539    | 191         | 55 |       |  |
| 56  | .95984     | 14 | .01381      | 156 | .96805     | 13         | .11730      | 191         | 56 |       |  |
| 57  | .95998     | 13 | .01537      | 157 | .96819     | 13         | .11922      | 192         | 57 |       |  |
| 58  | .96012     | 14 | .01694      | 157 | .96832     | 13         | .12114      | 193         | 58 |       |  |
| 59  | .96026     | 13 | .01852      | 158 | .96846     | 13         | .12307      | 193         | 59 |       |  |
| 60  | 9.96039    |    | 11.02010    |     | 9.96859    |            | 11.12501    |             | 60 |       |  |
|     | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D          | Log. Exsec. | D           |    | P. P. |  |

190 180

|    |       |       |
|----|-------|-------|
| 6  | 19.0  | 18.0  |
| 7  | 22.1  | 21.0  |
| 8  | 25.3  | 24.0  |
| 9  | 28.5  | 27.0  |
| 10 | 31.7  | 30.0  |
| 20 | 63.3  | 60.0  |
| 30 | 95.0  | 90.0  |
| 40 | 126.6 | 120.0 |
| 50 | 158.3 | 150.0 |

170 160

|    |       |       |
|----|-------|-------|
| 6  | 17.0  | 16.0  |
| 7  | 19.0  | 18.6  |
| 8  | 22.0  | 21.3  |
| 9  | 25.0  | 24.0  |
| 10 | 28.0  | 26.6  |
| 20 | 56.6  | 53.3  |
| 30 | 85.0  | 80.0  |
| 40 | 113.3 | 106.6 |
| 50 | 141.6 | 133.3 |

150 140

|    |       |       |
|----|-------|-------|
| 6  | 15.0  | 14.0  |
| 7  | 17.5  | 16.3  |
| 8  | 20.0  | 18.6  |
| 9  | 22.5  | 21.0  |
| 10 | 25.0  | 23.3  |
| 20 | 50.0  | 46.6  |
| 30 | 75.0  | 70.0  |
| 40 | 100.0 | 93.3  |
| 50 | 125.0 | 116.6 |

130 9 8

|    |       |     |     |
|----|-------|-----|-----|
| 6  | 13.0  | 0.9 | 0.8 |
| 7  | 15.1  | 1.6 | 0.9 |
| 8  | 17.3  | 1.2 | 1.0 |
| 9  | 19.5  | 1.3 | 1.2 |
| 10 | 21.6  | 1.5 | 1.3 |
| 20 | 43.3  | 3.0 | 2.6 |
| 30 | 65.0  | 4.5 | 4.0 |
| 40 | 86.6  | 6.0 | 5.3 |
| 50 | 108.3 | 7.5 | 6.6 |

7 6 5

|    |     |     |     |
|----|-----|-----|-----|
| 6  | 0.7 | 0.6 | 0.5 |
| 7  | 0.8 | 0.7 | 0.6 |
| 8  | 0.9 | 0.8 | 0.6 |
| 9  | 1.0 | 0.9 | 0.7 |
| 10 | 1.1 | 1.0 | 0.8 |
| 20 | 2.3 | 2.0 | 1.6 |
| 30 | 3.5 | 3.0 | 2.5 |
| 40 | 4.6 | 4.0 | 3.3 |
| 50 | 5.8 | 5.0 | 4.1 |

14 14 13

|    |      |      |      |
|----|------|------|------|
| 6  | 1.4  | 1.4  | 1.3  |
| 7  | 1.7  | 1.6  | 1.6  |
| 8  | 1.9  | 1.8  | 1.8  |
| 9  | 2.2  | 2.1  | 2.0  |
| 10 | 2.4  | 2.3  | 2.2  |
| 20 | 4.8  | 4.6  | 4.5  |
| 30 | 7.2  | 7.0  | 6.7  |
| 40 | 9.6  | 9.3  | 9.0  |
| 50 | 12.1 | 11.6 | 11.2 |

86°

87°

|    | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D  | Log. Exsec. | D   |    | P. P.          |
|----|------------|----|-------------|-----|------------|----|-------------|-----|----|----------------|
| 0  | 9.96859    | 13 | 11.12501    | 195 | 9.97665    | 13 | 11.25785    | 255 | 0  |                |
| 1  | .96873     | 14 | .12696      | 195 | .97679     | 13 | .26040      | 256 | 1  |                |
| 2  | .96887     | 13 | .12891      | 196 | .97692     | 13 | .26297      | 257 | 2  |                |
| 3  | .96900     | 13 | .13087      | 196 | .97705     | 13 | .26554      | 258 | 3  | 250 240        |
| 4  | .96914     | 13 | .13284      | 198 | .97718     | 13 | .26814      | 259 | 4  | 6 25.0 24.0    |
| 5  | 9.96927    | 13 | 11.13482    | 198 | 9.97732    | 13 | 11.27074    | 260 | 5  | 7 29.1 28.0    |
| 6  | .96941     | 13 | .13680      | 199 | .97745     | 13 | .27336      | 262 | 6  | 8 43.3 32.0    |
| 7  | .96954     | 13 | .13879      | 200 | .97758     | 13 | .27599      | 263 | 7  | 9 37.5 36.0    |
| 8  | .96968     | 13 | .14079      | 201 | .97772     | 13 | .27864      | 265 | 8  | 10 41.6 40.0   |
| 9  | .96981     | 13 | .14280      | 201 | .97785     | 13 | .28131      | 266 | 9  | 20 83.3 80.0   |
| 10 | 9.96995    | 13 | 11.14482    | 201 | 9.97798    | 13 | 11.28398    | 267 | 10 | 30 125.0 120.0 |
| 11 | .97008     | 13 | .14684      | 202 | .97811     | 13 | .28668      | 269 | 11 | 40 166.6 160.0 |
| 12 | .97022     | 13 | .14887      | 203 | .97825     | 13 | .28938      | 270 | 12 | 50 208.3 200.0 |
| 13 | .97035     | 13 | .15092      | 204 | .97838     | 13 | .29211      | 272 | 13 |                |
| 14 | .97049     | 13 | .15297      | 205 | .97851     | 13 | .29485      | 274 | 14 | 230 220        |
| 15 | 9.97062    | 13 | 11.15502    | 205 | 9.97864    | 13 | 11.29760    | 275 | 15 | 6 23.0 22.0    |
| 16 | .97076     | 13 | .15709      | 206 | .97878     | 13 | .30037      | 277 | 16 | 7 29.6 25.6    |
| 17 | .97089     | 13 | .15917      | 208 | .97891     | 13 | .30316      | 278 | 17 | 8 30.6 29.3    |
| 18 | .97103     | 13 | .16125      | 209 | .97904     | 13 | .30596      | 279 | 18 | 9 34.5 33.0    |
| 19 | .97116     | 13 | .16334      | 210 | .97917     | 13 | .30878      | 282 | 19 | 10 38.3 36.6   |
| 20 | 9.97130    | 13 | 11.16544    | 211 | 9.97931    | 13 | 11.31162    | 283 | 20 | 20 76.4 73.3   |
| 21 | .97143     | 13 | .16755      | 212 | .97944     | 13 | .31447      | 285 | 21 | 30 115.0 110.0 |
| 22 | .97157     | 13 | .16967      | 213 | .97957     | 13 | .31734      | 288 | 22 | 40 153.3 146.6 |
| 23 | .97170     | 13 | .17180      | 214 | .97970     | 13 | .32023      | 290 | 23 | 50 191.6 183.3 |
| 24 | .97183     | 13 | .17394      | 214 | .97984     | 13 | .32313      | 292 | 24 |                |
| 25 | 9.97197    | 13 | 11.17609    | 215 | 9.97997    | 13 | 11.32606    | 294 | 25 | 6 21.0 20.0    |
| 26 | .97210     | 13 | .17824      | 216 | .98010     | 13 | .32900      | 296 | 26 | 7 24.5 23.3    |
| 27 | .97224     | 13 | .18041      | 218 | .98023     | 13 | .33196      | 298 | 27 | 8 28.0 26.6    |
| 28 | .97237     | 13 | .18259      | 218 | .98036     | 13 | .33494      | 299 | 28 | 9 31.5 30.0    |
| 29 | .97251     | 13 | .18477      | 218 | .98050     | 13 | .33793      | 301 | 29 | 10 35.0 33.3   |
| 30 | 9.97264    | 13 | 11.18697    | 219 | 9.98063    | 13 | 11.34095    | 303 | 30 | 20 70.0 66.6   |
| 31 | .97277     | 13 | .18917      | 220 | .98076     | 13 | .34398      | 305 | 31 | 30 105.0 100.0 |
| 32 | .97291     | 13 | .19138      | 221 | .98089     | 13 | .34704      | 307 | 32 | 40 140.0 133.3 |
| 33 | .97304     | 13 | .19361      | 222 | .98102     | 13 | .35011      | 309 | 33 | 50 175.0 166.6 |
| 34 | .97318     | 13 | .19584      | 223 | .98116     | 13 | .35321      | 311 | 34 |                |
| 35 | 9.97331    | 13 | 11.19809    | 224 | 9.98129    | 13 | 11.35632    | 313 | 35 | 6 19.0 0.4     |
| 36 | .97345     | 13 | .20034      | 225 | .98142     | 13 | .35946      | 315 | 36 | 7 22.1 0.4     |
| 37 | .97358     | 13 | .20261      | 227 | .98155     | 13 | .36261      | 318 | 37 | 8 25.3 0.5     |
| 38 | .97371     | 13 | .20489      | 227 | .98168     | 13 | .36579      | 320 | 38 | 9 28.5 0.6     |
| 39 | .97385     | 13 | .20717      | 228 | .98181     | 13 | .36899      | 322 | 39 | 10 31.6 0.6    |
| 40 | 9.97398    | 13 | 11.20947    | 230 | 9.98195    | 13 | 11.37221    | 324 | 40 | 20 63.3 1.3    |
| 41 | .97412     | 13 | .21178      | 230 | .98208     | 13 | .37546      | 326 | 41 | 30 95.0 2.0    |
| 42 | .97425     | 13 | .21410      | 232 | .98221     | 13 | .37872      | 328 | 42 | 40 126.6 2.6   |
| 43 | .97438     | 13 | .21643      | 233 | .98234     | 13 | .38201      | 331 | 43 | 50 158.3 3.3   |
| 44 | .97452     | 13 | .21877      | 234 | .98247     | 13 | .38532      | 333 | 44 |                |
| 45 | 9.97465    | 13 | 11.22112    | 235 | 9.98260    | 13 | 11.38866    | 335 | 45 | 6 0.2 0.1      |
| 46 | .97478     | 13 | .22349      | 236 | .98273     | 13 | .39201      | 338 | 46 | 7 0.4 0.1      |
| 47 | .97492     | 13 | .22586      | 237 | .98287     | 13 | .39540      | 340 | 47 | 8 0.6 0.1      |
| 48 | .97505     | 13 | .22823      | 239 | .98300     | 13 | .39880      | 343 | 48 | 9 0.8 0.1      |
| 49 | .97519     | 13 | .23065      | 239 | .98313     | 13 | .40224      | 345 | 49 | 10 0.9 0.1     |
| 50 | 9.97532    | 13 | 11.23306    | 241 | 9.98326    | 13 | 11.40569    | 348 | 50 | 20 0.6 0.3     |
| 51 | .97545     | 13 | .23548      | 242 | .98339     | 13 | .40918      | 351 | 51 | 30 0.5 0.2     |
| 52 | .97559     | 13 | .23792      | 243 | .98352     | 13 | .41269      | 353 | 52 | 40 0.7 0.2     |
| 53 | .97572     | 13 | .24037      | 245 | .98365     | 13 | .41622      | 356 | 53 | 50 0.9 0.3     |
| 54 | .97585     | 13 | .24283      | 246 | .98378     | 13 | .41979      | 359 | 54 |                |
| 55 | 9.97599    | 13 | 11.24530    | 247 | 9.98392    | 13 | 11.42338    | 361 | 55 | 6 1.4 1.3      |
| 56 | .97612     | 13 | .24778      | 248 | .98405     | 13 | .42699      | 364 | 56 | 7 1.6 1.3      |
| 57 | .97625     | 13 | .25028      | 250 | .98418     | 13 | .43064      | 367 | 57 | 8 1.8 1.3      |
| 58 | .97639     | 13 | .25279      | 251 | .98431     | 13 | .43431      | 370 | 58 | 9 2.1 1.3      |
| 59 | .97652     | 13 | .25531      | 252 | .98444     | 13 | .43802      | 373 | 59 | 10 2.3 1.3     |
| 60 | 9.97665    | 13 | 11.25785    | 254 | 9.98457    | 13 | 11.44175    |     | 60 | 20 4.6 4.3     |
|    | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D  | Log. Exsec. | D   |    | P. P.          |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

88°

89°

|    | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D  | Log. Exsec. | D     |    | P. P. |
|----|------------|----|-------------|-----|------------|----|-------------|-------|----|-------|
| 0  | 9.98457    | 13 | 11.44173    | 376 | 9.99235    | 12 | 11.75050    | 742   | 0  |       |
| 1  | .98470     | 13 | .44551      | 370 | .99248     | 13 | .75792      | 755   | 1  |       |
| 2  | .98483     | 13 | .44931      | 382 | .99261     | 13 | .76547      | 768   | 2  |       |
| 3  | .98496     | 13 | .45313      | 386 | .99274     | 13 | .77316      | 781   | 3  |       |
| 4  | .98509     | 13 | .45699      | 389 | .99287     | 13 | .78097      | 795   | 4  |       |
| 5  | 9.98522    | 13 | 11.46088    | 392 | 9.99299    | 12 | 11.78892    | 809   | 5  |       |
| 6  | .98535     | 13 | .46480      | 395 | .99312     | 13 | .79702      | 825   | 6  |       |
| 7  | .98548     | 13 | .46876      | 399 | .99325     | 12 | .80527      | 840   | 7  |       |
| 8  | .98562     | 13 | .47275      | 402 | .99338     | 13 | .81367      | 856   | 8  |       |
| 9  | .98575     | 13 | .47677      | 406 | .99351     | 12 | .82223      | 872   | 9  |       |
| 10 | 9.98588    | 13 | 11.48083    | 409 | 9.99363    | 13 | 11.83095    | 890   | 10 |       |
| 11 | .98601     | 13 | .48493      | 413 | .99376     | 13 | .83986      | 908   | 11 |       |
| 12 | .98614     | 13 | .48906      | 417 | .99389     | 12 | .84894      | 927   | 12 |       |
| 13 | .98627     | 13 | .49323      | 420 | .99402     | 13 | .85821      | 947   | 13 |       |
| 14 | .98640     | 13 | .49743      | 425 | .99415     | 13 | .86768      | 967   | 14 |       |
| 15 | 9.98653    | 13 | 11.50168    | 428 | 9.99428    | 12 | 11.87735    | 989   | 15 |       |
| 16 | .98666     | 13 | .50597      | 432 | .99440     | 13 | .88724      | 1009  | 16 |       |
| 17 | .98679     | 13 | .51029      | 436 | .99453     | 12 | .89735      | 1034  | 17 |       |
| 18 | .98692     | 13 | .51466      | 440 | .99466     | 13 | .90769      | 1059  | 18 |       |
| 19 | .98705     | 13 | .51906      | 445 | .99479     | 12 | .91829      | 1085  | 19 |       |
| 20 | 9.98718    | 13 | 11.52351    | 449 | 9.99491    | 13 | 11.92914    | 1112  | 20 |       |
| 21 | .98731     | 13 | .52801      | 454 | .99504     | 13 | .94026      | 1140  | 21 |       |
| 22 | .98744     | 13 | .53255      | 458 | .99517     | 12 | .95167      | 1171  | 22 |       |
| 23 | .98757     | 13 | .53713      | 463 | .99530     | 13 | .96338      | 1203  | 23 |       |
| 24 | .98770     | 13 | .54176      | 467 | .99543     | 12 | .97541      | 1236  | 24 |       |
| 25 | 9.98783    | 13 | 11.54643    | 472 | 9.99555    | 13 | 11.98777    | 1271  | 25 |       |
| 26 | .98796     | 13 | .55116      | 477 | .99568     | 12 | 12.00048    | 1309  | 26 |       |
| 27 | .98809     | 13 | .55593      | 482 | .99581     | 13 | .01358      | 1349  | 27 |       |
| 28 | .98822     | 13 | .56076      | 487 | .99594     | 12 | .02707      | 1391  | 28 |       |
| 29 | .98835     | 13 | .56563      | 492 | .99606     | 13 | .04098      | 1436  | 29 |       |
| 30 | 9.98848    | 13 | 11.57056    | 498 | 9.99619    | 12 | 12.05535    | 1485  | 30 |       |
| 31 | .98861     | 13 | .57554      | 504 | .99632     | 13 | .07020      | 1537  | 31 |       |
| 32 | .98874     | 13 | .58058      | 509 | .99645     | 12 | .08557      | 1592  | 32 |       |
| 33 | .98887     | 13 | .58567      | 515 | .99657     | 13 | .10149      | 1652  | 33 |       |
| 34 | .98900     | 13 | .59082      | 520 | .99670     | 12 | .11801      | 1716  | 34 |       |
| 35 | 9.98913    | 12 | 11.59602    | 527 | 9.99683    | 13 | 12.13517    | 1785  | 35 |       |
| 36 | .98925     | 13 | .60129      | 533 | .99695     | 12 | .15302      | 1861  | 36 |       |
| 37 | .98938     | 13 | .60662      | 539 | .99708     | 13 | .17163      | 1943  | 37 |       |
| 38 | .98951     | 13 | .61202      | 545 | .99721     | 12 | .19106      | 2033  | 38 |       |
| 39 | .98964     | 13 | .61747      | 552 | .99734     | 13 | .21139      | 2131  | 39 |       |
| 40 | 9.98977    | 13 | 11.62300    | 559 | 9.99746    | 12 | 12.23271    | 2240  | 40 |       |
| 41 | .98990     | 13 | .62859      | 566 | .99759     | 13 | .25511      | 2361  | 41 |       |
| 42 | .99003     | 13 | .63425      | 573 | .99772     | 12 | .27872      | 2495  | 42 |       |
| 43 | .99016     | 12 | .63998      | 581 | .99784     | 13 | .30367      | 2645  | 43 |       |
| 44 | .99029     | 13 | .64579      | 588 | .99797     | 12 | .33013      | 2815  | 44 |       |
| 45 | 9.99042    | 13 | 11.65167    | 595 | 9.99810    | 13 | 12.35828    | 3009  | 45 |       |
| 46 | .99055     | 13 | .65762      | 604 | .99823     | 12 | .38837      | 3231  | 46 |       |
| 47 | .99068     | 13 | .66366      | 611 | .99835     | 12 | .42068      | 3489  | 47 |       |
| 48 | .99081     | 12 | .66978      | 620 | .99848     | 13 | .45557      | 3791  | 48 |       |
| 49 | .99093     | 13 | .67598      | 628 | .99861     | 12 | .49349      | 4152  | 49 |       |
| 50 | 9.99106    | 13 | 11.68227    | 638 | 9.99873    | 13 | 12.53501    | 4588  | 50 |       |
| 51 | .99119     | 13 | .68865      | 646 | .99886     | 12 | .58089      | 5127  | 51 |       |
| 52 | .99132     | 13 | .69511      | 656 | .99899     | 13 | .63217      | 5812  | 52 |       |
| 53 | .99145     | 12 | .70168      | 666 | .99911     | 12 | .69029      | 6707  | 53 |       |
| 54 | .99158     | 13 | .70834      | 675 | .99924     | 13 | .75736      | 7931  | 54 |       |
| 55 | 9.99171    | 13 | 11.71509    | 686 | 9.99937    | 12 | 12.83667    | 9704  | 55 |       |
| 56 | .99184     | 13 | .72196      | 696 | .99949     | 12 | .93371      | 12506 | 56 |       |
| 57 | .99197     | 12 | .72892      | 707 | .99962     | 13 | 13.05877    | 17621 | 57 |       |
| 58 | .99209     | 13 | .73600      | 719 | .99974     | 12 | .23499      | 30116 | 58 |       |
| 59 | .99222     | 13 | .74319      | 730 | .99987     | 12 | .53615      |       | 59 |       |
| 60 | 9.99235    |    | 11.75050    |     | 10.00000   |    | Infinity    |       | 60 |       |
|    | Log. Vers. | D  | Log. Exsec. | D   | Log. Vers. | D  | Log. Exsec. | D     |    | P. P. |

|    |      |      |
|----|------|------|
|    | 13   | 13   |
| 6  | 1.3  | 1.3  |
| 7  | 1.6  | 1.5  |
| 8  | 1.8  | 1.7  |
| 9  | 2.0  | 1.9  |
| 10 | 2.2  | 2.1  |
| 20 | 4.5  | 4.3  |
| 30 | 6.7  | 6.5  |
| 40 | 9.0  | 8.6  |
| 50 | 11.2 | 10.8 |

|    |      |
|----|------|
|    | 12   |
| 6  | 1.2  |
| 7  | 1.4  |
| 8  | 1.6  |
| 9  | 1.9  |
| 10 | 2.1  |
| 20 | 4.1  |
| 30 | 6.2  |
| 40 | 8.3  |
| 50 | 10.4 |



TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

0°-10°

| ° /         | Sin.          | d. | Tan.          | d. | Cot.           | d.     | Cos.          | d. |             | P. P.                     |
|-------------|---------------|----|---------------|----|----------------|--------|---------------|----|-------------|---------------------------|
| <b>0 0</b>  | <b>0.0000</b> |    | <b>0.0000</b> |    | <b>∞</b>       |        | <b>1.0000</b> |    | <b>0 90</b> |                           |
| 10          | 0.0029        | 29 | 0.0029        | 29 | 343.773        |        | 1.0000        | 0  | 50          |                           |
| 20          | 0.0058        | 29 | 0.0058        | 29 | 171.885        |        | 1.0000        | 0  | 40          |                           |
| 30          | 0.0087        | 29 | 0.0087        | 29 | 114.588        |        | 0.9999        | 0  | 30          |                           |
| 40          | 0.0116        | 29 | 0.0116        | 29 | 85.9398        |        | 0.9999        | 0  | 20          |                           |
| 50          | 0.0145        | 29 | 0.0145        | 29 | 68.7501        |        | 0.9999        | 0  | 10          |                           |
| <b>1 0</b>  | <b>0.0174</b> | 29 | <b>0.0174</b> | 29 | <b>57.2899</b> |        | <b>0.9998</b> | 0  | <b>0 89</b> | <b>30 29 29</b>           |
| 10          | 0.0203        | 29 | 0.0203        | 29 | 49.1039        | 8.1866 | 0.9998        | 0  | 50          | 1 3.0 2.9 2.9             |
| 20          | 0.0232        | 29 | 0.0233        | 29 | 42.9641        | 6.1398 | 0.9997        | 0  | 40          | 2 6.0 5.9 5.8             |
| 30          | 0.0262        | 29 | 0.0262        | 29 | 38.1884        | 4.7756 | 0.9996        | 0  | 30          | 3 9.0 8.8 8.7             |
| 40          | 0.0291        | 29 | 0.0291        | 29 | 34.3677        | 3.8217 | 0.9996        | 0  | 20          | 4 12.0 11.8 11.6          |
| 50          | 0.0320        | 29 | 0.0320        | 29 | 31.2416        | 3.1261 | 0.9995        | 0  | 10          | 5 15.0 14.7 14.5          |
| <b>2 0</b>  | <b>0.0349</b> | 29 | <b>0.0349</b> | 29 | <b>28.6362</b> | 2.6053 | <b>0.9994</b> | 0  | <b>0 88</b> | <b>6 18.0 17.7 17.4</b>   |
| 10          | 0.0378        | 29 | 0.0378        | 29 | 26.4316        | 2.2046 | 0.9993        | 0  | 50          | 7 21.0 20.6 20.3          |
| 20          | 0.0407        | 29 | 0.0407        | 29 | 24.5417        | 1.8898 | 0.9991        | 0  | 40          | 8 24.0 23.6 23.2          |
| 30          | 0.0436        | 29 | 0.0436        | 29 | 22.9037        | 1.6380 | 0.9990        | 0  | 30          | 9 27.0 26.5 26.1          |
| 40          | 0.0465        | 29 | 0.0466        | 29 | 21.4704        | 1.4333 | 0.9989        | 0  | 20          |                           |
| 50          | 0.0494        | 29 | 0.0495        | 29 | 20.2053        | 1.2648 | 0.9988        | 0  | 10          |                           |
| <b>3 0</b>  | <b>0.0523</b> | 29 | <b>0.0524</b> | 29 | <b>19.0811</b> | 1.1244 | <b>0.9986</b> | 0  | <b>0 87</b> |                           |
| 10          | 0.0552        | 29 | 0.0553        | 29 | 18.0750        | 1.0061 | 0.9984        | 0  | 50          |                           |
| 20          | 0.0581        | 29 | 0.0582        | 29 | 17.1693        | 9056   | 0.9983        | 0  | 40          | <b>28 5 4 4</b>           |
| 30          | 0.0610        | 29 | 0.0611        | 29 | 16.3498        | 8195   | 0.9981        | 0  | 30          | 1 2.8 0.5 0.4 0.4         |
| 40          | 0.0639        | 29 | 0.0641        | 29 | 15.6048        | 7450   | 0.9979        | 0  | 20          | 2 5.7 1.0 0.9 0.8         |
| 50          | 0.0668        | 29 | 0.0670        | 29 | 14.9244        | 6804   | 0.9977        | 0  | 10          | 3 8.5 1.5 1.3 1.2         |
| <b>4 0</b>  | <b>0.0697</b> | 29 | <b>0.0699</b> | 29 | <b>14.3006</b> | 6237   | <b>0.9975</b> | 0  | <b>0 86</b> | <b>4 11.4 2.0 1.8 1.6</b> |
| 10          | 0.0726        | 29 | 0.0728        | 29 | 13.7267        | 5739   | 0.9973        | 0  | 50          | 5 14.2 2.5 2.2 2.0        |
| 20          | 0.0755        | 29 | 0.0758        | 29 | 13.1969        | 5298   | 0.9971        | 0  | 40          | 6 17.1 3.0 2.7 2.4        |
| 30          | 0.0784        | 29 | 0.0787        | 29 | 12.7062        | 4907   | 0.9969        | 0  | 30          | 7 19.6 3.5 3.1 2.8        |
| 40          | 0.0813        | 29 | 0.0816        | 29 | 12.2505        | 4557   | 0.9967        | 0  | 20          | 8 22.8 4.0 3.6 3.2        |
| 50          | 0.0842        | 29 | 0.0845        | 29 | 11.8261        | 4243   | 0.9964        | 0  | 10          | 9 25.6 4.5 4.0 3.6        |
| <b>5 0</b>  | <b>0.0871</b> | 29 | <b>0.0875</b> | 29 | <b>11.4300</b> | 3961   | <b>0.9962</b> | 0  | <b>0 85</b> |                           |
| 10          | 0.0900        | 29 | 0.0904        | 29 | 11.0594        | 3706   | 0.9959        | 0  | 50          |                           |
| 20          | 0.0929        | 29 | 0.0933        | 29 | 10.7119        | 3475   | 0.9956        | 0  | 40          |                           |
| 30          | 0.0958        | 29 | 0.0963        | 29 | 10.3854        | 3265   | 0.9954        | 0  | 30          |                           |
| 40          | 0.0987        | 29 | 0.0992        | 29 | 10.0780        | 3073   | 0.9951        | 0  | 20          | <b>3 3 2 2</b>            |
| 50          | 0.1016        | 29 | 0.1021        | 29 | 9.7881         | 2899   | 0.9948        | 0  | 10          | 1 0.3 0.3 0.2 0.2         |
| <b>6 0</b>  | <b>0.1045</b> | 29 | <b>0.1051</b> | 29 | <b>9.5143</b>  | 2738   | <b>0.9945</b> | 0  | <b>0 84</b> | <b>2 0.7 0.6 0.5 0.4</b>  |
| 10          | 0.1074        | 28 | 0.1080        | 29 | 9.2553         | 2590   | 0.9942        | 0  | 50          | 3 1.0 0.9 0.7 0.6         |
| 20          | 0.1103        | 29 | 0.1110        | 29 | 9.0098         | 2454   | 0.9939        | 0  | 40          | 4 1.4 1.2 1.0 0.8         |
| 30          | 0.1132        | 29 | 0.1139        | 29 | 8.7769         | 2329   | 0.9935        | 0  | 30          | 5 1.7 1.5 1.2 1.0         |
| 40          | 0.1161        | 29 | 0.1169        | 29 | 8.5555         | 2213   | 0.9932        | 0  | 20          | 6 2.1 1.8 1.5 1.2         |
| 50          | 0.1190        | 29 | 0.1198        | 29 | 8.3449         | 2106   | 0.9929        | 0  | 10          | 7 2.4 2.1 1.7 1.4         |
| <b>7 0</b>  | <b>0.1218</b> | 28 | <b>0.1228</b> | 29 | <b>8.1443</b>  | 2006   | <b>0.9925</b> | 0  | <b>0 83</b> | <b>8 2.8 2.4 2.0 1.6</b>  |
| 10          | 0.1247        | 29 | 0.1257        | 29 | 7.9530         | 1913   | 0.9922        | 0  | 50          | 9 3.1 2.7 2.2 1.8         |
| 20          | 0.1276        | 29 | 0.1287        | 29 | 7.7703         | 1826   | 0.9918        | 0  | 40          |                           |
| 30          | 0.1305        | 28 | 0.1316        | 29 | 7.5957         | 1746   | 0.9914        | 0  | 30          |                           |
| 40          | 0.1334        | 29 | 0.1346        | 29 | 7.4287         | 1670   | 0.9910        | 0  | 20          |                           |
| 50          | 0.1363        | 29 | 0.1376        | 30 | 7.2687         | 1599   | 0.9906        | 0  | 10          | <b>1 1 0</b>              |
| <b>8 0</b>  | <b>0.1391</b> | 28 | <b>0.1405</b> | 29 | <b>7.1153</b>  | 1534   | <b>0.9902</b> | 0  | <b>0 82</b> | <b>10.1 0.1 0.0</b>       |
| 10          | 0.1420        | 29 | 0.1435        | 29 | 6.9682         | 1471   | 0.9898        | 0  | 50          | 2 0.3 0.2 0.1             |
| 20          | 0.1449        | 29 | 0.1465        | 30 | 6.8269         | 1413   | 0.9894        | 0  | 40          | 3 0.4 0.3 0.1             |
| 30          | 0.1478        | 28 | 0.1494        | 30 | 6.6911         | 1358   | 0.9890        | 0  | 30          | 4 0.6 0.4 0.2             |
| 40          | 0.1507        | 29 | 0.1524        | 30 | 6.5603         | 1306   | 0.9886        | 0  | 20          | 5 0.7 0.5 0.2             |
| 50          | 0.1535        | 28 | 0.1554        | 29 | 6.4348         | 1257   | 0.9881        | 0  | 10          | 6 0.9 0.6 0.3             |
| <b>9 0</b>  | <b>0.1564</b> | 29 | <b>0.1584</b> | 29 | <b>6.3137</b>  | 1211   | <b>0.9877</b> | 0  | <b>0 81</b> | <b>7 1.6 0.7 0.3</b>      |
| 10          | 0.1593        | 28 | 0.1613        | 29 | 6.1970         | 1167   | 0.9872        | 0  | 50          | 8 1.2 0.8 0.4             |
| 20          | 0.1622        | 29 | 0.1643        | 30 | 6.0844         | 1126   | 0.9867        | 0  | 40          | 9 1.3 0.9 0.4             |
| 30          | 0.1650        | 28 | 0.1673        | 30 | 5.9757         | 1087   | 0.9863        | 0  | 30          |                           |
| 40          | 0.1679        | 28 | 0.1703        | 30 | 5.8708         | 1049   | 0.9858        | 0  | 20          |                           |
| 50          | 0.1708        | 29 | 0.1733        | 30 | 5.7693         | 1014   | 0.9853        | 0  | 10          |                           |
| <b>10 0</b> | <b>0.1736</b> | 28 | <b>0.1763</b> | 30 | <b>5.6713</b>  | 980    | <b>0.9848</b> | 0  | <b>0 80</b> |                           |
|             | Cos.          | d. | Cot.          | d. | Tan.           | d.     | Sin.          | d. | °           | P. P.                     |

80°-90°

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

10°-20°

| ° /         | Sin.          | d. | Tan.          | d. | Cot.          | d.  | Cos.          | d. | P. P.       |
|-------------|---------------|----|---------------|----|---------------|-----|---------------|----|-------------|
| <b>10 0</b> | <b>0.1736</b> |    | <b>0.1763</b> |    | <b>5.6713</b> |     | <b>0.9848</b> |    | <b>0 80</b> |
| 10          | 0.1765        | 28 | 0.1793        | 30 | 5.5764        | 949 | 0.9843        | 5  | 50          |
| 20          | 0.1793        | 28 | 0.1823        | 30 | 5.4845        | 919 | 0.9838        | 5  | 40          |
| 30          | 0.1822        | 29 | 0.1853        | 30 | 5.3955        | 890 | 0.9832        | 5  | 30          |
| 40          | 0.1851        | 28 | 0.1883        | 30 | 5.3093        | 862 | 0.9827        | 5  | 20          |
| 50          | 0.1879        | 28 | 0.1913        | 30 | 5.2256        | 836 | 0.9822        | 5  | 10          |
| <b>11 0</b> | <b>0.1908</b> |    | <b>0.1944</b> |    | <b>5.1445</b> |     | <b>0.9816</b> |    | <b>0 79</b> |
| 10          | 0.1936        | 28 | 0.1974        | 30 | 5.0658        | 787 | 0.9810        | 6  | 50          |
| 20          | 0.1965        | 28 | 0.2004        | 30 | 4.9894        | 764 | 0.9805        | 6  | 40          |
| 30          | 0.1993        | 28 | 0.2034        | 30 | 4.9151        | 742 | 0.9799        | 6  | 30          |
| 40          | 0.2022        | 28 | 0.2065        | 30 | 4.8430        | 721 | 0.9793        | 6  | 20          |
| 50          | 0.2050        | 28 | 0.2095        | 30 | 4.7728        | 701 | 0.9787        | 6  | 10          |
| <b>12 0</b> | <b>0.2079</b> |    | <b>0.2125</b> |    | <b>4.7046</b> |     | <b>0.9781</b> |    | <b>0 78</b> |
| 10          | 0.2107        | 28 | 0.2156        | 30 | 4.6382        | 664 | 0.9775        | 6  | 50          |
| 20          | 0.2136        | 28 | 0.2186        | 30 | 4.5736        | 646 | 0.9769        | 6  | 40          |
| 30          | 0.2164        | 28 | 0.2217        | 30 | 4.5107        | 629 | 0.9763        | 6  | 30          |
| 40          | 0.2193        | 28 | 0.2247        | 30 | 4.4494        | 613 | 0.9756        | 6  | 20          |
| 50          | 0.2221        | 28 | 0.2278        | 30 | 4.3897        | 597 | 0.9750        | 6  | 10          |
| <b>13 0</b> | <b>0.2249</b> |    | <b>0.2308</b> |    | <b>4.3315</b> |     | <b>0.9743</b> |    | <b>0 77</b> |
| 10          | 0.2278        | 28 | 0.2339        | 31 | 4.2747        | 582 | 0.9737        | 6  | 50          |
| 20          | 0.2306        | 28 | 0.2370        | 30 | 4.2193        | 568 | 0.9730        | 6  | 40          |
| 30          | 0.2334        | 28 | 0.2401        | 31 | 4.1653        | 553 | 0.9723        | 7  | 30          |
| 40          | 0.2362        | 28 | 0.2431        | 30 | 4.1125        | 540 | 0.9717        | 7  | 20          |
| 50          | 0.2391        | 28 | 0.2462        | 31 | 4.0610        | 527 | 0.9710        | 7  | 10          |
| <b>14 0</b> | <b>0.2419</b> |    | <b>0.2493</b> |    | <b>4.0108</b> |     | <b>0.9703</b> |    | <b>0 76</b> |
| 10          | 0.2447        | 28 | 0.2524        | 30 | 3.9616        | 502 | 0.9696        | 7  | 50          |
| 20          | 0.2475        | 28 | 0.2555        | 31 | 3.9136        | 491 | 0.9690        | 7  | 40          |
| 30          | 0.2504        | 28 | 0.2586        | 31 | 3.8667        | 480 | 0.9688        | 7  | 30          |
| 40          | 0.2532        | 28 | 0.2617        | 31 | 3.8208        | 469 | 0.9681        | 7  | 20          |
| 50          | 0.2560        | 28 | 0.2648        | 32 | 3.7759        | 458 | 0.9674        | 7  | 10          |
| <b>15 0</b> | <b>0.2588</b> |    | <b>0.2679</b> |    | <b>3.7320</b> |     | <b>0.9659</b> |    | <b>0 75</b> |
| 10          | 0.2616        | 28 | 0.2710        | 31 | 3.6891        | 439 | 0.9651        | 7  | 50          |
| 20          | 0.2644        | 28 | 0.2742        | 31 | 3.6470        | 429 | 0.9644        | 8  | 40          |
| 30          | 0.2672        | 28 | 0.2773        | 31 | 3.6059        | 420 | 0.9636        | 8  | 30          |
| 40          | 0.2700        | 28 | 0.2804        | 31 | 3.5655        | 411 | 0.9628        | 8  | 20          |
| 50          | 0.2728        | 28 | 0.2836        | 31 | 3.5261        | 403 | 0.9620        | 8  | 10          |
| <b>16 0</b> | <b>0.2756</b> |    | <b>0.2867</b> |    | <b>3.4874</b> |     | <b>0.9612</b> |    | <b>0 74</b> |
| 10          | 0.2784        | 28 | 0.2899        | 31 | 3.4495        | 387 | 0.9604        | 8  | 50          |
| 20          | 0.2812        | 28 | 0.2930        | 31 | 3.4123        | 379 | 0.9596        | 8  | 40          |
| 30          | 0.2840        | 27 | 0.2962        | 31 | 3.3759        | 371 | 0.9588        | 8  | 30          |
| 40          | 0.2868        | 28 | 0.2994        | 32 | 3.3402        | 364 | 0.9580        | 8  | 20          |
| 50          | 0.2896        | 28 | 0.3025        | 31 | 3.3052        | 357 | 0.9571        | 8  | 10          |
| <b>17 0</b> | <b>0.2923</b> |    | <b>0.3057</b> |    | <b>3.2708</b> |     | <b>0.9563</b> |    | <b>0 73</b> |
| 10          | 0.2951        | 28 | 0.3089        | 31 | 3.2371        | 343 | 0.9554        | 8  | 50          |
| 20          | 0.2979        | 28 | 0.3121        | 32 | 3.2040        | 337 | 0.9546        | 8  | 40          |
| 30          | 0.3007        | 27 | 0.3153        | 32 | 3.1716        | 331 | 0.9537        | 9  | 30          |
| 40          | 0.3035        | 28 | 0.3185        | 32 | 3.1397        | 324 | 0.9528        | 9  | 20          |
| 50          | 0.3062        | 27 | 0.3217        | 32 | 3.1084        | 319 | 0.9519        | 9  | 10          |
| <b>18 0</b> | <b>0.3090</b> |    | <b>0.3249</b> |    | <b>3.0777</b> |     | <b>0.9510</b> |    | <b>0 72</b> |
| 10          | 0.3118        | 28 | 0.3281        | 32 | 3.0475        | 307 | 0.9501        | 9  | 50          |
| 20          | 0.3145        | 27 | 0.3313        | 32 | 3.0178        | 302 | 0.9492        | 9  | 40          |
| 30          | 0.3173        | 27 | 0.3346        | 32 | 2.9887        | 296 | 0.9483        | 9  | 30          |
| 40          | 0.3200        | 27 | 0.3378        | 32 | 2.9600        | 291 | 0.9474        | 9  | 20          |
| 50          | 0.3228        | 27 | 0.3411        | 32 | 2.9319        | 286 | 0.9464        | 9  | 10          |
| <b>19 0</b> | <b>0.3255</b> |    | <b>0.3443</b> |    | <b>2.9042</b> |     | <b>0.9455</b> |    | <b>0 71</b> |
| 10          | 0.3283        | 27 | 0.3476        | 32 | 2.8770        | 277 | 0.9445        | 9  | 50          |
| 20          | 0.3310        | 27 | 0.3508        | 32 | 2.8502        | 272 | 0.9436        | 9  | 40          |
| 30          | 0.3338        | 27 | 0.3541        | 33 | 2.8239        | 267 | 0.9426        | 9  | 30          |
| 40          | 0.3365        | 27 | 0.3574        | 33 | 2.7980        | 263 | 0.9416        | 10 | 20          |
| 50          | 0.3393        | 27 | 0.3607        | 32 | 2.7725        | 259 | 0.9407        | 9  | 10          |
| <b>20 0</b> | <b>0.3420</b> |    | <b>0.3639</b> |    | <b>2.7475</b> |     | <b>0.9397</b> |    | <b>0 70</b> |
|             | Cos.          | d. | Cot.          | d. | Tan.          | d.  | Sin.          | d. | P. P.       |

70°-80°



TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

20°-30°

| ° /         | Sin.          | d. | Tan.          | d. | Cot.          | d.  | Cos.          | d. |             | P. P. |
|-------------|---------------|----|---------------|----|---------------|-----|---------------|----|-------------|-------|
| <b>20 0</b> | <b>0.3420</b> |    | <b>0.3639</b> |    | <b>2.7475</b> |     | <b>0.9397</b> |    | <b>0 70</b> |       |
| 10          | 0.3447        | 27 | 0.3672        | 33 | 2.7228        | 247 | 0.9387        | 10 | 50          |       |
| 20          | 0.3475        | 27 | 0.3703        | 33 | 2.6983        | 242 | 0.9377        | 10 | 40          |       |
| 30          | 0.3502        | 27 | 0.3739        | 33 | 2.6746        | 239 | 0.9366        | 10 | 30          |       |
| 40          | 0.3529        | 27 | 0.3772        | 33 | 2.6511        | 235 | 0.9356        | 10 | 20          |       |
| 50          | 0.3556        | 27 | 0.3803        | 33 | 2.6279        | 232 | 0.9346        | 10 | 10          |       |
| <b>21 0</b> | <b>0.3583</b> |    | <b>0.3838</b> |    | <b>2.6051</b> |     | <b>0.9336</b> |    | <b>0 69</b> |       |
| 10          | 0.3611        | 27 | 0.3872        | 33 | 2.5826        | 225 | 0.9325        | 10 | 50          |       |
| 20          | 0.3638        | 27 | 0.3903        | 33 | 2.5604        | 221 | 0.9315        | 10 | 40          |       |
| 30          | 0.3665        | 27 | 0.3939        | 33 | 2.5386        | 218 | 0.9304        | 11 | 30          |       |
| 40          | 0.3692        | 27 | 0.3972        | 34 | 2.5171        | 215 | 0.9293        | 11 | 20          |       |
| 50          | 0.3719        | 27 | 0.4006        | 34 | 2.4959        | 212 | 0.9282        | 11 | 10          |       |
| <b>22 0</b> | <b>0.3746</b> |    | <b>0.4040</b> |    | <b>2.4751</b> |     | <b>0.9272</b> |    | <b>0 68</b> |       |
| 10          | 0.3773        | 27 | 0.4074        | 34 | 2.4545        | 206 | 0.9261        | 11 | 50          |       |
| 20          | 0.3800        | 27 | 0.4108        | 34 | 2.4342        | 203 | 0.9250        | 11 | 40          |       |
| 30          | 0.3827        | 27 | 0.4142        | 34 | 2.4142        | 200 | 0.9239        | 11 | 30          |       |
| 40          | 0.3853        | 26 | 0.4176        | 34 | 2.3945        | 197 | 0.9227        | 11 | 20          |       |
| 50          | 0.3880        | 27 | 0.4210        | 34 | 2.3750        | 194 | 0.9216        | 11 | 10          |       |
| <b>23 0</b> | <b>0.3907</b> |    | <b>0.4244</b> |    | <b>2.3558</b> |     | <b>0.9205</b> |    | <b>0 67</b> |       |
| 10          | 0.3934        | 26 | 0.4279        | 34 | 2.3369        | 192 | 0.9193        | 11 | 50          |       |
| 20          | 0.3961        | 26 | 0.4313        | 34 | 2.3182        | 189 | 0.9182        | 11 | 40          |       |
| 30          | 0.3987        | 26 | 0.4348        | 34 | 2.2998        | 187 | 0.9170        | 11 | 30          |       |
| 40          | 0.4014        | 27 | 0.4383        | 35 | 2.2816        | 184 | 0.9159        | 11 | 20          |       |
| 50          | 0.4041        | 26 | 0.4417        | 35 | 2.2637        | 182 | 0.9147        | 12 | 10          |       |
| <b>24 0</b> | <b>0.4067</b> |    | <b>0.4452</b> |    | <b>2.2460</b> |     | <b>0.9135</b> |    | <b>0 66</b> |       |
| 10          | 0.4094        | 26 | 0.4487        | 34 | 2.2283        | 177 | 0.9123        | 11 | 50          |       |
| 20          | 0.4120        | 26 | 0.4522        | 35 | 2.2213        | 175 | 0.9111        | 12 | 40          |       |
| 30          | 0.4147        | 26 | 0.4557        | 35 | 2.1943        | 172 | 0.9099        | 12 | 30          |       |
| 40          | 0.4173        | 26 | 0.4592        | 35 | 2.1775        | 170 | 0.9087        | 12 | 20          |       |
| 50          | 0.4200        | 26 | 0.4627        | 35 | 2.1609        | 168 | 0.9075        | 12 | 10          |       |
| <b>25 0</b> | <b>0.4226</b> |    | <b>0.4663</b> |    | <b>2.1445</b> |     | <b>0.9063</b> |    | <b>0 65</b> |       |
| 10          | 0.4252        | 26 | 0.4698        | 35 | 2.1283        | 164 | 0.9050        | 12 | 50          |       |
| 20          | 0.4279        | 26 | 0.4734        | 35 | 2.1123        | 162 | 0.9038        | 12 | 40          |       |
| 30          | 0.4305        | 26 | 0.4770        | 36 | 2.0965        | 159 | 0.9026        | 12 | 30          |       |
| 40          | 0.4331        | 26 | 0.4805        | 36 | 2.0809        | 158 | 0.9013        | 12 | 20          |       |
| 50          | 0.4357        | 26 | 0.4841        | 36 | 2.0655        | 156 | 0.9000        | 13 | 10          |       |
| <b>26 0</b> | <b>0.4383</b> |    | <b>0.4877</b> |    | <b>2.0503</b> |     | <b>0.8988</b> |    | <b>0 64</b> |       |
| 10          | 0.4410        | 26 | 0.4913        | 36 | 2.0352        | 154 | 0.8975        | 13 | 50          |       |
| 20          | 0.4436        | 26 | 0.4949        | 36 | 2.0204        | 152 | 0.8962        | 13 | 40          |       |
| 30          | 0.4462        | 26 | 0.4986        | 36 | 2.0057        | 150 | 0.8949        | 13 | 30          |       |
| 40          | 0.4488        | 26 | 0.5022        | 36 | 1.9911        | 148 | 0.8936        | 13 | 20          |       |
| 50          | 0.4514        | 26 | 0.5058        | 37 | 1.9768        | 147 | 0.8923        | 13 | 10          |       |
| <b>27 0</b> | <b>0.4540</b> |    | <b>0.5095</b> |    | <b>1.9626</b> |     | <b>0.8910</b> |    | <b>0 63</b> |       |
| 10          | 0.4566        | 25 | 0.5132        | 36 | 1.9486        | 144 | 0.8897        | 13 | 50          |       |
| 20          | 0.4591        | 25 | 0.5169        | 37 | 1.9347        | 142 | 0.8883        | 13 | 40          |       |
| 30          | 0.4617        | 25 | 0.5205        | 36 | 1.9210        | 140 | 0.8870        | 13 | 30          |       |
| 40          | 0.4643        | 25 | 0.5242        | 37 | 1.9074        | 139 | 0.8856        | 13 | 20          |       |
| 50          | 0.4669        | 25 | 0.5280        | 37 | 1.8940        | 137 | 0.8843        | 13 | 10          |       |
| <b>28 0</b> | <b>0.4694</b> |    | <b>0.5317</b> |    | <b>1.8807</b> |     | <b>0.8829</b> |    | <b>0 62</b> |       |
| 10          | 0.4720        | 25 | 0.5354        | 37 | 1.8676        | 136 | 0.8816        | 13 | 50          |       |
| 20          | 0.4746        | 25 | 0.5392        | 37 | 1.8546        | 134 | 0.8802        | 14 | 40          |       |
| 30          | 0.4771        | 25 | 0.5429        | 38 | 1.8417        | 132 | 0.8788        | 14 | 30          |       |
| 40          | 0.4797        | 25 | 0.5467        | 37 | 1.8290        | 131 | 0.8774        | 14 | 20          |       |
| 50          | 0.4822        | 25 | 0.5505        | 38 | 1.8165        | 129 | 0.8760        | 14 | 10          |       |
| <b>29 0</b> | <b>0.4848</b> |    | <b>0.5543</b> |    | <b>1.8040</b> |     | <b>0.8746</b> |    | <b>0 61</b> |       |
| 10          | 0.4873        | 25 | 0.5581        | 38 | 1.7917        | 128 | 0.8732        | 14 | 50          |       |
| 20          | 0.4899        | 25 | 0.5619        | 38 | 1.7793        | 127 | 0.8718        | 14 | 40          |       |
| 30          | 0.4924        | 25 | 0.5657        | 38 | 1.7675        | 126 | 0.8703        | 14 | 30          |       |
| 40          | 0.4949        | 25 | 0.5696        | 39 | 1.7555        | 124 | 0.8689        | 14 | 20          |       |
| 50          | 0.4975        | 25 | 0.5735        | 38 | 1.7437        | 123 | 0.8675        | 14 | 10          |       |
| <b>30 0</b> | <b>0.5000</b> |    | <b>0.5773</b> |    | <b>1.7320</b> |     | <b>0.8660</b> |    | <b>0 60</b> |       |
|             | Cos.          | d. | Cot.          | d. | Tan.          | d.  | Sin.          | d. | °           | P. P. |

60°-70°

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

30°-40°

| ° / |    | Sin.   | d. | Tan.   | d. | Cot.   | d.  | Cos.   | d. | P. P. |  |  |  |       |
|-----|----|--------|----|--------|----|--------|-----|--------|----|-------|--|--|--|-------|
| 30  | 0  | 0.5000 |    | 0.5773 |    | 1.7320 |     | 0.8660 |    | 0 60  |  |  |  |       |
|     | 10 | 0.5025 | 25 | 0.5812 | 39 | 1.7204 | 116 | 0.8643 | 15 | 50    |  |  |  |       |
|     | 20 | 0.5050 | 25 | 0.5851 | 39 | 1.7090 | 114 | 0.8631 | 14 | 40    |  |  |  |       |
|     | 30 | 0.5073 | 25 | 0.5890 | 39 | 1.6976 | 113 | 0.8616 | 14 | 30    |  |  |  |       |
|     | 40 | 0.5100 | 25 | 0.5929 | 39 | 1.6864 | 112 | 0.8601 | 15 | 20    |  |  |  |       |
|     | 50 | 0.5125 | 25 | 0.5969 | 39 | 1.6753 | 111 | 0.8586 | 15 | 10    |  |  |  |       |
| 31  | 0  | 0.5150 | 25 | 0.6008 | 39 | 1.6643 | 110 | 0.8571 | 15 | 0 59  |  |  |  |       |
|     | 10 | 0.5175 | 25 | 0.6048 | 40 | 1.6533 | 109 | 0.8556 | 15 | 50    |  |  |  |       |
|     | 20 | 0.5200 | 24 | 0.6088 | 39 | 1.6423 | 108 | 0.8541 | 15 | 40    |  |  |  |       |
|     | 30 | 0.5225 | 25 | 0.6128 | 40 | 1.6318 | 107 | 0.8526 | 15 | 30    |  |  |  |       |
|     | 40 | 0.5250 | 25 | 0.6168 | 40 | 1.6212 | 106 | 0.8511 | 15 | 20    |  |  |  |       |
|     | 50 | 0.5274 | 24 | 0.6208 | 40 | 1.6107 | 105 | 0.8496 | 15 | 10    |  |  |  |       |
| 32  | 0  | 0.5299 | 24 | 0.6248 | 40 | 1.6003 | 104 | 0.8480 | 15 | 0 58  |  |  |  |       |
|     | 10 | 0.5324 | 25 | 0.6289 | 40 | 1.5900 | 103 | 0.8465 | 15 | 50    |  |  |  |       |
|     | 20 | 0.5348 | 24 | 0.6330 | 41 | 1.5798 | 102 | 0.8449 | 15 | 40    |  |  |  |       |
|     | 30 | 0.5373 | 24 | 0.6370 | 40 | 1.5697 | 101 | 0.8434 | 15 | 30    |  |  |  |       |
|     | 40 | 0.5397 | 24 | 0.6411 | 41 | 1.5596 | 100 | 0.8418 | 16 | 20    |  |  |  |       |
|     | 50 | 0.5422 | 24 | 0.6453 | 41 | 1.5497 | 99  | 0.8402 | 15 | 10    |  |  |  |       |
| 33  | 0  | 0.5446 | 24 | 0.6494 | 41 | 1.5398 | 98  | 0.8386 | 16 | 0 57  |  |  |  |       |
|     | 10 | 0.5471 | 24 | 0.6535 | 41 | 1.5301 | 97  | 0.8371 | 15 | 50    |  |  |  |       |
|     | 20 | 0.5495 | 24 | 0.6577 | 41 | 1.5204 | 96  | 0.8355 | 16 | 40    |  |  |  |       |
|     | 30 | 0.5519 | 24 | 0.6619 | 42 | 1.5108 | 96  | 0.8339 | 16 | 30    |  |  |  |       |
|     | 40 | 0.5543 | 24 | 0.6661 | 42 | 1.5013 | 95  | 0.8323 | 16 | 20    |  |  |  |       |
|     | 50 | 0.5568 | 24 | 0.6703 | 42 | 1.4919 | 94  | 0.8306 | 16 | 10    |  |  |  |       |
| 34  | 0  | 0.5592 | 24 | 0.6745 | 42 | 1.4825 | 93  | 0.8290 | 16 | 0 56  |  |  |  |       |
|     | 10 | 0.5616 | 24 | 0.6787 | 42 | 1.4733 | 92  | 0.8274 | 16 | 50    |  |  |  |       |
|     | 20 | 0.5640 | 24 | 0.6830 | 42 | 1.4641 | 92  | 0.8257 | 16 | 40    |  |  |  |       |
|     | 30 | 0.5664 | 24 | 0.6873 | 43 | 1.4550 | 91  | 0.8241 | 16 | 30    |  |  |  |       |
|     | 40 | 0.5688 | 24 | 0.6915 | 42 | 1.4460 | 90  | 0.8225 | 16 | 20    |  |  |  |       |
|     | 50 | 0.5712 | 24 | 0.6959 | 43 | 1.4370 | 89  | 0.8208 | 17 | 10    |  |  |  |       |
| 35  | 0  | 0.5736 | 24 | 0.7002 | 43 | 1.4281 | 89  | 0.8191 | 16 | 0 55  |  |  |  |       |
|     | 10 | 0.5759 | 23 | 0.7045 | 43 | 1.4193 | 88  | 0.8175 | 16 | 50    |  |  |  |       |
|     | 20 | 0.5783 | 24 | 0.7089 | 43 | 1.4106 | 87  | 0.8158 | 17 | 40    |  |  |  |       |
|     | 30 | 0.5807 | 23 | 0.7133 | 44 | 1.4019 | 86  | 0.8141 | 17 | 30    |  |  |  |       |
|     | 40 | 0.5830 | 23 | 0.7177 | 44 | 1.3933 | 86  | 0.8124 | 17 | 20    |  |  |  |       |
|     | 50 | 0.5854 | 24 | 0.7221 | 44 | 1.3848 | 85  | 0.8107 | 17 | 10    |  |  |  |       |
| 36  | 0  | 0.5878 | 23 | 0.7265 | 44 | 1.3764 | 84  | 0.8090 | 17 | 0 54  |  |  |  |       |
|     | 10 | 0.5901 | 23 | 0.7310 | 44 | 1.3680 | 84  | 0.8073 | 17 | 50    |  |  |  |       |
|     | 20 | 0.5925 | 23 | 0.7354 | 44 | 1.3597 | 83  | 0.8056 | 17 | 40    |  |  |  |       |
|     | 30 | 0.5948 | 23 | 0.7399 | 45 | 1.3514 | 83  | 0.8038 | 17 | 30    |  |  |  |       |
|     | 40 | 0.5971 | 23 | 0.7444 | 45 | 1.3432 | 81  | 0.8021 | 17 | 20    |  |  |  |       |
|     | 50 | 0.5995 | 23 | 0.7490 | 45 | 1.3351 | 81  | 0.8004 | 17 | 10    |  |  |  |       |
| 37  | 0  | 0.6018 | 23 | 0.7535 | 45 | 1.3270 | 80  | 0.7986 | 17 | 0 53  |  |  |  |       |
|     | 10 | 0.6041 | 23 | 0.7581 | 45 | 1.3190 | 80  | 0.7969 | 17 | 50    |  |  |  |       |
|     | 20 | 0.6064 | 23 | 0.7627 | 46 | 1.3111 | 79  | 0.7951 | 18 | 40    |  |  |  |       |
|     | 30 | 0.6087 | 23 | 0.7673 | 46 | 1.3032 | 78  | 0.7933 | 17 | 30    |  |  |  |       |
|     | 40 | 0.6110 | 23 | 0.7719 | 46 | 1.2954 | 78  | 0.7916 | 17 | 20    |  |  |  |       |
|     | 50 | 0.6133 | 23 | 0.7766 | 46 | 1.2876 | 77  | 0.7898 | 18 | 10    |  |  |  |       |
| 38  | 0  | 0.6156 | 23 | 0.7813 | 47 | 1.2799 | 77  | 0.7880 | 18 | 0 52  |  |  |  |       |
|     | 10 | 0.6179 | 23 | 0.7860 | 47 | 1.2723 | 76  | 0.7862 | 18 | 50    |  |  |  |       |
|     | 20 | 0.6202 | 22 | 0.7907 | 47 | 1.2647 | 76  | 0.7844 | 18 | 40    |  |  |  |       |
|     | 30 | 0.6225 | 23 | 0.7954 | 47 | 1.2571 | 75  | 0.7826 | 18 | 30    |  |  |  |       |
|     | 40 | 0.6248 | 22 | 0.8002 | 47 | 1.2497 | 74  | 0.7808 | 18 | 20    |  |  |  |       |
|     | 50 | 0.6270 | 22 | 0.8050 | 48 | 1.2422 | 74  | 0.7789 | 18 | 10    |  |  |  |       |
| 39  | 0  | 0.6293 | 22 | 0.8098 | 48 | 1.2349 | 73  | 0.7771 | 18 | 0 51  |  |  |  |       |
|     | 10 | 0.6316 | 22 | 0.8146 | 48 | 1.2276 | 73  | 0.7753 | 18 | 50    |  |  |  |       |
|     | 20 | 0.6338 | 22 | 0.8194 | 48 | 1.2203 | 73  | 0.7734 | 18 | 40    |  |  |  |       |
|     | 30 | 0.6361 | 22 | 0.8243 | 49 | 1.2131 | 72  | 0.7716 | 18 | 30    |  |  |  |       |
|     | 40 | 0.6383 | 22 | 0.8292 | 49 | 1.2059 | 71  | 0.7697 | 18 | 20    |  |  |  |       |
|     | 50 | 0.6405 | 22 | 0.8341 | 49 | 1.1988 | 71  | 0.7679 | 18 | 10    |  |  |  |       |
| 40  | 0  | 0.6428 | 22 | 0.8391 | 49 | 1.1917 | 70  | 0.7660 | 18 | 0 50  |  |  |  |       |
|     |    | Cos.   | d. | Cot.   | d. | Tan.   | d.  | Sin.   | d. | °     |  |  |  | P. P. |

50°-60°

40°-45°

| °    | Sin.   | d. | Tan.   | d. | Cot.   | d. | Cos.   | d. |      | P. P. |
|------|--------|----|--------|----|--------|----|--------|----|------|-------|
| 40 0 | 0.6428 |    | 0.8391 |    | 1.1917 |    | 0.7666 |    | 0 50 |       |
| 10   | 0.6450 | 22 | 0.8440 | 49 | 1.1847 | 70 | 0.7641 | 19 | 50   |       |
| 20   | 0.6472 | 22 | 0.8490 | 50 | 1.1777 | 69 | 0.7623 | 18 | 50   |       |
| 30   | 0.6494 | 22 | 0.8541 | 50 | 1.1708 | 68 | 0.7604 | 19 | 40   |       |
| 40   | 0.6516 | 22 | 0.8591 | 51 | 1.1640 | 68 | 0.7585 | 19 | 30   |       |
| 50   | 0.6538 | 22 | 0.8642 | 51 | 1.1571 | 68 | 0.7566 | 19 | 20   |       |
| 41 0 | 0.6560 |    | 0.8693 |    | 1.1503 |    | 0.7547 |    | 0 49 |       |
| 10   | 0.6582 | 22 | 0.8744 | 51 | 1.1436 | 67 | 0.7528 | 19 | 50   |       |
| 20   | 0.6604 | 21 | 0.8795 | 52 | 1.1369 | 66 | 0.7509 | 19 | 40   |       |
| 30   | 0.6626 | 22 | 0.8847 | 51 | 1.1303 | 66 | 0.7489 | 19 | 30   |       |
| 40   | 0.6648 | 21 | 0.8899 | 52 | 1.1237 | 65 | 0.7470 | 19 | 20   |       |
| 50   | 0.6669 | 22 | 0.8951 | 52 | 1.1171 | 65 | 0.7451 | 19 | 10   |       |
| 42 0 | 0.6691 |    | 0.9004 |    | 1.1106 |    | 0.7431 |    | 0 48 |       |
| 10   | 0.6713 | 21 | 0.9057 | 53 | 1.1041 | 64 | 0.7412 | 19 | 50   |       |
| 20   | 0.6734 | 21 | 0.9110 | 53 | 1.0977 | 64 | 0.7392 | 19 | 40   |       |
| 30   | 0.6756 | 21 | 0.9163 | 53 | 1.0913 | 63 | 0.7373 | 20 | 30   |       |
| 40   | 0.6777 | 21 | 0.9217 | 54 | 1.0849 | 63 | 0.7353 | 19 | 20   |       |
| 50   | 0.6798 | 21 | 0.9271 | 54 | 1.0786 | 63 | 0.7333 | 20 | 10   |       |
| 43 0 | 0.6820 |    | 0.9325 |    | 1.0723 |    | 0.7313 |    | 0 47 |       |
| 10   | 0.6841 | 21 | 0.9379 | 54 | 1.0661 | 62 | 0.7293 | 20 | 50   |       |
| 20   | 0.6862 | 21 | 0.9434 | 55 | 1.0599 | 61 | 0.7273 | 20 | 40   |       |
| 30   | 0.6883 | 21 | 0.9489 | 55 | 1.0538 | 61 | 0.7253 | 20 | 30   |       |
| 40   | 0.6904 | 21 | 0.9545 | 56 | 1.0476 | 60 | 0.7233 | 20 | 20   |       |
| 50   | 0.6925 | 21 | 0.9601 | 56 | 1.0416 | 60 | 0.7213 | 20 | 10   |       |
| 44 0 | 0.6946 |    | 0.9657 |    | 1.0355 |    | 0.7193 |    | 0 46 |       |
| 10   | 0.6967 | 21 | 0.9713 | 56 | 1.0295 | 60 | 0.7173 | 20 | 50   |       |
| 20   | 0.6988 | 21 | 0.9770 | 56 | 1.0235 | 59 | 0.7153 | 20 | 40   |       |
| 30   | 0.7009 | 20 | 0.9827 | 57 | 1.0176 | 59 | 0.7132 | 20 | 30   |       |
| 40   | 0.7030 | 21 | 0.9884 | 57 | 1.0117 | 59 | 0.7112 | 20 | 20   |       |
| 50   | 0.7050 | 20 | 0.9942 | 57 | 1.0058 | 58 | 0.7091 | 20 | 10   |       |
| 45 0 | 0.7071 |    | 1.0000 |    | 1.0000 |    | 0.7071 |    | 0 45 |       |
|      | Cos.   | d. | Cot.   | d. | Tan.   | d. | Sin.   | d. | °    |       |

| 65     | 64   | 64   | 63   | 62   | 61   | 60   | 59   | 59   | 58   | 58   | 57   | 57   | 56   | 56   | 55   | 54   | 54   | 53   | 53   | 52   | 52   |
|--------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 6.3  | 6.4  | 6.4  | 6.3  | 6.2  | 6.1  | 6.0  | 5.9  | 5.9  | 5.8  | 5.8  | 5.7  | 5.7  | 5.6  | 5.6  | 5.5  | 5.4  | 5.4  | 5.3  | 5.3  | 5.2  | 5.2  |
| 2 13.1 | 12.9 | 12.8 | 12.6 | 12.4 | 12.3 | 12.1 | 11.9 | 11.8 | 11.7 | 11.6 | 11.5 | 11.4 | 11.3 | 11.2 | 11.0 | 10.9 | 10.8 | 10.7 | 10.6 | 10.5 | 10.4 |
| 3 19.6 | 19.3 | 19.2 | 18.9 | 18.6 | 18.4 | 18.1 | 17.8 | 17.7 | 17.5 | 17.4 | 17.2 | 17.1 | 16.9 | 16.8 | 16.5 | 16.3 | 16.2 | 16.0 | 15.9 | 15.7 | 15.6 |
| 4 26.2 | 25.8 | 25.6 | 25.2 | 24.8 | 24.6 | 24.2 | 23.8 | 23.6 | 23.4 | 23.2 | 23.0 | 22.8 | 22.6 | 22.4 | 22.0 | 21.8 | 21.6 | 21.4 | 21.2 | 21.0 | 20.8 |
| 5 32.7 | 32.2 | 32.0 | 31.5 | 31.0 | 30.7 | 30.2 | 29.7 | 29.5 | 29.2 | 29.0 | 28.7 | 28.5 | 28.2 | 28.0 | 27.5 | 27.2 | 27.0 | 26.7 | 26.5 | 26.2 | 26.0 |
| 6 39.3 | 38.7 | 38.4 | 37.8 | 37.2 | 36.9 | 36.3 | 35.7 | 35.4 | 35.1 | 34.8 | 34.5 | 34.2 | 33.9 | 33.6 | 33.0 | 32.7 | 32.4 | 32.1 | 31.8 | 31.5 | 31.2 |
| 7 45.8 | 45.1 | 44.8 | 44.1 | 43.4 | 43.0 | 42.3 | 41.6 | 41.3 | 40.9 | 40.6 | 40.2 | 39.9 | 39.5 | 39.2 | 38.5 | 38.1 | 37.8 | 37.4 | 37.1 | 36.7 | 36.4 |
| 8 52.4 | 51.6 | 51.2 | 50.4 | 49.6 | 49.2 | 48.4 | 47.6 | 47.2 | 46.8 | 46.4 | 46.0 | 45.6 | 45.2 | 44.8 | 44.0 | 43.6 | 43.2 | 42.8 | 42.4 | 42.0 | 41.6 |
| 9 58.9 | 58.0 | 57.6 | 56.7 | 55.8 | 55.3 | 54.4 | 53.5 | 53.1 | 52.6 | 52.2 | 51.7 | 51.3 | 50.8 | 50.4 | 49.5 | 49.0 | 48.6 | 48.1 | 47.7 | 47.2 | 46.8 |

Table for passing from Sexagesimal to Circular Measure.

| °   | Circular Meas. | '  | Circular Meas. | "  | Circular Meas. |
|-----|----------------|----|----------------|----|----------------|
| 100 | 1.74 532 9     | 10 | 0.00 290 9     | 10 | 0.00 004 8     |
| 200 | 3.49 065 8     | 20 | 0.00 581 8     | 20 | 0.00 009 7     |
| 300 | 5.23 598 8     | 30 | 0.00 872 6     | 30 | 0.00 014 5     |
|     |                | 40 | 0.01 163 5     | 40 | 0.00 019 4     |
| 40  | 0.69 813 1     |    |                |    |                |
| 50  | 0.87 266 4     | 50 | 0.01 454 4     | 50 | 0.00 024 2     |
| 60  | 1.04 719 7     |    |                |    |                |
|     |                | 6  | 0.00 174 5     | 6  | 0.00 002 9     |
| 70  | 1.22 173 0     | 7  | 0.00 203 6     | 7  | 0.00 003 4     |
| 80  | 1.39 626 3     | 8  | 0.00 232 7     | 8  | 0.00 003 9     |
| 90  | 1.57 079 6     | 9  | 0.00 261 8     | 9  | 0.00 004 3     |

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

0°-10°

10°-20°

| °    | Vers.  | d. | Exsec. | d. | °    | Vers.  | d. | Exsec. | d.  | P. P.                         |
|------|--------|----|--------|----|------|--------|----|--------|-----|-------------------------------|
| 0 0  | .00000 | 0  | .00000 | 0  | 10 0 | .01519 | 51 | .01542 | 52  |                               |
| 10   | .00000 | 1  | .00000 | 1  | 10   | .01570 | 52 | .01595 | 53  |                               |
| 20   | .00001 | 2  | .00001 | 2  | 20   | .01622 | 52 | .01648 | 54  |                               |
| 30   | .00004 | 3  | .00004 | 3  | 30   | .01674 | 53 | .01703 | 55  |                               |
| 40   | .00007 | 3  | .00007 | 3  | 40   | .01728 | 53 | .01758 | 56  |                               |
| 50   | .00010 | 4  | .00010 | 4  | 50   | .01782 | 54 | .01814 | 57  |                               |
| 1 0  | .00015 | 5  | .00015 | 5  | 11 0 | .01837 | 55 | .01871 | 58  |                               |
| 10   | .00020 | 6  | .00020 | 6  | 10   | .01893 | 55 | .01929 | 59  | 110 100 90 80 70 60 50 40     |
| 20   | .00027 | 7  | .00027 | 7  | 20   | .01950 | 57 | .01988 | 60  | 1 11 10 9 8 7 6 5 4 3 2 1     |
| 30   | .00034 | 8  | .00034 | 8  | 30   | .02007 | 58 | .02048 | 61  | 3 33 30 27 24 21 18 15 12     |
| 40   | .00042 | 8  | .00042 | 8  | 40   | .02066 | 59 | .02109 | 62  | 4 44 40 36 32 28 24 20 16     |
| 50   | .00051 | 10 | .00051 | 10 | 50   | .02125 | 60 | .02171 | 62  | 5 55 50 45 40 35 30 25 20     |
| 2 0  | .00061 | 10 | .00061 | 10 | 12 0 | .02185 | 61 | .02234 | 63  | 6 66 60 54 48 42 36 30 24     |
| 10   | .00071 | 11 | .00071 | 11 | 10   | .02246 | 62 | .02297 | 65  | 7 77 70 63 56 49 42 35 28     |
| 20   | .00083 | 12 | .00083 | 12 | 20   | .02308 | 62 | .02362 | 65  | 8 88 80 72 64 56 48 40 32     |
| 30   | .00095 | 13 | .00095 | 13 | 30   | .02370 | 63 | .02428 | 66  | 9 99 90 81 72 63 54 45 36     |
| 40   | .00108 | 13 | .00108 | 14 | 40   | .02434 | 64 | .02494 | 67  |                               |
| 50   | .00122 | 15 | .00122 | 14 | 50   | .02498 | 65 | .02562 | 68  |                               |
| 3 0  | .00137 | 15 | .00137 | 16 | 13 0 | .02563 | 66 | .02630 | 69  |                               |
| 10   | .00152 | 16 | .00153 | 16 | 10   | .02629 | 66 | .02700 | 70  |                               |
| 20   | .00169 | 17 | .00169 | 17 | 20   | .02693 | 67 | .02770 | 71  | 30 20 10 0 9 8 7 6 5 4 3 2 1  |
| 30   | .00186 | 18 | .00187 | 18 | 30   | .02763 | 68 | .02841 | 72  | 1 3 2 1 0.9 0.9 0.8 0.8 0.7   |
| 40   | .00204 | 19 | .00205 | 19 | 40   | .02831 | 69 | .02914 | 73  | 2 6 4 3 2 1.9 1.8 1.7 1.6 1.5 |
| 50   | .00223 | 20 | .00224 | 20 | 50   | .02900 | 70 | .02987 | 74  | 3 9 6 3 2.8 2.7 2.5 2.4 2.2   |
| 4 0  | .00243 | 21 | .00244 | 21 | 14 0 | .02970 | 70 | .03061 | 75  | 4 12 8 4 3.8 3.6 3.4 3.2 3.0  |
| 10   | .00264 | 21 | .00265 | 21 | 10   | .03041 | 72 | .03136 | 76  | 5 15 10 5 4.7 4.5 4.2 4.0 3.7 |
| 20   | .00286 | 22 | .00286 | 22 | 20   | .03113 | 72 | .03213 | 77  | 6 18 12 6 5.7 5.4 5.1 4.8 4.5 |
| 30   | .00308 | 23 | .00309 | 23 | 30   | .03185 | 72 | .03290 | 78  | 7 21 14 7 6.6 6.3 5.9 5.6 5.2 |
| 40   | .00331 | 24 | .00332 | 24 | 40   | .03258 | 74 | .03368 | 79  | 8 24 16 8 7.6 7.2 6.8 6.4 6.0 |
| 50   | .00355 | 25 | .00357 | 25 | 50   | .03332 | 75 | .03447 | 80  | 9 27 18 9 8.5 8.1 7.6 7.2 6.7 |
| 5 0  | .00380 | 26 | .00382 | 26 | 15 0 | .03407 | 75 | .03527 | 81  |                               |
| 10   | .00406 | 26 | .00408 | 27 | 10   | .03483 | 76 | .03609 | 82  |                               |
| 20   | .00433 | 27 | .00435 | 27 | 20   | .03559 | 77 | .03691 | 83  |                               |
| 30   | .00460 | 28 | .00462 | 28 | 30   | .03637 | 78 | .03774 | 84  |                               |
| 40   | .00488 | 29 | .00491 | 29 | 40   | .03715 | 79 | .03858 | 85  |                               |
| 50   | .00518 | 30 | .00520 | 30 | 50   | .03794 | 80 | .03943 | 86  |                               |
| 6 0  | .00548 | 30 | .00551 | 31 | 16 0 | .03874 | 80 | .04030 | 87  |                               |
| 10   | .00578 | 32 | .00582 | 32 | 10   | .03954 | 81 | .04117 | 88  |                               |
| 20   | .00610 | 32 | .00614 | 33 | 20   | .04036 | 82 | .04205 | 89  |                               |
| 30   | .00643 | 33 | .00647 | 34 | 30   | .04118 | 83 | .04295 | 90  |                               |
| 40   | .00676 | 33 | .00681 | 34 | 40   | .04201 | 84 | .04385 | 91  |                               |
| 50   | .00710 | 35 | .00715 | 35 | 50   | .04285 | 84 | .04476 | 92  |                               |
| 7 0  | .00745 | 36 | .00751 | 36 | 17 0 | .04369 | 85 | .04569 | 93  |                               |
| 10   | .00781 | 36 | .00787 | 37 | 10   | .04455 | 86 | .04662 | 95  |                               |
| 20   | .00818 | 37 | .00824 | 38 | 20   | .04541 | 87 | .04757 | 95  |                               |
| 30   | .00855 | 38 | .00863 | 39 | 30   | .04628 | 87 | .04853 | 96  |                               |
| 40   | .00894 | 39 | .00902 | 40 | 40   | .04716 | 89 | .04949 | 98  |                               |
| 50   | .00933 | 40 | .00942 | 41 | 50   | .04805 | 89 | .05047 | 98  |                               |
| 8 0  | .00973 | 41 | .00983 | 41 | 18 0 | .04894 | 90 | .05146 | 100 |                               |
| 10   | .01014 | 42 | .01024 | 42 | 10   | .04984 | 91 | .05246 | 101 |                               |
| 20   | .01056 | 42 | .01067 | 43 | 20   | .05076 | 91 | .05347 | 102 |                               |
| 30   | .01098 | 43 | .01110 | 44 | 30   | .05167 | 93 | .05449 | 103 |                               |
| 40   | .01142 | 44 | .01155 | 45 | 40   | .05260 | 93 | .05552 | 104 |                               |
| 50   | .01186 | 44 | .01200 | 46 | 50   | .05354 | 94 | .05656 | 105 |                               |
| 9 0  | .01231 | 45 | .01246 | 47 | 19 0 | .05448 | 95 | .05762 | 106 |                               |
| 10   | .01277 | 46 | .01293 | 48 | 10   | .05543 | 95 | .05868 | 107 |                               |
| 20   | .01324 | 47 | .01341 | 49 | 20   | .05639 | 97 | .05976 | 109 |                               |
| 30   | .01371 | 47 | .01390 | 50 | 30   | .05736 | 97 | .06085 | 109 |                               |
| 40   | .01420 | 48 | .01440 | 50 | 40   | .05833 | 98 | .06194 | 111 |                               |
| 50   | .01469 | 49 | .01491 | 51 | 50   | .05931 | 99 | .06305 | 112 |                               |
| 10 0 | .01519 | 50 | .01542 |    | 20 0 | .06036 |    | .06418 |     |                               |
| °    | Vers.  | d. | Exsec. | d. | °    | Vers.  | d. | Exsec. | d.  | P. P.                         |

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

20°-30°

30°-40°

| °  | '  | Vers. | d. | Exsec. | d. | °  | '  | Vers. | d. | Exsec. | d. | P. P. |  |  |  |
|----|----|-------|----|--------|----|----|----|-------|----|--------|----|-------|--|--|--|
| 20 | 0  | .0603 |    | .0642  |    | 30 | 0  | .1339 |    | .1547  |    |       |  |  |  |
|    | 10 | .0613 | 10 | .0653  | 11 |    | 10 | .1354 | 15 | .1566  | 19 |       |  |  |  |
|    | 20 | .0623 | 10 | .0664  | 11 |    | 20 | .1369 | 14 | .1586  | 19 |       |  |  |  |
|    | 30 | .0633 | 10 | .0676  | 12 |    | 30 | .1383 | 14 | .1606  | 20 |       |  |  |  |
|    | 40 | .0643 | 10 | .0688  | 11 |    | 40 | .1398 | 15 | .1626  | 20 |       |  |  |  |
|    | 50 | .0654 | 10 | .0699  | 12 |    | 50 | .1413 | 15 | .1646  | 20 |       |  |  |  |
| 21 | 0  | .0664 | 10 | .0711  | 12 | 31 | 0  | .1428 | 15 | .1666  | 20 |       |  |  |  |
|    | 10 | .0674 | 10 | .0723  | 12 |    | 10 | .1443 | 15 | .1687  | 20 |       |  |  |  |
|    | 20 | .0685 | 11 | .0733  | 12 |    | 20 | .1458 | 15 | .1707  | 21 |       |  |  |  |
|    | 30 | .0696 | 10 | .0748  | 12 |    | 30 | .1473 | 15 | .1728  | 21 |       |  |  |  |
|    | 40 | .0706 | 11 | .0760  | 12 |    | 40 | .1489 | 15 | .1749  | 21 |       |  |  |  |
|    | 50 | .0717 | 10 | .0772  | 13 |    | 50 | .1504 | 15 | .1770  | 21 |       |  |  |  |
| 22 | 0  | .0728 | 11 | .0785  | 12 | 32 | 0  | .1519 | 15 | .1792  | 21 |       |  |  |  |
|    | 10 | .0739 | 11 | .0798  | 13 |    | 10 | .1535 | 15 | .1813  | 21 |       |  |  |  |
|    | 20 | .0750 | 11 | .0811  | 13 |    | 20 | .1550 | 15 | .1835  | 22 |       |  |  |  |
|    | 30 | .0761 | 11 | .0824  | 13 |    | 30 | .1566 | 16 | .1857  | 22 |       |  |  |  |
|    | 40 | .0772 | 11 | .0837  | 13 |    | 40 | .1582 | 15 | .1879  | 22 |       |  |  |  |
|    | 50 | .0783 | 11 | .0850  | 13 |    | 50 | .1597 | 16 | .1901  | 22 |       |  |  |  |
| 23 | 0  | .0795 | 11 | .0863  | 13 | 33 | 0  | .1613 | 15 | .1923  | 23 |       |  |  |  |
|    | 10 | .0806 | 11 | .0877  | 13 |    | 10 | .1629 | 16 | .1946  | 23 |       |  |  |  |
|    | 20 | .0818 | 11 | .0890  | 14 |    | 20 | .1645 | 16 | .1969  | 23 |       |  |  |  |
|    | 30 | .0829 | 11 | .0904  | 14 |    | 30 | .1661 | 16 | .1992  | 23 |       |  |  |  |
|    | 40 | .0841 | 12 | .0918  | 14 |    | 40 | .1677 | 16 | .2015  | 23 |       |  |  |  |
|    | 50 | .0853 | 11 | .0932  | 14 |    | 50 | .1693 | 16 | .2038  | 24 |       |  |  |  |
| 24 | 0  | .0864 | 12 | .0946  | 14 | 34 | 0  | .1709 | 16 | .2062  | 24 |       |  |  |  |
|    | 10 | .0876 | 12 | .0960  | 14 |    | 10 | .1726 | 16 | .2086  | 24 |       |  |  |  |
|    | 20 | .0888 | 12 | .0975  | 14 |    | 20 | .1742 | 16 | .2110  | 24 |       |  |  |  |
|    | 30 | .0900 | 12 | .0989  | 14 |    | 30 | .1758 | 16 | .2134  | 24 |       |  |  |  |
|    | 40 | .0912 | 12 | .1004  | 15 |    | 40 | .1775 | 17 | .2158  | 24 |       |  |  |  |
|    | 50 | .0924 | 12 | .1019  | 15 |    | 50 | .1792 | 16 | .2183  | 24 |       |  |  |  |
| 25 | 0  | .0937 | 12 | .1034  | 15 | 35 | 0  | .1808 | 16 | .2207  | 25 |       |  |  |  |
|    | 10 | .0949 | 12 | .1049  | 15 |    | 10 | .1825 | 17 | .2232  | 25 |       |  |  |  |
|    | 20 | .0961 | 12 | .1064  | 15 |    | 20 | .1842 | 17 | .2258  | 25 |       |  |  |  |
|    | 30 | .0974 | 12 | .1079  | 15 |    | 30 | .1859 | 17 | .2283  | 25 |       |  |  |  |
|    | 40 | .0986 | 12 | .1094  | 16 |    | 40 | .1876 | 17 | .2309  | 25 |       |  |  |  |
|    | 50 | .0999 | 12 | .1110  | 15 |    | 50 | .1893 | 17 | .2334  | 26 |       |  |  |  |
| 26 | 0  | .1012 | 13 | .1126  | 16 | 36 | 0  | .1910 | 17 | .2360  | 26 |       |  |  |  |
|    | 10 | .1025 | 13 | .1142  | 16 |    | 10 | .1927 | 17 | .2387  | 26 |       |  |  |  |
|    | 20 | .1037 | 13 | .1158  | 16 |    | 20 | .1944 | 17 | .2413  | 26 |       |  |  |  |
|    | 30 | .1050 | 13 | .1174  | 16 |    | 30 | .1961 | 17 | .2440  | 27 |       |  |  |  |
|    | 40 | .1063 | 13 | .1190  | 16 |    | 40 | .1979 | 17 | .2467  | 27 |       |  |  |  |
|    | 50 | .1077 | 13 | .1206  | 17 |    | 50 | .1996 | 17 | .2494  | 27 |       |  |  |  |
| 27 | 0  | .1090 | 13 | .1223  | 16 | 37 | 0  | .2013 | 17 | .2521  | 27 |       |  |  |  |
|    | 10 | .1103 | 13 | .1240  | 17 |    | 10 | .2031 | 18 | .2549  | 27 |       |  |  |  |
|    | 20 | .1116 | 13 | .1257  | 17 |    | 20 | .2049 | 18 | .2576  | 27 |       |  |  |  |
|    | 30 | .1130 | 13 | .1274  | 17 |    | 30 | .2066 | 18 | .2604  | 28 |       |  |  |  |
|    | 40 | .1143 | 13 | .1291  | 17 |    | 40 | .2084 | 18 | .2633  | 28 |       |  |  |  |
|    | 50 | .1157 | 13 | .1308  | 17 |    | 50 | .2102 | 18 | .2661  | 28 |       |  |  |  |
| 28 | 0  | .1170 | 13 | .1325  | 18 | 38 | 0  | .2120 | 18 | .2690  | 28 |       |  |  |  |
|    | 10 | .1184 | 14 | .1343  | 18 |    | 10 | .2138 | 18 | .2719  | 29 |       |  |  |  |
|    | 20 | .1198 | 14 | .1361  | 18 |    | 20 | .2156 | 18 | .2748  | 29 |       |  |  |  |
|    | 30 | .1212 | 14 | .1379  | 18 |    | 30 | .2174 | 18 | .2778  | 29 |       |  |  |  |
|    | 40 | .1225 | 14 | .1397  | 18 |    | 40 | .2192 | 18 | .2807  | 30 |       |  |  |  |
|    | 50 | .1239 | 14 | .1415  | 18 |    | 50 | .2210 | 18 | .2837  | 30 |       |  |  |  |
| 29 | 0  | .1254 | 14 | .1433  | 18 | 39 | 0  | .2228 | 18 | .2867  | 30 |       |  |  |  |
|    | 10 | .1268 | 14 | .1452  | 18 |    | 10 | .2247 | 18 | .2898  | 30 |       |  |  |  |
|    | 20 | .1282 | 14 | .1470  | 19 |    | 20 | .2265 | 18 | .2928  | 31 |       |  |  |  |
|    | 30 | .1296 | 14 | .1489  | 19 |    | 30 | .2284 | 18 | .2959  | 31 |       |  |  |  |
|    | 40 | .1311 | 14 | .1508  | 19 |    | 40 | .2302 | 18 | .2991  | 31 |       |  |  |  |
|    | 50 | .1325 | 14 | .1527  | 19 |    | 50 | .2321 | 18 | .3022  | 31 |       |  |  |  |
| 30 | 0  | .1339 | 14 | .1547  | 19 | 40 | 0  | .2339 | 18 | .3054  | 31 |       |  |  |  |
| °  | '  | Vers. | d. | Exsec. | d. | °  | '  | Vers. | d. | Exsec. | d. | P. P. |  |  |  |

31 30 29 28

|   |      |      |      |      |
|---|------|------|------|------|
| 1 | 3.1  | 3.0  | 2.9  | 2.8  |
| 2 | 6.2  | 6.0  | 5.8  | 5.6  |
| 3 | 9.3  | 9.0  | 8.7  | 8.4  |
| 4 | 12.4 | 12.0 | 11.6 | 11.2 |
| 5 | 15.5 | 15.0 | 14.5 | 14.0 |
| 6 | 18.6 | 18.0 | 17.4 | 16.8 |
| 7 | 21.7 | 21.0 | 20.3 | 19.6 |
| 8 | 24.8 | 24.0 | 23.2 | 22.4 |
| 9 | 27.9 | 27.0 | 26.1 | 25.2 |

27 26 25 24

|   |      |      |      |      |
|---|------|------|------|------|
| 1 | 2.7  | 2.6  | 2.5  | 2.4  |
| 2 | 5.4  | 5.2  | 5.0  | 4.8  |
| 3 | 8.1  | 7.8  | 7.5  | 7.2  |
| 4 | 10.8 | 10.4 | 10.0 | 9.6  |
| 5 | 13.5 | 13.0 | 12.5 | 12.0 |
| 6 | 16.2 | 15.6 | 15.0 | 14.4 |
| 7 | 18.9 | 18.2 | 17.5 | 16.8 |
| 8 | 21.6 | 20.8 | 20.0 | 19.2 |
| 9 | 24.3 | 23.4 | 22.5 | 21.6 |

23 22 21 20

|   |      |      |      |      |
|---|------|------|------|------|
| 1 | 2.3  | 2.2  | 2.1  | 2.0  |
| 2 | 4.6  | 4.4  | 4.2  | 4.0  |
| 3 | 6.9  | 6.6  | 6.3  | 6.0  |
| 4 | 9.2  | 8.8  | 8.4  | 8.0  |
| 5 | 11.5 | 11.0 | 10.5 | 10.0 |
| 6 | 13.8 | 13.2 | 12.6 | 12.0 |
| 7 | 16.1 | 15.4 | 14.7 | 14.0 |
| 8 | 18.4 | 17.6 | 16.8 | 16.0 |
| 9 | 20.7 | 19.8 | 18.9 | 18.0 |

19 18 17 16

|   |      |      |      |      |
|---|------|------|------|------|
| 1 | 1.9  | 1.8  | 1.7  | 1.6  |
| 2 | 3.8  | 3.6  | 3.4  | 3.2  |
| 3 | 5.7  | 5.4  | 5.1  | 4.8  |
| 4 | 7.6  | 7.2  | 6.8  | 6.4  |
| 5 | 9.5  | 9.0  | 8.5  | 8.0  |
| 6 | 11.4 | 10.8 | 10.2 | 9.6  |
| 7 | 13.3 | 12.6 | 11.9 | 11.2 |
| 8 | 15.2 | 14.4 | 13.6 | 12.8 |
| 9 | 17.1 | 16.2 | 15.3 | 14.4 |

15 14 13 12

|   |      |      |      |      |
|---|------|------|------|------|
| 1 | 1.5  | 1.4  | 1.3  | 1.2  |
| 2 | 3.0  | 2.8  | 2.6  | 2.4  |
| 3 | 4.5  | 4.2  | 3.9  | 3.6  |
| 4 | 6.0  | 5.6  | 5.2  | 4.8  |
| 5 | 7.5  | 7.0  | 6.5  | 6.0  |
| 6 | 9.0  | 8.4  | 7.8  | 7.2  |
| 7 | 10.5 | 9.8  | 9.1  | 8.4  |
| 8 | 12.0 | 11.2 | 10.4 | 9.6  |
| 9 | 13.5 | 12.6 | 11.7 | 10.8 |

11 10 0

|   |     |     |     |     |
|---|-----|-----|-----|-----|
| 1 | 1.1 | 1.0 | 0.9 | 0.8 |
| 2 | 2.2 | 2.0 | 1.8 | 1.6 |
| 3 | 3.3 | 3.0 | 2.7 | 2.4 |
| 4 | 4.4 | 4.0 | 3.6 | 3.2 |
| 5 | 5.5 | 5.0 | 4.5 | 4.0 |
| 6 | 6.6 | 6.0 | 5.4 | 4.8 |
| 7 | 7.7 | 7.0 | 6.3 | 5.6 |
| 8 | 8.8 | 8.0 | 7.2 | 6.4 |
| 9 | 9.9 | 9.0 | 8.1 | 7.2 |

40°-50°

50°-60°

| °    | Vers. | d. | Exsec. | d. | °    | Vers. | d. | Exsec. | d.  | P. P.                                                                                                                                                                                                                                                                                                                  |
|------|-------|----|--------|----|------|-------|----|--------|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 40 0 | .2339 | 19 | .3054  | 32 | 50 0 | .3572 | 22 | .5557  | 53  | 9 8 7 6 5 4<br>1 0.9 0.8 0.7 0.6 0.5 0.4<br>2 1.8 1.6 1.4 1.2 1.0 0.8<br>3 2.7 2.4 2.1 1.8 1.5 1.2<br>4 3.6 3.2 2.8 2.4 2.0 1.6<br>5 4.5 4.0 3.5 3.0 2.5 2.0<br>6 5.4 4.8 4.2 3.6 3.0 2.4<br>7 6.3 5.6 4.9 4.2 3.5 2.8<br>8 7.2 6.4 5.6 4.8 4.0 3.2<br>9 8.1 7.2 6.3 5.4 4.5 3.6                                       |
| 10   | .2358 | 18 | .3086  | 32 | 10   | .3594 | 22 | .5611  | 54  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .2377 | 19 | .3118  | 32 | 20   | .3617 | 22 | .5666  | 54  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .2396 | 19 | .3151  | 32 | 30   | .3639 | 22 | .5721  | 55  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .2415 | 19 | .3183  | 33 | 40   | .3661 | 22 | .5777  | 55  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .2434 | 19 | .3217  | 33 | 50   | .3684 | 23 | .5833  | 56  |                                                                                                                                                                                                                                                                                                                        |
| 41 0 | .2453 | 19 | .3250  | 34 | 51 0 | .3707 | 22 | .5890  | 56  | 3 2 1 9 8 7<br>1 0.3 0.2 0.1 0.0 0.8 0.7<br>2 0.6 0.4 0.2 1.9 1.7 1.5<br>3 0.9 0.6 0.3 2.8 2.5 2.2<br>4 1.2 0.8 0.4 3.8 3.4 3.0<br>5 1.5 1.0 0.5 4.7 4.2 3.7<br>6 1.8 1.2 0.6 5.7 5.1 4.5<br>7 2.1 1.4 0.7 6.6 5.9 5.2<br>8 2.4 1.6 0.8 7.6 6.8 6.0<br>9 2.7 1.8 0.9 8.5 7.6 6.7                                       |
| 10   | .2472 | 19 | .3284  | 33 | 10   | .3729 | 22 | .5947  | 57  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .2491 | 19 | .3317  | 34 | 20   | .3752 | 23 | .6005  | 58  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .2510 | 19 | .3352  | 34 | 30   | .3775 | 22 | .6064  | 58  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .2529 | 19 | .3386  | 34 | 40   | .3797 | 23 | .6123  | 59  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .2549 | 19 | .3421  | 35 | 50   | .3820 | 23 | .6182  | 59  |                                                                                                                                                                                                                                                                                                                        |
| 42 0 | .2568 | 19 | .3456  | 35 | 52 0 | .3843 | 23 | .6242  | 60  | 6 5 4 3 2 1<br>1 0.6 0.5 0.4 0.3 0.2 0.1<br>2 1.3 1.1 0.9 0.7 0.5 0.3<br>3 1.9 1.6 1.3 1.0 0.7 0.4<br>4 2.6 2.2 1.8 1.4 1.0 0.6<br>5 3.2 2.7 2.2 1.7 1.2 0.7<br>6 3.9 3.3 2.7 2.1 1.5 0.9<br>7 4.5 3.8 3.1 2.4 1.7 1.0<br>8 5.2 4.4 3.6 2.8 2.0 1.2<br>9 5.8 4.9 4.0 3.1 2.2 1.3                                       |
| 10   | .2588 | 19 | .3491  | 36 | 10   | .3866 | 23 | .6303  | 61  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .2607 | 19 | .3527  | 36 | 20   | .3889 | 23 | .6365  | 61  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .2627 | 20 | .3553  | 36 | 30   | .3913 | 23 | .6427  | 62  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .2647 | 19 | .3599  | 37 | 40   | .3935 | 23 | .6489  | 62  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .2666 | 20 | .3636  | 37 | 50   | .3958 | 23 | .6552  | 63  |                                                                                                                                                                                                                                                                                                                        |
| 43 0 | .2686 | 20 | .3673  | 37 | 53 0 | .3982 | 23 | .6616  | 64  | 25 25 24 24 23 23<br>1 2.5 2.5 2.4 2.4 2.3 2.3<br>2 5.1 5.0 4.9 4.8 4.7 4.6<br>3 7.6 7.5 7.3 7.2 7.0 6.9<br>4 10.2 10.0 9.8 9.6 9.4 9.2<br>5 12.7 12.5 12.2 12.0 11.7 11.5<br>6 15.3 15.0 14.7 14.4 14.1 13.8<br>7 17.8 17.5 17.1 16.8 16.4 16.1<br>8 20.4 20.0 19.6 19.2 18.8 18.4<br>9 22.9 22.5 22.0 21.6 21.1 20.7 |
| 10   | .2706 | 20 | .3710  | 37 | 10   | .4005 | 23 | .6681  | 64  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .2726 | 20 | .3748  | 38 | 20   | .4028 | 23 | .6746  | 65  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .2746 | 20 | .3786  | 38 | 30   | .4052 | 23 | .6811  | 65  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .2766 | 20 | .3824  | 39 | 40   | .4075 | 23 | .6878  | 66  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .2786 | 20 | .3863  | 38 | 50   | .4098 | 23 | .6945  | 67  |                                                                                                                                                                                                                                                                                                                        |
| 44 0 | .2806 | 20 | .3901  | 39 | 54 0 | .4122 | 23 | .7013  | 68  | 22 22 21 21 20 20<br>1 2.2 2.2 2.1 2.1 2.0 2.0<br>2 4.5 4.4 4.3 4.2 4.1 4.0<br>3 6.7 6.6 6.4 6.3 6.1 6.0<br>4 9.0 8.8 8.6 8.4 8.2 8.0<br>5 11.2 11.0 10.7 10.5 10.2 10.0<br>6 13.5 13.2 12.9 12.6 12.3 12.0<br>7 15.7 15.4 15.0 14.7 14.3 14.0<br>8 18.0 17.6 17.2 16.8 16.4 16.0<br>9 20.2 19.8 19.3 18.9 18.4 18.0   |
| 10   | .2827 | 20 | .3941  | 39 | 10   | .4145 | 24 | .7081  | 68  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .2847 | 20 | .3980  | 40 | 20   | .4169 | 23 | .7150  | 69  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .2867 | 20 | .4020  | 40 | 30   | .4193 | 23 | .7220  | 70  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .2888 | 20 | .4060  | 40 | 40   | .4216 | 24 | .7291  | 70  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .2908 | 20 | .4101  | 41 | 50   | .4240 | 23 | .7362  | 71  |                                                                                                                                                                                                                                                                                                                        |
| 45 0 | .2929 | 20 | .4142  | 41 | 55 0 | .4264 | 24 | .7434  | 72  | 19 19 18<br>1 1.9 1.8 1.7 1.6 1.5 1.4<br>2 3.9 3.8 3.7 3.6 3.5 3.4<br>3 5.8 5.7 5.6 5.5 5.4 5.3<br>4 7.8 7.6 7.5 7.4 7.3 7.2<br>5 9.7 9.5 9.4 9.3 9.2 9.1<br>6 11.7 11.4 11.2 11.0 10.8 10.6<br>7 13.6 13.3 13.1 12.9 12.7 12.5<br>8 15.6 15.2 15.0 14.8 14.6 14.4<br>9 17.5 17.1 16.9 16.6 16.4 16.2                  |
| 10   | .2949 | 20 | .4183  | 41 | 10   | .4288 | 24 | .7507  | 73  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .2970 | 21 | .4225  | 42 | 20   | .4312 | 24 | .7581  | 73  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .2991 | 20 | .4267  | 42 | 30   | .4336 | 24 | .7655  | 74  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .3011 | 21 | .4309  | 43 | 40   | .4360 | 24 | .7730  | 75  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .3032 | 21 | .4352  | 43 | 50   | .4384 | 24 | .7806  | 75  |                                                                                                                                                                                                                                                                                                                        |
| 46 0 | .3053 | 21 | .4395  | 43 | 56 0 | .4408 | 24 | .7883  | 77  | 22 22 21 21 20 20<br>1 2.2 2.2 2.1 2.1 2.0 2.0<br>2 4.5 4.4 4.3 4.2 4.1 4.0<br>3 6.7 6.6 6.4 6.3 6.1 6.0<br>4 9.0 8.8 8.6 8.4 8.2 8.0<br>5 11.2 11.0 10.7 10.5 10.2 10.0<br>6 13.5 13.2 12.9 12.6 12.3 12.0<br>7 15.7 15.4 15.0 14.7 14.3 14.0<br>8 18.0 17.6 17.2 16.8 16.4 16.0<br>9 20.2 19.8 19.3 18.9 18.4 18.0   |
| 10   | .3074 | 21 | .4439  | 44 | 10   | .4432 | 24 | .7960  | 77  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .3095 | 21 | .4483  | 44 | 20   | .4456 | 24 | .8039  | 78  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .3116 | 21 | .4527  | 44 | 30   | .4480 | 24 | .8118  | 79  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .3137 | 21 | .4572  | 45 | 40   | .4505 | 24 | .8198  | 80  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .3157 | 21 | .4617  | 45 | 50   | .4529 | 24 | .8279  | 81  |                                                                                                                                                                                                                                                                                                                        |
| 47 0 | .3180 | 21 | .4663  | 45 | 57 0 | .4553 | 24 | .8361  | 82  | 19 19 18<br>1 1.9 1.8 1.7 1.6 1.5 1.4<br>2 3.9 3.8 3.7 3.6 3.5 3.4<br>3 5.8 5.7 5.6 5.5 5.4 5.3<br>4 7.8 7.6 7.5 7.4 7.3 7.2<br>5 9.7 9.5 9.4 9.3 9.2 9.1<br>6 11.7 11.4 11.2 11.0 10.8 10.6<br>7 13.6 13.3 13.1 12.9 12.7 12.5<br>8 15.6 15.2 15.0 14.8 14.6 14.4<br>9 17.5 17.1 16.9 16.6 16.4 16.2                  |
| 10   | .3201 | 21 | .4708  | 46 | 10   | .4578 | 24 | .8443  | 82  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .3222 | 21 | .4755  | 47 | 20   | .4602 | 24 | .8527  | 83  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .3244 | 21 | .4802  | 47 | 30   | .4627 | 24 | .8611  | 84  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .3265 | 21 | .4849  | 47 | 40   | .4651 | 24 | .8697  | 85  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .3287 | 21 | .4896  | 48 | 50   | .4676 | 25 | .8783  | 86  |                                                                                                                                                                                                                                                                                                                        |
| 48 0 | .3308 | 22 | .4945  | 48 | 58 0 | .4701 | 24 | .8871  | 88  | 19 19 18<br>1 1.9 1.8 1.7 1.6 1.5 1.4<br>2 3.9 3.8 3.7 3.6 3.5 3.4<br>3 5.8 5.7 5.6 5.5 5.4 5.3<br>4 7.8 7.6 7.5 7.4 7.3 7.2<br>5 9.7 9.5 9.4 9.3 9.2 9.1<br>6 11.7 11.4 11.2 11.0 10.8 10.6<br>7 13.6 13.3 13.1 12.9 12.7 12.5<br>8 15.6 15.2 15.0 14.8 14.6 14.4<br>9 17.5 17.1 16.9 16.6 16.4 16.2                  |
| 10   | .3330 | 21 | .4993  | 48 | 10   | .4725 | 24 | .8959  | 88  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .3352 | 22 | .5042  | 49 | 20   | .4750 | 25 | .9048  | 89  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .3374 | 21 | .5091  | 50 | 30   | .4775 | 25 | .9139  | 90  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .3395 | 22 | .5141  | 50 | 40   | .4800 | 24 | .9230  | 91  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .3417 | 22 | .5192  | 50 | 50   | .4824 | 25 | .9322  | 92  |                                                                                                                                                                                                                                                                                                                        |
| 49 0 | .3439 | 22 | .5242  | 51 | 59 0 | .4849 | 25 | .9416  | 93  | 19 19 18<br>1 1.9 1.8 1.7 1.6 1.5 1.4<br>2 3.9 3.8 3.7 3.6 3.5 3.4<br>3 5.8 5.7 5.6 5.5 5.4 5.3<br>4 7.8 7.6 7.5 7.4 7.3 7.2<br>5 9.7 9.5 9.4 9.3 9.2 9.1<br>6 11.7 11.4 11.2 11.0 10.8 10.6<br>7 13.6 13.3 13.1 12.9 12.7 12.5<br>8 15.6 15.2 15.0 14.8 14.6 14.4<br>9 17.5 17.1 16.9 16.6 16.4 16.2                  |
| 10   | .3461 | 22 | .5294  | 51 | 10   | .4874 | 25 | .9510  | 94  |                                                                                                                                                                                                                                                                                                                        |
| 20   | .3483 | 22 | .5345  | 52 | 20   | .4899 | 25 | .9606  | 95  |                                                                                                                                                                                                                                                                                                                        |
| 30   | .3505 | 22 | .5397  | 53 | 30   | .4924 | 25 | .9703  | 97  |                                                                                                                                                                                                                                                                                                                        |
| 40   | .3527 | 22 | .5450  | 53 | 40   | .4949 | 25 | .9801  | 98  |                                                                                                                                                                                                                                                                                                                        |
| 50   | .3550 | 22 | .5503  | 53 | 50   | .4975 | 25 | .9900  | 99  |                                                                                                                                                                                                                                                                                                                        |
| 50 0 | .3572 |    | .5557  | 53 | 60 0 | .5000 |    | I 0000 | 100 | 19 19 18<br>1 1.9 1.8 1.7 1.6 1.5 1.4<br>2 3.9 3.8 3.7 3.6 3.5 3.4<br>3 5.8 5.7 5.6 5.5 5.4 5.3<br>4 7.8 7.6 7.5 7.4 7.3 7.2<br>5 9.7 9.5 9.4 9.3 9.2 9.1<br>6 11.7 11.4 11.2 11.0 10.8 10.6<br>7 13.6 13.3 13.1 12.9 12.7 12.5<br>8 15.6 15.2 15.0 14.8 14.6 14.4<br>9 17.5 17.1 16.9 16.6 16.4 16.2                  |
| °    | Vers. | d. | Exsec. | d. | °    | Vers. | d. | Exsec. | d.  | P. P.                                                                                                                                                                                                                                                                                                                  |



TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

60°-70°

70°-80°

| °  | '  | Vers. | d. | Exsec. | d.  | °  | '  | Vers. | d. | Exsec. | d.  | P. P. |
|----|----|-------|----|--------|-----|----|----|-------|----|--------|-----|-------|
| 60 | 0  | .5000 |    | 1.0000 | 101 | 70 | 0  | .6580 |    | 1.9238 |     |       |
|    | 10 | .5025 | 25 | 1.0101 | 102 |    | 10 | .6607 | 27 | 1.9473 | 235 |       |
|    | 20 | .5050 | 23 | 1.0204 | 103 |    | 20 | .6634 | 27 | 1.9713 | 240 |       |
|    | 30 | .5076 | 25 | 1.0307 | 103 |    | 30 | .6662 | 27 | 1.9957 | 244 |       |
|    | 40 | .5101 | 25 | 1.0413 | 105 |    | 40 | .6689 | 27 | 2.0203 | 248 |       |
|    | 50 | .5126 | 23 | 1.0519 | 106 |    | 50 | .6717 | 27 | 2.0458 | 253 |       |
| 61 | 0  | .5152 | 23 | 1.0626 | 107 | 71 | 0  | .6744 | 27 | 2.0715 | 257 |       |
|    | 10 | .5177 | 23 | 1.0733 | 109 |    | 10 | .6772 | 27 | 2.0977 | 262 |       |
|    | 20 | .5203 | 23 | 1.0846 | 110 |    | 20 | .6799 | 27 | 2.1244 | 267 |       |
|    | 30 | .5228 | 23 | 1.0957 | 111 |    | 30 | .6827 | 27 | 2.1513 | 270 |       |
|    | 40 | .5254 | 23 | 1.1070 | 113 |    | 40 | .6854 | 27 | 2.1792 | 276 |       |
|    | 50 | .5279 | 23 | 1.1184 | 114 |    | 50 | .6882 | 27 | 2.2073 | 281 |       |
| 62 | 0  | .5305 | 26 | 1.1300 | 116 | 72 | 0  | .6910 | 28 | 2.2360 | 287 |       |
|    | 10 | .5331 | 23 | 1.1418 | 117 |    | 10 | .6937 | 27 | 2.2653 | 292 |       |
|    | 20 | .5356 | 26 | 1.1536 | 118 |    | 20 | .6965 | 27 | 2.2951 | 298 |       |
|    | 30 | .5382 | 26 | 1.1657 | 120 |    | 30 | .6993 | 28 | 2.3255 | 304 |       |
|    | 40 | .5408 | 26 | 1.1778 | 121 |    | 40 | .7020 | 27 | 2.3565 | 310 |       |
|    | 50 | .5434 | 23 | 1.1902 | 123 |    | 50 | .7048 | 28 | 2.3881 | 316 |       |
| 63 | 0  | .5460 | 26 | 1.2027 | 125 | 73 | 0  | .7076 | 28 | 2.4203 | 322 |       |
|    | 10 | .5486 | 26 | 1.2153 | 126 |    | 10 | .7104 | 27 | 2.4531 | 328 |       |
|    | 20 | .5512 | 26 | 1.2281 | 128 |    | 20 | .7132 | 28 | 2.4867 | 335 |       |
|    | 30 | .5538 | 26 | 1.2411 | 130 |    | 30 | .7160 | 28 | 2.5209 | 342 |       |
|    | 40 | .5564 | 26 | 1.2543 | 131 |    | 40 | .7187 | 27 | 2.5558 | 349 |       |
|    | 50 | .5590 | 26 | 1.2676 | 133 |    | 50 | .7215 | 28 | 2.5915 | 356 |       |
| 64 | 0  | .5616 | 26 | 1.2811 | 135 | 74 | 0  | .7243 | 28 | 2.6279 | 364 |       |
|    | 10 | .5642 | 26 | 1.2948 | 137 |    | 10 | .7271 | 28 | 2.6651 | 372 |       |
|    | 20 | .5668 | 26 | 1.3087 | 139 |    | 20 | .7299 | 28 | 2.7031 | 380 |       |
|    | 30 | .5695 | 26 | 1.3228 | 140 |    | 30 | .7327 | 28 | 2.7420 | 388 |       |
|    | 40 | .5721 | 26 | 1.3371 | 143 |    | 40 | .7355 | 28 | 2.7816 | 396 |       |
|    | 50 | .5747 | 26 | 1.3515 | 144 |    | 50 | .7383 | 28 | 2.8222 | 406 |       |
| 65 | 0  | .5774 | 26 | 1.3662 | 146 | 75 | 0  | .7412 | 28 | 2.8637 | 414 |       |
|    | 10 | .5800 | 26 | 1.3810 | 148 |    | 10 | .7440 | 28 | 2.9061 | 424 |       |
|    | 20 | .5826 | 26 | 1.3961 | 151 |    | 20 | .7468 | 28 | 2.9495 | 434 |       |
|    | 30 | .5853 | 26 | 1.4114 | 152 |    | 30 | .7496 | 28 | 2.9939 | 444 |       |
|    | 40 | .5879 | 26 | 1.4269 | 155 |    | 40 | .7524 | 28 | 3.0394 | 454 |       |
|    | 50 | .5906 | 26 | 1.4426 | 157 |    | 50 | .7552 | 28 | 3.0859 | 465 |       |
| 66 | 0  | .5932 | 26 | 1.4586 | 159 | 76 | 0  | .7581 | 28 | 3.1335 | 476 |       |
|    | 10 | .5959 | 26 | 1.4747 | 161 |    | 10 | .7609 | 28 | 3.1824 | 488 |       |
|    | 20 | .5986 | 27 | 1.4912 | 164 |    | 20 | .7637 | 28 | 3.2324 | 500 |       |
|    | 30 | .6012 | 26 | 1.5078 | 166 |    | 30 | .7665 | 28 | 3.2836 | 512 |       |
|    | 40 | .6039 | 26 | 1.5247 | 169 |    | 40 | .7694 | 28 | 3.3362 | 525 |       |
|    | 50 | .6066 | 27 | 1.5419 | 171 |    | 50 | .7722 | 28 | 3.3901 | 539 |       |
| 67 | 0  | .6092 | 26 | 1.5593 | 174 | 77 | 0  | .7750 | 28 | 3.4454 | 553 |       |
|    | 10 | .6119 | 27 | 1.5770 | 177 |    | 10 | .7779 | 28 | 3.5021 | 567 |       |
|    | 20 | .6146 | 27 | 1.5949 | 179 |    | 20 | .7807 | 28 | 3.5604 | 582 |       |
|    | 30 | .6173 | 26 | 1.6131 | 182 |    | 30 | .7835 | 28 | 3.6202 | 598 |       |
|    | 40 | .6200 | 27 | 1.6316 | 185 |    | 40 | .7864 | 28 | 3.6816 | 614 |       |
|    | 50 | .6227 | 27 | 1.6504 | 188 |    | 50 | .7892 | 28 | 3.7448 | 631 |       |
| 68 | 0  | .6254 | 27 | 1.6694 | 190 | 78 | 0  | .7921 | 28 | 3.8097 | 649 |       |
|    | 10 | .6281 | 27 | 1.6888 | 194 |    | 10 | .7949 | 28 | 3.8765 | 667 |       |
|    | 20 | .6308 | 27 | 1.7085 | 196 |    | 20 | .7978 | 28 | 3.9451 | 686 |       |
|    | 30 | .6335 | 27 | 1.7285 | 200 |    | 30 | .8006 | 28 | 4.0158 | 707 |       |
|    | 40 | .6362 | 27 | 1.7488 | 203 |    | 40 | .8035 | 28 | 4.0886 | 728 |       |
|    | 50 | .6389 | 27 | 1.7694 | 206 |    | 50 | .8063 | 28 | 4.1636 | 749 |       |
| 69 | 0  | .6416 | 27 | 1.7904 | 210 | 79 | 0  | .8092 | 28 | 4.2408 | 772 |       |
|    | 10 | .6443 | 27 | 1.8117 | 213 |    | 10 | .8120 | 28 | 4.3205 | 796 |       |
|    | 20 | .6470 | 27 | 1.8334 | 216 |    | 20 | .8149 | 28 | 4.4026 | 821 |       |
|    | 30 | .6498 | 27 | 1.8554 | 220 |    | 30 | .8177 | 28 | 4.4874 | 847 |       |
|    | 40 | .6525 | 27 | 1.8778 | 224 |    | 40 | .8206 | 29 | 4.5749 | 875 |       |
|    | 50 | .6552 | 27 | 1.9006 | 227 |    | 50 | .8235 | 28 | 4.6653 | 904 |       |
| 70 | 0  | .6580 | 27 | 1.9238 | 232 | 80 | 0  | .8263 | 28 | 4.7587 | 934 |       |
| °  | '  | Vers. | d. | Exsec. | d.  | °  | '  | Vers. | d. | Exsec. | d.  | P. P. |

9 8 7 6 5 4

|   |     |     |     |     |     |     |
|---|-----|-----|-----|-----|-----|-----|
| 1 | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 |
| 2 | 1.8 | 1.0 | 1.4 | 1.2 | 1.0 | 0.8 |
| 3 | 2.7 | 2.4 | 2.1 | 1.8 | 1.5 | 1.2 |
| 4 | 3.6 | 3.2 | 2.8 | 2.4 | 2.0 | 1.6 |
| 5 | 4.5 | 4.0 | 3.5 | 3.0 | 2.5 | 2.0 |
| 6 | 5.4 | 4.8 | 4.2 | 3.6 | 3.0 | 2.4 |
| 7 | 6.3 | 5.6 | 4.9 | 4.2 | 3.5 | 2.8 |
| 8 | 7.2 | 6.4 | 5.6 | 4.8 | 4.0 | 3.2 |
| 9 | 8.1 | 7.2 | 6.3 | 5.4 | 4.5 | 3.6 |

3 2 1 9 8 7

|   |     |     |     |     |     |     |
|---|-----|-----|-----|-----|-----|-----|
| 1 | 0.3 | 0.2 | 0.1 | 0.9 | 0.8 | 0.7 |
| 2 | 0.6 | 0.4 | 0.2 | 1.9 | 1.7 | 1.5 |
| 3 | 0.9 | 0.6 | 0.3 | 2.8 | 2.5 | 2.2 |
| 4 | 1.2 | 0.8 | 0.4 | 3.8 | 3.4 | 3.0 |
| 5 | 1.5 | 1.0 | 0.5 | 4.7 | 4.2 | 3.7 |
| 6 | 1.8 | 1.2 | 0.6 | 5.7 | 5.1 | 4.5 |
| 7 | 2.1 | 1.4 | 0.7 | 6.6 | 5.9 | 5.2 |
| 8 | 2.4 | 1.6 | 0.8 | 7.6 | 6.8 | 6.0 |
| 9 | 2.7 | 1.8 | 0.9 | 8.5 | 7.6 | 6.7 |

6 5 4 3 2 1

|   |     |     |     |     |     |     |
|---|-----|-----|-----|-----|-----|-----|
| 1 | 0.6 | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 |
| 2 | 1.3 | 1.1 | 0.9 | 0.7 | 0.5 | 0.3 |
| 3 | 1.9 | 1.6 | 1.3 | 1.0 | 0.7 | 0.4 |
| 4 | 2.6 | 2.2 | 1.8 | 1.4 | 1.0 | 0.6 |
| 5 | 3.2 | 2.7 | 2.2 | 1.7 | 1.2 | 0.7 |
| 6 | 3.9 | 3.3 | 2.7 | 2.1 | 1.5 | 0.9 |
| 7 | 4.5 | 3.8 | 3.1 | 2.4 | 1.7 | 1.0 |
| 8 | 5.2 | 4.4 | 3.6 | 2.8 | 2.0 | 1.2 |
| 9 | 5.8 | 4.9 | 4.0 | 3.1 | 2.2 | 1.3 |

29 28 28 27

|   |     |     |     |     |
|---|-----|-----|-----|-----|
| 1 | 2.9 | 2.8 | 2.8 | 2.7 |
| 2 | 5.8 | 5.7 | 5.6 | 5.5 |
| 3 | 8.7 | 8.5 | 8.4 | 8.2 |

|   |      |      |      |      |
|---|------|------|------|------|
| 4 | 11.6 | 11.4 | 11.2 | 11.0 |
| 5 | 14.5 | 14.2 | 14.0 | 13.7 |
| 6 | 17.4 | 17.1 | 16.8 | 16.5 |

|   |      |      |      |      |
|---|------|------|------|------|
| 7 | 20.3 | 19.9 | 19.6 | 19.2 |
| 8 | 23.2 | 22.8 | 22.4 | 22.0 |
| 9 | 26.1 | 25.6 | 25.2 | 24.7 |

27 26 26 25

|   |     |     |     |     |
|---|-----|-----|-----|-----|
| 1 | 2.7 | 2.6 | 2.6 | 2.5 |
| 2 | 5.4 | 5.3 | 5.2 | 5.1 |
| 3 | 8.1 | 7.9 | 7.8 | 7.6 |

|   |      |      |      |      |
|---|------|------|------|------|
| 4 | 10.8 | 10.6 | 10.4 | 10.2 |
| 5 | 13.5 | 13.2 | 13.0 | 12.7 |
| 6 | 16.2 | 15.9 | 15.6 | 15.3 |

|   |      |      |      |      |
|---|------|------|------|------|
| 7 | 18.9 | 18.5 | 18.2 | 17.8 |
| 8 | 21.6 | 21.2 | 20.8 | 20.4 |
| 9 | 24.3 | 23.8 | 23.4 | 22.9 |

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

80°-85°

85°-90°

| ° /  | Vers. | d. | Exsec.  | d.   | ° /  | Vers.  | d. | Exsec.   | d.     | P. P. |
|------|-------|----|---------|------|------|--------|----|----------|--------|-------|
| 80 0 | .8263 | 28 | 4.7587  | 966  | 85 0 | .9128  | 29 | 10.4737  |        |       |
| 10   | .8292 | 29 | 4.8554  | 999  | 10   | .9157  | 29 | 10.8683  | .3946  |       |
| 20   | .8321 | 28 | 4.9553  | 1035 | 20   | .9186  | 29 | 11.2912  | .4229  |       |
| 30   | .8349 | 28 | 5.0588  | 1072 | 30   | .9215  | 29 | 11.7455  | .4542  |       |
| 40   | .8378 | 29 | 5.1660  | 1111 | 40   | .9244  | 29 | 12.2347  | .4892  |       |
| 50   | .8407 | 28 | 5.2772  | 1152 | 50   | .9273  | 29 | 12.7631  | .5284  |       |
| 81 0 | .8435 | 29 | 5.3924  | 1196 | 86 0 | .9302  | 29 | 13.3356  | .5725  |       |
| 10   | .8464 | 28 | 5.5121  | 1242 | 10   | .9331  | 29 | 13.9579  | .6223  |       |
| 20   | .8493 | 29 | 5.6363  | 1291 | 20   | .9360  | 29 | 14.6368  | .6789  |       |
| 30   | .8522 | 28 | 5.7654  | 1343 | 30   | .9389  | 29 | 15.3804  | .7436  |       |
| 40   | .8550 | 29 | 5.8998  | 1398 | 40   | .9418  | 29 | 16.1984  | .8186  |       |
| 50   | .8579 | 29 | 6.0396  | 1456 | 50   | .9447  | 29 | 17.1026  | .9041  |       |
| 82 0 | .8608 | 28 | 6.1853  | 1519 | 87 0 | .9476  | 29 | 18.1073  | 1.0047 |       |
| 10   | .8637 | 29 | 6.3372  | 1585 | 10   | .9505  | 29 | 19.2303  | 1.1230 |       |
| 20   | .8666 | 28 | 6.4957  | 1656 | 20   | .9534  | 29 | 20.4937  | 1.2634 |       |
| 30   | .8694 | 29 | 6.6613  | 1731 | 30   | .9564  | 29 | 21.9256  | 1.4319 |       |
| 40   | .8723 | 29 | 6.8344  | 1812 | 40   | .9593  | 29 | 23.5621  | 1.6365 |       |
| 50   | .8752 | 29 | 7.0156  | 1898 | 50   | .9622  | 29 | 25.4505  | 1.8884 |       |
| 83 0 | .8781 | 28 | 7.2055  | 1991 | 88 0 | .9651  | 29 | 27.6537  | 2.2032 |       |
| 10   | .8810 | 29 | 7.4046  | 2091 | 10   | .9680  | 29 | 30.2576  | 2.6039 |       |
| 20   | .8839 | 29 | 7.6138  | 2198 | 20   | .9709  | 29 | 33.3823  | 3.1247 |       |
| 30   | .8868 | 29 | 7.8336  | 2315 | 30   | .9738  | 29 | 37.2015  | 3.8192 |       |
| 40   | .8897 | 29 | 8.0651  | 2440 | 40   | .9767  | 29 | 41.9757  | 4.7741 |       |
| 50   | .8926 | 28 | 8.3091  | 2576 | 50   | .9796  | 29 | 48.1140  | 6.1383 |       |
| 84 0 | .8954 | 29 | 8.5667  | 2723 | 89 0 | .9825  | 29 | 56.2987  | 8.1846 |       |
| 10   | .8983 | 29 | 8.8391  | 2884 | 10   | .9854  | 29 | 67.7573  |        |       |
| 20   | .9012 | 29 | 9.1275  | 3059 | 20   | .9883  | 29 | 84.9456  |        |       |
| 30   | .9041 | 29 | 9.4334  | 3250 | 30   | .9912  | 29 | 113.5930 |        |       |
| 40   | .9070 | 29 | 9.7585  | 3460 | 40   | .9942  | 29 | 170.8883 |        |       |
| 50   | .9099 | 29 | 10.1045 | 3691 | 50   | .9971  | 29 | 342.7752 |        |       |
| 85 0 | .9128 | 29 | 10.4737 |      | 90 0 | 1.0000 |    | ∞        |        |       |
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29 29 28

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| 1 | 2.9 | 2.9 | 2.8 |
| 2 | 5.9 | 5.8 | 5.7 |
| 3 | 8.8 | 8.7 | 8.5 |

|   |      |      |      |
|---|------|------|------|
| 4 | 11.8 | 11.6 | 11.4 |
| 5 | 14.7 | 14.5 | 14.2 |
| 6 | 17.7 | 17.4 | 17.1 |

|   |      |      |      |
|---|------|------|------|
| 7 | 20.6 | 20.3 | 19.6 |
| 8 | 23.6 | 23.2 | 22.8 |
| 9 | 26.5 | 26.1 | 25.6 |



|    |                                                                                                                                                                                                                                                                                                                                                                           |
|----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1  | $\sin a = \frac{1}{\operatorname{cosec} a} = \frac{\tan a}{\sqrt{1 + \tan^2 a}} = \sqrt{\frac{1 - \cos 2a}{2}} = \frac{1}{\sqrt{1 + \cot^2 a}}$ $= \cos a \tan a = \sqrt{1 - \cos^2 a} = 2 \sin \frac{1}{2}a \cos \frac{1}{2}a$ $= \frac{1 + \cos a}{\cot \frac{1}{2}a} = \frac{2 \tan \frac{1}{2}a}{1 + \tan^2 \frac{1}{2}a} = \operatorname{vers} a \cot \frac{1}{2}a.$ |
| 2  | $\cos a = \frac{1}{\sec a} = \frac{\cot a}{\sqrt{1 + \cot^2 a}} = \frac{1}{\sqrt{1 + \tan^2 a}}$ $= 1 - \operatorname{vers} a = \sin a \cot a = \sqrt{1 - \sin^2 a} = 2 \cos^2 \frac{1}{2}a - 1$ $= \sin a \cot \frac{1}{2}a - 1 = \cos^2 \frac{1}{2}a - \sin^2 \frac{1}{2}a = 1 - 2 \sin^2 \frac{1}{2}a.$                                                                |
| 3  | $\tan a = \frac{1}{\cot a} = \frac{\sin a}{\cos a} = \frac{\sec a}{\operatorname{cosec} a} = \frac{1}{\sqrt{\operatorname{cosec}^2 a - 1}}$ $= \operatorname{vers} 2a \operatorname{cosec} 2a = \cot a - 2 \cot 2a = \sin a \sec a$ $= \frac{\sin 2a}{1 + \cos 2a} = \operatorname{exsec} a \cot \frac{1}{2}a = \operatorname{exsec} 2a \cot 2a.$                         |
| 4  | $\cot a = \frac{1}{\tan a} = \frac{\cos a}{\sin a} = \frac{\sin 2a}{1 - \cos 2a} = \frac{1 + \cos 2a}{\sin 2a}$ $= \sqrt{\operatorname{cosec}^2 a - 1} = \cot \frac{1}{2}a - \operatorname{cosec} a.$                                                                                                                                                                     |
| 5  | $\operatorname{vers} a = 1 - \cos a = \sin a \tan \frac{1}{2}a = 2 \sin^2 \frac{1}{2}a = \cos a \operatorname{exsec} a.$                                                                                                                                                                                                                                                  |
| 6  | $\operatorname{exsec} a = \sec a - 1 = \tan a \tan \frac{1}{2}a = \operatorname{vers} a \sec a.$                                                                                                                                                                                                                                                                          |
| 7  | $\sin \frac{1}{2}a = \sqrt{\frac{\operatorname{vers} a}{2}} = \frac{\sin a}{2 \cos \frac{1}{2}a} = \frac{\operatorname{vers} a \cos \frac{1}{2}a}{\sin a}.$                                                                                                                                                                                                               |
| 8  | $\cos \frac{1}{2}a = \sqrt{\frac{1 + \cos a}{2}} = \frac{\sin a}{2 \sin \frac{1}{2}a} = \frac{\sin a \sin \frac{1}{2}a}{\operatorname{vers} a}.$                                                                                                                                                                                                                          |
| 9  | $\tan \frac{1}{2}a = \operatorname{vers} a \operatorname{cosec} a = \operatorname{cosec} a - \cot a = \frac{\tan a}{1 + \sec a}.$                                                                                                                                                                                                                                         |
| 10 | $\cot \frac{1}{2}a = \frac{1 + \cos a}{\sin a} = \operatorname{cosec} a + \cot a = \frac{\tan a}{\operatorname{exsec} a} = \frac{1}{\operatorname{cosec} a - \cot a}.$                                                                                                                                                                                                    |
| 11 | $\operatorname{vers} \frac{1}{2}a = 1 - \sqrt{\frac{1}{2}(1 + \cos a)}.$                                                                                                                                                                                                                                                                                                  |
| 12 | $\operatorname{exsec} \frac{1}{2}a = \frac{1}{\sqrt{\frac{1}{2}(1 + \cos a)}} - 1.$                                                                                                                                                                                                                                                                                       |

- 13  $\sin 2a = 2 \sin a \cos a = \frac{2 \tan a}{1 + \tan^2 a}.$
- 14  $\cos 2a = \cos^2 a - \sin^2 a = 1 - 2 \sin^2 a = 2 \cos^2 a - 1$   
 $= \frac{1 - \tan^2 a}{1 + \tan^2 a}.$
- 15  $\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}.$
- 16  $\cot 2a = \frac{1}{2} \cot a - \frac{1}{2} \tan a = \frac{\cot^2 a - 1}{2 \cot a} = \frac{1 - \tan^2 a}{2 \tan a}.$
- 17  $\text{vers } 2a = 2 \sin^2 a = 1 - \cos 2a = 2 \sin a \cos a \tan a.$
- 18  $\text{exsec } 2a = \frac{\tan 2a}{\cot a} = \frac{2 \tan^2 a}{1 - \tan^2 a} = \frac{2 \sin^2 a}{1 - 2 \sin^2 a}.$
- 19  $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b.$
- 20  $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b.$
- 21  $\sin a + \sin b = 2 \sin \frac{1}{2}(a + b) \cos \frac{1}{2}(a - b).$
- 22  $\sin a - \sin b = 2 \sin \frac{1}{2}(a - b) \cos \frac{1}{2}(a + b).$
- 23  $\cos a + \cos b = 2 \cos \frac{1}{2}(a + b) \cos \frac{1}{2}(a - b).$
- 24  $\cos a - \cos b = -2 \sin \frac{1}{2}(a + b) \sin \frac{1}{2}(a - b).$

Call the sides of any triangle  $A, B, C$ , and the opposite angles  $a, b$ , and  $c$ . Call  $s = \frac{1}{2}(A + B + C)$ .

- 25  $\tan \frac{1}{2}(a - b) = \frac{A - B}{A + B} \tan \frac{1}{2}(a + b) = \frac{A - B}{A + B} \cot \frac{1}{2}c.$
- 26  $C = (A + B) \frac{\cos \frac{1}{2}(a + b)}{\cos \frac{1}{2}(a - b)} = (A - B) \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}(a - b)}.$
- 27  $\sin \frac{1}{2}a = \sqrt{\frac{(s - B)(s - C)}{BC}}.$
- 28  $\cos \frac{1}{2}a = \sqrt{\frac{s(s - A)}{BC}}.$
- 29  $\text{vers } A = \frac{2(s - B)(s - C)}{BC}.$
- 30  $\text{Area} = \sqrt{s(s - A)(s - B)(s - C)} = A^2 \frac{\sin b \sin c}{2 \sin a}.$

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